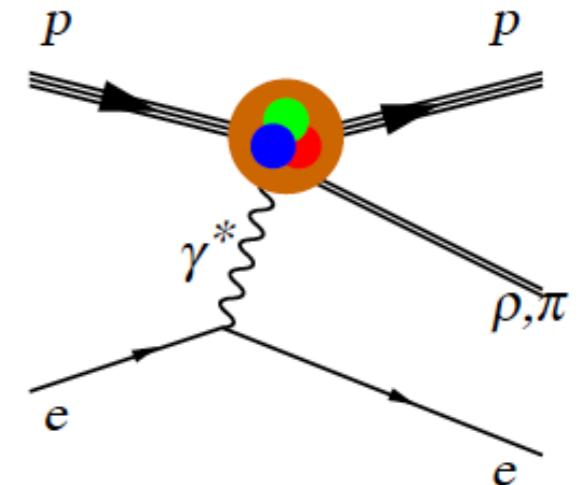
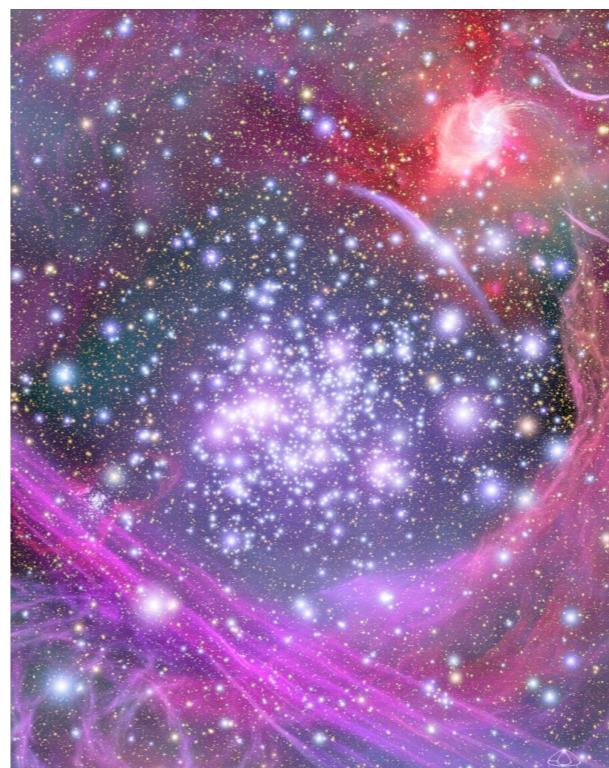
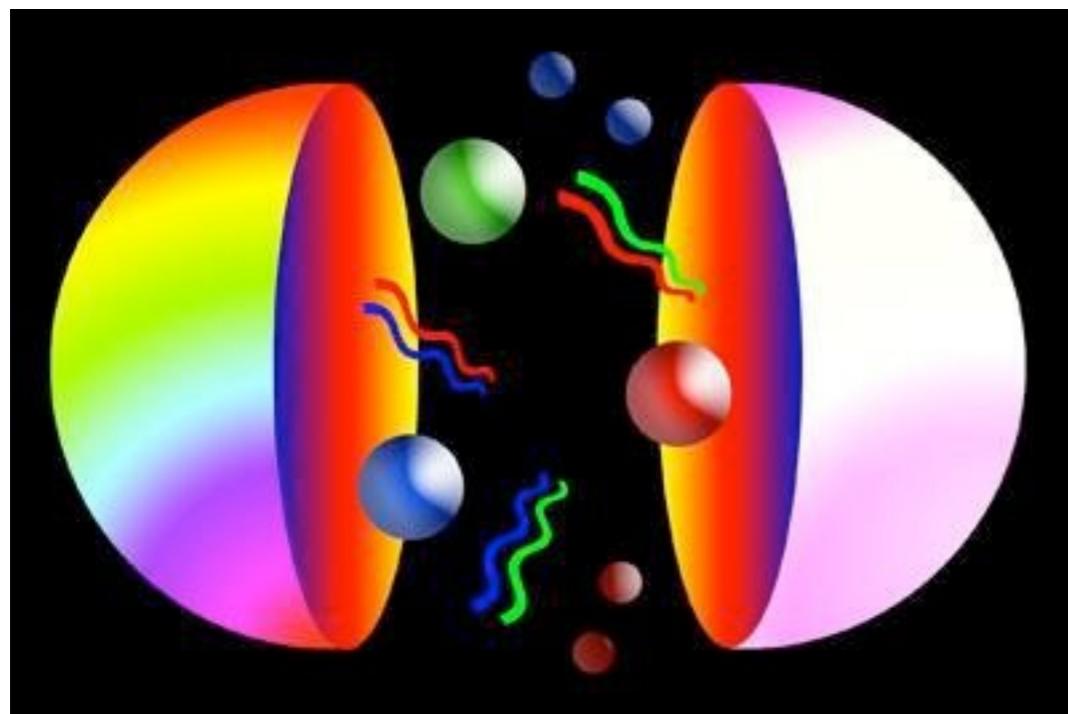


Pion and Kaon Properties from Dyson-Schwinger Eons



Peter C. Tandy

Dept of Physics
Kent State University USA

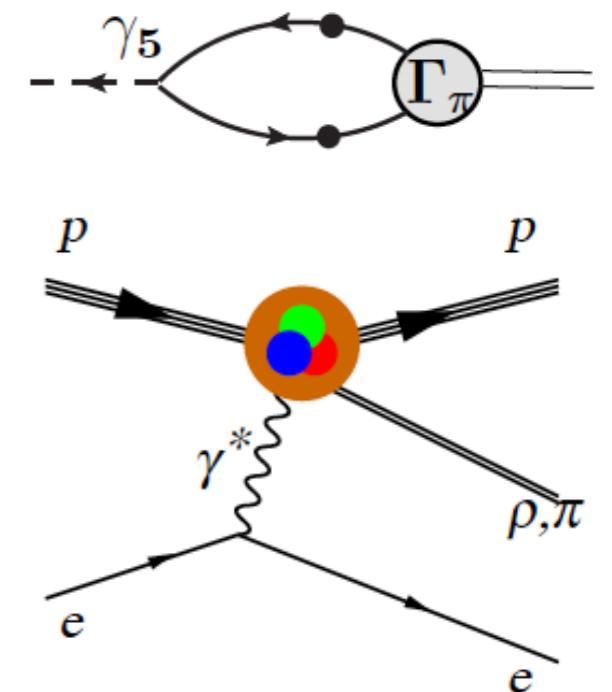
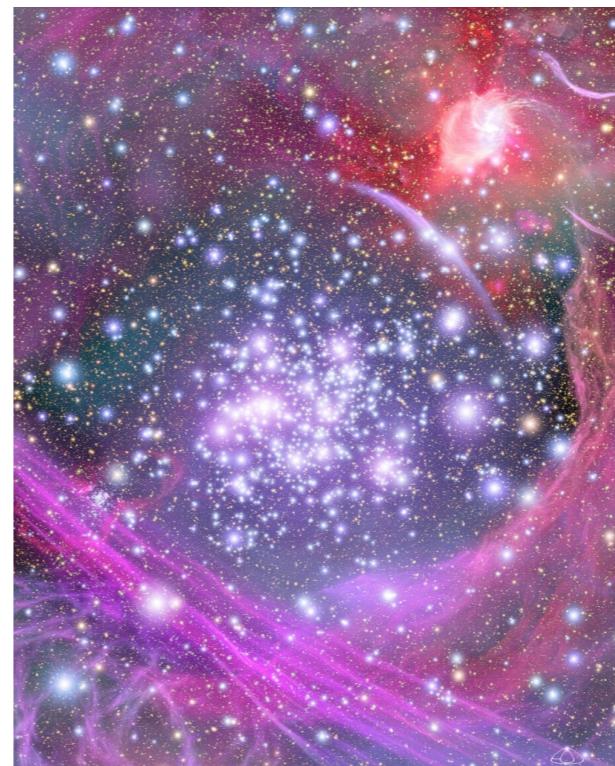
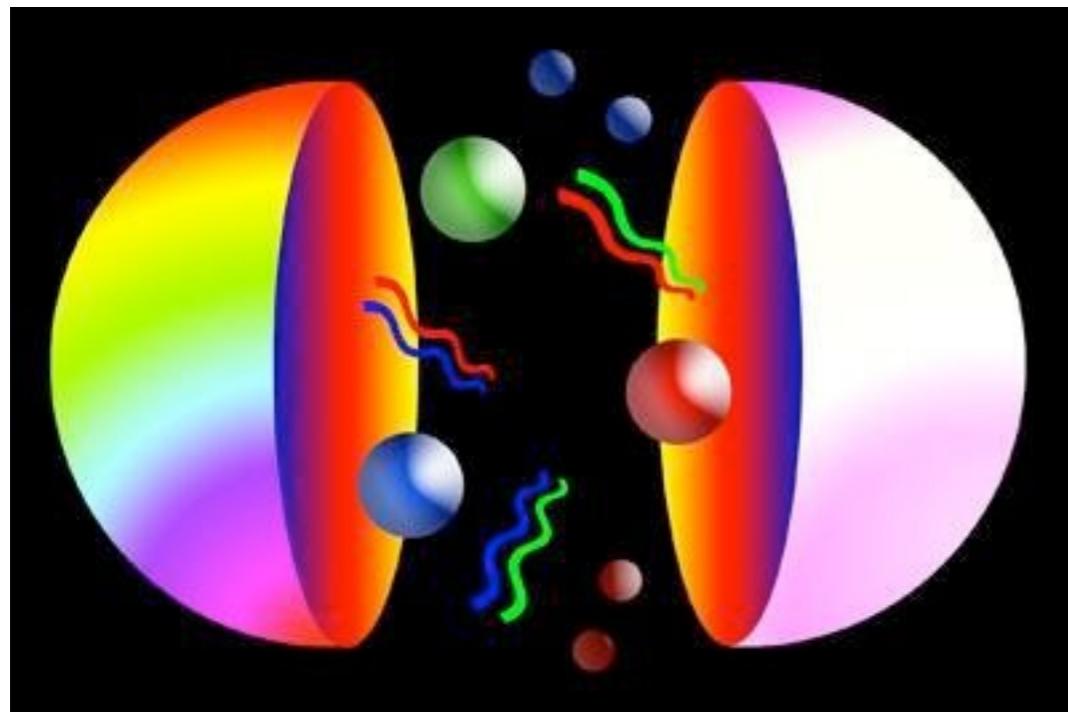


Pion and Kaon Properties from Dyson-Schwinger Eons



Peter C. Tandy

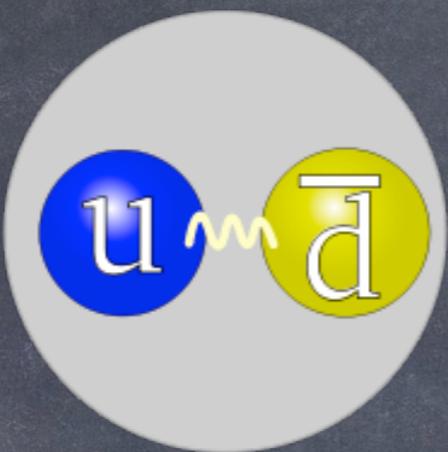
Dept of Physics
Kent State University USA



Topics

- DSE continuum approach to QCD
- Old work on pion and kaon properties and decays
- Parton distribution amplitudes and PDFs—mainly mesons as an example. DSE-model calculations with direct connection to QCD. Comparison to LQCD.
- Some applications to uv physics (Form Factors, HS behavior)
- PDFs including X. Ji's space-like correlator approximation for LQCD—a model investigation.

The Pion (1947)



The Pion (1947)



- ⦿ Cecil Powell (U. Bristol)



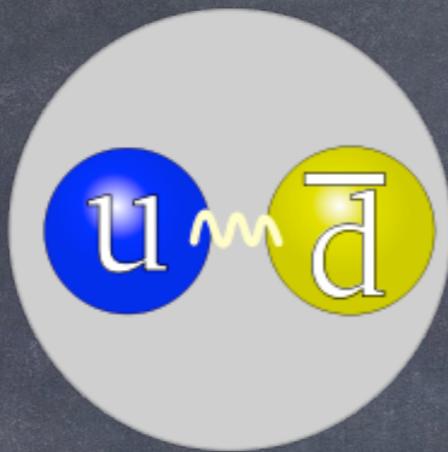
- ⦿ Giuseppe Occhialini (Italy)

- ⦿ Cesar Lattes (student,
Brazil)



- ⦿ Counting grains in
emulsions

The Pion (1947)



The Pion (1947)



The Pion (1947)

- Nobel Prize 1950 to Cecil Powell
- Now the simplest, lightest hadron--a test system for calibrating our npQCD understanding & tools
- Similar to the 1960-80s role of deuteron for nuclear structure/reaction tools...



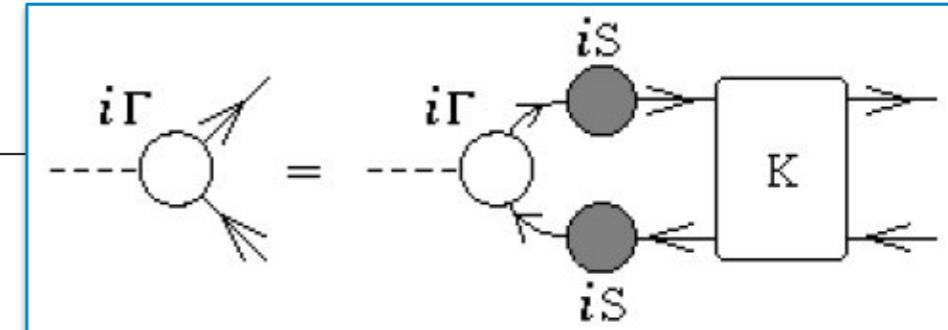
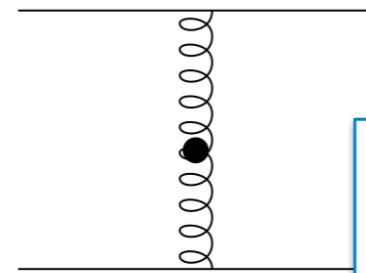
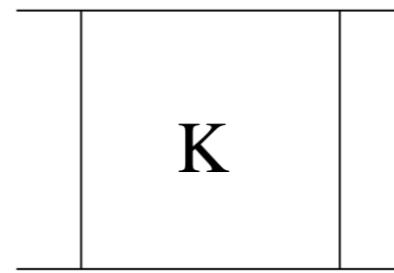
DSE Modeling of Hadron Physics

- Most common: Rainbow-ladder truncation of QCD's eqns of motion. Approximation to full BSE kernel now starting to produce results.....
- Constrain modeling by preserving AV-Ward-Takahashi Id, V-WTI. [Color singlet] Naturally implements DCSB, conserved vector current, Goldstone Thm, PCAC...
- RL truncation only good for ground state vector & pseudoscalar mesons, q-qq descriptions of baryons with AV and S diquarks.
- At the very least: DSE continuum QCD modeling suited for surveying the landscape quickly from large to small scales; finding out which underlying mechanisms are dominant. Applicable to all scales, high Q^2 form factors, etc. Do not expect ab initio final-precision QCD results, except in special cases. [pion, kaon..]
- Unifying DSE treatment of light front quantities (PDFs, GPDs, DA) with other aspects of hadron structure: masses, decays, charge form factors, transition form factors.....
- Pion & kaon $q\bar{q}$ Bethe-Salpeter wavefn is very well known

$$\text{AV - WTI : } m_q \rightarrow 0, P \rightarrow 0 \Rightarrow \Gamma_{\pi q\bar{q}}(k^2) = i\gamma_5 \frac{\frac{1}{4} \text{tr} S_0^{-1}(k)}{f_\pi^0} + \mathcal{O}(P)$$

Ladder-Rainbow Model

Landau gauge only



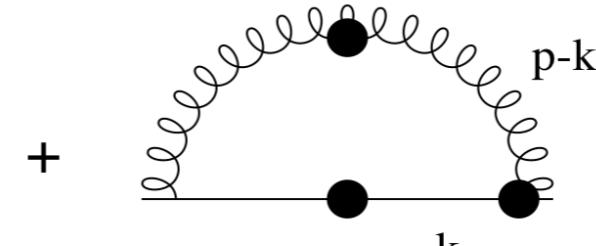
- $K_{\text{BSE}} \rightarrow -\gamma_\mu \frac{\lambda^a}{2} 4\pi \alpha_{\text{eff}}(q^2) D_{\mu\nu}^{\text{free}}(q) \gamma_\nu \frac{\lambda^a}{2}$

1 true phen
parameter

- $\alpha_{\text{eff}}(q^2) \xrightarrow{IR} \langle \bar{q}q \rangle_{\mu=1 \text{ GeV}} = -(240 \text{ MeV})^3$, incl vertex dressing

- $\alpha_{\text{eff}}(q^2) \xrightarrow{UV} \alpha_s^{\text{1-loop}}(q^2)$

$$p \xrightarrow{-1} = p \xrightarrow{-1}$$



modern π, K qDSE-BSE strategy: Maris & Roberts, PRC56, 3369 (1997)

- P. Maris & P.C. Tandy, PRC60, 055214 (1999)

M_ρ, M_ϕ, M_{K^*} good to 5%, f_ρ, f_ϕ, f_{K^*} good to 10% [fit : m_π, m_K, f_π], f_K (2%)

An Ansatz for the FULL QCD kernel:
L. Chang, C.D. Roberts, PRL103,
081601 (2009), + S. Qin (2015).

A more modern RL kernel: S. Qin, L.
Chang, C.D. Roberts, D.J. Wilson, PRC84,
042202 (2011).

