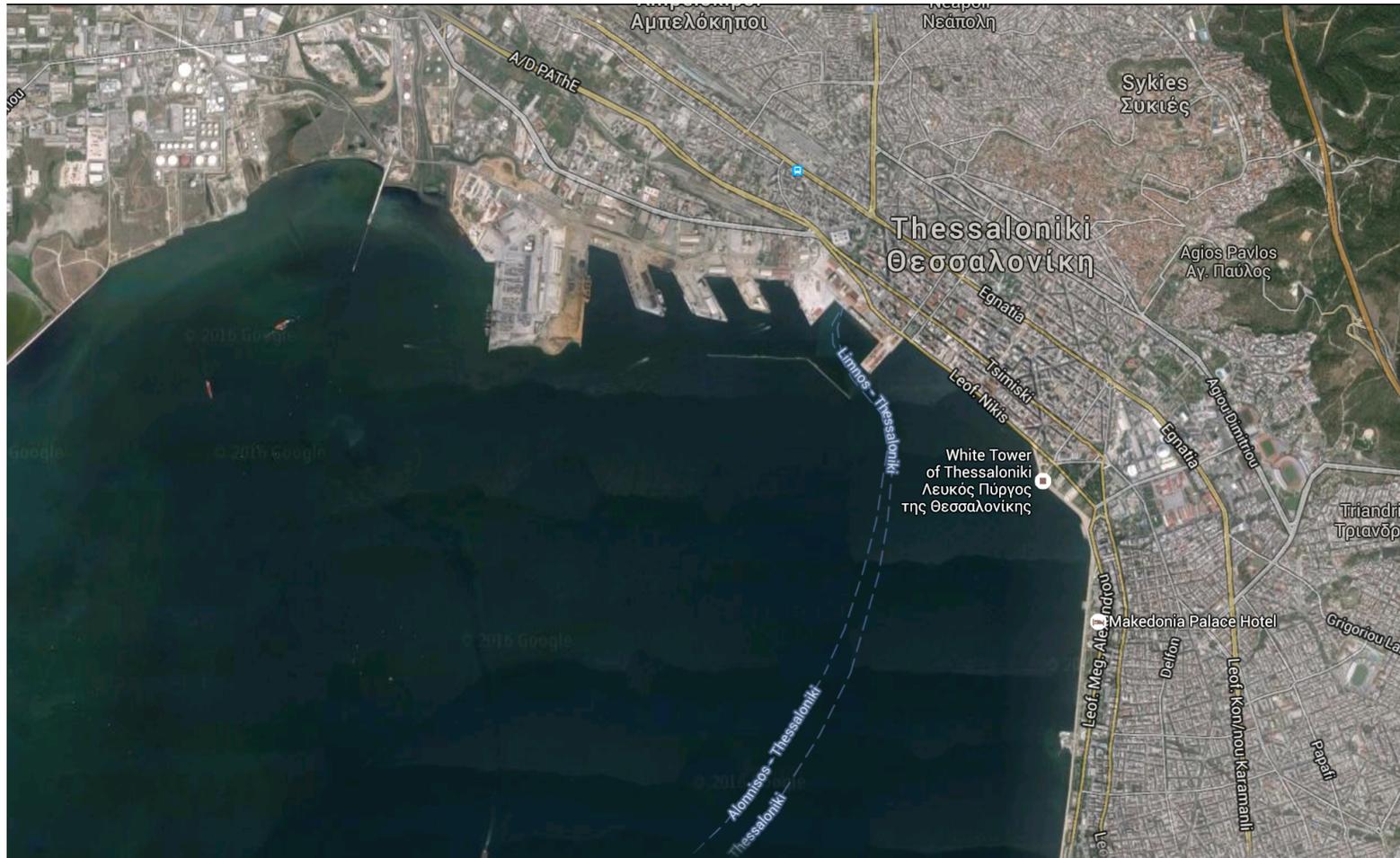


Exotic Hadrons and Large N_c QCD



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An Overview



- QCD is the underlying theory of hadronic physics. But the simple quark model remains a basic way hadronic physicists think about states.
- Exotic do not fit into the simple constituent quark model and clarify the difference of QCD and the quark model .
- The discovery of exotic hadrons—both tetraquarks (eg. $Z(4430)$) and pentaquarks ($P_c^+(4380)$ and $P_c^+(4450)$)—is the most exciting thing in hadron spectroscopy in a very long time.

There are two types of exotics

- **Quantum number exotics.** Hadrons whose quantum numbers **cannot** be made in the simplest quark model.
 - **Cryptoexotics.** States whose quantum numbers are allowed in the simple quark model but which dynamically are dominated by components which are not of the quark model type.
- **Recently discovered exotics are formally cryptoexotic: they contain charm-anticharm pairs.** However, charmed quarks are heavy and thus, the number of charm and anticharm quarks are each “quasi-conserved”.
- **Key question: Is the heaviness of the charmed quarks essential to the formation of exotic quarks?** There are deep theoretical reasons why heavy quarks are more likely to resonate.
- **Do exotics composed only of light quarks exist?** Experimentally, the situation is at best ambiguous
 - **We can use large N_c QCD to try to get theoretical insights about exotics in the light-quark sector.**

The Conventional Wisdom for Tetraquarks



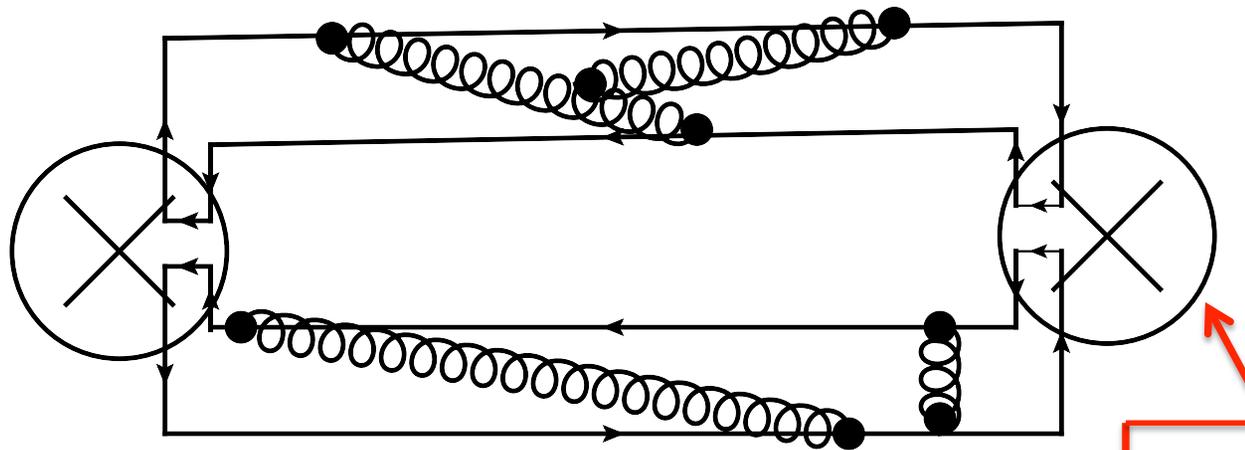
- **Tetraquarks cannot exist at large N_c .** (Witten 1979; Coleman 1985)

Basic argument:

Hadrons at large N_c are studied via the correlation functions for sources with appropriate quantum numbers. Leading order diagrams for minimal tetraquark source (two bilinears at the same point) is $\mathcal{O}(N_c^2)$; it is just a disconnected diagram which behaves like two non-interacting mesons. It does not act like a tetraquark.

