Effective Field Theories for heavy probes in a hot QCD plasma and in the early universe

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Based on works in collaboration with Nora Brambilla, Simone Biondini, Jacopo Ghiglieri, Antonio Vairo and Joan Soto.

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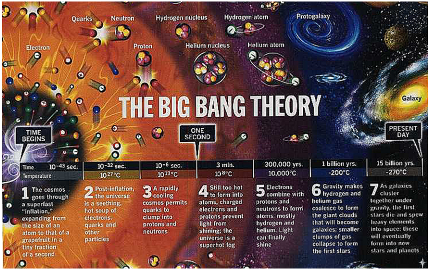
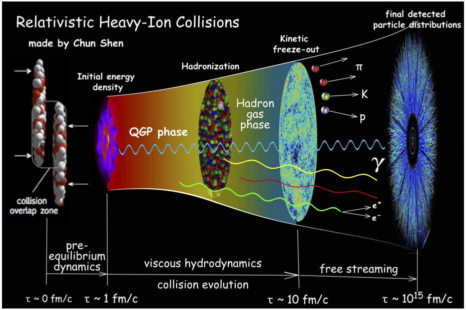
The big bang and the little bangs

The big bang of the universe



The little bangs created in particle accelerators



Both systems can be seen as a very hot medium that expands and cools down. A thermal history.

Why heavy particles?

In general

- They leave very clear signals and their properties are modified in the presence of a medium. Therefore they are a good probe of the medium.
- They can form non-relativistic bound states which have different separated energy scales that are useful to test the plasma.

Examples

- Quarkonium in heavy ion collisions.
- Many models for baryogenesis or dark matter involve yet undiscovered heavy particles. Ex: Decay of heavy Majorana neutrinos in the hot early universe might help to explain baryon asymmetry.

Quantum field theory in the presence of separated energy scales

Difficulties may arise if QFT is applied naively to problems in which very separated energy scales are present

- Large suppressions.
- Unexpected enhancements.
- Large logarithmic corrections.

A way to deal with these problems is to use Effective field theories.

Perturbation theory in the presence of separated energy scales

Example: Heavy particle with mass M interacting with a medium with temperature $T \ll M$. For any quantity of dimension D the perturbative expansion can be written as

$$M^D \sum_{n=0}^{\infty} g^n A_n \left(\frac{T}{M}\right)$$

If $A_n \to \infty$ as $\frac{T}{M} \to 0$ we might lose control of the perturbative expansion. If we use EFT the computation is an expansion in both $\frac{T}{M}$ and g.

$$M^D \sum_{n,m} g^n \left(\frac{T}{M}\right)^m B_{n,m}$$

now $B_{n,m}$ is of order 1.

Advantage of using EFT for heavy particles at finite T

- Allow to study the effects of the scale of the heavy mass M forgetting about the medium. $T = 0$ computations are simpler.
- The scales affected by the medium can be studied with the EFT, which is simpler than the full theory. Example: Heavy particles are bispinors in the full theory while in the EFT they are only spinors.
- The contributions that need to be resummed and the ones that are suppressed can be seen at the Lagrangian level by applying a systematic methodology called power counting. Specially important for bound states.

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Why heavy Majorana neutrinos?

Adding Majorana neutrinos to the standard model can solve three of its problems.

- Active neutrinos could be light because Majorana (or sterile) neutrinos are heavy. See-saw mechanism.
- Heavy and/or very weakly interacting particles could form part of dark matter.
- Majorana masses violate lepton number conservation and that can help to explain the higher abundance of particles over antiparticles.
- It does not need the inclusion of new particles in the SM, as opposed to supersymmetry scenarios.

EFT for heavy Majorana neutrinos

To compute thermal corrections to the decay width or to study the lepton asymmetry involves high order computations which are very challenging in thermal field theory. Applying an EFT is convenient because

- All the physics happening at scales bigger than the temperature can be computed using $T = 0$ techniques.
- A perturbative computation in the full theory gives all thermal corrections to a given order in the coupling constant. But this contains terms with very different powers in $\frac{T}{M}$. With an EFT it is easier to extract the leading order thermal corrections.
- Because $\frac{T}{M}$ is small the power in the coupling constant can be misleading when considering the size of a contribution.