Summary of light meson results

$m_{u=d} = 5.5 \text{ MeV}$, $m_s = 125 \text{ MeV}$ at $\mu = 1 \text{ GeV}$

Pseudoscalar (PM, Roberts, PRC56, 3369)

	expt.	calc.
$-\langle \bar{q}q \rangle_\mu^0$	$(0.236 \text{ GeV})^3$	$(0.241^\dagger)^3$
m_π	0.1385 GeV	0.138^\dagger
f_π	0.0924 GeV	0.093^\dagger
m_K	0.496 GeV	0.497^\dagger
f_K	0.113 GeV	0.109

Charge radii (PM, Tandy, PRC62, 055204)

r_π^2	0.44 fm^2	0.45
$r_{K^+}^2$	0.34 fm^2	0.38
$r_{K^0}^2$	-0.054 fm^2	-0.086

$\gamma\pi\gamma$ transition (PM, Tandy, PRC65, 045211)

$g_{\pi\gamma\gamma}$	0.50	0.50
$r_{\pi\gamma\gamma}^2$	0.42 fm^2	0.41

Weak K_{l3} decay (PM, Ji, PRD64, 014032)

$\lambda_+(e3)$	0.028	0.027
$\Gamma(K_{e3})$	$7.6 \cdot 10^6 \text{ s}^{-1}$	7.38
$\Gamma(K_{\mu 3})$	$5.2 \cdot 10^6 \text{ s}^{-1}$	4.90

Vector mesons

(PM, Tandy, PRC60, 055214)

$m_{\rho/\omega}$	0.770 GeV	0.742
$f_{\rho/\omega}$	0.216 GeV	0.207
m_{K^*}	0.892 GeV	0.936
f_{K^*}	0.225 GeV	0.241
m_ϕ	1.020 GeV	1.072
f_ϕ	0.236 GeV	0.259

Strong decay (Jarecke, PM, Tandy, PRC67, 035202)

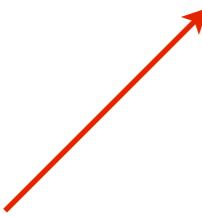
$g_{\rho\pi\pi}$	6.02	5.4
$g_{\phi KK}$	4.64	4.3
$g_{K^* K\pi}$	4.60	4.1

Radiative decay (PM, nucl-th/0112022)

$g_{\rho\pi\gamma}/m_\rho$	0.74	0.69
$g_{\omega\pi\gamma}/m_\omega$	2.31	2.07
$(g_{K^* K\gamma}/m_K)^+$	0.83	0.99
$(g_{K^* K\gamma}/m_K)^0$	1.28	1.19

Scattering length (PM, Cotanch, PRD66, 116010)

a_0^0	0.220	0.170
a_0^2	0.044	0.045
a_1^1	0.038	0.036



Bridging a gap between continuum-QCD and ab initio predictions of hadron observables

Daniele Binosi (ECT, Trento & Fond. Bruno Kessler, Trento), Lei Chang (Adelaide U., Sch. Chem. Phys.), Joannis Papavassiliou (Valencia U. & Valencia U., IFIC), Craig D. Roberts (Argonne, PHY). Dec 15, 2014. 6 pp.
Published in **Phys.Lett. B742 (2015) 183-188**

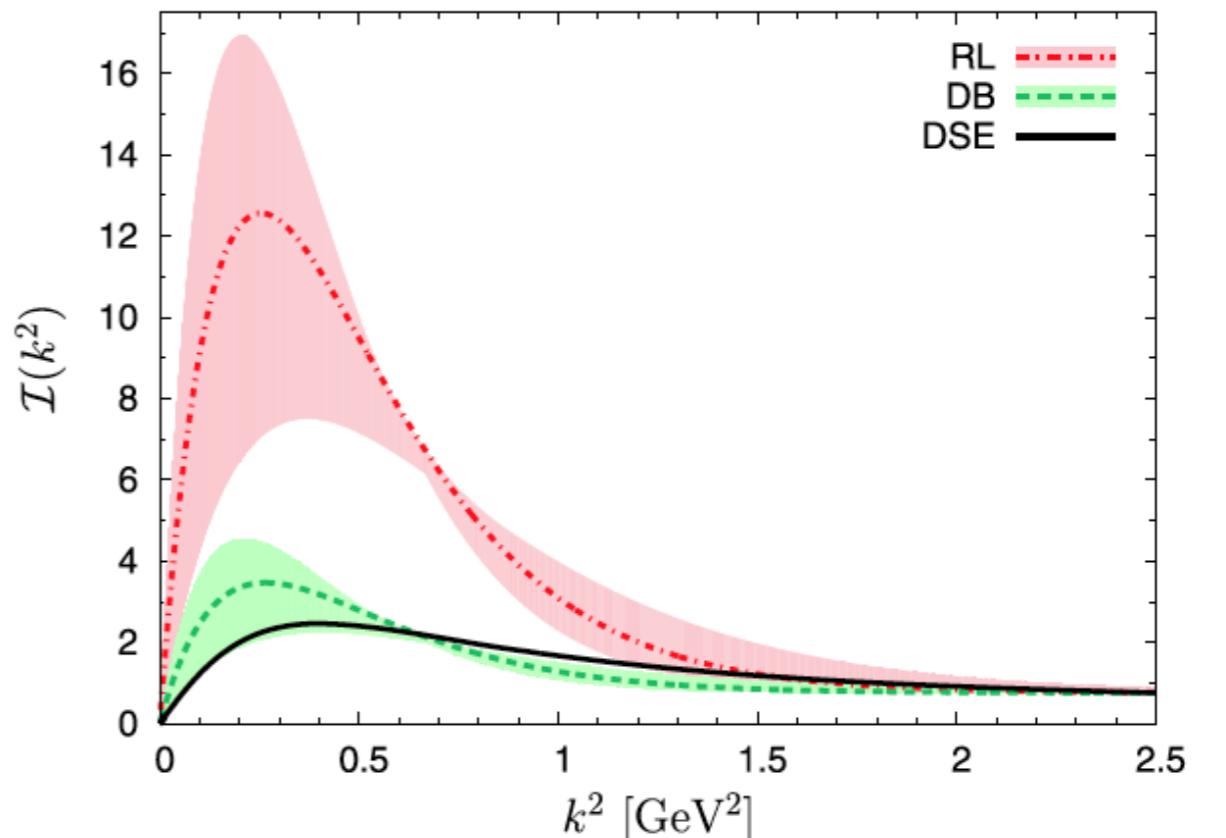


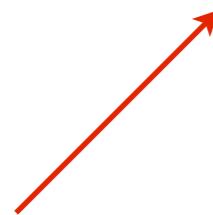
Table 1

Row 1 – Computed values determined from the interaction tension in Eq. (23), quoted in GeV; and Row 2 – the difference: $\varepsilon_S := \xi_I / \xi_{I_d} - 1$. So as to represent the domain of constant ground-state physics, described in connection with Eq. (5), we list values obtained with bottom-up interactions using $\omega = 0.5, 0.6$ GeV.

I	I_d	$I_{DB}^{\omega=0.5}$	$I_{DB}^{\omega=0.6}$	$I_{RL}^{\omega=0.5}$	$I_{RL}^{\omega=0.6}$
ξ_I	1.86	1.91	1.82	3.14	2.90
ε_S	0	2.8%	-2.4%	68.5%	55.8%

Modern Context for DSE Interaction Kernel

Landau gauge, lattice – QCD gluon propagator,
I.L.Bogolubisky *etal.*, PosLAT2007, 290 (2007)



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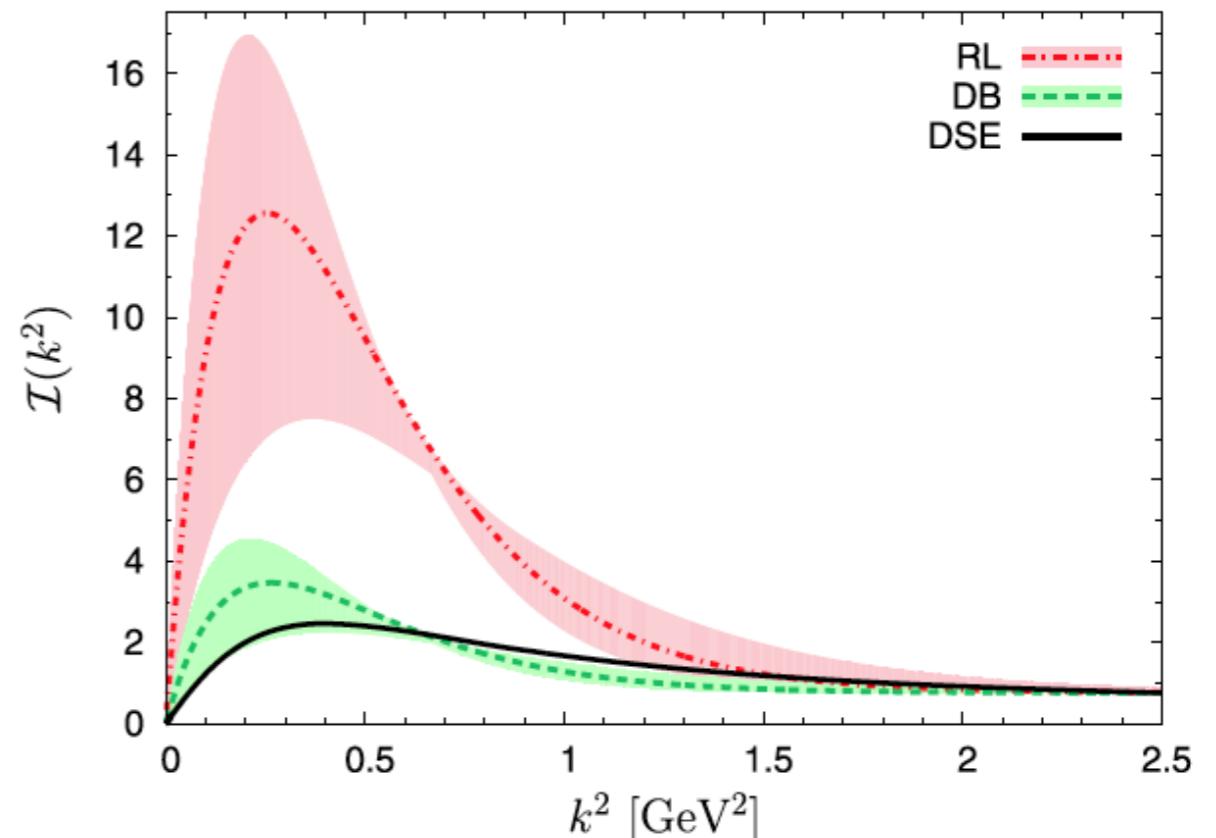
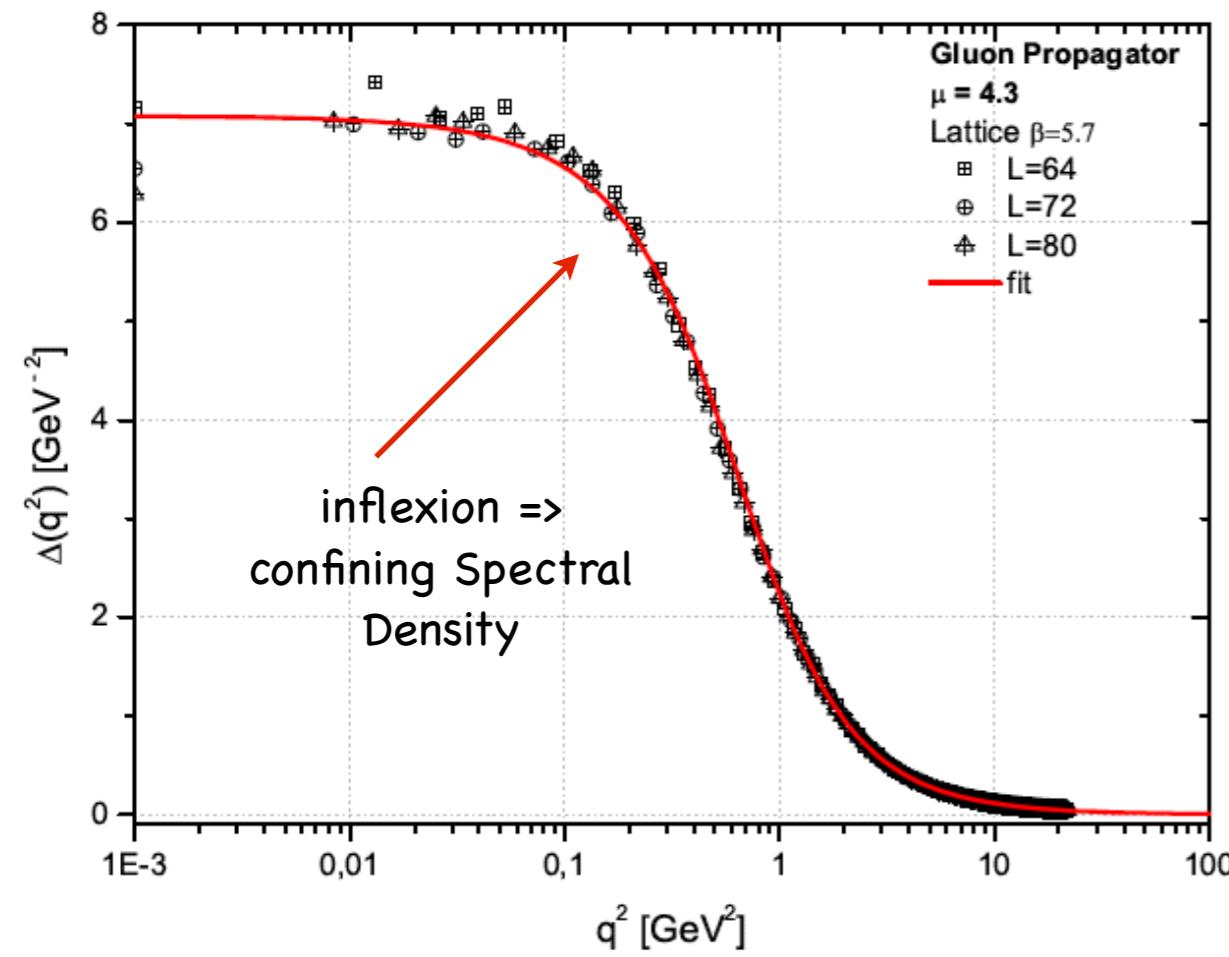


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Landau gauge, lattice – QCD gluon propagator,
 I.L.Bogolubisky *etal.*, PoS(LAT2007), 290 (2007)

$$\Rightarrow m_G(k^2) \quad m_G(0) \sim 0.38 \text{ GeV}$$

Bridging a gap between continuum-QCD and ab initio predictions
 of hadron observables

Daniele Binosi (ECT, Trento & Fond. Bruno Kessler, Trento), Lei
 Chang (Adelaide U., Sch. Chem. Phys.), Joannis Papavassiliou (Valencia
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 Published in **Phys.Lett. B742 (2015) 183-188**

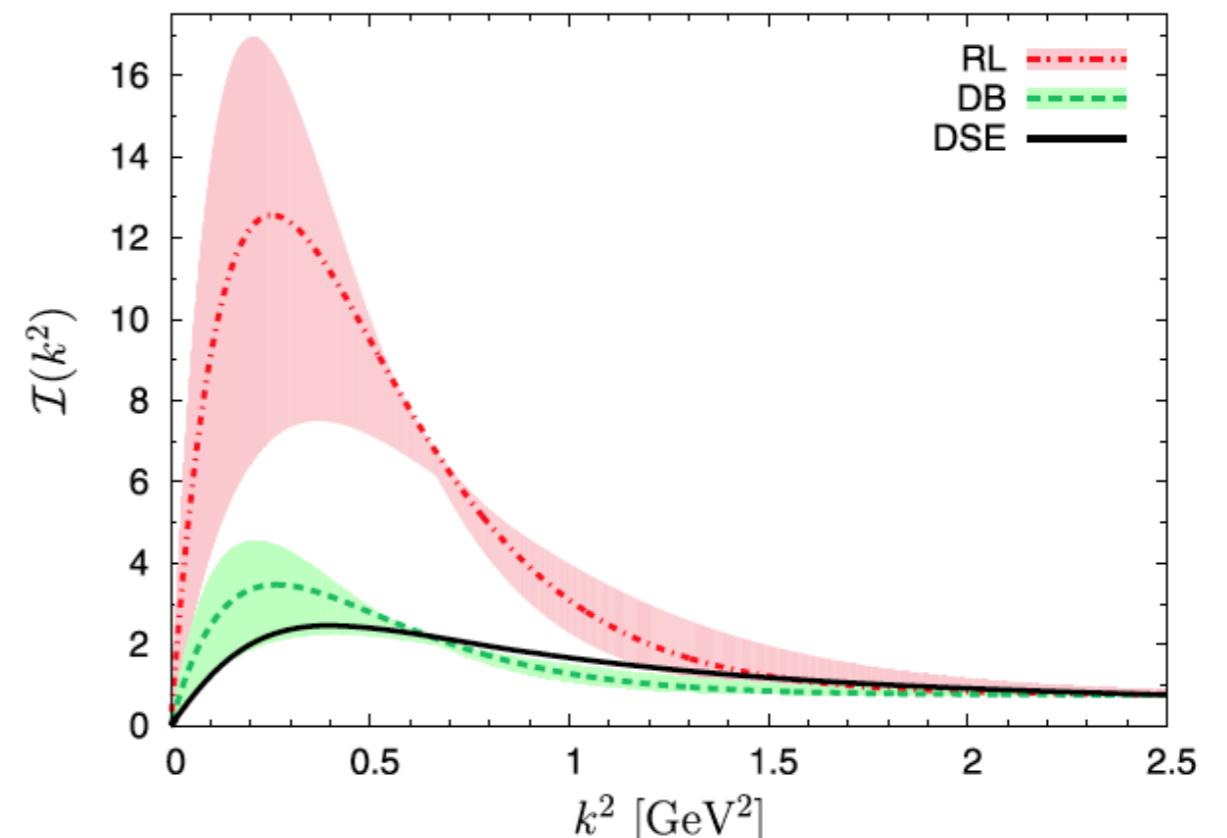


Table 1

Row 1 – Computed values determined from the interaction tension in Eq. (23), quoted in GeV; and Row 2 – the difference: $\varepsilon_\zeta := \zeta_I / \zeta_{I_d} - 1$. So as to represent the domain of constant ground-state physics, described in connection with Eq. (5), we list values obtained with bottom-up interactions using $\omega = 0.5, 0.6$ GeV.