Disconnected graphs $\mathcal{O}(N_c^2)$

a typical diagram at
quark/gluon level:
dominated by loops
with planar gluons
inside



Source

$$J = \bar{q}^a(x)q_a(x)\bar{q}^a(x)q_b(x)$$

a, b are color indices



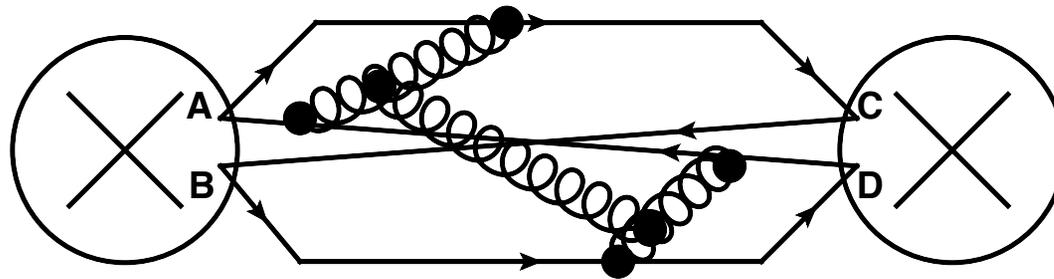
hadronic level
two mesons

Weinberg's Critique

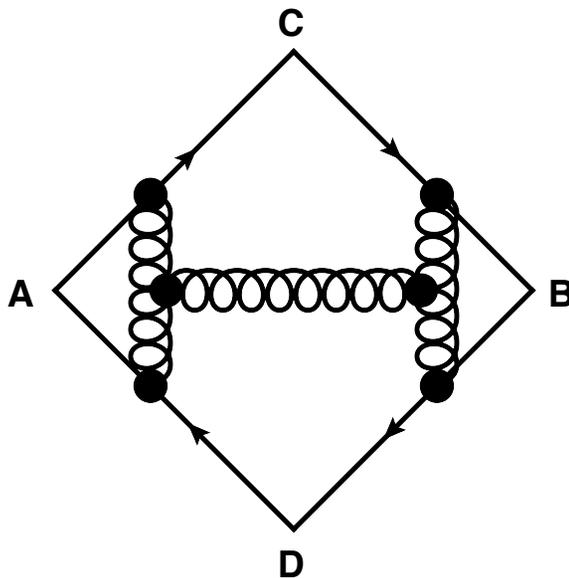
- Weinberg in a 2013 PRL pointed out that the standard argument is not valid.
- That the leading order correlator has the tetraquark operator make “two meson and nothing else” is irrelevant. To see states which could look tetraquark, one needs to diagrams in which the four quarks all interact —i.e. the leading connected diagram.

Whether or not these resonate into tetraquarks is separate question from whether the leading diagrams only make noninteracting mesons.

Connected graphs $\mathcal{O}(N_c)$



A typical diagram at quark/gluon level : dominated by a single loop with planar gluons inside. Written as a sensible looking space-time type diagram, it does not seem to be by a single loop with planar gluons inside.

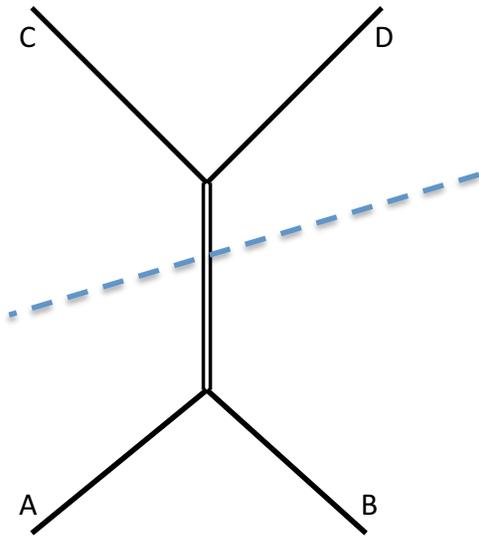


But topologically it is, and the N_c counting only depends on the topology

- Weinberg's critique is correct. Whether tetraquarks do exist depends on the dynamics of these connected diagrams. the argument to disprove the existence of tetraquark is wrong.
- However, he has **NOT** shown that tetraquarks do exist as narrow resonances at large N_c .
- Indeed, there is a somewhat subtle argument based on the topology of diagrams that despite Weinberg's critique, the conclusion of Witten and Coleman is nevertheless correct: **tetraquarks do not exist in the standard version of large N_c QCD** (TDC & R.F. Lebed 2014)

A sketch of the argument why there are no tetraquarks in QCD(F) at Large N_c

If narrow exotic tetraquarks exist they will couple to ordinary meson with a coupling strength $\sim N_c^{-1/2}$. It **must** yield a singularity in the s-channel scattering of incident mesons.



Follows from standard Mandelstam type dispersion analysis. At fixed t , the dispersion relation is

$$T(s, t) = \text{pole terms} + \frac{1}{\pi^2} \int_{\text{threshold}}^{\infty} ds' \frac{\rho(s', t)}{s - s' + i\epsilon}$$