EFT for heavy Majorana neutrinos

S. Biondini, N. Brambilla, M. A. E and A. Vairo, JHEP 1312 (2013) 028

Write the more general Lagrangian compatible with the symmetries and the degrees of freedom

$$\mathcal{L} = N^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) N + sub - leading$$

- Interaction is always given by higher order operators, always suppressed by powers of M.
- LO thermal corrections will always come from operators whose dimension is smaller.
- We have to respect the symmetries.

LO interaction with Higgs

$$N^\dagger N\phi^\dagger\phi$$

Operator of dimension 5

$$\delta\Gamma \propto \frac{T^2}{M}$$

LO interaction with relativistic fermions

$$N^\dagger N \bar{L} L$$

Operator of dimension 6. But in thermal equilibrium $\langle L\bar{L}\rangle = 0$.
Need to include a derivative D_0.

$$\delta\Gamma \propto \frac{T^4}{M^3}$$

LO interaction with gauge bosons

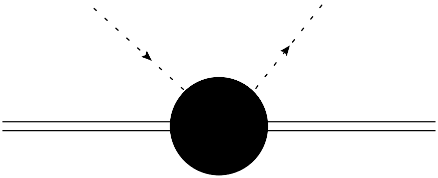
$$N^\dagger N F^{\mu\nu} F_{\mu\nu}$$

Operator of dimension 7.

$$\delta\Gamma \propto \frac{T^4}{M^3}$$

Easy way to compute thermal corrections to a process

Compute in the full theory the scattering of a heavy neutrino with a Higgs particle. Matching computation at T=0 to find out the Wilson coefficient.



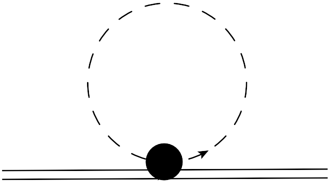
$$\delta\mathcal{L} = \frac{1}{M}\left(\Re a - \frac{3i}{8\pi}|F|^2\lambda\right)N^\dagger N\phi^\dagger\phi$$

Easy way to compute thermal corrections to a process

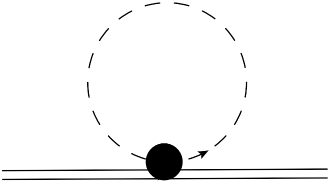
See how this operator contributes to the process you are interested

$$\delta\mathcal{L} = \frac{1}{M}\left(\Re a - \frac{3i}{8\pi}|F|^2\lambda\right) N^\dagger N\phi^\dagger\phi$$

Example, Corrections to the decay width. Tadpole diagram.



LO correction to the decay width



$$\delta\Gamma = -\frac{3|F|^2\lambda}{4\pi M}\langle\phi^\dagger\phi\rangle = -\frac{\lambda|F|^2T^2}{8\pi M} + \mathcal{O}\left(\frac{T^4}{M^3}\right)$$

Agree with Savio, Lodone and Strumia (2011) and Laine and Schroeder (2012).

Two loop computation in thermal field theory $\rightarrow$ One loop matching computation at $T = 0$ and trivial tadpole diagram at finite temperature.

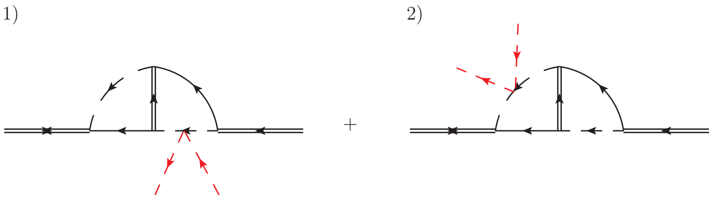
Direct Lepton asymmetry. Thermal corrections

Information needed to compute the size of the baryonasymmetry created during the evolution of the universe

$$\epsilon_i = \sum_i \frac{\Gamma(N_i \to leptons) - \Gamma(N_i \to antileptons)}{\Gamma(N_i \to leptons) + \Gamma(N_i \to antileptons)}$$

- Compute scattering of heavy neutrinos with Higgs.
- Using the cutting rules (at $T = 0$) keep track of which diagram contribute to decay into leptons and which to antileptons.
- Compute the same tadpole diagram in the EFT.