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ε_ζ	0	2.8%	-2.4%	68.5%	55.8%

Exact Mass Relation for Flavor Non-Singlet PS Mesons

PCAC $\Rightarrow \langle \bar{q}(x)q(y) (\partial_\mu J_5)_\mu = 2m_q J_5 \rangle \Rightarrow AV - WTI :$

$$-iP_\mu \Gamma_{5\mu}(k; P) = S^{-1}(k+P/2) \gamma_5 \frac{\tau}{2} + \gamma_5 \frac{\tau}{2} S^{-1}(k-P/2) - 2m_q \Gamma_5(k; P)$$

$$\Gamma_\pi(k; P) \frac{f_\pi P_\mu}{P^2 + m_\pi^2}$$

$$\Gamma_\pi(k; P) \frac{i \rho_\pi}{P^2 + m_\pi^2}$$

- $m_q = 0 : S_0^{-1}(k) = i k A_0(k^2) + B_0(k^2)$
- $m_q = 0, P = 0 \Rightarrow GT_q : \Gamma_\pi(k^2; 0) = i \gamma_5 \tau \frac{B_0(k^2)}{f_\pi^0} + \dots$ ie, Goldstone Thm
- $m_q \neq 0 : \Rightarrow f_\pi m_\pi^2 = 2 m_q \rho_\pi(m_q)$ [for all m_q , all ps mesons]

$$\bullet \quad \rho_{ps}(\mu) = -\langle 0 | \bar{q} \gamma_5 q | ps \rangle \quad \text{---} \quad \begin{array}{c} \gamma_5 \\ \swarrow \quad \searrow \\ \Gamma_\pi \end{array} \quad \rightarrow \frac{|\langle \bar{q}q \rangle|}{f_\pi^0} + \mathcal{O}(m_q) \quad (\text{GMOR})$$

Maris, Roberts, PCT, Phys. Lett. B420, 267(1998) — an exact result in QCD

Pion $F(Q^2)$: Low Q^2

(P Maris & PCT, PRC 61, 045202 (2000))

(P. Maris & PCT, PRC 62, 0555204 (2000))

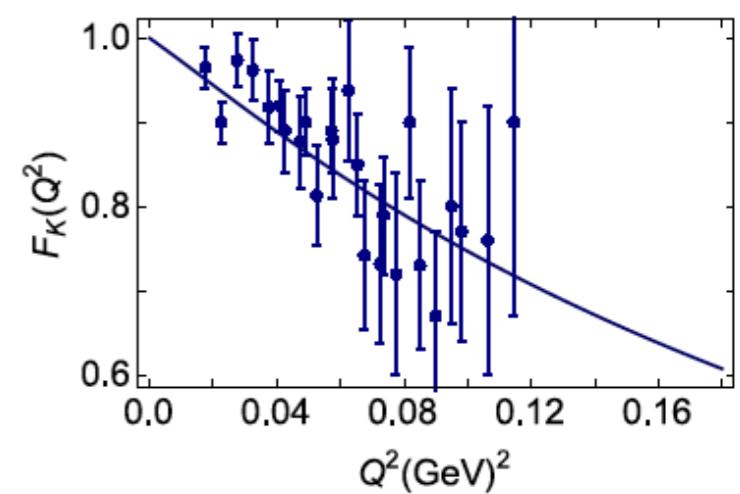
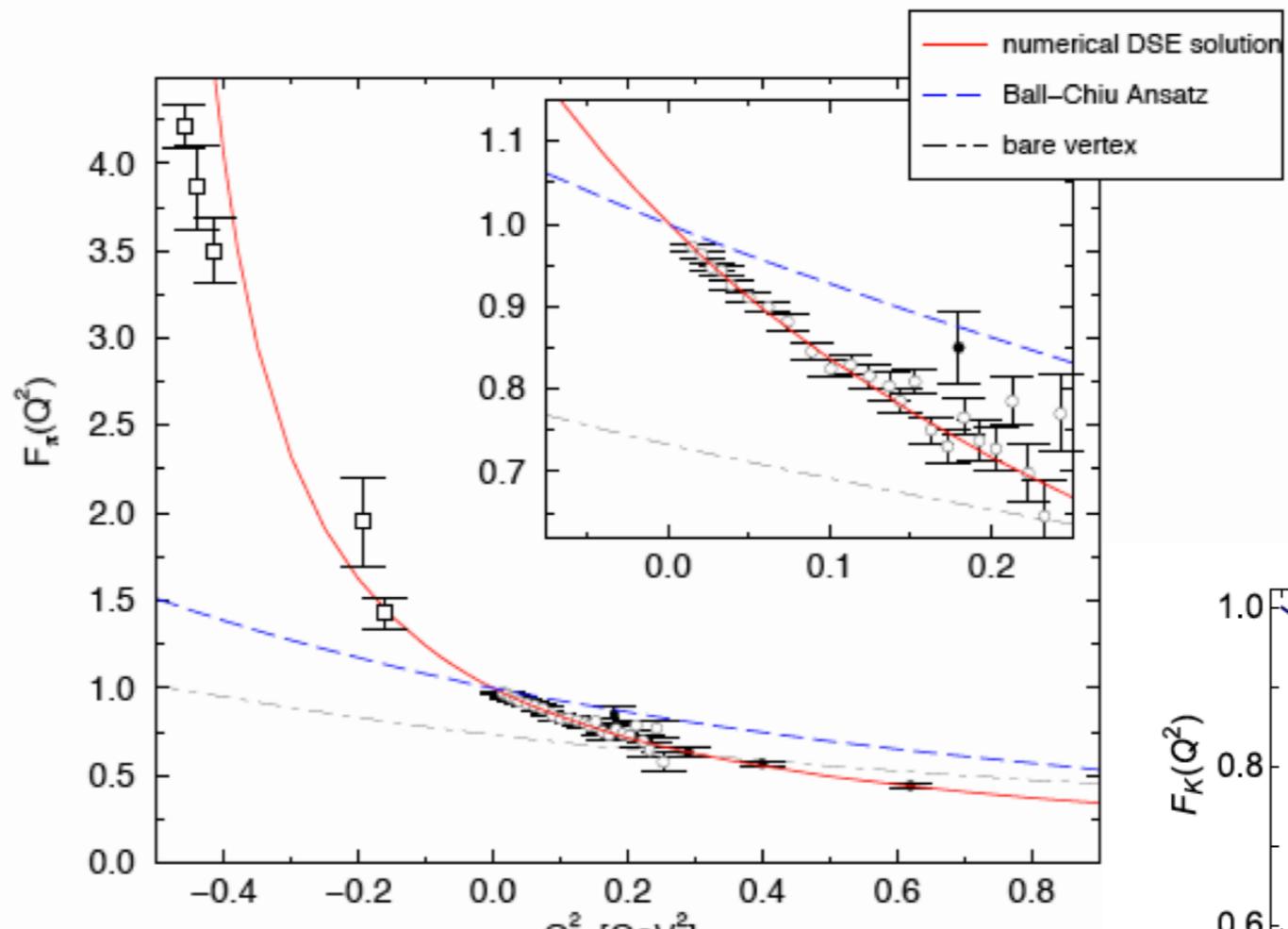
$r_\pi^{\text{DSE}} = 0.68 \text{ fm}$	$r_\pi^{\text{expt}} = 0.663 \pm .006 \text{ fm}$
--	---

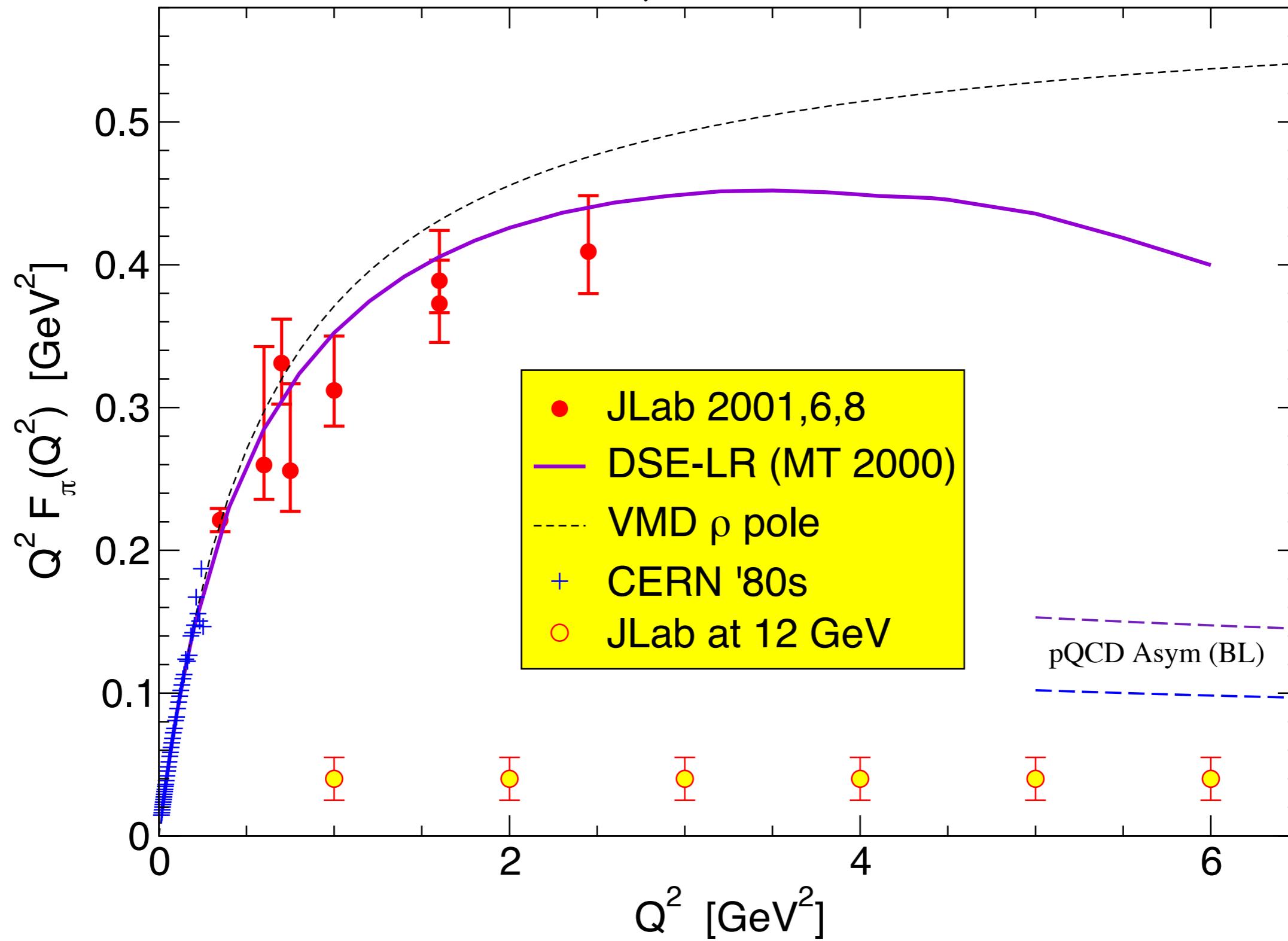
$$\Gamma_\pi = \sqrt{1 - \alpha^2} \Gamma_{q\bar{q}}^{\text{RL}} + \alpha \Gamma_{\pi q\bar{q}}$$

CPT: 18% effect

$$r_{\text{ch}}^2 = (1 - \alpha^2) r_{\text{RL}}^2 + \alpha^2 r_{\pi-\text{lp}}^2$$

$$\text{DSE-RL: } r_{\text{RL}}^2 = r_{\text{ch}}^2 \Rightarrow \alpha^2 = 18\%$$





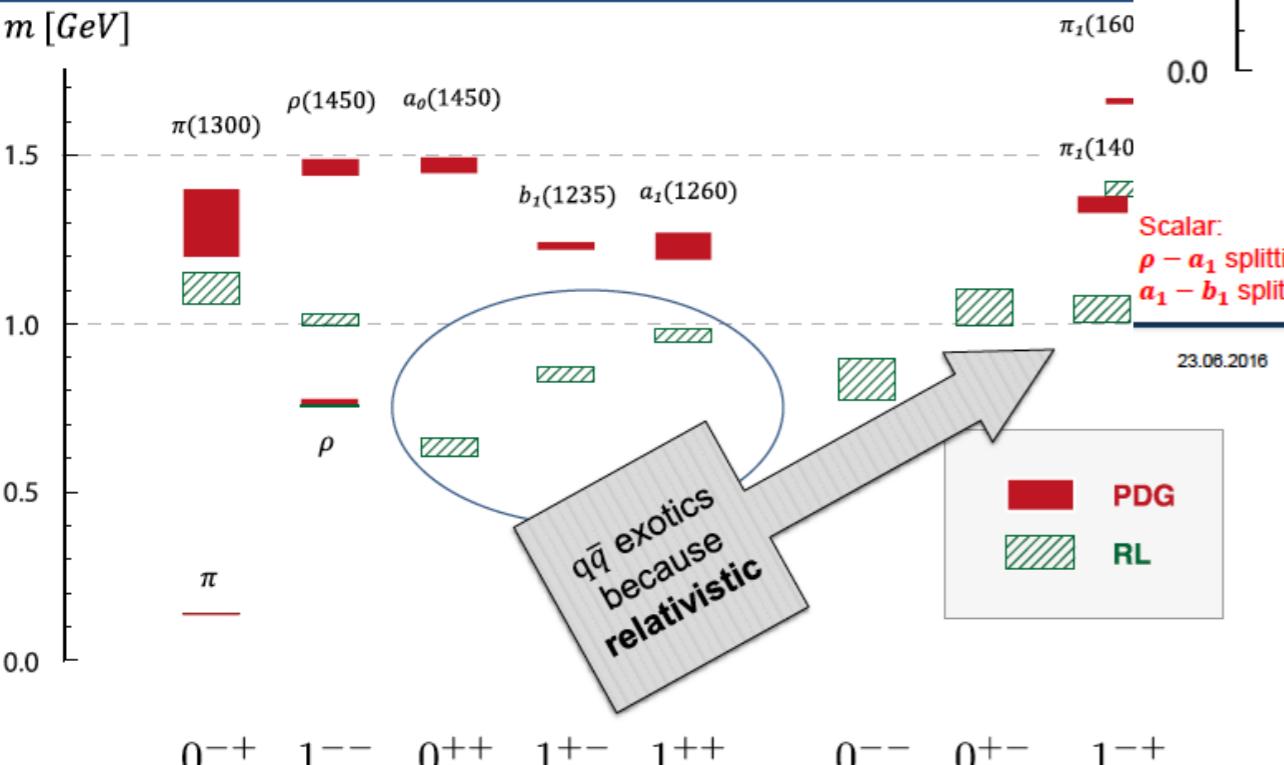
Jlab data: G. Huber et al., PRC78, 045203 (2008)

Much More Work on Mesons and Baryons...

Results: beyond rainbow-ladder

18 of 28

Light mesons: ground and excited states

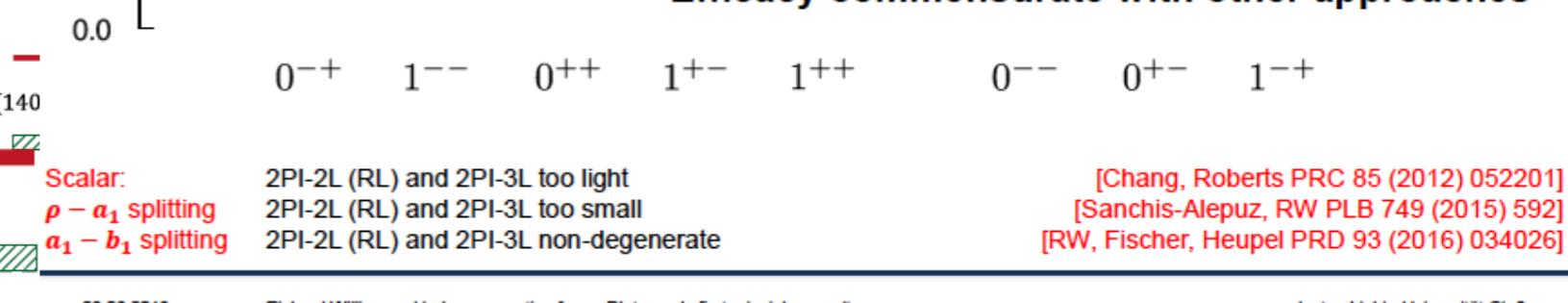


- Sensitivity to interaction exasperated in light sector
- Deficiencies in many channels

[Kubrak, Fischer, RW EPJA 50 (2014) 126]
[Hilger, Gomez-Rocha, Krassnigg arXiv:1508.07183]

Justus-Liebig-Universität Gießen

Efficacy commensurate with other approaches



[Chang, Roberts PRC 85 (2012) 052201]
[Sanchis-Alepuz, RW PLB 749 (2015) 592]
[RW, Fischer, Heupel PRD 93 (2016) 034026]

Justus-Liebig-Universität Gießen

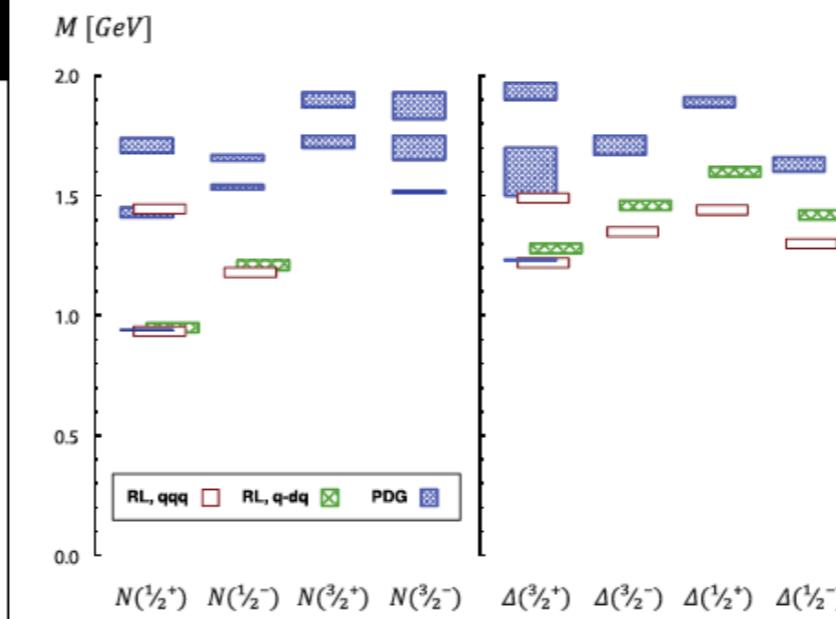
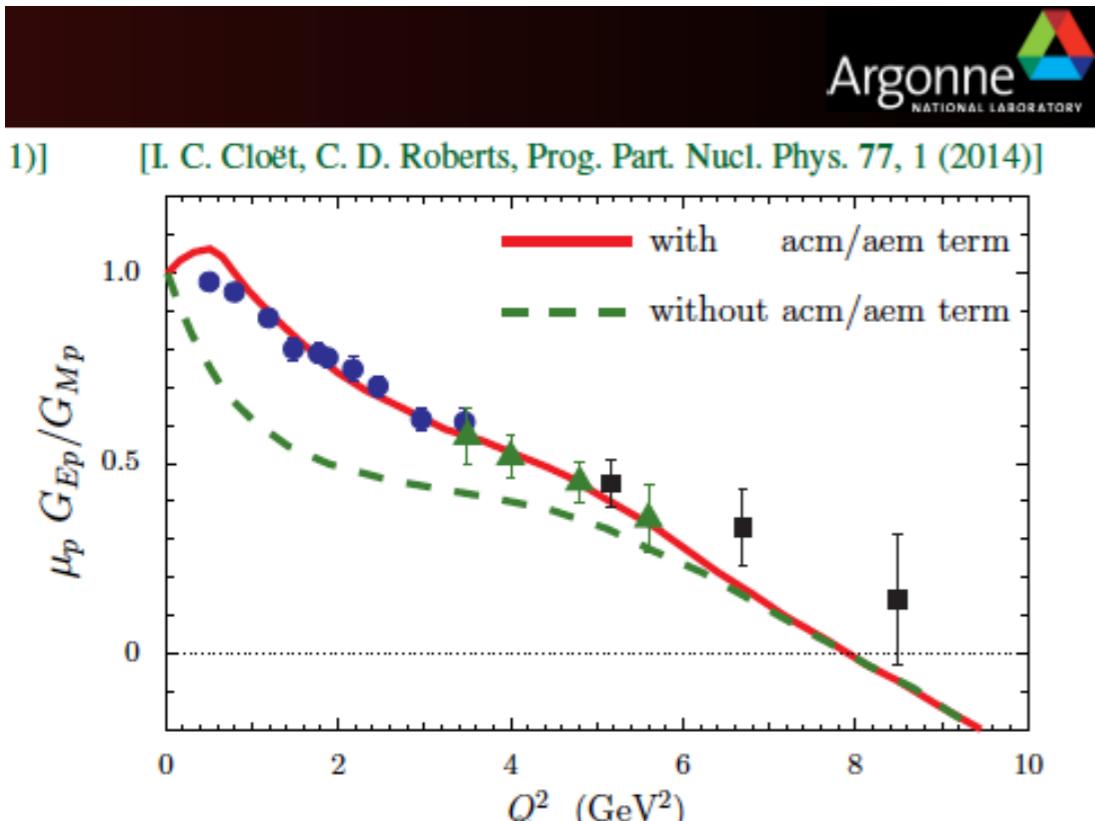
Richard Williams – Hadron properties from nPI: towards first principles results

23.06.2016

Much More Work on Mesons and Baryons...