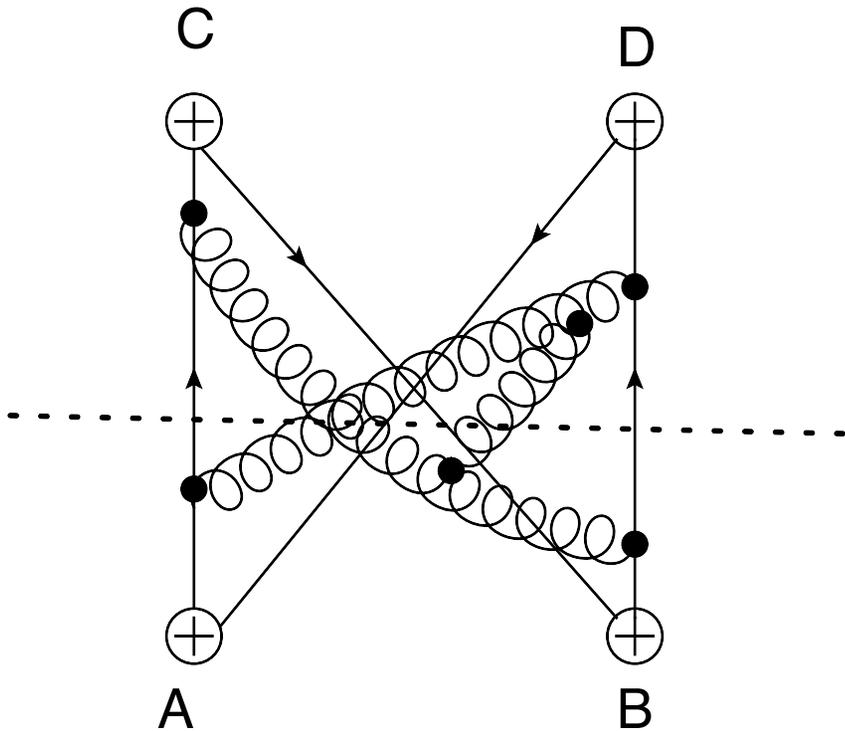
A tetraquark must appear as a sharp structure in ρ (i.e. in the s-channel cut; it will become a δ function at large N_c).

- To proceed use standard assumptions
 - Scaling with N_c of physical observables will match the N_c scaling of the leading order family of diagrams.
 - A cut in the diagram corresponds to intermediate particles going on-shell
- Focus on the the scattering amplitude and in particular the spectral function
 - A key point is that the LSZ reduction relates the scattering amplitude to the **amputated** 4-point function—not the 4-pt function itself.
 - That is it multiplies the 4-pt function by inverse propagators to eliminate singularities associated with the incident and final particles

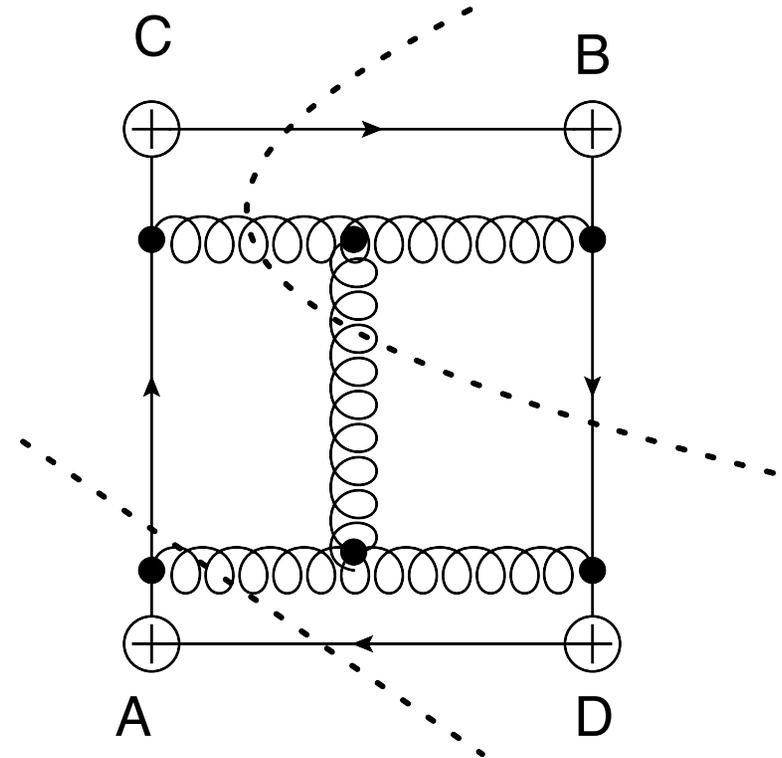
There is a topological argument that amputated 4-point functions at leading order for **every diagram** in an exotic channel only has singularities in the s-channel associated with the asymptotic mesons (either initial or final) in the sense that the cut has two color singlets carry the initial four momenta of each; thus there are no singularities associated with intermediate object.

Key point is to distinguish between a space-time description of the process $A+B \rightarrow C+D$ from the topology of the color flow

A typical contribution to the full 4-pt function



Space-time diagram

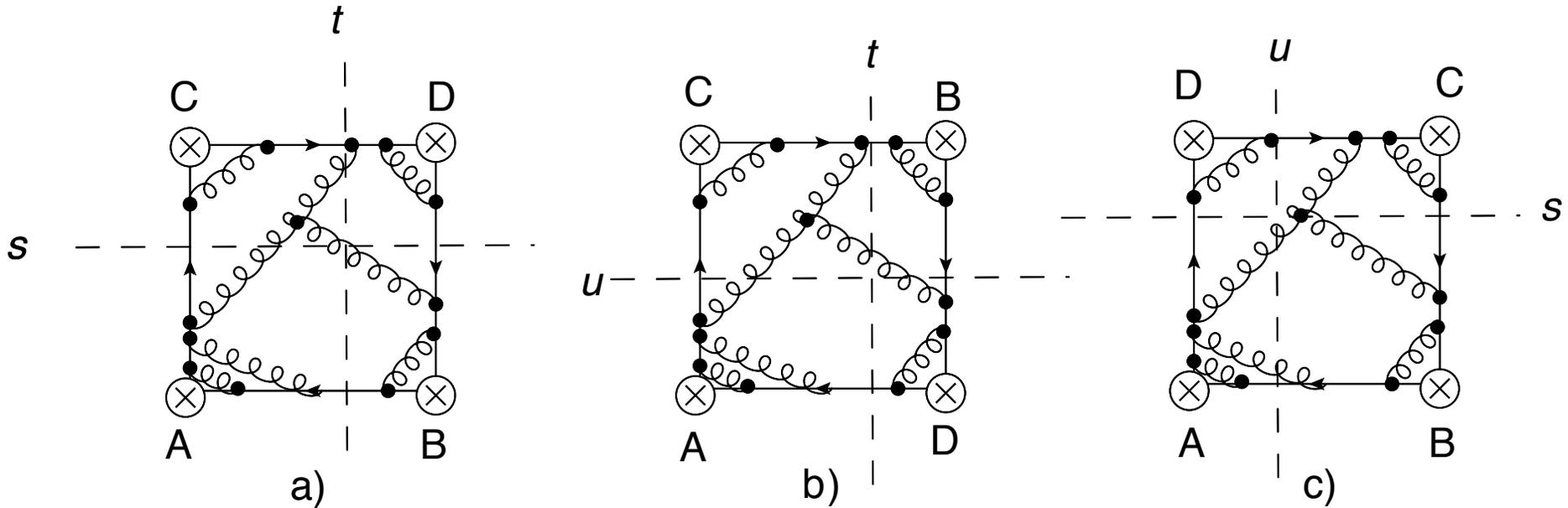


Topologically equivalent planar graph. Note that the cut has broken in two and each part carries the 4-momenta inserted at A or B

- The nature of the cut when drawn as a space-time diagram might lead you to believe that it is associated with a tetraquark.
- However from the diagram drawn in planar form, it is clear that the cut merely cuts corners carrying exactly the momentum brought in at A & B and thus correspond to on-shell incident or mesons.
 - Hence when going to the scattering amplitude from the 4-point function (i.e. amputating the external legs) this cut of diagram will vanish.
- **Simple to show by looking at all topologically distinct classes of diagram that this behavior is generic for exotic channels. They do not have tetraquark cuts in the s-channel and hence there are no exotic tetraquarks.**

What about non-exotic channels?