Direct Lepton asymmetry. Thermal corrections



- Compute scattering of heavy neutrinos with Higgs.
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Direct Lepton asymmetry. Thermal corrections

Needed terms in the EFT Lagrangian are

$$\delta\mathcal{L} = \frac{i}{2}(\Gamma_I^{l} + \Gamma_I^{\bar{l}})N_I^\dagger N_I + \frac{i}{M_I}(\mathrm{Im}\, a_I^{l} + \mathrm{Im}\, a_I^{\bar{l}})N_I^\dagger N_I \phi^\dagger\phi$$

l means decay into leptons and $\bar{l}$ to antileptons

$$\delta\epsilon_I = \frac{4\langle\phi^\dagger\phi\rangle}{M_I(\Gamma_I^{l} + \Gamma_I^{\bar{l}})^2}(\mathrm{Im}\, a_I^{l}\Gamma_I^{\bar{l}} - \mathrm{Im}\, a_I^{\bar{l}}\Gamma_I^{l})$$

- Γ_I is proportional to M_I and can be computed using $T = 0$ techniques.
- $\mathrm{Im}\, a_I$ is adimensional and only contains $T = 0$ physics.
- $\langle\phi^\dagger\phi\rangle$ contains all the needed information from the medium.

Degenerate case

S. Biondini, N. Brambilla, M. A. E and A. Vairo, JHEP 1603 (2016) 191

Two heavy neutrinos with a degenerate mass

$$M_1 = M$$
$$M_2 = M + \Lambda$$
$$M \gg \Lambda$$
$$\Lambda > 0$$

In this scenario baryogenesis is enhanced. Flanz, Pachos, Sarkar and Weiss (1996).

Direct Lepton asymmetry. Thermal corrections

S. Biondini, N. Brambilla, M. A. E and A. Vairo, JHEP 1603 (2016) 191

$$\epsilon^{T}_{1,\mathrm{direct}} = \frac{\mathrm{Im}\left[(F_1^* F_2)^2\right]}{8\pi|F_1|^2}\left(\frac{T}{M}\right)^2\left\{\lambda\left[2-\ln 2+(1-3\ln 2)\frac{\Delta}{M}\right]\right.$$
$$\left.-\frac{g^2}{16}\left[2-\ln 2+(3-5\ln 2)\frac{\Delta}{M}\right]-\frac{g'^2}{48}\left[4-\ln 2+(1-5\ln 2)\frac{\Delta}{M}\right]\right\}$$
$$\epsilon^{T}_{2,\mathrm{direct}} = -\frac{\mathrm{Im}\left[(F_1^* F_2)^2\right]}{8\pi|F_2|^2}\left(\frac{T}{M}\right)^2\left\{\lambda\left[2-\ln 2-(9-5\ln 2)\frac{\Delta}{M}\right]\right.$$
$$\left.-\frac{g^2}{16}\left[2-\ln 2-7(1-\ln 2)\frac{\Delta}{M}\right]-\frac{g'^2}{48}\left[4-\ln 2-(9-7\ln 2)\frac{\Delta}{M}\right]\right\}$$

Instead of three loop in thermal field theory $\rightarrow$ two loop in normal quantum field theory (but using optical theorem)+ tad-pole in thermal field theory

The hierarchical case

S. Biondini, N. Brambilla and A. Vairo, arXiv:1608.01979

Case $M_2 \gg M_1$, also called *vanilla leptogenesis*

$$\epsilon_T = -\frac{3}{16\pi}\frac{M_1}{M_2}\frac{\mathrm{Im}\left[(F_1^* F_2)^2\right]}{|F_1|^2}\left[\left(-\frac{5}{3}\lambda + \frac{2g^2+g'^2}{12}\right)\left(\frac{T}{M_1}\right)^2 + \frac{7\pi^2}{20}|\lambda_t|^2\left(\frac{T}{M_1}\right)^4\right].$$