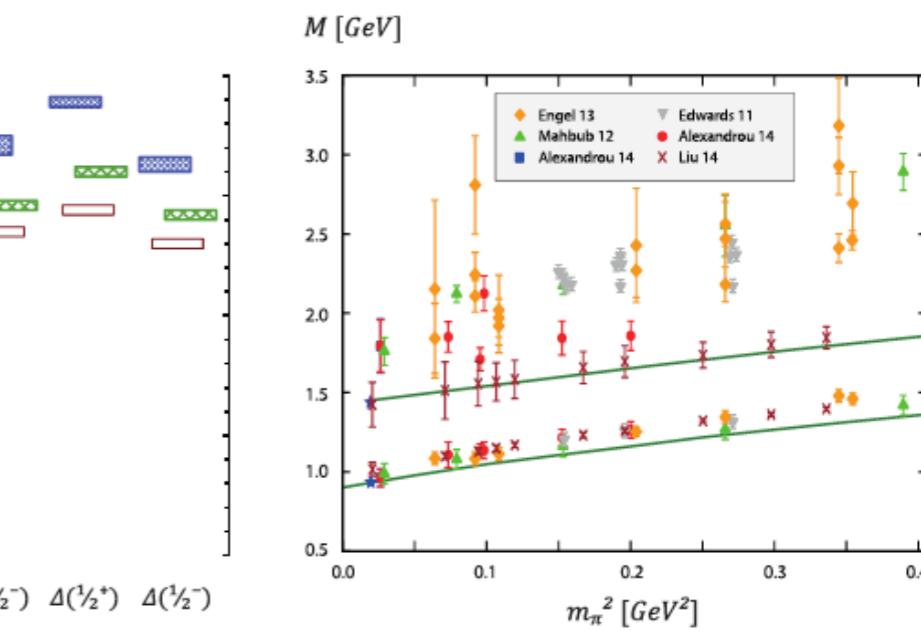
Light baryons: ground and excited states

9 of 28



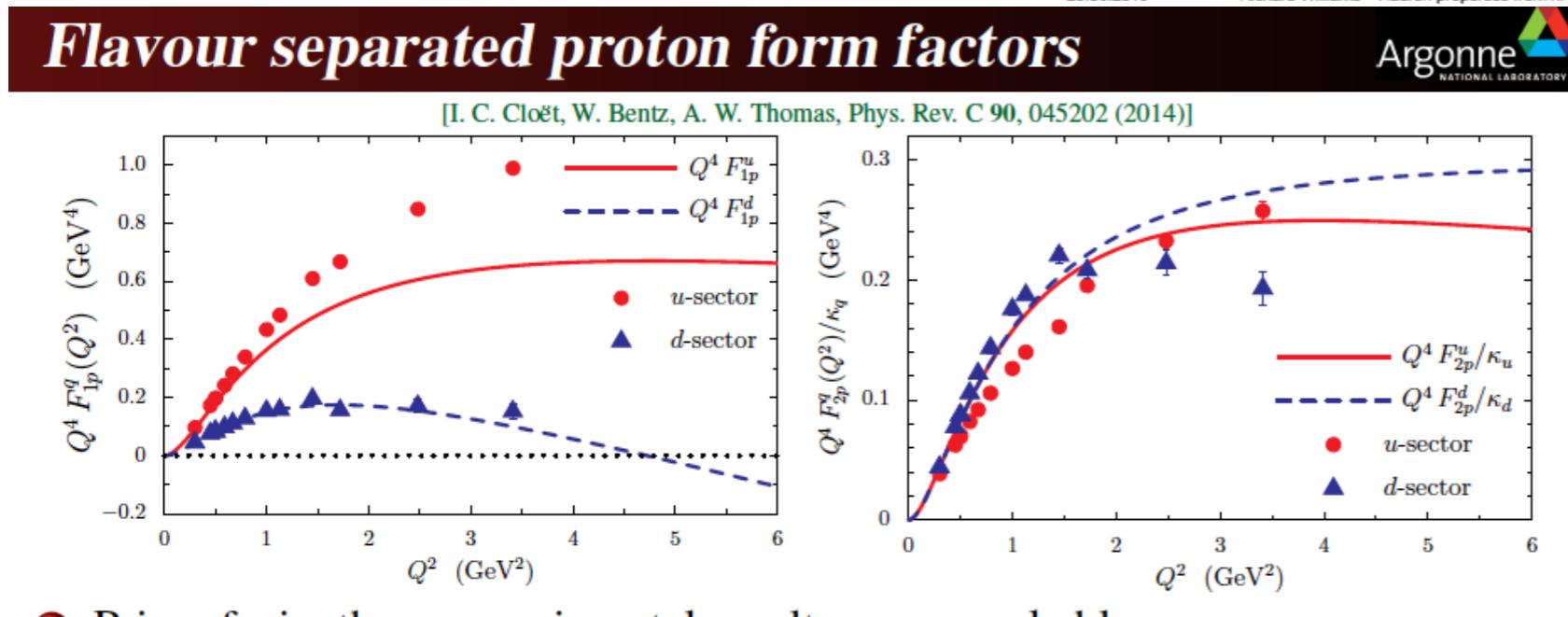
Nucleon/Delta ground + excited states good

Expected deficiencies in diquarks/meson analogs



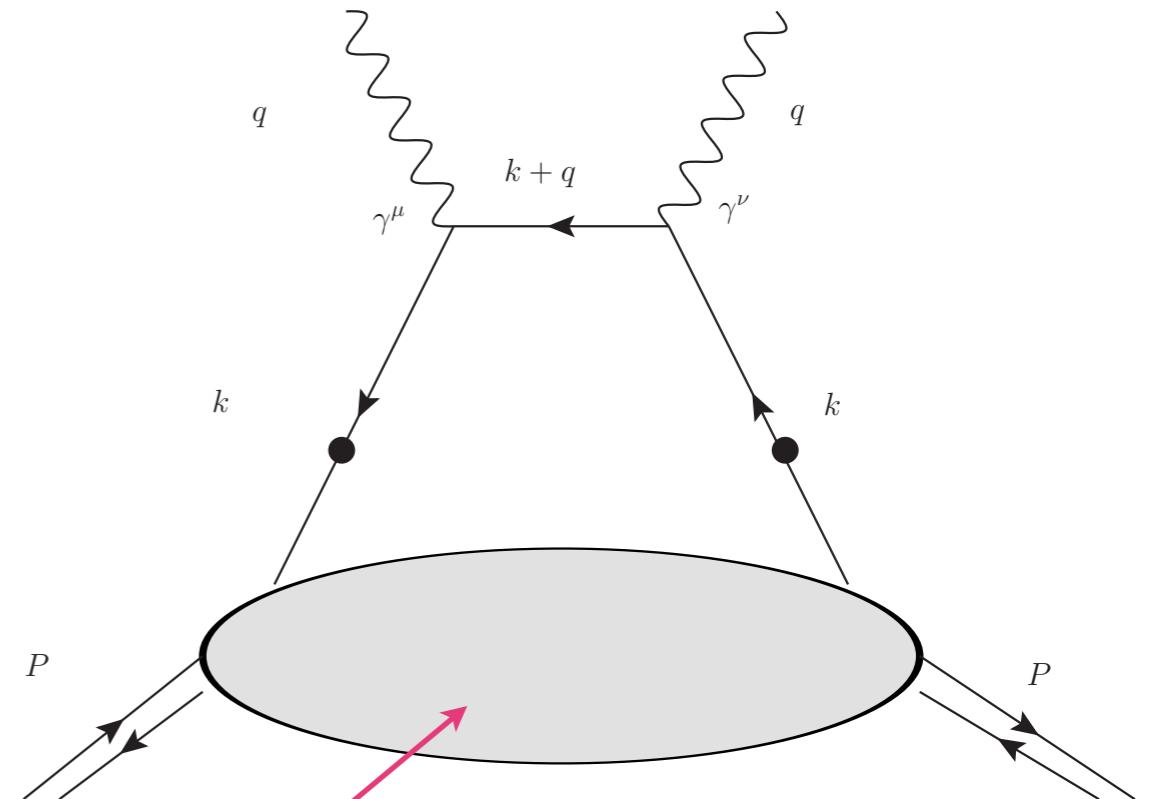
[Eichmann, Alkofer, Krassnigg, Nicmorus PRL 104 (2010) 201601]
 [Sanchis-Alepuz, Eichmann, Villalba-Chavez, Alkofer, PRD 84 (2011) 096003]
 [Sanchis-Alepuz, Eichmann, Fischer *in preparation*]

[Roberts, Chang, Cloet, Roberts FBS 51 (2011) 1]
 [Chen, Chang, Lei, Roberts, Wan, Wilson FBS 53 (2012) 293]
 [Segovia, El-Bennich, Rojas, Cloet, Roberts, Xu, Zong PRL 115 (2015) 171801]



Prima facie, these experimental results are remarkable

Parton Distribution Functions

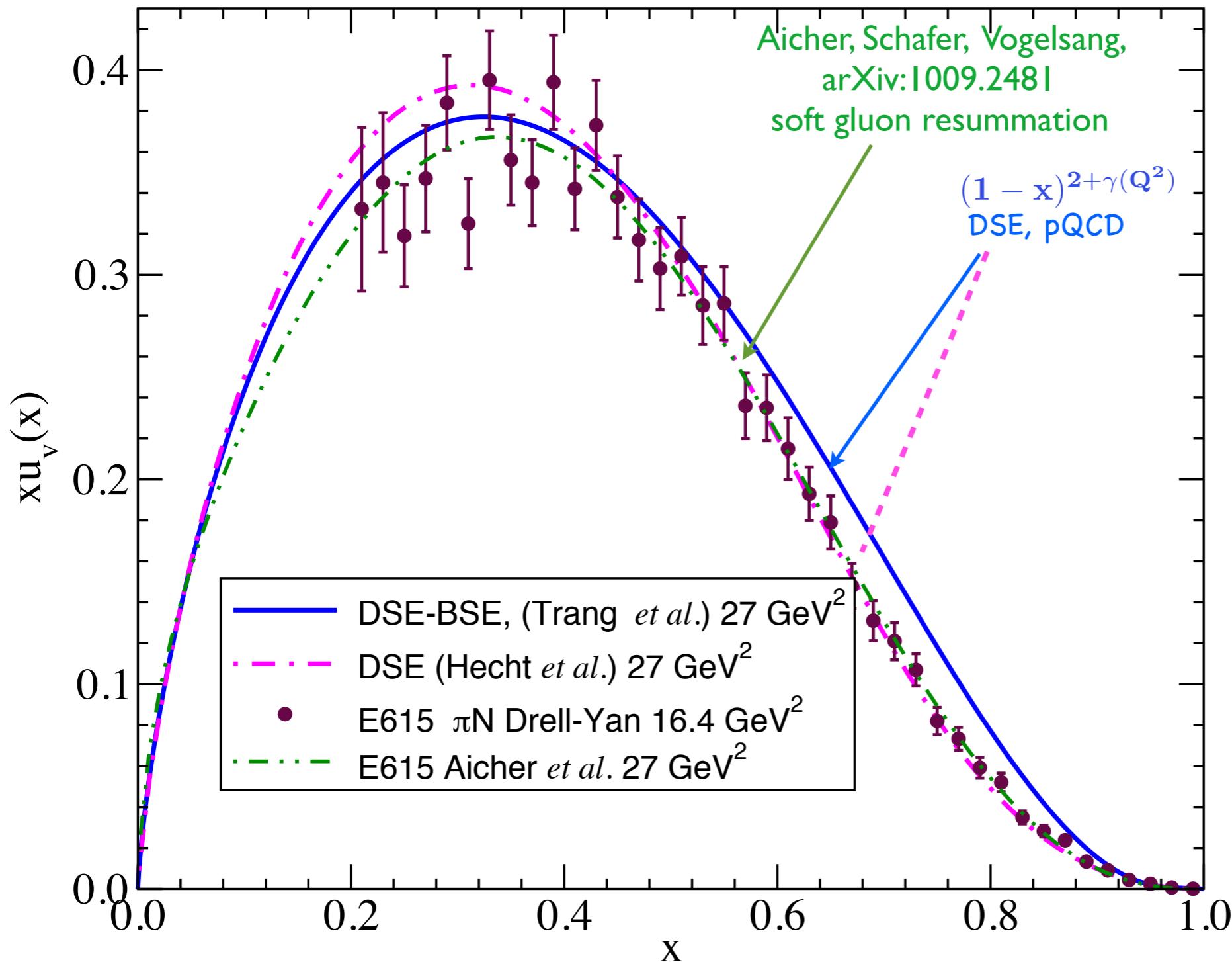


Covariant formulation
and calculation

$$\int d^4q \ F(q^2, q \cdot P, q \cdot k, k^2)$$

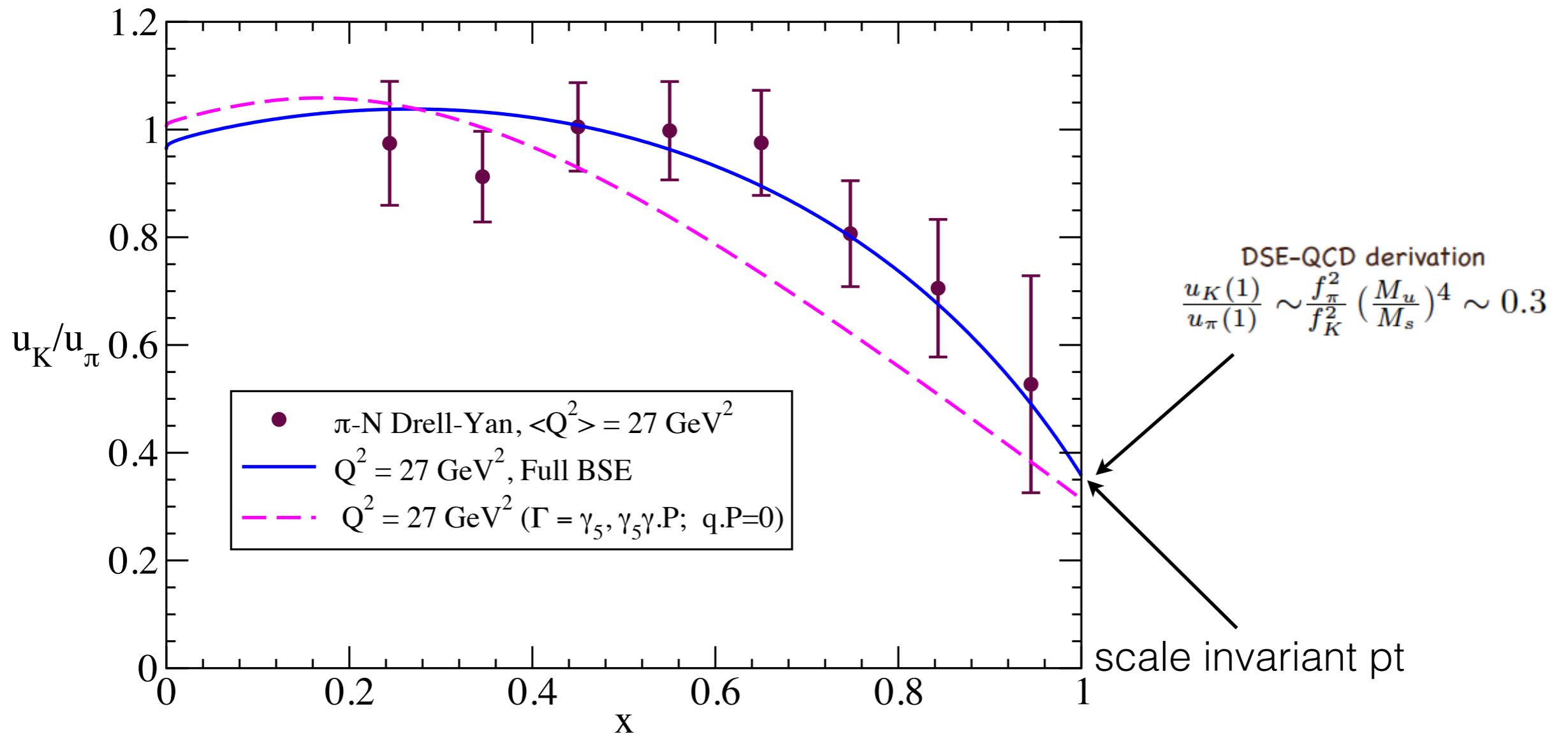
Pion Valence PDF

Nguyen, Bashir, Roberts, PCT, PRC 83 062201 (2011); arXiv:1102.2448



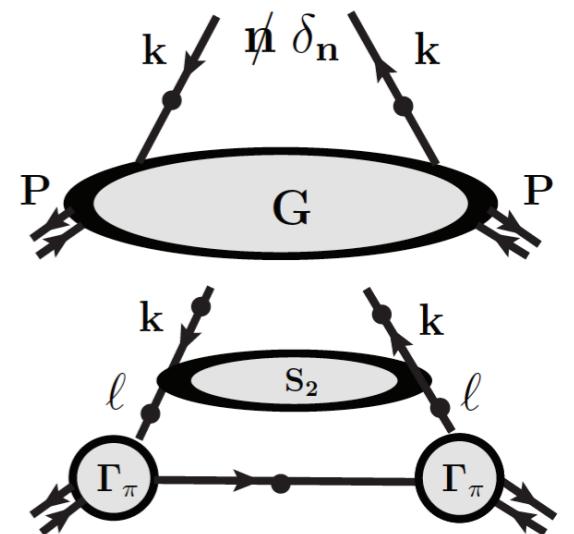
Environmental Dependence of Valence $u(x)$

Nguyen, Bashir, Roberts, PCT, arXiv : 1102.2448 (2011).



The Leading Order PDF

$$q_f(x) = \frac{1}{4\pi} \int d\lambda e^{-ixP \cdot n\lambda} \langle \pi(P) | \bar{\psi}_f(\lambda n) \not{p} \psi_f(0) | \pi(P) \rangle_c$$



RL DSE:

$q(x)$ From Directly Obtained Moments

$$\langle x^m \rangle_v^{RL} = \frac{-N_c}{2P \cdot n} \text{tr} \int_\ell \Gamma_\pi(\ell - \frac{P}{2}) \left[\left(\frac{\ell \cdot n}{P \cdot n} \right)^m n \cdot \partial_\ell S(\ell) \right] \Gamma_\pi(\ell - \frac{P}{2}) S(\ell - P)$$

Method can easily exceed the Lattice – QCD practical limit : $m = 3$

Fit numerical DSE-BSE solns to PTIRs (Nakanishi)

EG: $\Gamma_\pi(q^2, q \cdot P) = \gamma_5 \{ E_\pi(q^2, q \cdot P) + \not{P} F_\pi(..) + \not{q} q \cdot P G_\pi(..) + \sigma : q P H_\pi(..) \}$

Use Nakanishi Repn (or PTIR) (1965) :- $\mathcal{F} = E, F, G, \text{ or } H$

$$\mathcal{F}(q^2; q \cdot P) = \int_{-1}^1 d\alpha \int_0^\infty d\Lambda \left\{ \frac{\rho_{\text{IR}}(\alpha; \Lambda)}{(q^2 + \alpha q \cdot P + \Lambda^2)^{m+n}} + \frac{\rho_{\text{UV}}(\alpha; \Lambda)}{(q^2 + \alpha q \cdot P + \Lambda^2)^n} \right\}$$

npQCD info is in the variables and constants that are not momenta
---Wick rotation is trivial as in pert thy.