Look at cuts that do more than “just cut corners” in color-flow diagrams for the space-time process AB goes to CD



Non-exotic channels **do** have s -channel singularities since a) and c) are not forbidden by quantum numbers (as they are for exotic channels). But they cut exactly one quark-antiquark pair; they are ordinary mesons.

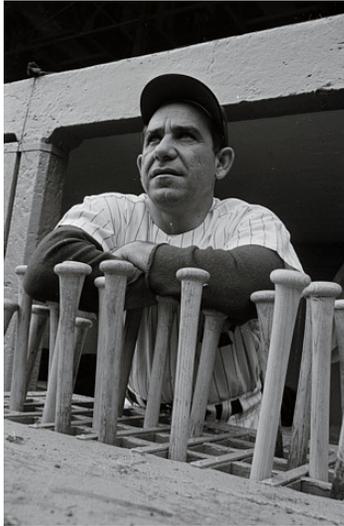
Implication

- This seems to imply that if large N_c QCD is a useful guide to behavior at $N_c=3$, that the existence of tetraquarks requires heavy quarks and exploits a non-commutativity of the heavy quark and large N_c limits.
 - From this perspective, that observed tetraquarks contain heavy quarks is not a surprise.
- However, things are a bit more complicated since there is more than one way to take the large N_c limit.

QCD (AS)

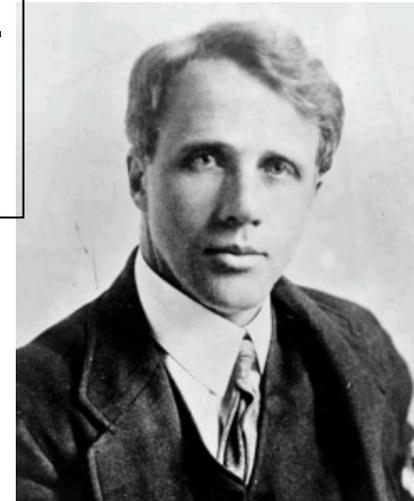
- **The large N_c limit of QCD is not unique**
 - **For gluons there is a unique prescription $SU(3) \rightarrow SU(N_c)$** However for quarks, we can choose different representations of the gauge group: **the fundamental (F) two index anti-symmetric (AS).**
 - **AS transforms like two colors (eg fundamental quarks) with indices antisymmetrized; dimension $\frac{1}{2}N_c(N_c-1)$; 3 for $N_c=3$**
Note that $N_c=3$ quarks in the AS representation are indistinguishable from the (anti-)fundamental. (In essence antisymmetric $r b$ is the same as anti g .)
 - QCD(AS) and QCD(F) extrapolate to large N_c in **different ways.**
 - **The large N_c limits are physically different**
 - The $1/N_c$ expansions are different.
 - A priori it is not obvious which expansion is better
 - It may well depend on the observable in question

Two Roads to Large Nc QCD



Quarks in
Fundamental

Quarks in 2-
index anti-
symmetric



“When you come to a
fork in the road, take it.”

---Yogi Berra,
American baseball
player, coach and part-
time philosopher

“Two roads diverged in a
wood, and I—
I took the one less traveled
by And that has made all the
difference.”

---Robert Frost,
American poet

Large Nc QCD

- QCD(AS) differs from QCD(F) at large N_c in the role of quarks loops
- QCD(AS) naturally includes quark loops at leading order. Thus one might expect that in non-quantum number exotic channels tetraquarks will mix with ordinary mesons at leading order.
 - This can be shown to be correct.
- More interestingly, in quantum number exotic channels, QCD(AS) **MUST** have narrow tetraquarks at large N_c (i.e. narrow states which have at least 2 quarks and 2 antiquarks)Cohen&Lebed 2014.

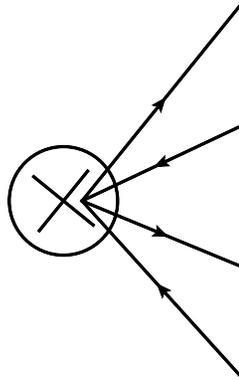
Key ingredient: there are single color trace tetraquark sources in QCD(AS). That is the source cannot be broken up into two separate color singlets (except for N_c^{-2} contributions). **This cannot be done in QCD(F)**

$$J(x) = \sum_{\substack{A,B \\ a,b,c,d}} C_{AB} \bar{q}^{ab}(x) \Gamma_A q_{bc}(x) \bar{q}^{cd}(x) \Gamma_B q_{da}(x)$$

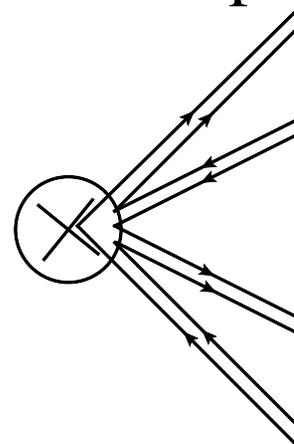
Γ_A, Γ_B are matrices in Dirac-flavor space.