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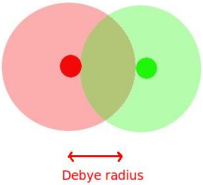
The original idea of Matsui and Satz (1986)

- Quarkonia is quite stable in the vacuum.
- Phenomena of colour screening, quantities measurable in Lattice QCD at finite temperature (static) support this. For example Polyakov loop.
- Dissociation of heavy quarkonium in heavy-ion collisions due to colour screening signals the creation of a quark-gluon plasma.

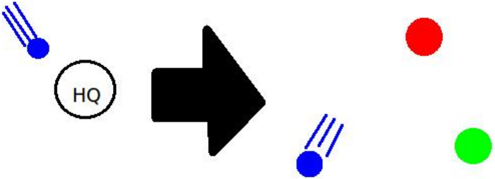
Colour screening

$$V(r) = -\alpha_s \frac{e^{-m_D r}}{r}$$

At finite temperature



Another mechanism, the decay width



Laine et al. perturbative potential (2007)

$$V(r) = -\alpha_s C_F \left[m_D + \frac{e^{-m_D r}}{r} \right] - i\alpha_s T C_F \phi(m_D r)$$

with

$$\phi(x) = 2\int_0^\infty \frac{dzz}{(z^2+1)^2}\left(1 - \frac{\sin(zx)}{zx}\right)$$

- This potential was obtained through the Wilson loop in Minkowski space at finite temperature.
- It has an imaginary part that has to be related with a decay width.
- This result is valid for $T \gg 1/r \sim m_D$. Other regimes have been explored using EFTs (Brambilla, Ghiglieri, Petreczky And Vairo, M. A. E and Soto).

Questions

- What is the correct definition of the potential? Free energy or internal energy.
- Is the potential model picture valid? If not, what kind of picture is expected.
- Predict the dissociation pattern.

Effective field theories at $T = 0$

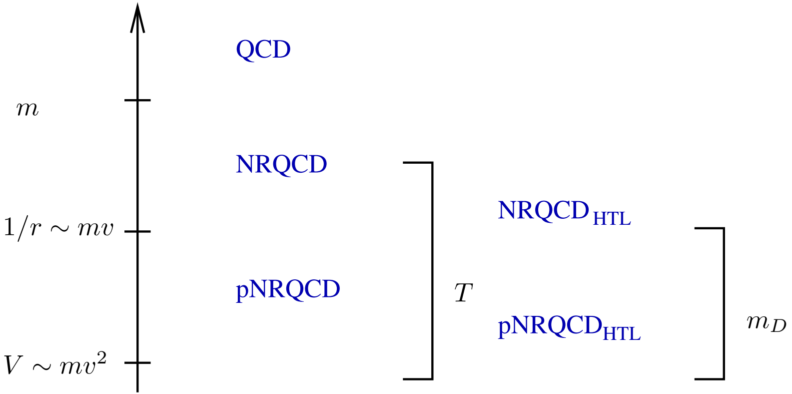
- QCD
  Quarks and gluons.
- NRQCD
  Caswell and Lepage (1986), Bodwin, Braaten and Lepage (1994). Integrate out hard scale. Non-relativistic quarks and gluons.

$$\mathcal{L}_{NRQCD} = \sum_n \frac{1}{m_Q^n} \mathcal{L}_n$$

- pNRQCD
  Pineda and Soto (1998), Brambilla, Pineda, Soto and Vairo (2005). Integrate out also soft scale. Color singlet field, color octet field and gluons.

$$\mathcal{L}_{pNRQCD} = \sum_{n,m} \frac{1}{m_Q^n} r^m \mathcal{L}_{nm}$$

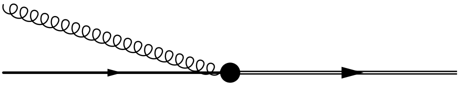
Effective field theories at finite temperature



