On properties of the exotic hadrons from QCD sum rules

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I will discuss specific features of the description of exotic hadrons compared to normal hadrons within the method of QCD sum rules.

1. Strong decays

2. Structure of exotic states and mixing

3. Open problems

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Properties of individual resonances from OPE

• **The basic object**

T**-product of a number of the interpolating currents** $j(x)$**:**

$$\langle\Omega|j(0)|M\rangle = f_M \neq 0.$$

(E.g. $j(x) = \bar{q}_1(x)Oq_2(x)$ **for "normal" mesons, 4-quark currents for exotic mesons).**

The simplest object—2-point function

$$\Pi(p^2) = i\int d^4x\, e^{ipx}\left\langle\Omega\left|T\left(j(x)j^\dagger(0)\right)\right|\Omega\right\rangle$$

• **Wilsonian OPE - separation of distances:**

$$T\left(j(x)j^\dagger(0)\right) = C_0(x^2,\mu)\hat{1} + \sum_n C_n(x^2,\mu) : \hat{O}_n(x=0,\mu) :$$

$$\Pi(p^2) = \Pi_{\rm pert}(p^2,\mu) + \sum_n \frac{C_n}{(p^2)^n}\langle\Omega| : \hat{O}_n(x=0,\mu) : |\Omega\rangle$$

• **Physical QCD vacuum** $|\Omega\rangle$ **is complicated and differs from perturbative QCD vacuum** $|0\rangle$**.**

Condensates – nonzero expectation values of gauge-invariant operators over physical vacuum:

$$\boxed{\langle\Omega| : \hat{O}(0,\mu) : |\Omega\rangle \neq 0}$$

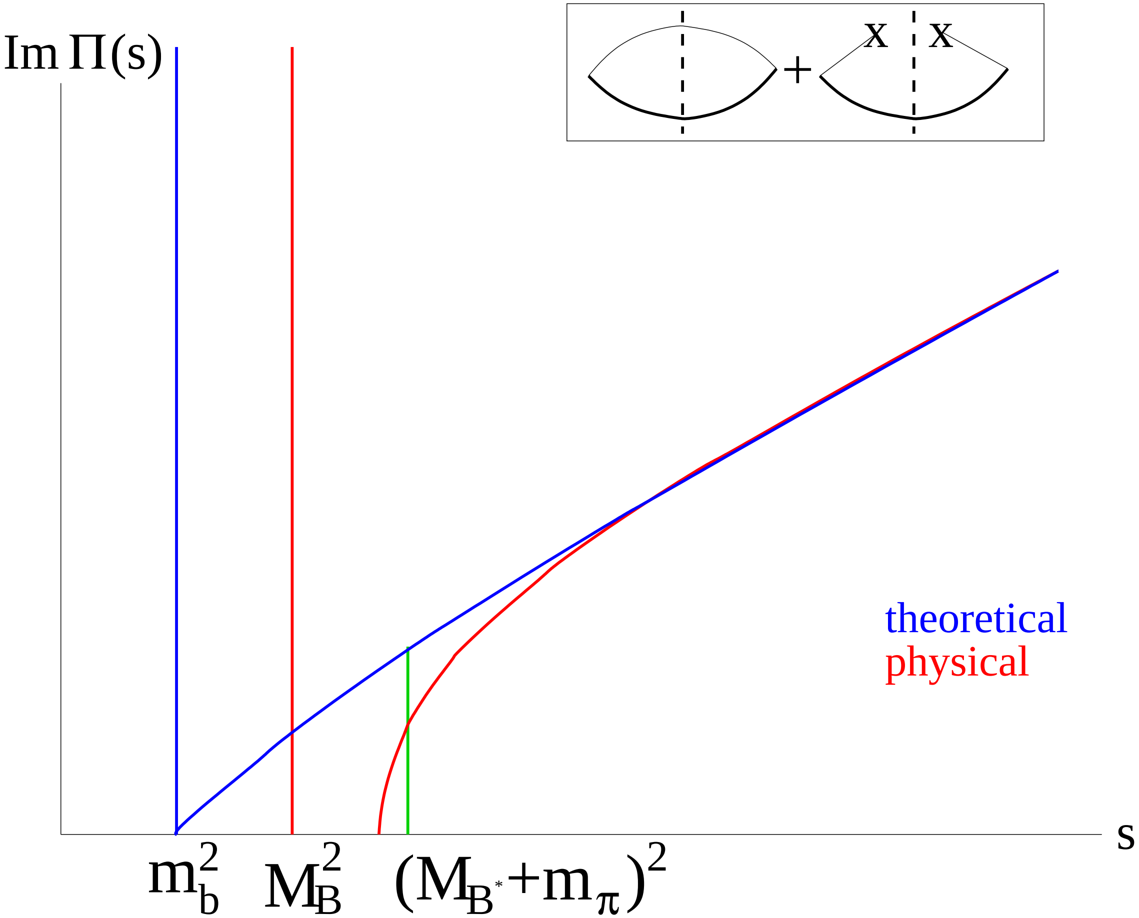
$$\langle\Omega|\bar{q}q(2\ {\rm GeV}|\Omega\rangle = (271\pm 3\ {\rm MeV})^3, \qquad \langle\Omega|\alpha_s/\pi GG|\Omega\rangle = 0.012\pm 0.006\ {\rm GeV}.$$