a, b, c, d are fundamental color indices

choice of C_{AB} fixes quantum #s; for simplicity chose an exotic

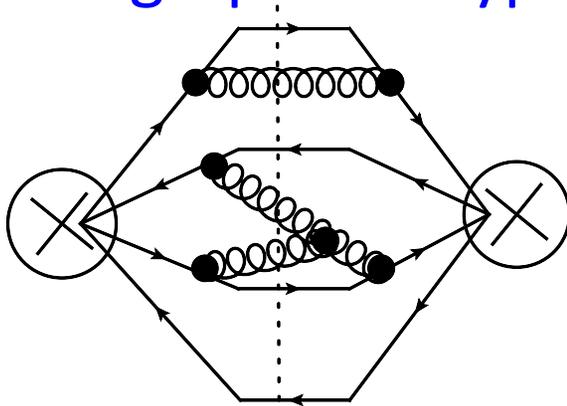


Source as a Feynman diagram

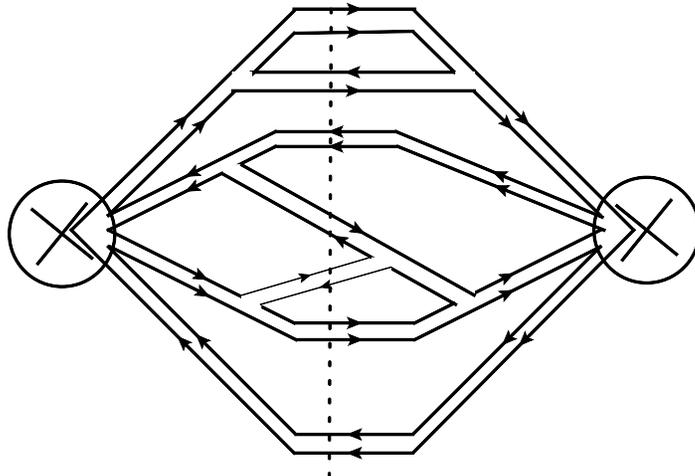


Source as a color-flow diagram

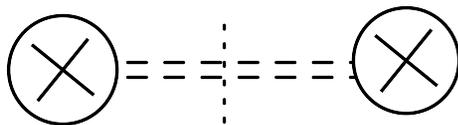
Look at the JJ correlation function. It is dominated by planar graphs. A typical diagram scales as N_c^4



Feynman diagram

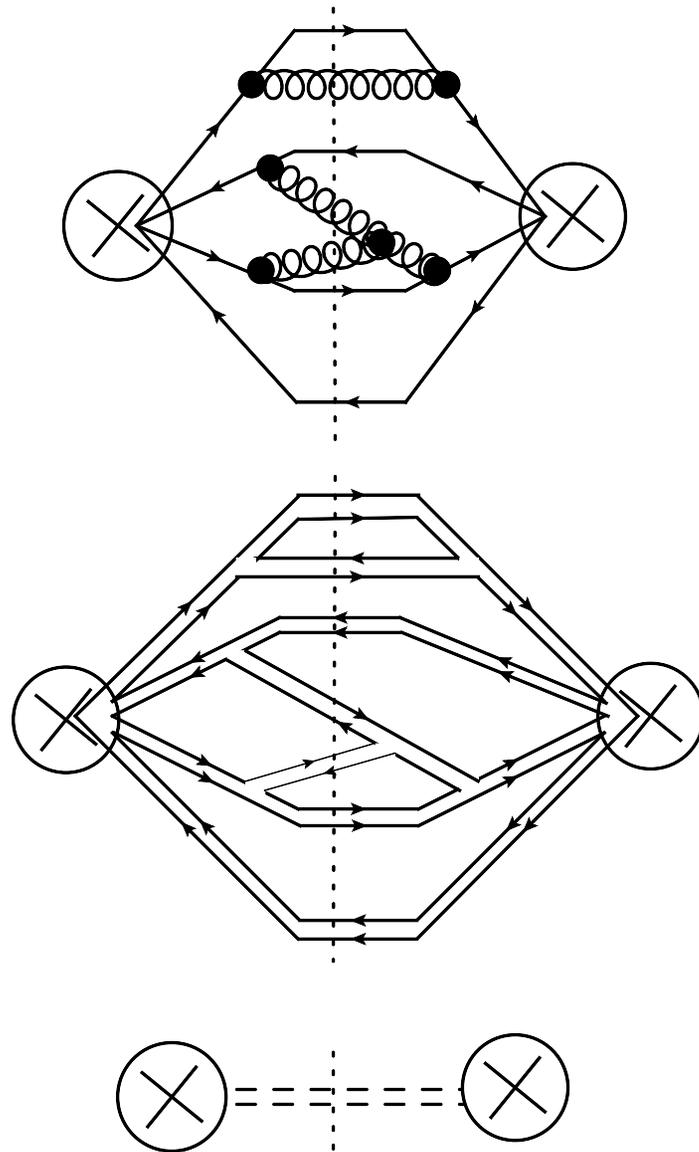


Color-flow diagram; 7 color loops $\sim N_c^7$; 6 factors of $g \sim N_c^{-3}$; overall scaling $\sim N_c^4$



Hadronic level diagram: propagation of a single tetraquark

The reason this corresponds to a single tetraquark can be understood in terms of a cut of the diagram.



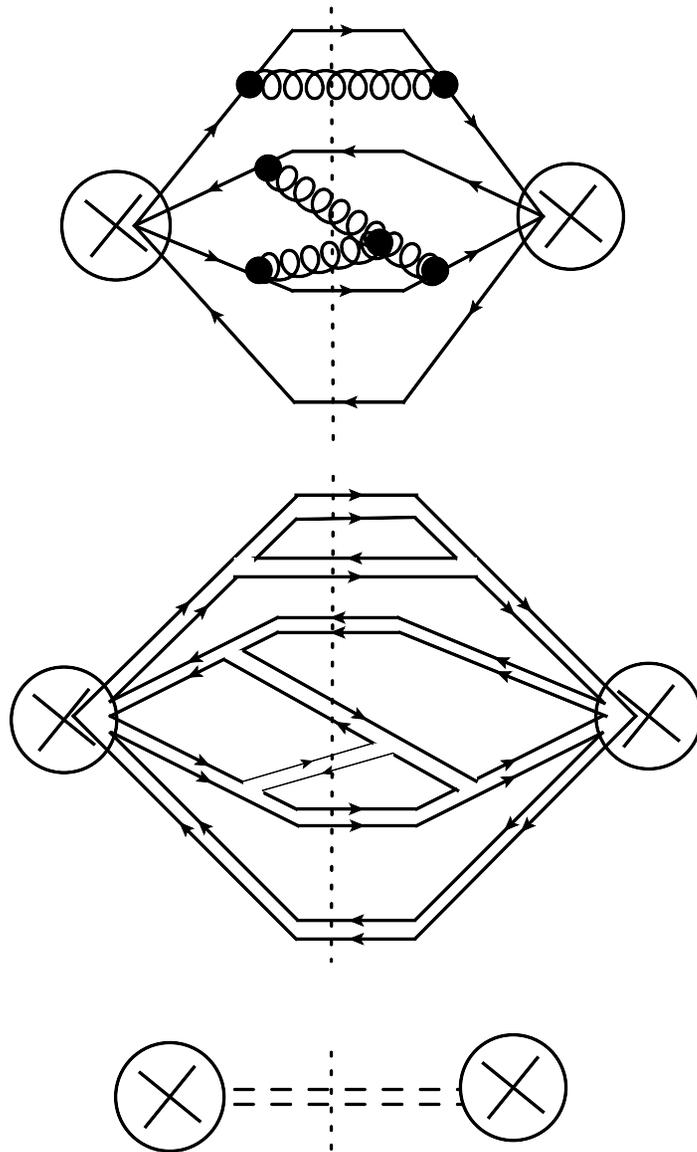
Short dashed line indicates a cut which reveals the intermediate state structure of the diagram.