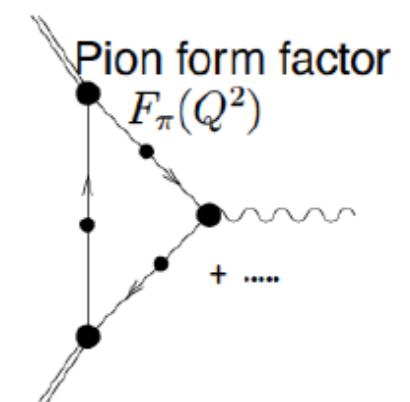
$$\rho_{\text{IR}}(\alpha; \Lambda) \rightarrow \rho_1(\alpha) \delta(\Lambda - \Lambda_{\text{IR}_1}) + \dots$$

$$S(q) = \sum_{k=1}^3 \left(\frac{z_k}{i \not{q} + m_k} + \frac{z_k^*}{i \not{q} + m_k^*} \right)$$

Works for u-, d-, s-, c-, b-quarks.
Also for lattice-QCD propagators.

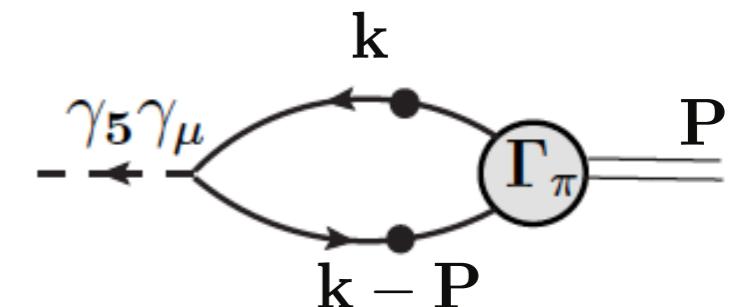
N. Souchlas, PhD thesis KSU, (2009), J. Phys. G37, 115001 (2010)

EG: $q_A(x) = i N_c \text{tr} \int \frac{dk^+ dk^- d^2 k_\perp}{(2\pi)^4} \delta(k^+ - x P^+) \text{tr}[\Gamma_\pi S (i\gamma^+) S \Gamma_\pi S]$



Pion Distribution Amplitude (leading twist)

$$f_\pi \phi_\pi(x) = \int \frac{d\lambda}{2\pi} e^{-ixP \cdot n \lambda} \langle 0 | \bar{q}(0) \gamma_5 q(\lambda n) | \pi(P) \rangle$$



$$f_\pi \langle x^m \rangle_\phi = \frac{Z_2 N_c}{P \cdot n} \text{tr} \int_k \left(\frac{k \cdot n}{P \cdot n} \right)^m \gamma_5 \not{n} [S(k) \Gamma_\pi(k - \frac{P}{2}; P) S(k - P)]$$

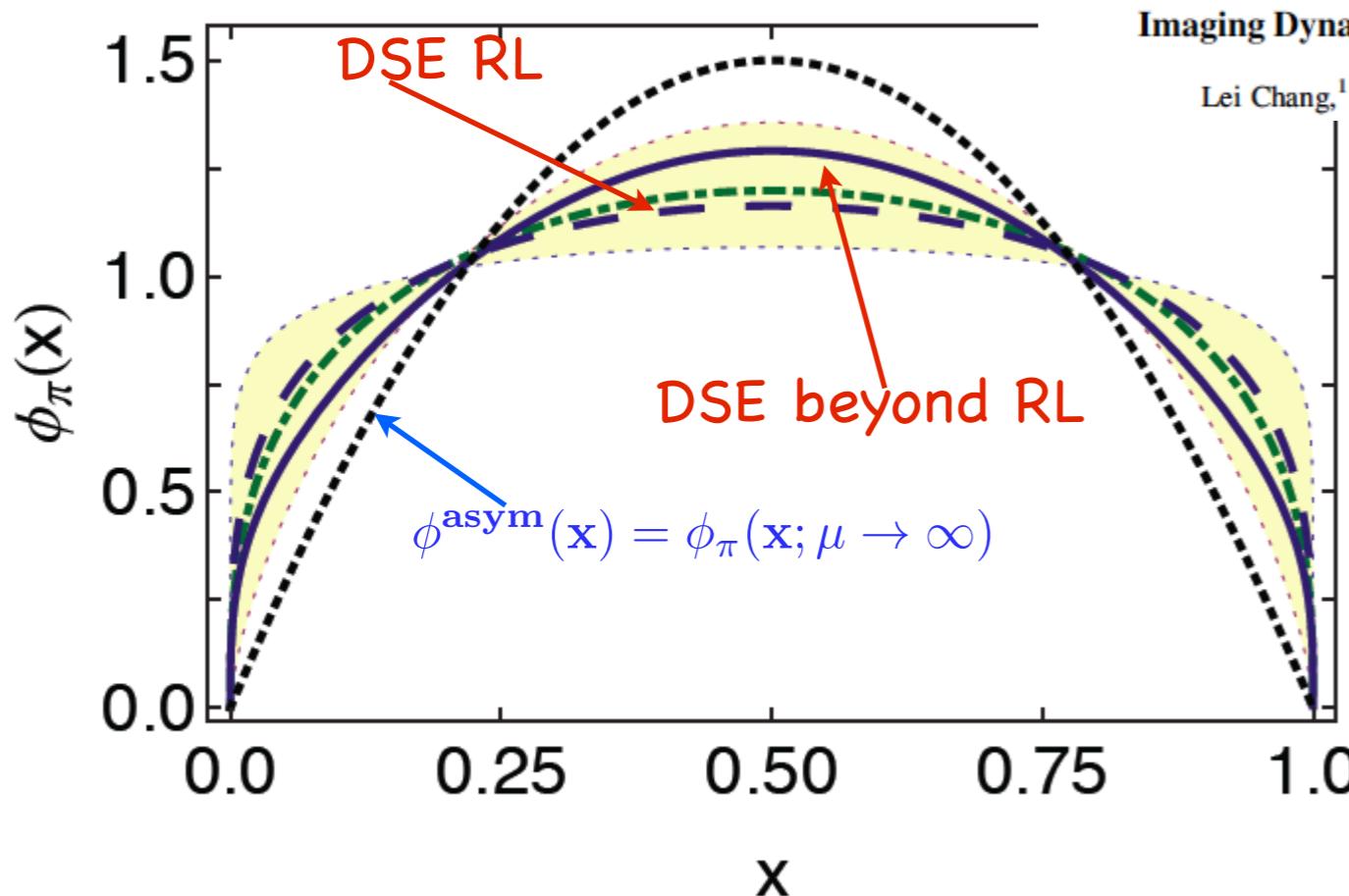
← BS Wavefn

$\mu = 2 \text{ GeV}$

PRL 110, 132001 (2013)

PHYSICAL REVIEW LETTERS

week ending
29 MARCH 2013



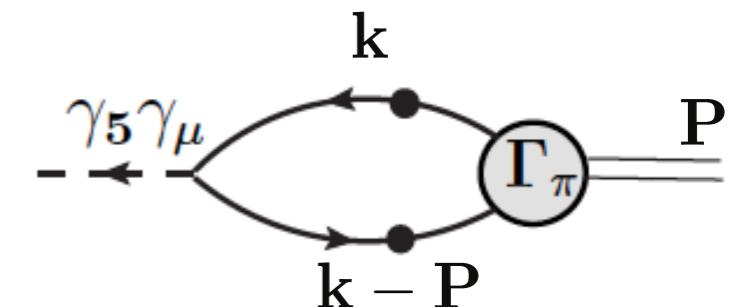
Imaging Dynamical Chiral-Symmetry Breaking: Pion Wave Function on the Light Front

Lei Chang,¹ I.C. Cloët,^{2,3} J.J. Cobos-Martinez,^{4,5} C.D. Roberts,^{3,6} S.M. Schmidt,⁷ and P.C. Tandy⁴

Broadening of PDA is an expression of DCSB
---long sought after in LF QFT

Pion Distribution Amplitude (leading twist)

$$f_\pi \phi_\pi(x) = \int \frac{d\lambda}{2\pi} e^{-ixP \cdot n \lambda} \langle 0 | \bar{q}(0) \gamma_5 q(\lambda n) | \pi(P) \rangle$$



$$f_\pi \langle x^m \rangle_\phi = \frac{Z_2 N_c}{P \cdot n} \text{tr} \int_k \left(\frac{k \cdot n}{P \cdot n} \right)^m \gamma_5 \not{n} [S(k) \Gamma_\pi(k - \frac{P}{2}; P) S(k - P)]$$

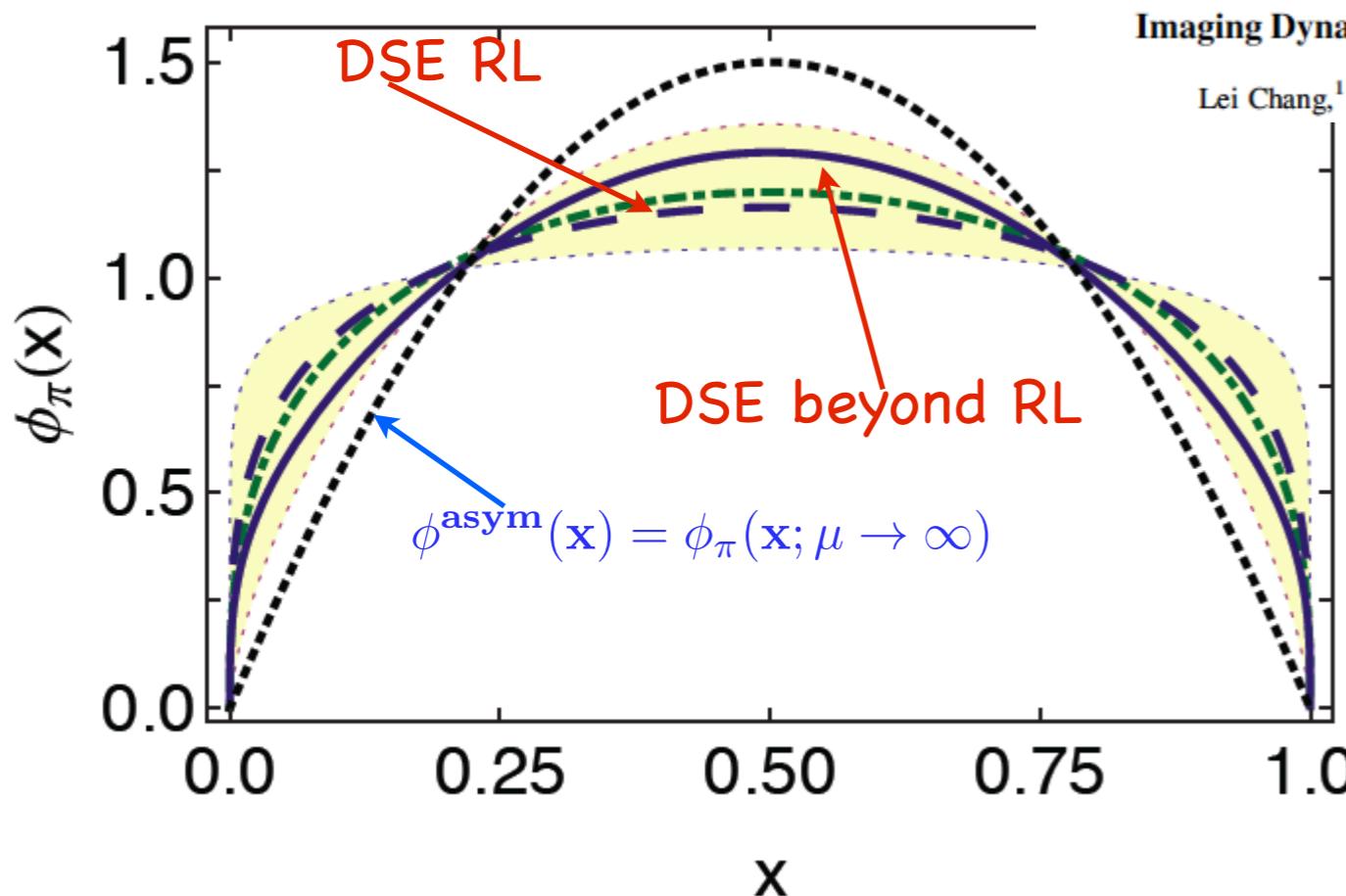
BS Wavefn

$\mu = 2 \text{ GeV}$

PRL 110, 132001 (2013)

PHYSICAL REVIEW LETTERS

week ending
29 MARCH 2013



Imaging Dynamical Chiral-Symmetry Breaking: Pion Wave Function on the Light Front

Lei Chang,¹ I.C. Cloët,^{2,3} J.J. Cobos-Martinez,^{4,5} C.D. Roberts,^{3,6} S.M. Schmidt,⁷ and P.C. Tandy⁴

Broadening of PDA is an expression of DCSB
---long sought after in LF QFT

Pion Distribution Amplitude

ERBL (~ 1980): $\phi_\pi(x; \mu) = 6x(1-x) \left\{ 1 + \sum_{n=2,4,\dots} a_n(\mu) C_n^{3/2}(2x-1) \right\}$

$$a_n(\mu) = a_n(\mu_0) \left[\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)} \right]^{\gamma_n^{(0)}/\beta_0}$$

Evolution to higher scales is
EXTREMELY SLOW
Not much change up to LHC energy

Conformal limit: $a_n(\mu \rightarrow \infty) = 0$

Efficient representation of DSE results:

$$\phi_\pi(x; \mu) = N_\alpha x^\alpha (1-x)^\alpha \left\{ 1 + \sum_{n=2}^{\infty} \tilde{a}_n(\mu) C_n^{\alpha+1/2}(2x-1) \right\}$$

$$\phi_K(x; \mu) = N_\alpha x^\alpha (1-x)^\alpha \left\{ 1 + \sum_{n=2,4,\dots} \tilde{a}_n(\mu) C_n^{\alpha+1/2}(2x-1) \right\}$$

$$+ N_\beta x^\beta (1-x)^\beta \left\{ \sum_{n=1,3,\dots} \tilde{a}_n(\mu) C_n^{\beta+1/2}(2x-1) \right\}$$

Low Order Truncation of ERBL-Gegenbauer Expn of PDA

$$\phi_\pi(x; \mu) = 6x(1-x) \left\{ 1 + \sum_{n=2,4,\dots} a_n(\mu) C_n^{3/2}(2x-1) \right\}$$

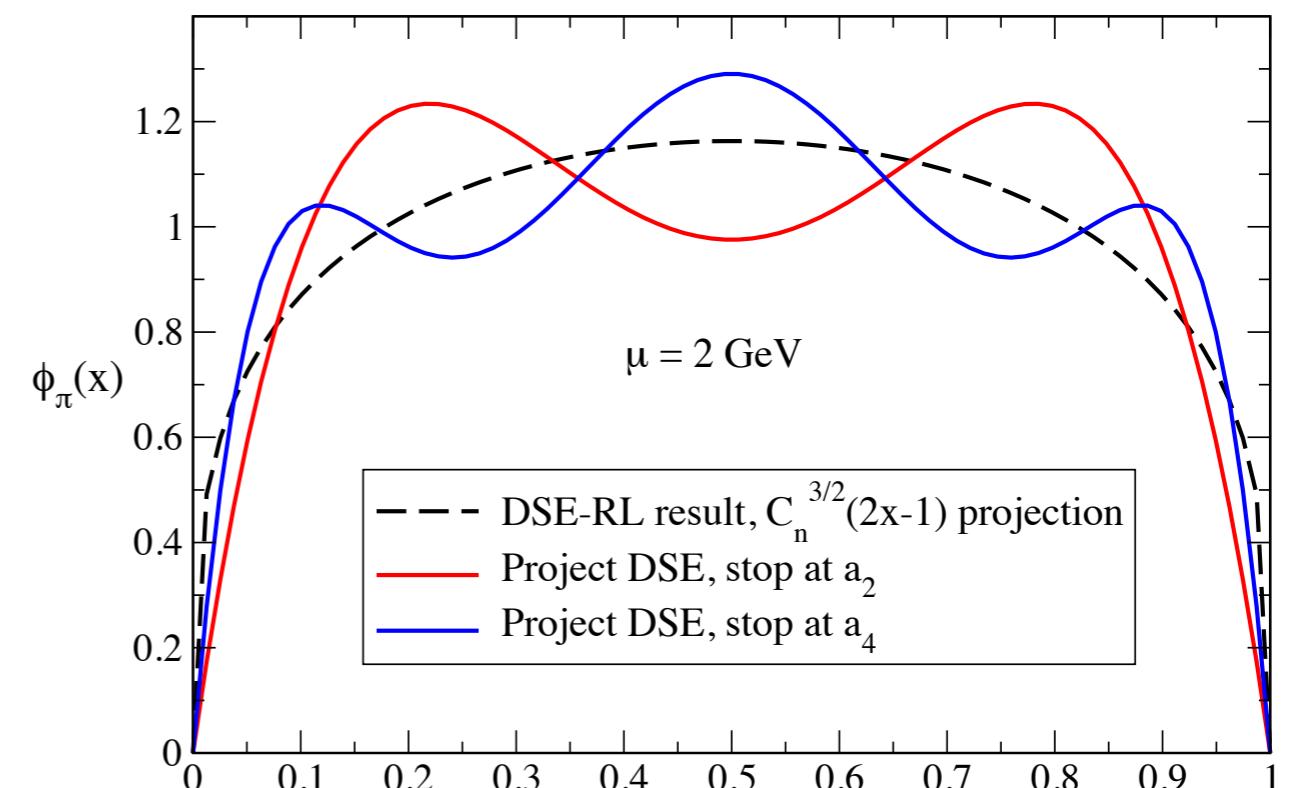
DSE soln

$\{(0, 1.), \{2, 0.233104\}, \{4, 0.112135\},$
 $\{6, 0.0683202\}, \{8, 0.0469145\},$
 $\{10, 0.0346469\}, \{12, 0.0268732\},$
 $\{14, 0.0215933\}, \{16, 0.0178199\},$
 $\{18, 0.0150159\}, \{20, 0.0128672\},$
 $\{22, 0.0111788\}, \{24, 0.00982438\},$
 $\{26, 0.00871886\}, \{28, 0.00780296\},$
 $\{30, 0.00703438\}, \{32, 0.0063823\},$
 $\{34, 0.00582279\}, \{36, 0.00534272\},$
 $\{38, 0.00493277\}, \{40, 0.00447911\}\}$

10%

+.....