2-point function is analytic function of p^2

$$\Pi(p^2) = \int \frac{ds}{s - p^2}\rho(s),$$

One calculates the spectral densities using OPE and using hadron states

$$\rho_{\text{theor}}(s) = \left[\rho_{\text{pert}}(s,\mu) + \sum_n C_n \delta^{(n)}(s)\langle\Omega|O_n(\mu)|\Omega\rangle\right], \qquad \rho_{\text{hadr}}(s) = f^2\delta(s - M^2) + \rho_{\text{cont}}(s)$$



How to relate to each other truncated $\Pi_{\rm OPE}(p^2)$ and $\Pi_{\rm hadron}(p^2)$?

Borel transform $p^2 \to \tau$ $[\frac{1}{s-p^2} \to \exp(-\tau p^2)]$

$$\Pi(\tau) = \int ds\,\exp(-s\tau)\rho(s) = f^2 e^{-M_B^2\tau} + \int\limits_{s_{\rm phys}}^{\infty} ds\,e^{-s\tau}\rho_{\rm hadr}(s) = \int\limits_{(m_b+m)^2}^{\infty} ds\,e^{-s\tau}\rho_{\rm pert}(s,\mu) + \Pi_{\rm power}(\tau,\mu).$$

Here $s_{\rm phys}$ is the physical threshold, and f is the decay constant defined by

$$\langle 0|\bar{q}Ob|B\rangle = f.$$

To get rid of the excited-state contributions, one adopts the *duality Ansatz*: all contributions of excited states are counterbalanced by the perturbative contribution above an *effective continuum threshold*, $s_{\rm eff}(\tau)$ which differs from the physical continuum threshold.

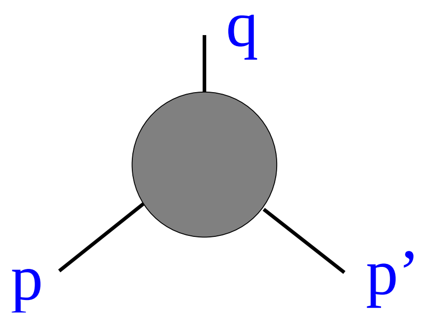
Applying the duality assumption yields:

$$f^2 e^{-M_B^2\tau} = \int\limits_{(m_b+m)^2}^{s_{\rm eff}(\tau)} ds\,e^{-s\tau}\rho_{\rm pert}(s,\mu) + \Pi_{\rm power}(\tau,\mu).$$