The cut shown here corresponds to a state of the form

$$\bar{q}^{ab} q_{bc} A_d^c A_e^d \bar{q}^{ef} A_f^g q_{ga}$$

This is a single color-trace object. It can not be divided into two separate color singlets (except by a $1/N_c^2$ contribution)

This is generic: all cuts yield single-color trace objects



If one includes confinement, this implies that the state must be a single hadron at leading order. It cannot break up into two color singlet hadrons since all intermediate states consist of a single indivisible color singlet.

It must be narrow as components with more than one hadron are suppressed in the $1/N_c$ expansion.

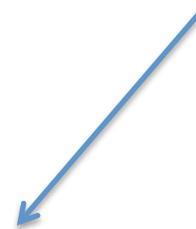
Exotics in QCD(AS)

Analysis of this type allows one to deduce that:

- Along with tetraquarks higher multi-quark hadrons (eg. heptaquarks) exist as narrow resonances in the large N_c limit of QCD(AS).
- Non-exotic tetraquarks exist and mix with ordinary mesons.
- The generic n -hadronic vertex will (if allowed by quantum numbers) scale as N_c^{2-n} .
- The width of all hadrons with phase space to decay will scale as N_c^{-2} .

- Thus whether tetraquarks with light quarks exist in the real world depends on whether the real world is closer (in this aspect) to QCD(F) at large N_c or to QCD(AS) at large N_c . This is a dynamical question.

The bottom line

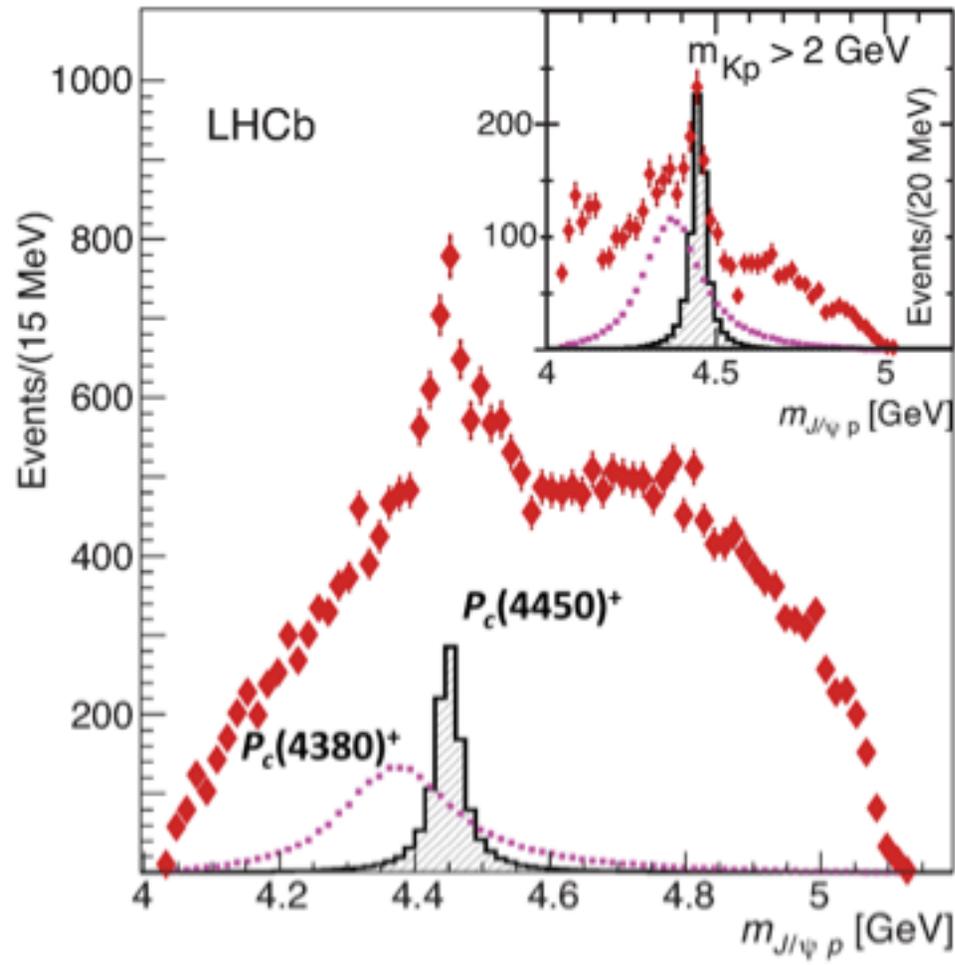


Generic large N_c analysis cannot give unambiguous insight as to whether narrow tetraquarks made of light quarks are likely to exist in the real world since QCD(AS) differs from QCD(F).

However, large N_c arguments based on QCD(AS) show such exotics cannot be excluded as incompatible with QCDlike theories

Back ups

What about Pentaquarks?



Ancient History

- The Θ^+ pentaquark was predicted by Diakonov, Petrov and Polyakov (DPP) in 1997 on the basis of large N_c considerations
 - It was done in the context of a chiral soliton model, but the prediction only depended on the collective quantization which *appears* to depend only on the large N_c structure and not on any details of the model and hence might be believed to be a model-independent prediction of large N_c .
 - Numerous experiments in the early part of this century designed “discovered” or “confirmed” the Θ^+ at masses near that predicted by DPP when looking over data taken for other purposes.

- However

- On the theory side it was shown (TDC (2003); Klebanov and Oyang 2004)) that the collective quantization procedure used by DPP was inconsistent with large N_c counting rules.

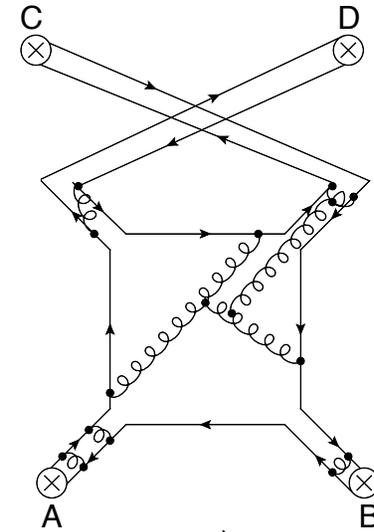
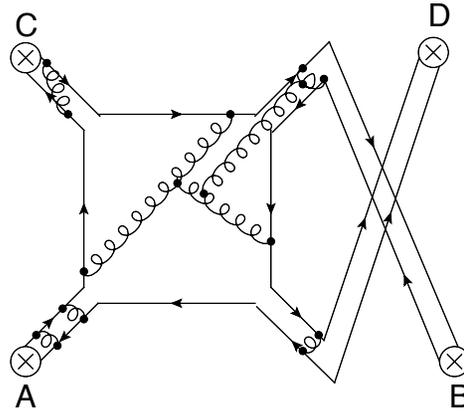
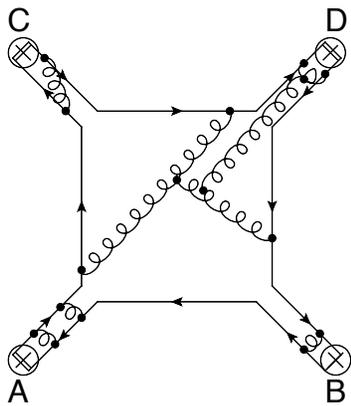
- Thus the detailed prediction of where a pentaquark should be at large N_c was wrong.
 - It was also shown that large N_c considerations alone neither require nor exclude pentaquark resonances; it is a matter of dynamical detail. If they exist, Pentaquark widths are of order N_c^0 .