2%



A double-humped PDA is almost ruled out by
 V. Braun, I. Filyanov, Z. Phys. C44, 157 (1989)

$\phi_\pi^{\text{QCDSR}}(x = 1/2; \mu = 2) = 1.2 \pm 0.3$

One Lattice-QCD Moment Almost Determines Pion DA

PRL 111, 092001 (2013)

PHYSICAL REVIEW LETTERS

week ending
30 AUGUST 2013

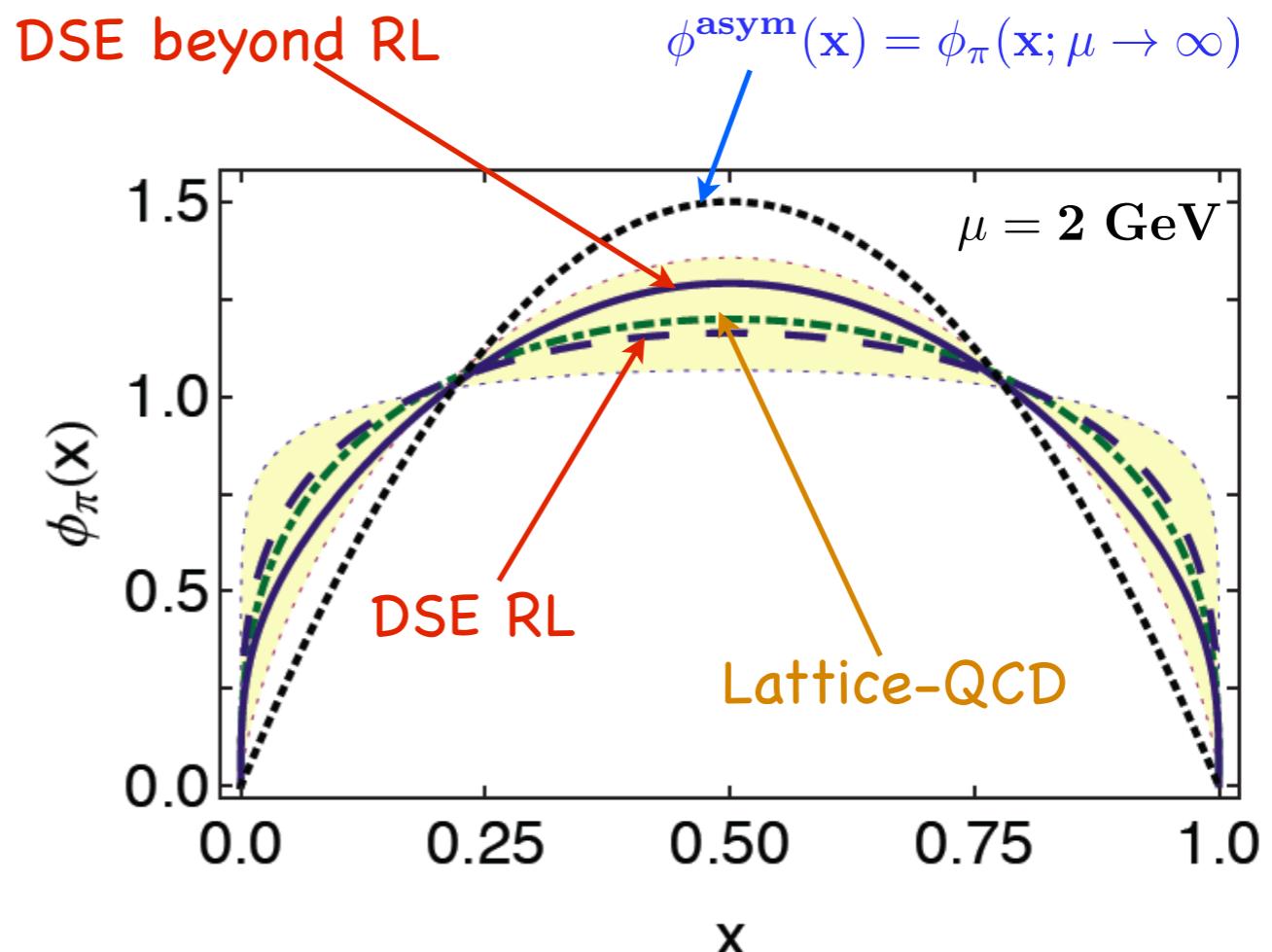
Pion Distribution Amplitude from Lattice QCD

I. C. Cloët,¹ L. Chang,² C. D. Roberts,¹ S. M. Schmidt,³ and P. C. Tandy⁴

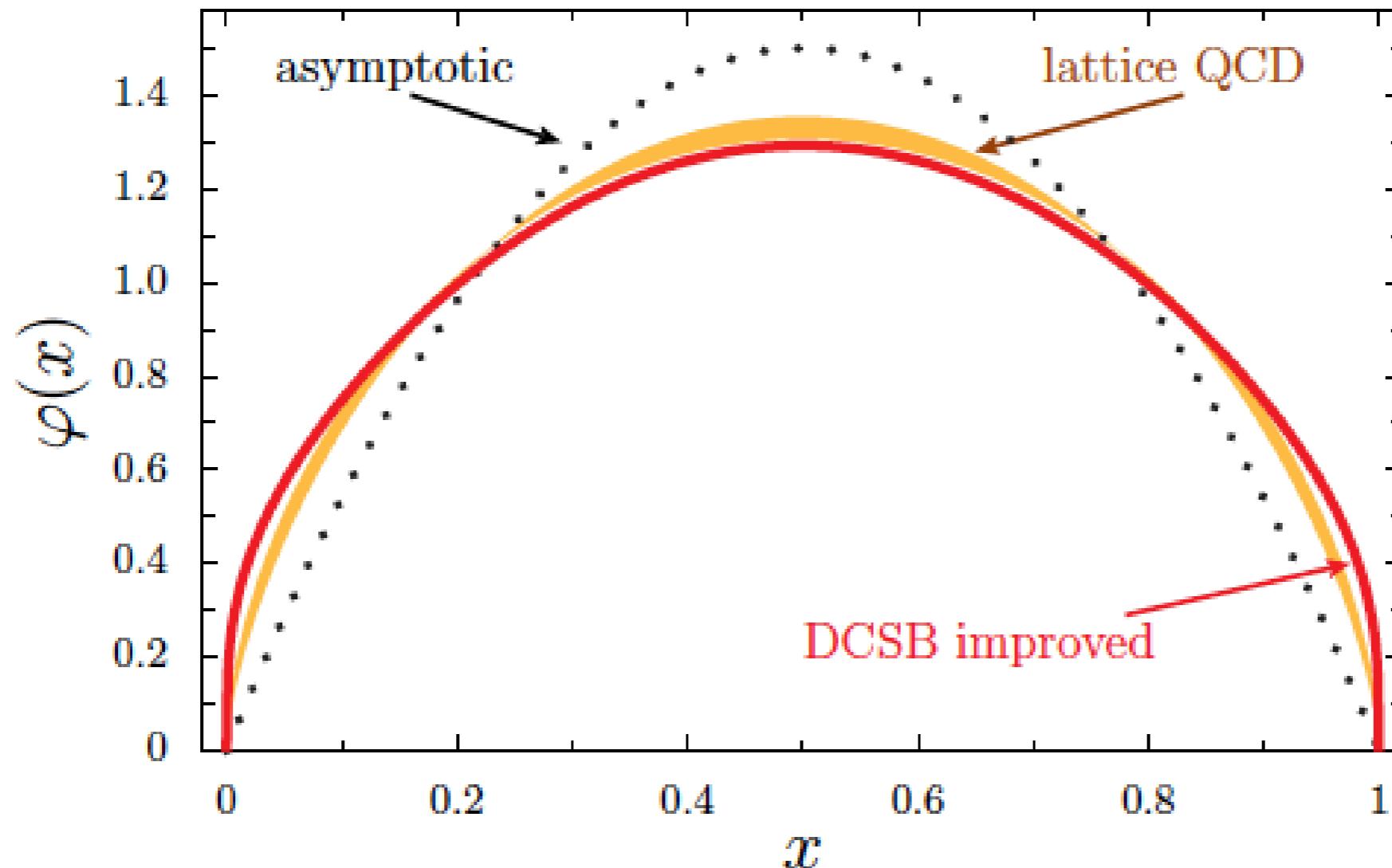
$$\phi_{\pi}^{\text{LQCD}}(x; \mu = 2) = N x^{\alpha} (1 - x)^{\alpha}$$
$$\alpha = 0.35 + 0.32 - 0.24$$

$$\langle (2x - 1)^2 \rangle_{\mu=2}^{\text{LQCD}} = 0.27 \pm 0.04$$

V. Braun et al., PRD74, 074501 (2006)



Pion Distribution Amplitude



$$\langle (2x - 1)^2 \rangle_{\mu=2 \text{ GeV}}^{\text{LQCD}} = 0.2361(41)(39)$$

V. Braun et al., arXiv:1503.03656 [hep-lat]

DSE prediction: 0.251

Kaon Distribution Amplitude

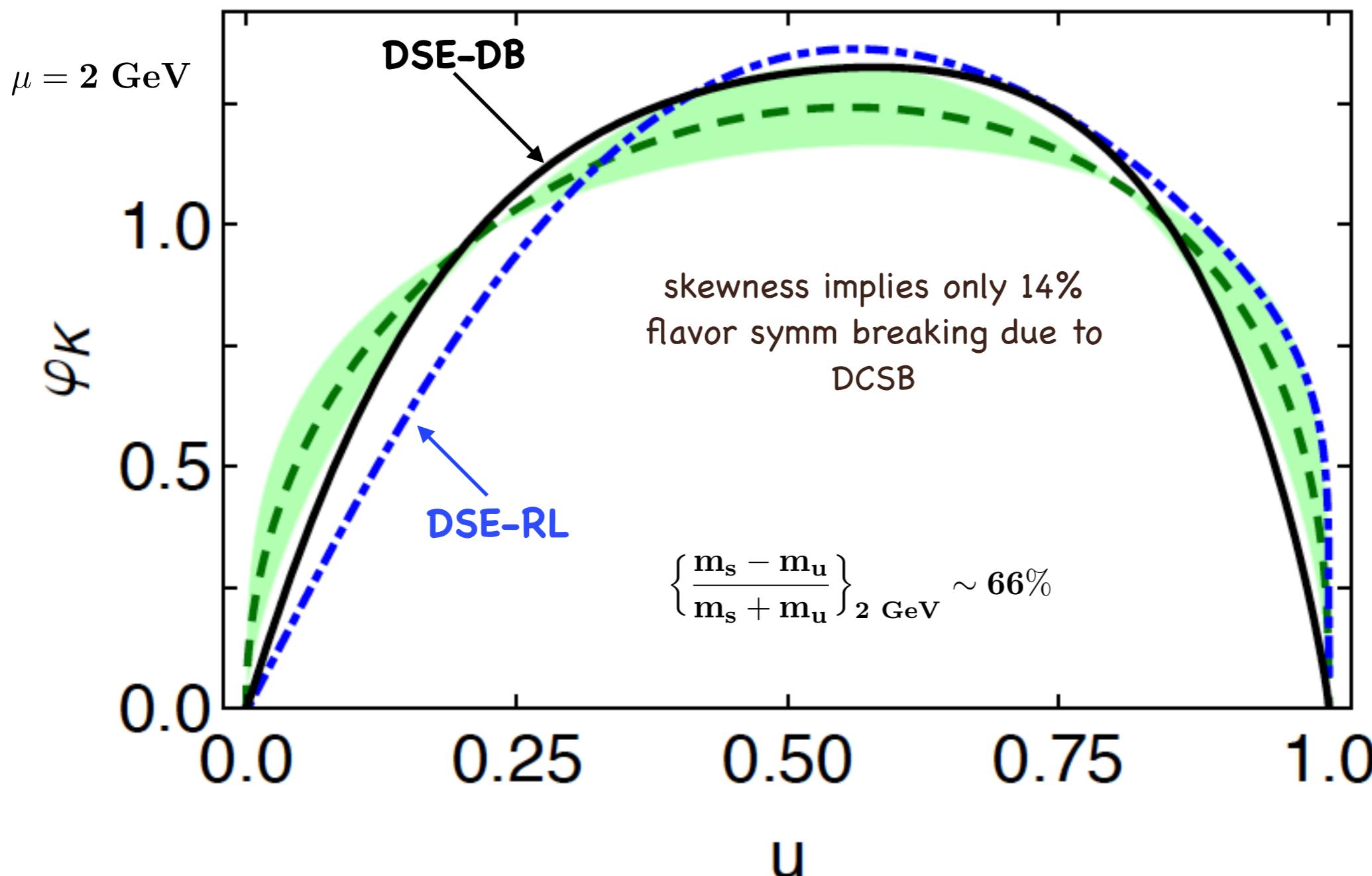
Size of $SU(2) \times SU(3)$ spin-flavor symmetry-breaking?

that, as strong interaction bound states whose decay is mediated only by the weak interaction, so that they have a relatively long lifetime, kaons have been instrumental in establishing the foundation and properties of the Standard Model; notably, the physics of CP violation. In this connection the nonleptonic decays of B mesons are crucial because, e.g., the transitions $B^\pm \rightarrow (\pi K)^\pm$ and $B^\pm \rightarrow \pi^\pm \pi^0$ provide access to the imaginary part of the CKM matrix element V_{ub} : $\gamma = \text{Arg}(V_{ub}^*)$ [4]. Factorisation theorems have been derived and are applicable to such decays [5]. However, the formulae involve a certain class of so-called “non-factorisable” corrections because the parton distribution amplitudes (PDAs) of strange mesons are not symmetric with respect to quark and antiquark momenta. Therefore, any derived estimate of γ is only as accurate as the evaluation of both the difference between K and π PDAs and also their respective differences from the asymptotic distribution, $\varphi^{\text{asy}}(u) = 6u(1-u)$. Amplitudes of twist-two and -three are involved. With this motivation, we focus on the twist-two amplitudes herein.

C. Shi, L. Chang, C.D. Roberts, S.Schmidt, PCT, H-S. Zong, PLB738, 512 (2014)

Kaon Distribution Amplitude

C. Shi, L. Chang, C.D. Roberts, S.Schmidt, PCT, H-S. Zong, PLB738, 512 (2014)



R. Arthur, P. Boyle, D. Brommel, M. Donnellan, J. Flynn et al, PRD83, 074505 (2011)

Kaon DA Moments

$\mu = 2 \text{ GeV}$

Table 1

Moments ($u_\Delta = 2u - 1$) of the K -meson PDA computed using Eqs. (11) and (12), compared with selected results obtained elsewhere: Refs. [40,41], lattice-QCD; Ref. [10], analysis of lattice-QCD results in Ref. [41]; Refs. [42–46], compilation of results from QCD sum rules; and Ref. [47], holographic soft-wall Ansatz for the kaon's light-front wave function. We also list values obtained with $\varphi = \varphi^{\text{asy}}$, Eq. (14), and $\varphi = \varphi_{\text{ms}}$, Eq. (16), because they represent lower and upper bounds, respectively, for concave distribution amplitudes.

	$\langle u_\Delta^m \rangle$	$m = 1$	2	3	4	5	6
DSE-QCD:	RL	0.11	0.24	0.064	0.12	0.045	0.076
	DB	0.040	0.23	0.021	0.11	0.013	0.063
Lattice-QCD:	[40]	0.027(2)	0.26(2)				
	[41]	0.036(2)	0.26(2)				
	[10]	0.036(2)	0.26(2)	0.020(2)	0.13(2)	0.014(2)	0.085(15)
QCD Sum Rules:	[42–46]	0.035(8)					
	[47]	0.04(2)	0.24(1)				
	$\varphi = \varphi_{\text{ms}}$	0.33	0.33	0.2	0.2	0.14	0.14
	$\varphi = \varphi^{\text{asy}}$	0	0.2	0	0.086	0	0.048

Shi Chao, L. Chang, C.D. Roberts, P.C. Tandy, PLB738, 512 (2014)

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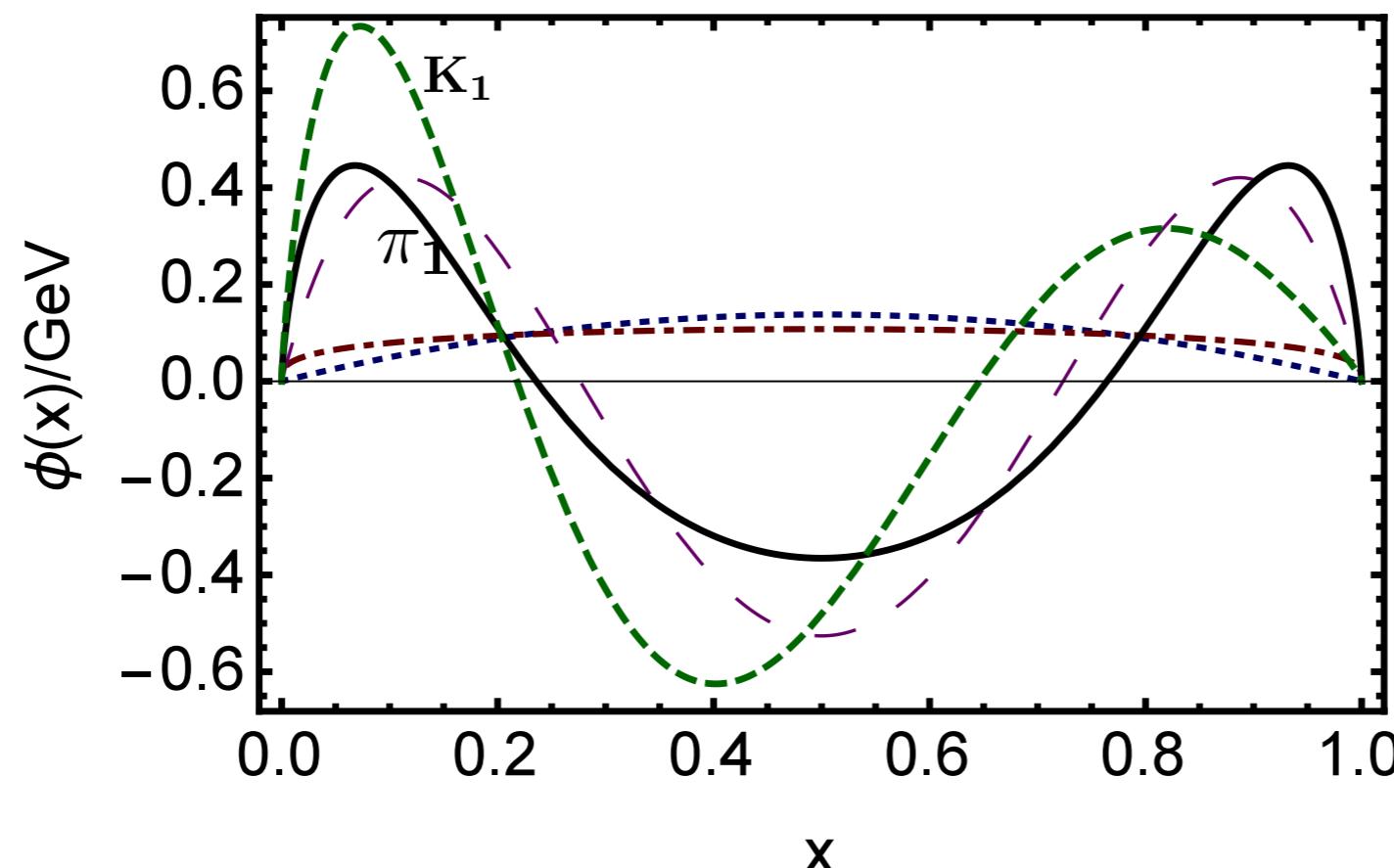
	$\langle u_\Delta^m \rangle$	$m = 1$	2	3	4	5	6
DSE-QCD:	RL	0.11	0.24	0.064	0.12	0.045	0.076
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Shi Chao, L. Chang, C.D. Roberts, P.C. Tandy, PLB738, 512 (2014)

1st Excited State Pion & Kaon DAs

$\mu = 2 \text{ GeV}$

Bo-Lin, L. Chang, F. Gao, C.D.Roberts, S.M.Schmidt, H-S., Zong,
Phys. Rev. D93 114033 (2016).



Spacelike Correlator Approximation for PDFs

To help lattice-QCD be more applicable to hadron
PDFs and GPDs than just the first 3 moments ?