Strong decays from 3 – point vertex functions

• **The basic object:**

$$\Gamma(p,p',q)=\int\langle 0|T(J(x)j(0)j'(x')|0\rangle\exp(ipx-ip'x')dxdx'$$



This correlator contains the triple-pole in the Minkowski region: namely

$$\Gamma(p,p',q)=\frac{ff'}{(p^2-M^2)(p'^2-M'^2)}F(q^2)+\cdots$$

where the form factor $F(q^2)$ **contains pole at** $q^2=M_q^2$**:**

$$F(q^2)=\frac{f_q g_{MM'M_q}}{(q^2-M_q^2)}+\cdots$$

$g_{MM'M_q}$ **describes the** $M\to M_1M_2$ **strong transition;**

f**,** f'**, and** f_{M_q} **are the decay constants of the mesons** $\langle 0|j(0)|M\rangle=f_M$**.**

The three-point function satisfies the double spectral representation

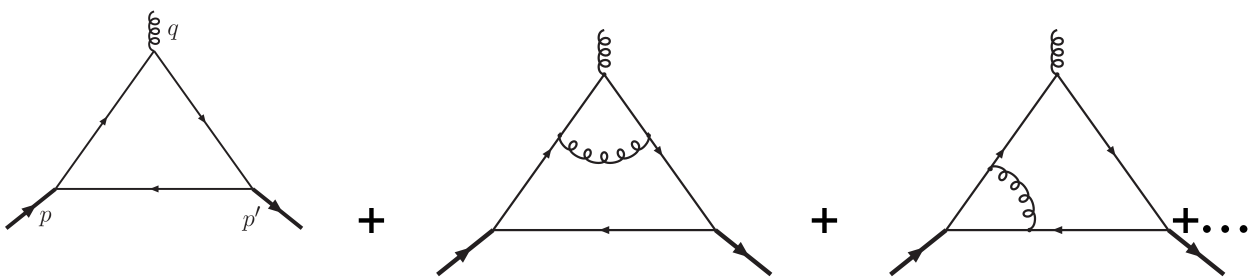
$$\Gamma(p,p',q)=\int \frac{ds}{s-p^2}\frac{ds'}{s'-p'^2}\Delta(s,s',q^2)$$

Perform double Borel transform $p^2\to\tau$, $p'^2\to\tau'$ **and applying duality we obtain**

$$\exp(-M^2\tau)\exp(-M'^2\tau')F(q^2)=\int^{s_{\rm eff}} ds\exp(-s\tau)\int^{s'_{\rm eff}} ds'\exp(-s'\tau')\Delta_{\rm OPE}(s,s',q^2)$$

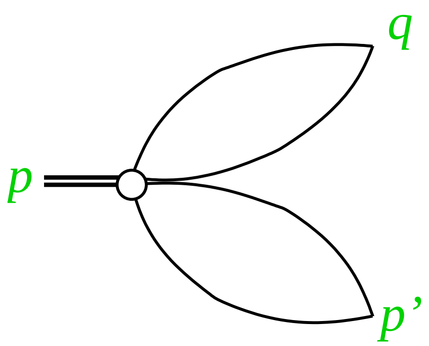
• **Normal hadrons:**

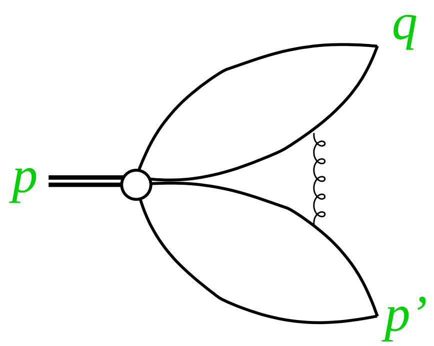
$$\Gamma_{\rm OPE}(p^2,p'^2,q^2)=\Gamma_0(p^2,p'^2,q^2)+\alpha_s\Gamma_1(p^2,p'^2,q^2)+\dots$$



Already one-loop zero-order diagram has a nonzero double-spectral density. And therefore provides a nonzero contribution to the form factor at small and intermediate momentum transfers (and to the coupling). Radiative corrections and are crucial for large q^2 **and improve the result.**

• Exotic hadrons:





$$\Gamma_{\rm OPE}(p^2, p'^2, q^2) = \Pi(p'^2)\Pi(q^2) + \alpha_s \Gamma_{\rm connected}(p^2, p'^2, q^2)$$

After the Borel transform $p^2 \to \tau$, the disconnected leading-order contribution vanishes.