- The experimental discovery of the Θ^+ was discredited when a high statistics dedicated experiments at Jefferson lab failed to see it, even though they had similar conditions and much better statistics to claimed discoveries.

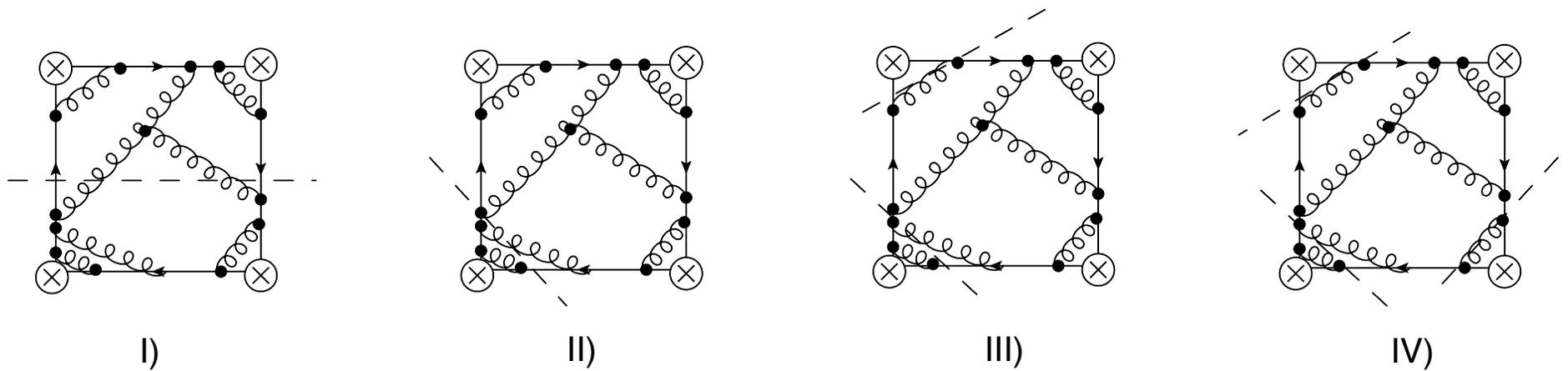
- It was also shown theoretically that heavy pentaquarks (i.e. containing one heavy antiquark) must exist in the combined large N_c and heavy quark limits (TDC, P. Hohler and R.F. Lebed (2005)) such states are stable in this limit and have a spectrum given by $SU(4) \times O(8) \times SU(2)$.
 - The reason these bind is essentially the one given earlier in the talk; heavy particles see more effective attraction. The group theory follows from spin-flavor + a nearly harmonic spectrum.
 - It was argued, however, that real world parameters were far enough from the combined limit that group structure is unlikely to be seen and it is an open question as to whether they would bind.

What about Pentaquarks in QCD(AS)?

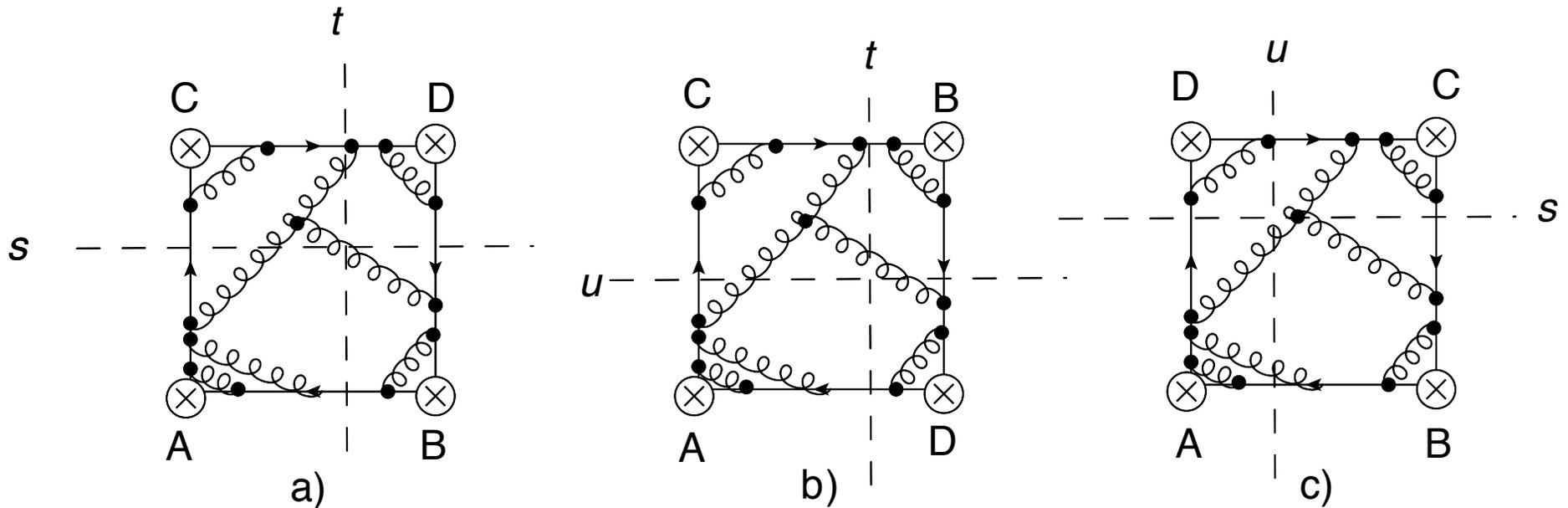
- Preceding arguments about pentaquarks were all in the context of QCD(F). Does anything significant change for QCD(AS).
 - Given the radical differences between QCD(AS) and QCD(F) for tetraquarks one might imagine a similar thing here.
 - However as far as I can see that this not the case here: appart from a change in scaling rules ($1/N_c \rightarrow 1/N_c^2$) the qualitative results are the same as in QCD(F)
 - Collective quantization a la DPP is not valid; no Θ^+ predicted
 - Whether pentaquarks exist is a matter of dynamical detail; if the do they have widths of order unity.
 - In extreme heavy quark and large N_c limit, pentaquarks, exist with by $SU(4) \times O(8) \times SU(2)$ symmetry.