Brambilla, Ghiglieri, Vairo and Petreczky (PRD78 (2008) 014017)
M. A. E and Soto (PRA78 (2008) 032520)

The case $\frac{1}{r} \gg T \gg E \gg m_D$

N. Brambilla, M. A. E., J. Ghiglieri, J. Soto and A. Vairo, JHEP 1009 (2010) 038

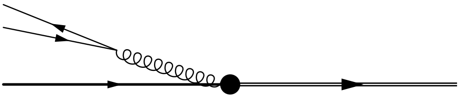


Gluo-dissociation, parton + singlet $\rightarrow$ octet

- Thermal effects are proportional to r^2 because the medium sees the singlet as a small color dipole.
- The process called gluo-dissociation dominates the decay width.
- Both the binding energy and the decay width contain terms that can not be encoded in a potential model.

The case $\frac{1}{r} \gg T \gg m_D \gg E$

N. Brambilla, M. A. E, J. Ghiglieri and A. Vairo, JHEP 1305 (2013) 130



Inelastic parton scattering, parton + singlet $\to$ parton + octet.

- Thermal effects are proportional to r^2 because the medium sees the singlet as a small color dipole.
- Inelastic parton scattering dominates the decay width.
- All terms can be encoded in a potential model.

The case $T \gtrsim \frac{1}{r}$

The case $T \sim \frac{1}{r}$

- Potential with both a real and an imaginary part.
- The medium no longer sees the singlet as a color dipole. The potential is not a polynomial of rT.
- In perturbation theory thermal effects are suppressed by an additional α_s.

Case studied in M. A. E and Soto (2010)

The case $T \gg m_D \sim \frac{1}{r}$

- EFT potential coincides with the one first studied by Laine et al.
- Thermal corrections give a leading order contribution.

Case studied in M. A. E and Soto (2008) and Brambilla, Ghiglieri, Vairo and Petreczky (2008)

Towards a computation of R_{AA} in EFT

- Until now we have discussed the decay width and corrections to the binding energy.
- Both quantities can be found in the EFT by studying the pole of the singlet time-ordered propagator.
- To compute R_{AA} we need to know how the state of the heavy quarks in the medium changes with time.
- The natural way to formulate this problem is in terms of density matrices $\rightarrow$ open quantum system.

Open quantum systems and the time evolution of quarkonium

Open quantum systems introduced in the context of quarkonium by Akamatsu and Rothkopf.

- $\mathcal{H} = \mathcal{H}_{System} \otimes \mathcal{H}_{Environment}$, the heavy particles are the system and the thermal bath is the environment.
- Wave-function $\rightarrow$ reduced density matrix. Trace over environment degrees of freedom.
- All the information of the system is not encoded in time-ordered correlators.

Evolution of the number of singlets

In the $\frac{1}{r} \gg T$ regime

$$f_s(x,y) = Tr(\rho S^\dagger(x) S(y))$$

We can use perturbation theory but expanding in r instead of α_s. In the interaction picture

$$i\partial_t S = [S, H_0]$$
$$i\partial_t \rho = [H_I, \rho]$$

Evolution of the number of singlets and octets

In the $\frac{1}{r} \gg T$ regime

$$\partial_t f_s = -i[H_s, f_s] - \frac{1}{2}\{\Gamma_s, f_s\} + \mathcal{F}(f_o)$$

- $\mathcal{F}(f_o)$ is a new term that takes into account the process $O \to g + S$. It ensures that the total number of heavy quarks is conserved.

$$f_o^{ab}(x,y) = Tr(\rho O^{\dagger,a}(x) O^b(y))$$

$$\partial_t f_o = -i[H_o, f_o] - \frac{1}{2}\{\Gamma_o, f_o\} + \mathcal{F}_1(f_s) + \mathcal{F}_2(f_o)$$