Thus the LO contribution is not connected with exotic-state decay.
Clear from the factorization property $\Gamma(p, p', q) = \Pi(p'^2)\Pi(q^2)$
and from the large-N_c behaviour of the QCD diagrams.

The "fall-apart" decay mechanism of exotic hadrons differs from the decay mechanism of the ordinary hadrons and requires the appropriate treatment within QCD sum rules. The calculation of the radiative corrections is mandatory for a reliable analysis of the properties of the exotic states.

Outlook

Normal vs Exotic

• The 3-point (vertex) Green functions for the usual bilinear quark currents have crucial difference from the 3-point functions involving exotic (4-quark) current: the latter is described at leading order by disconnected diagrams, not related to strong decays of exotic hadrons. The relevant diagrams start at order α_s. This requires the calculation of α_s-corrections.

• Not all contributions to the 2-point functions of the exotic currents may relate to the exotic bound state: the N_c-scaling may be the guiding principle for selecting the appropriate contributions.

• Our experience from the 2-point functions of the bilinear currents shows that it is virtually impossible to study at the same time both the existence of the isolated ground state and of its coupling. However, if the mass is measured one can obtain reliable predictions e.g. for the decay constants. The observed exotic states are narrow, the procedure of extracting their parameters from the OPE has the same features and the same challenges as for the normal hadrons.

• **Structure and mixing**

Whereas the usual hadron (meson) is described by one (or few) decay constants

$$\langle 0|\bar{q}_1 O_A q_2|M\rangle = f_A$$

exotic states have many constants related to possible different structures of the 4-quark currents: "meson-meson" form

$$\langle 0|M_{12}(x)M_{34}(x)|\theta\rangle = f_{MM}, \qquad M_{12} = \bar{q}_1 O q_2$$

or in "diquark-diquark" form

$$\langle 0|\bar{D}^a_{13}(x)D^a_{24}(x)|\theta\rangle = f_{DD}, \qquad D^a_{24} = \epsilon^{abd}(q^b_{c4})^T q^d_2.$$

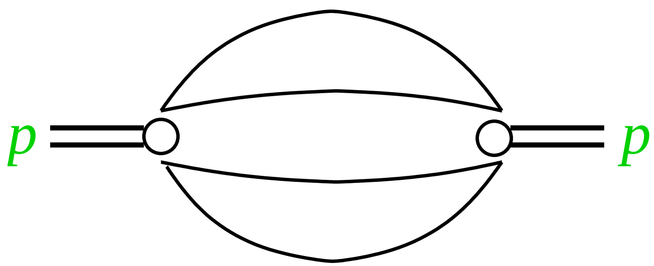
If in some limits certain class of the decay constants vanish, then one is eligible to say that the existing exotic state is a pure molecular or pure diquark-diguark state.

In general, mixing of different "components" seems unavoidable.

• **Structure of exotic states**

The form factors of the exotic states are the appropriate quantities to describe their properties.

Many efforts for otaining reliable predictions are necessary!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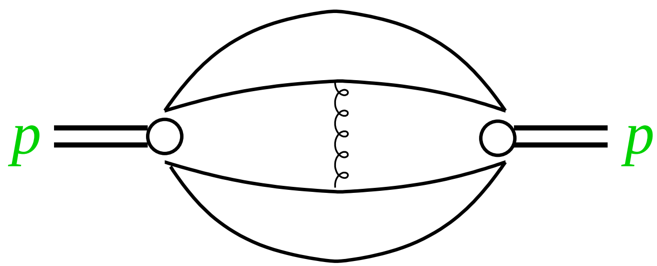
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