Parton Physics on a Euclidean Lattice

Xiangdong Ji^{1,2}

Standard light-cone correlator, leading twist: $x = k \cdot n / P \cdot n = k^+ / P^+ \in [0, 1]$

$$q_f(x) = \frac{1}{4\pi} \int d\lambda e^{-ixP \cdot n \lambda} \langle \pi(P) | \bar{\psi}_f(\lambda n) \not{p} \psi_f(0) | \pi(P) \rangle_c$$

$$\mathbf{n}^2 = \mathbf{0} ; \quad \mathbf{z}^- = \lambda \mathbf{n} ; \quad \mathbf{z}^+ = \mathbf{0} = \mathbf{z}_\perp$$

Ji: Take large P_z limit of frame-dependent equal-time correlator: $x = kz/Pz \in [-\infty, +\infty]$

$$\tilde{q}_f(x; P_z) = \frac{1}{4\pi} \int dz e^{-ixP_z z} \langle \pi(P) | \bar{\psi}_f(z) \gamma_z \psi_f(0) | \pi(P) \rangle_c$$

$\rightarrow q_f(x)$ as $P_z \rightarrow \infty$

How fast?



Quark Distribution

§ Back to the continuum

Xiangdong Ji, Phys. Rev. Lett. 111, 039103 (2013)

$$q(x, \mu) = \tilde{q}(x, \mu, P_z) + \mathcal{O}(M_N^2/P_z^2) + \mathcal{O}(\alpha_s) + \mathcal{O}(\Lambda_{\text{QCD}}^2/P_z^2)$$

Finite $P_z \rightarrow \infty P_z$ perturbative matching

$$\tilde{q}(x, \mu, P_z) = \int_{-1}^1 \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}\right) q(y, \mu)$$

$$Z(x, \mu/P_z) = \delta(x - 1) + \frac{\alpha_s}{2\pi} Z^{(1)}(x, \mu/P_z)$$

Non-singlet case only

X. Xiong, X. Ji, J. Zhang,
1310.7471 [hep-ph]; Y. Zhao, this workshop



Huey-Wen Lin — Light Cone 2014



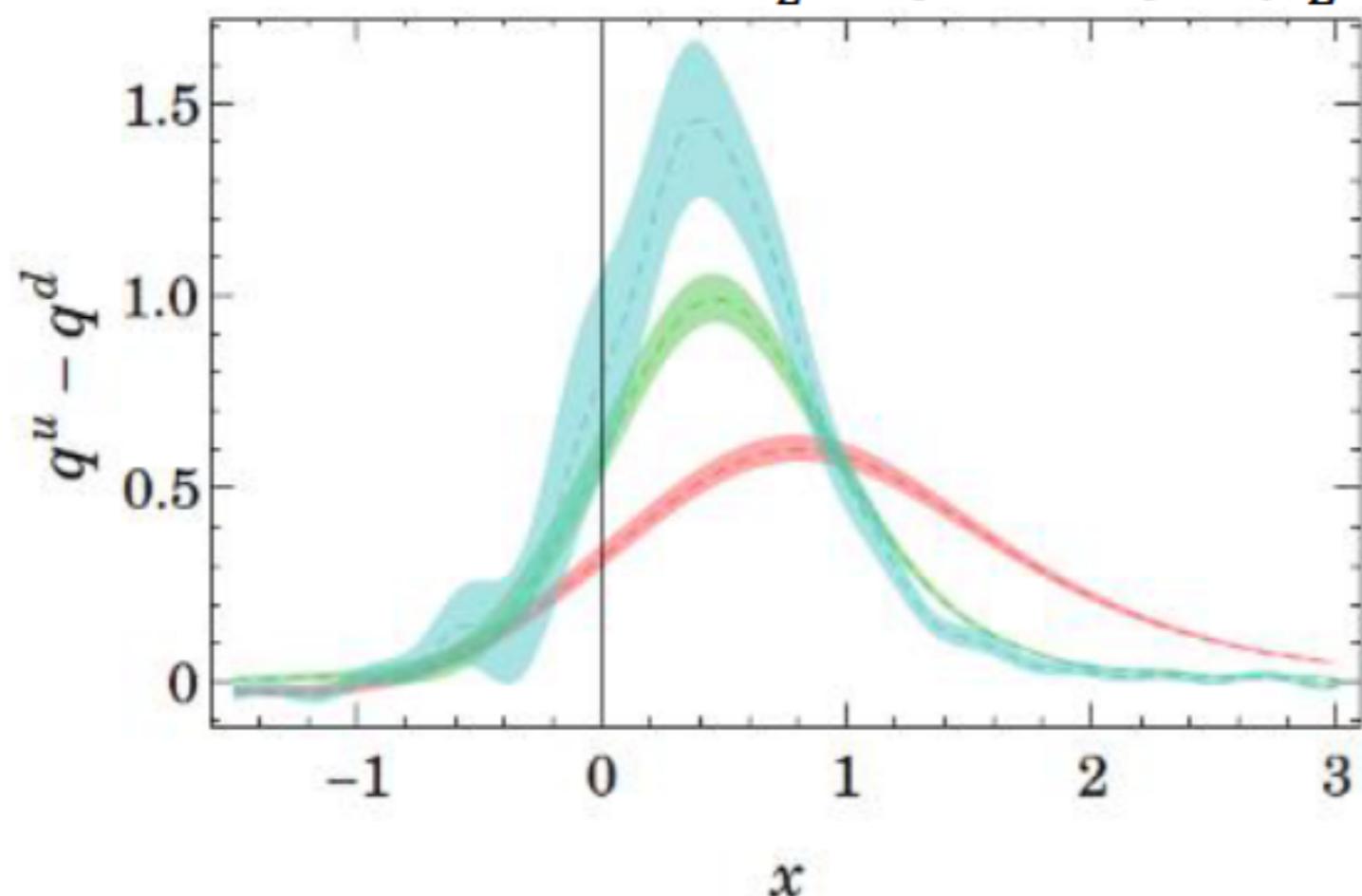
FIG. 1: One loop corrections to quasi quark distribution.

Quark Distribution

§ Exploratory study

$$\int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \bar{\psi}(z) \gamma_z \exp\left(-ig \int_0^z dz' A_z(z')\right) \psi(0) \right| P \right\rangle$$

$$P_z \in \{1, 2, 3\}^{2\pi/L}$$



Distribution gets sharper as P_z increases
Artifacts due to finite P_z on the lattice
Improvement?
Work out leading- P_z corrections

Simple model for pion PDF & Quasi-PDF

$$S(k) = 1/(ik + M), \quad M = 0.4 \text{ GeV}$$

$$\Gamma_\pi(q, P) = \gamma_5 N_\pi \int_{-1}^1 d\alpha \frac{\rho(\alpha)}{q^2 + \alpha q \cdot P + \Lambda^2}, \quad \rho(\alpha) = \text{even}$$

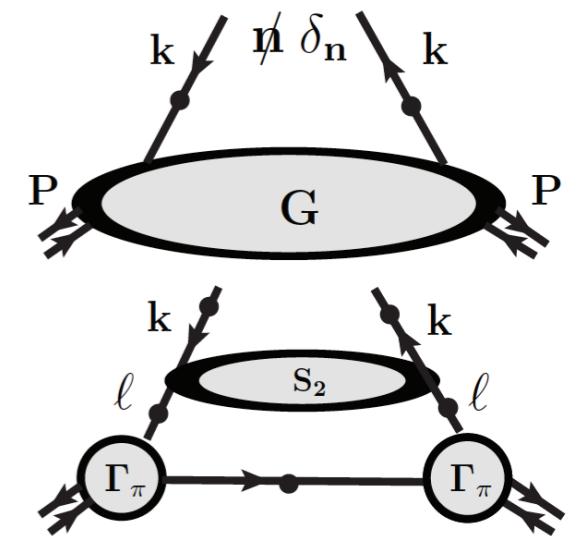
Euclidean to Minkowski:-

Evaluate $q(x)$ directly using Cauchy Residue Thm for $\int_{-\infty}^{\infty} dk^-$

$$q_A(x) = i N_c \text{tr} \int \frac{dk^+ dk^- d^2 k_\perp}{(2\pi)^4} \delta(k^+ - x P^+) \text{tr}[\Gamma_\pi S (i\gamma^+) S \Gamma_\pi S]$$

Evaluate $\tilde{q}(x; P_z)$ directly using Cauchy Residue Thm for $\int_{-\infty}^{\infty} dk^0$

$$\tilde{q}_A(x) = i N_c \text{tr} \int \frac{dk^0 dk_z d^2 k_\perp}{(2\pi)^4} \delta(k_z - x P_z) \text{tr}[\Gamma_\pi S (i\gamma^z) S \Gamma_\pi S]$$

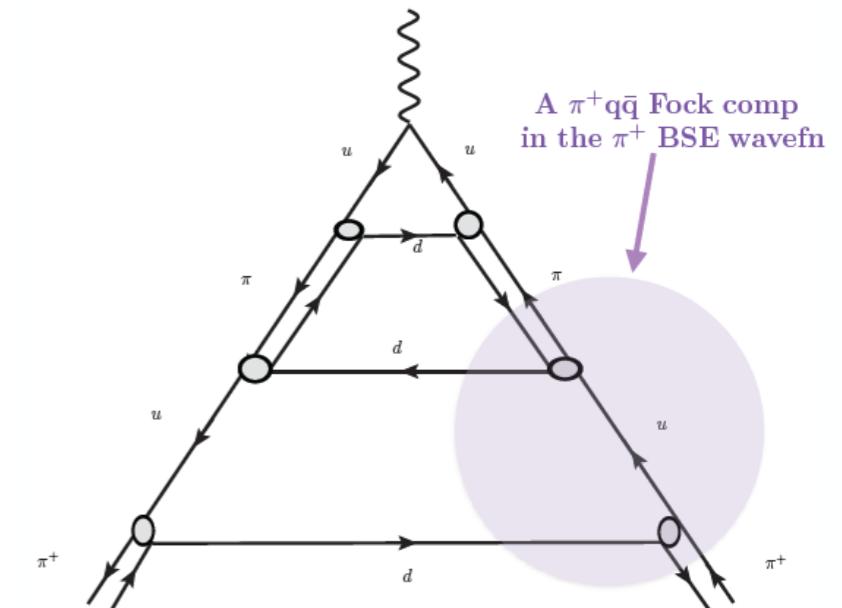
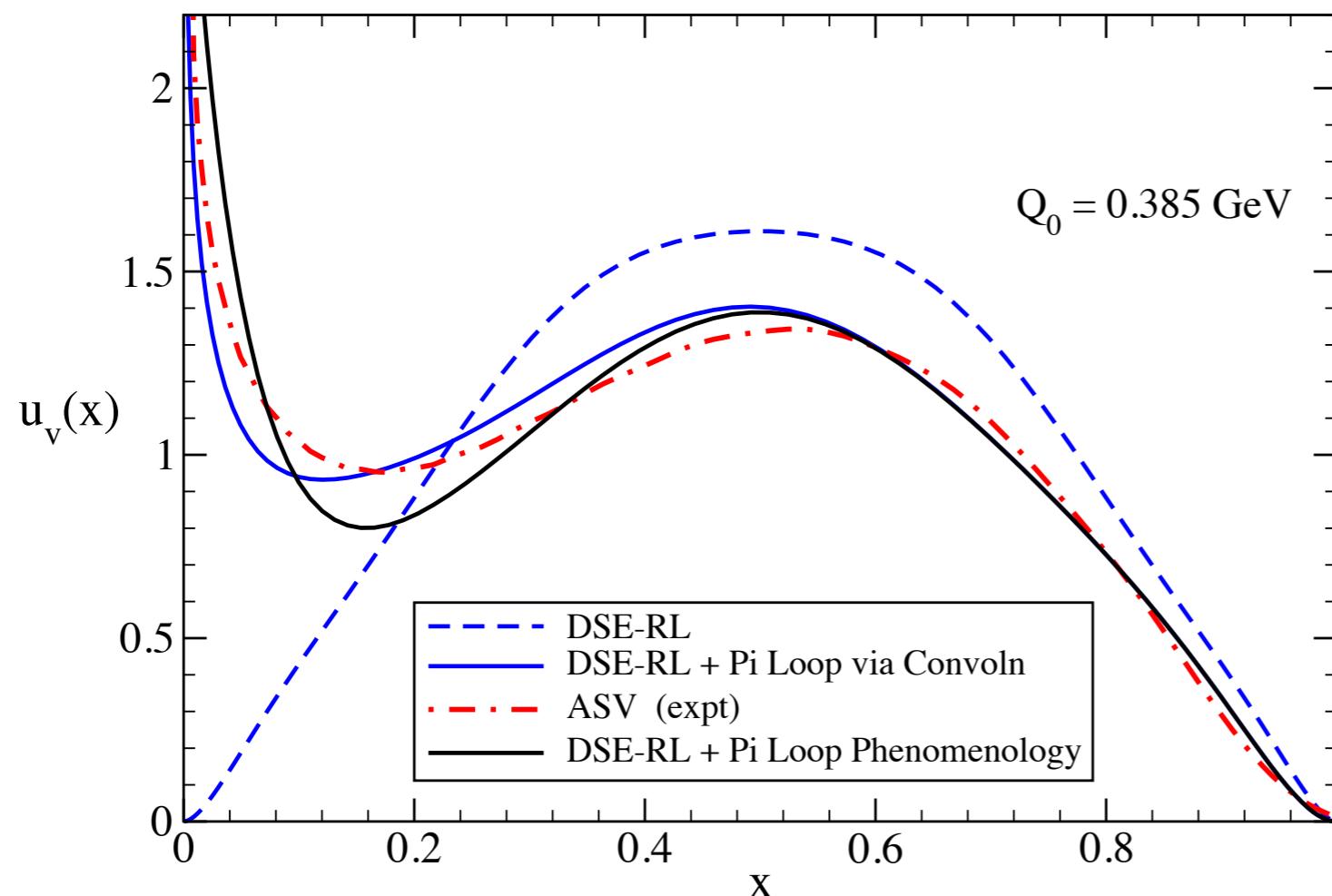


DIVERSION—A full DSE calculation of the true pion valence PDF

TABLE II: Momentum fraction sum rule from this work at scale $Q_0 = 0.630$ GeV corresponding to the ASV [13] compilation.

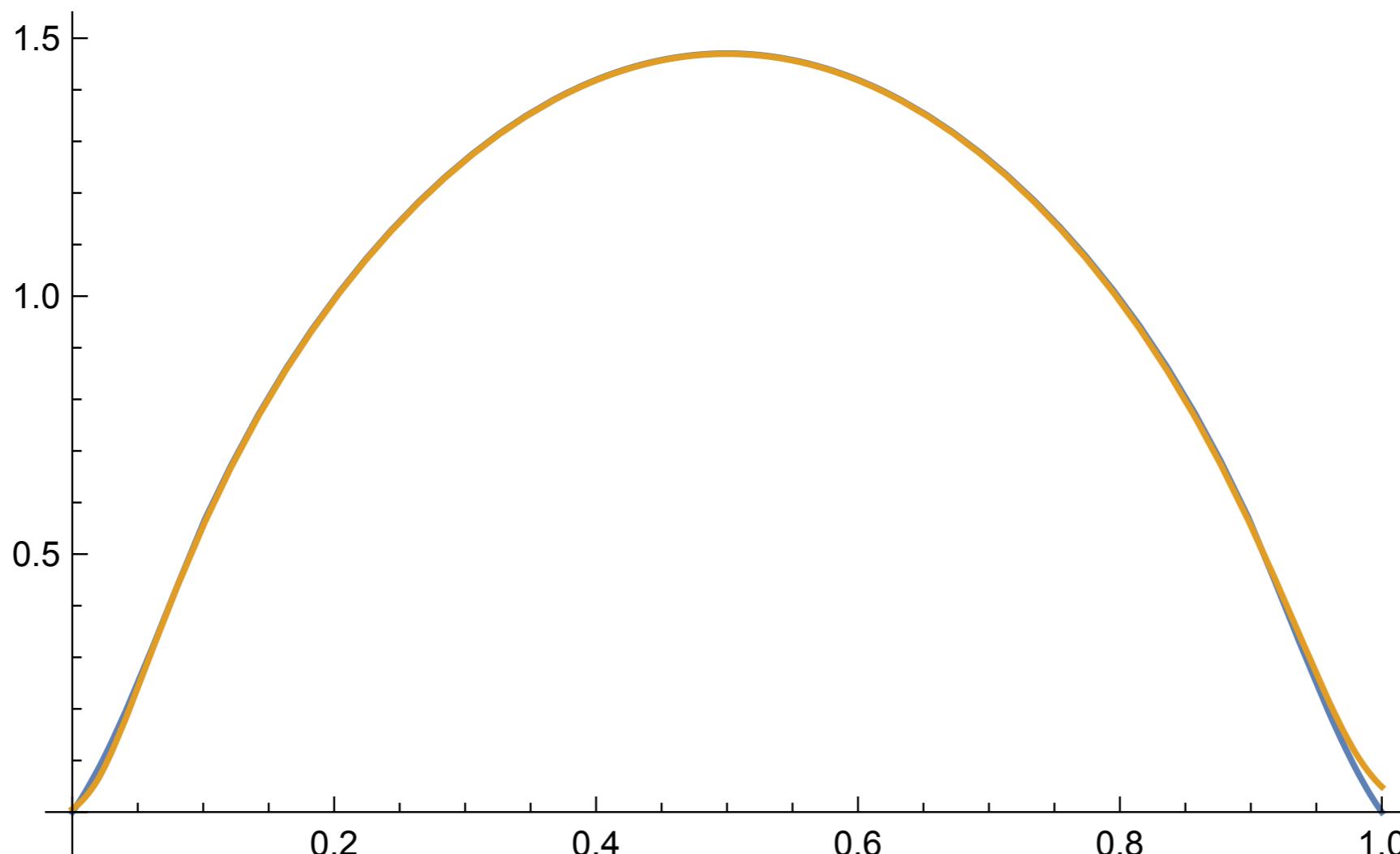
K. Khitrit, P. Tandy, in progress (2015)

Modern empirical expt parameterization:
Aicher, Shafer, Vogelsang, (ASV) PRL 105, 252003 (2010)