Relating leading space-time diagrams to topological ones. Topologically all that matters is order of the four corners. Moreover, time-reversal invariance means **ABCD** is identical to **ADCB**. Thus, there are only three classes of diagrams **ABCD**, **ADBC** and **ABDC**.

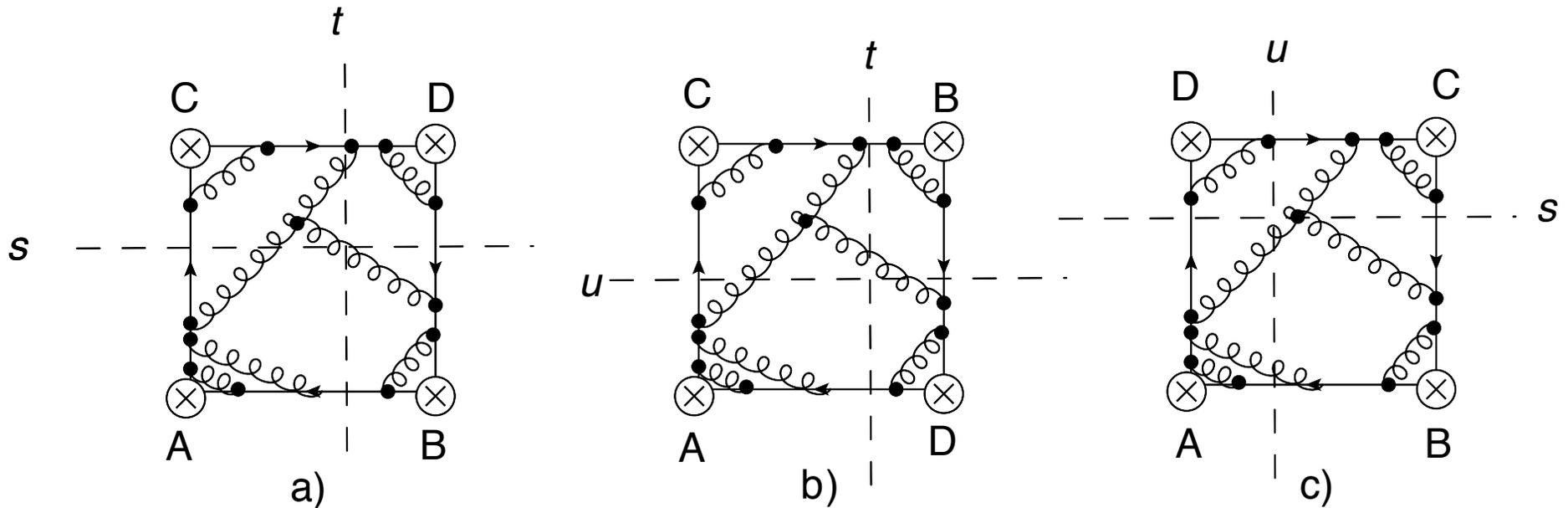


Broad categories of cuts. Note that except for category I) these all cut through a corner. These corner cuts all are associated with the momentum carried in at the corner and are eliminated when looking at the amputated diagram, AKA the scattering amplitude. Thus the only singularities in category I that could be associated with scattering going through a tetraquark. We will show below that this is not possible for exotic channels by looking at these in detail



Type I cuts. The three topological classes in terms of ordering are given here. Note that for each there are cuts in only two of the tree Mandelstam variables.

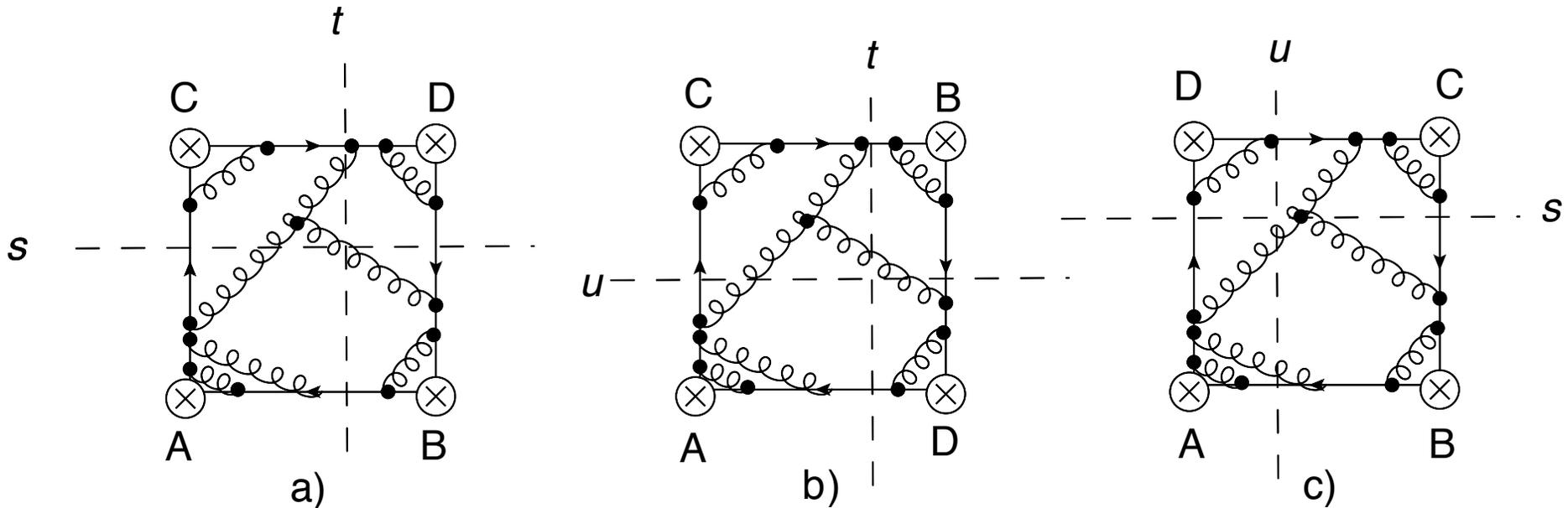
S-channel cuts exist in type a) and c) but not b). If we can show that in exotic channels only have topology b) then there is no s-channel cut at leading order in $1/N_c$ expansion and hence no tetra quark



Note that in a) and c) type diagrams A & B are adjacent to each other. If the channel is a flavor exotic (say isospin 2), then a quark line (isospin $\frac{1}{2}$) cannot run past an adjacent A&B since doing so must change its isospin to $\frac{3}{2}$ or $\frac{5}{2}$ but cannot keep it as $\frac{1}{2}$.

Thus, as advertised only b) is possible and it has no s-channel cut.

What about non-exotic channels in QCD(F ?)



The non-exotic channels **do** have s -channel singularities a) and c) are not forbidden by quantum numbers. But note they cut exactly one quark-antiquark pair. Thus they are associated with ordinary mesons. They are NOT tetraquarks.