The $\frac{1}{r} \gg T \gg m_D \gg E$ regime

Because all the thermal scales are smaller than $\frac{1}{r}$ but bigger than E the evolution equation is of the Lindblad form.

$$\partial_t \rho = -i[H(\gamma), \rho] + \sum_k (C_k \rho C_k^\dagger - \frac{1}{2}\{C_k^\dagger C_k, \rho\})$$

there is a transition singlet-octet

$$C_i^{so} = 2\sqrt{\frac{2T_F\kappa}{N_c(N_c^2-1)}}\, r \begin{pmatrix} 0 & \frac{1}{\sqrt{N_c^2-1}} \\ 1 & 0 \end{pmatrix}$$

and octet to octet

$$C_i^{oo} = \sqrt{\frac{2(N_c^2-4)\kappa}{N_c(N_c^2-1)}}\, r \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$\frac{1}{r} \gg T_{eff} \gg m_D \gg E$

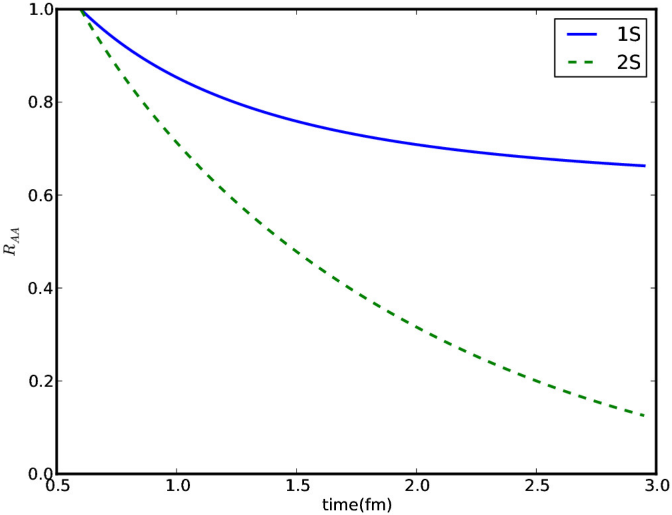
Preliminary study, see which value of κ reproduces best the experimental results.

			30 − 50% centrality			50 − 100% centrality		
$\frac{\kappa}{T^3}$	γ	δ	$R_{AA}(1S)$	$\frac{R_{AA}(2S)}{R_{AA}(1S)}$	$\frac{R_{AA}(3S)}{R_{AA}(1S)}$	$R_{AA}(1S)$	$\frac{R_{AA}(2S)}{R_{AA}(1S)}$	$\frac{R_{AA}(3S)}{R_{AA}(1S)}$
2.5	0.144	1	0.138	0.216	0.188	0.722	0.512	0.458
25	0.144	1	0.0104	4.02	4.10	0.153	0.622	0.591
0.25	0.144	1	0.643	0.195	0.154	0.929	0.910	0.896
2.5	1.44	1	0.0191	0.542	0.485	0.528	0.657	0.609
2.5	0.0144	1	0.156	0.276	0.262	0.742	0.497	0.443
2.5	0.144	0.1	0.133	0.214	0.183	0.721	0.513	0.458
2.5	0.144	10	0.143	0.241	0.227	0.730	0.508	0.455

- γ is a constant that control the corrections to the real part of the potential.
- δ depend of the initial probabilities of having a singlet P_s or and octet P_o. $\delta = \frac{\alpha_s P_o}{P_s}$.

$\frac{1}{r} \gg T_{eff} \gg m_D \gg E$

Example with $\kappa = 0.25\,T^3$, $\gamma = 0.144$ and $\delta = 1$



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Conclusions

- EFT gives new and valuable insights into relevant problems regarding heavy particles in a medium in both the early universe and heavy ion collisions.
- Using EFT the effect of the energy scale M can always be computed using $T = 0$ techniques, which is simpler.
- pNRQCD gives contact with potential models but also indicates when it is needed to go beyond.
- Good understanding on how to compute corrections to the binding energy and the decay width. Recent progress on how to understand the time evolution of the density matrix.