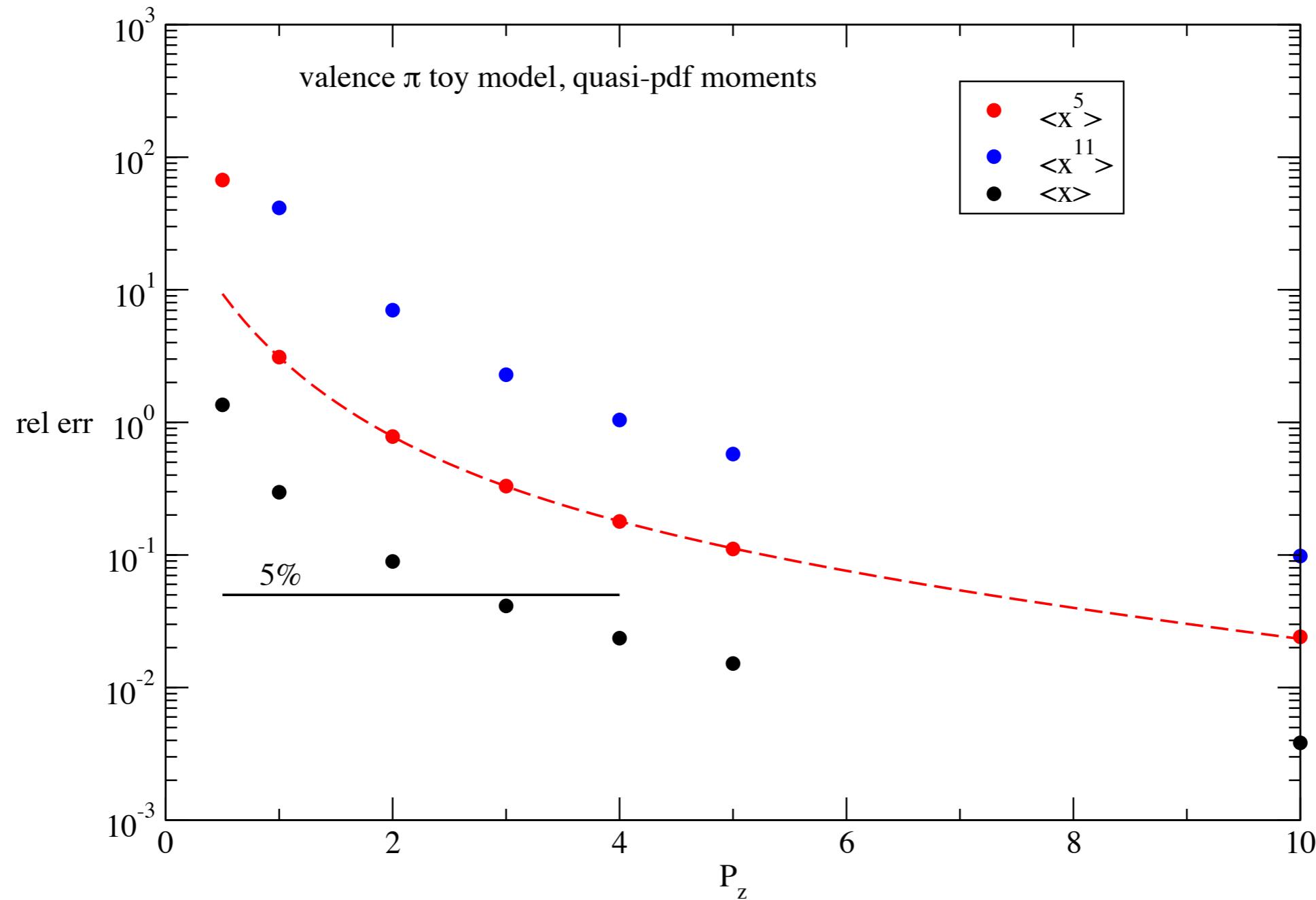


Back to: Spacelike Correlator Approximation for PDFs

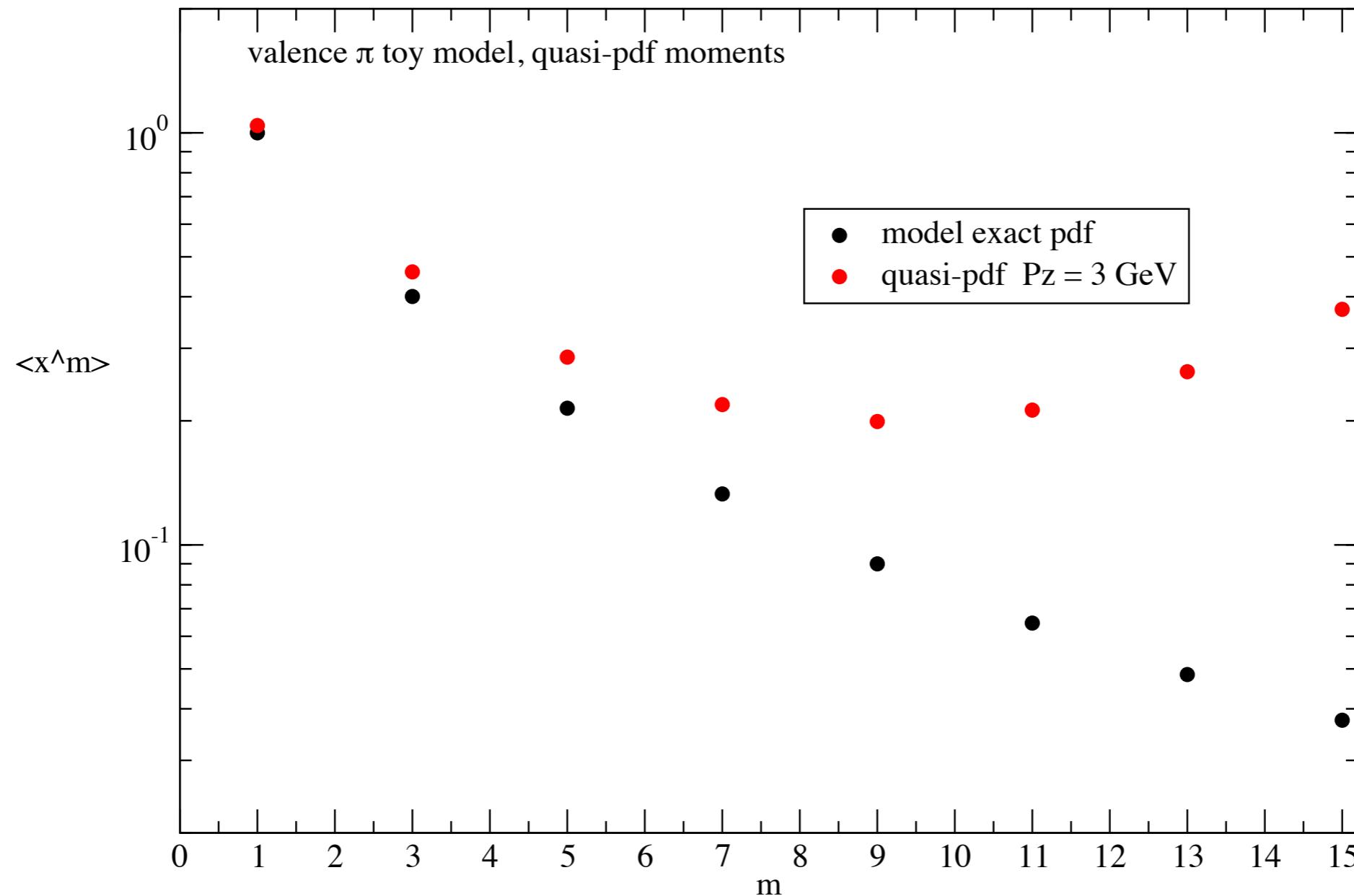
Model-exact PDF & Quasi-PDF @ $P_z=10 \text{ GeV}$



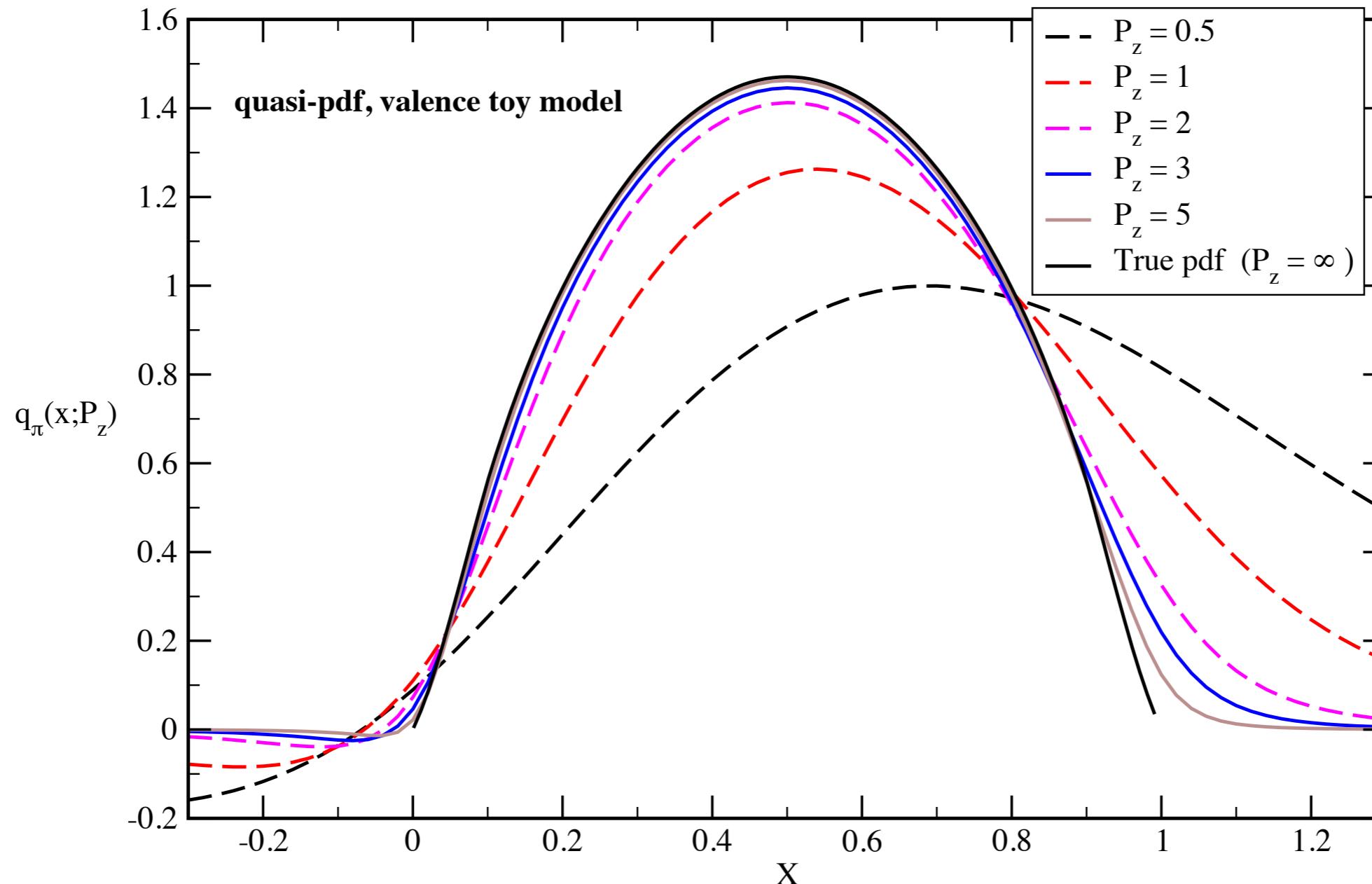
Pz Dependence of quasi-pdf of valence model pion



$\langle x^m \rangle$ for toy model pion at $P_z = 3$ GeV



Pz Dependence of quasi-pdf of u-ubar "pion"



---I.Cloet, Lei Chang, PCT, in progress (2015).....

Applications:-
eg: Form Factors

The Pion Charge Form Factor: Transition from npQCD to pQCD

$$F_\pi(Q^2 = uv) = \int_0^1 dx \int_0^1 dy \phi_\pi^*(x; Q) [T_H(x, y; Q^2)] \phi_\pi(y; Q) + \text{NLO/higher twist....}$$

---LFQCD, Brodsky, LePage PRD (1980)

$$Q^2 \gg \Lambda_{\text{QCD}}^2 : Q^2 F_\pi(Q^2) \rightarrow 16 \pi f_\pi^2 \alpha_s(Q^2) \omega_\phi^2(Q^2) + \mathcal{O}(1/Q^2)$$

$\omega_\phi(Q^2) = \frac{1}{3} \int_0^1 dx \frac{\phi_\pi(x; Q)}{x}$

$\rightarrow 1, Q^2 \rightarrow \infty$

at $Q^2 \sim 3 - 4 \text{ GeV}^2, \Rightarrow 0.1$

JLab expt, Theory $\Rightarrow 0.45$

But, recent DSE theory $\Rightarrow \phi_\pi(x; \mu = 2 \text{ GeV}) \Rightarrow \omega_\phi^2 = 3.3$

PRL 111, 141802 (2013)

PHYSICAL REVIEW LETTERS

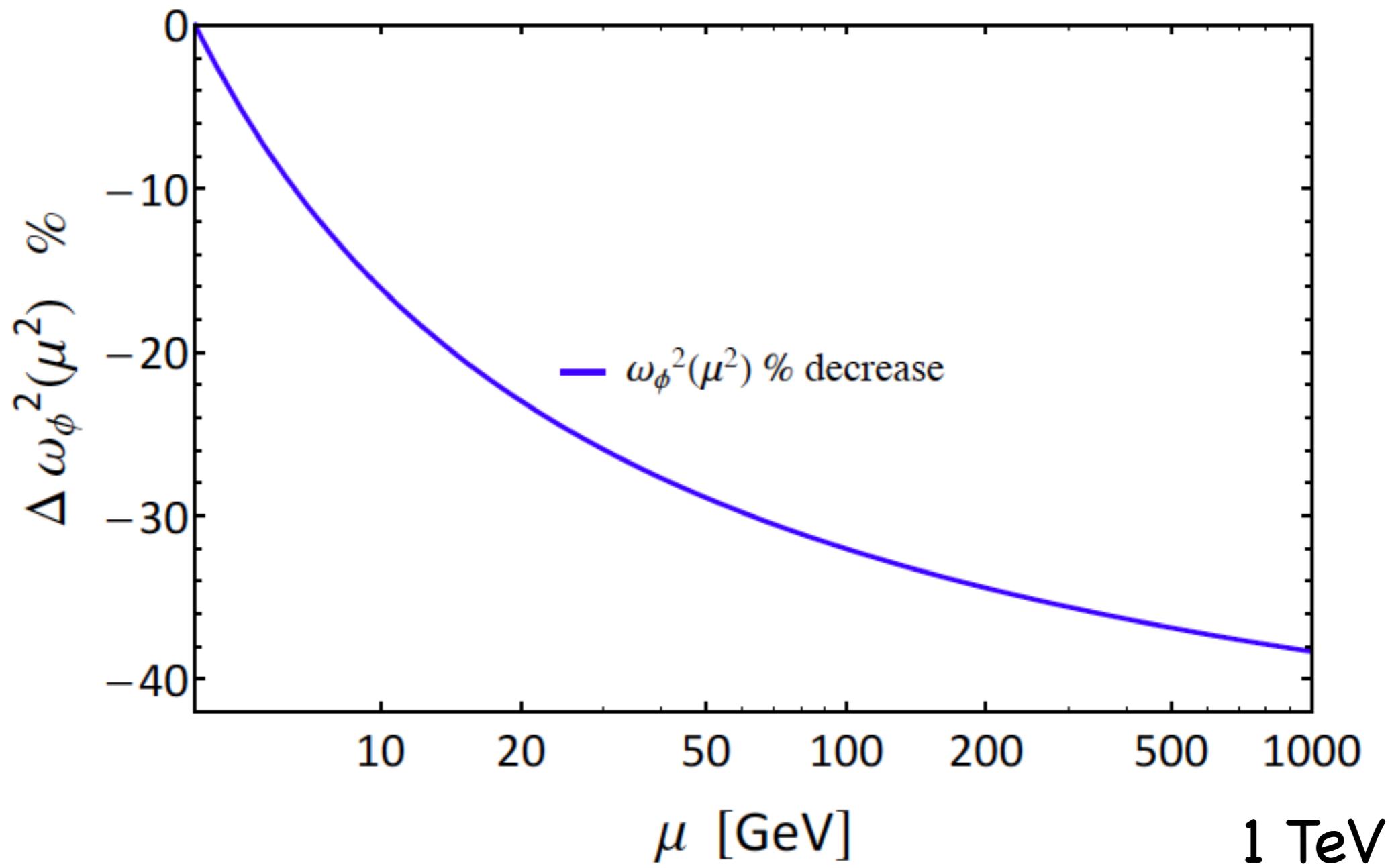
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4 OCTOBER 2013

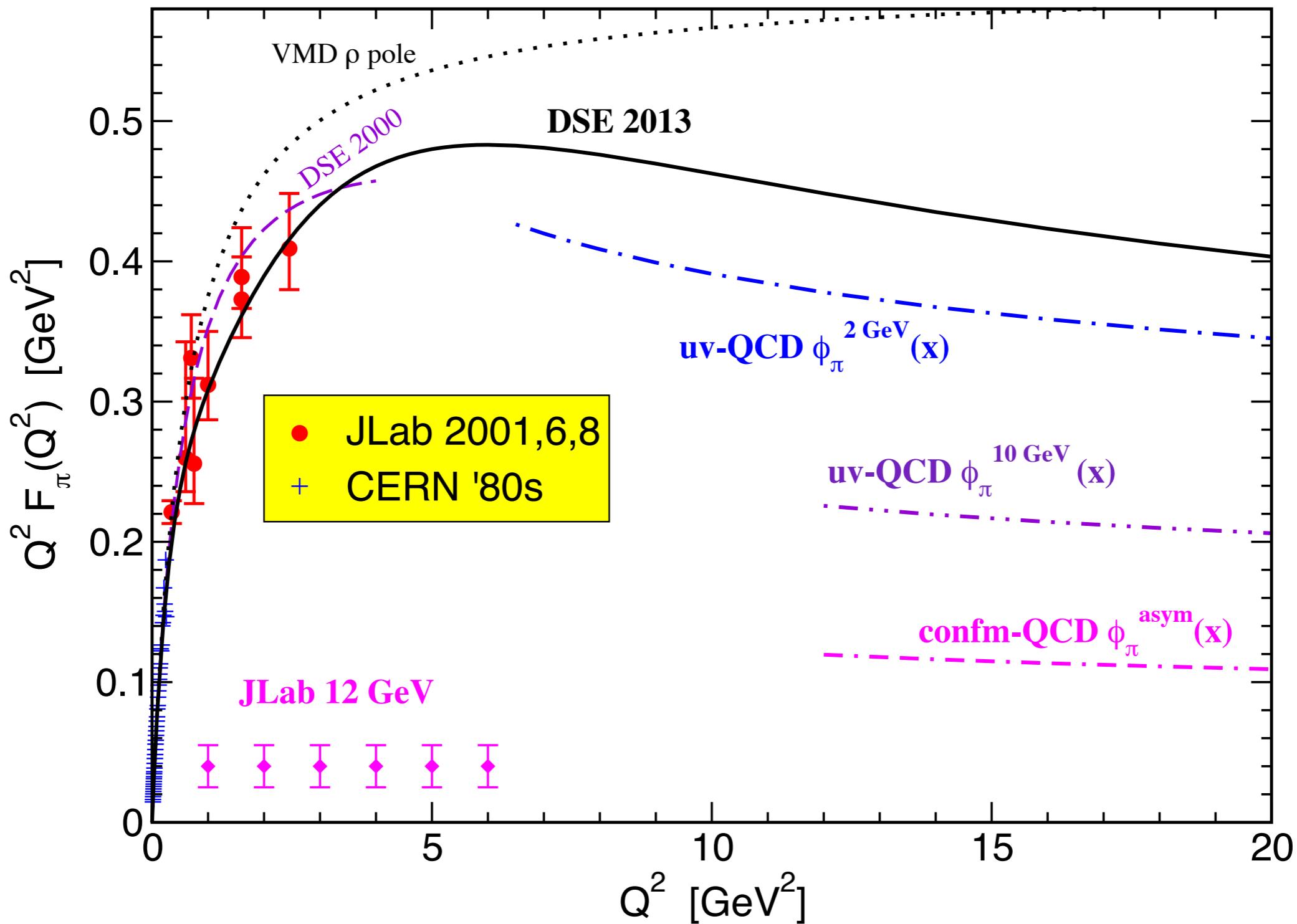
Pion Electromagnetic Form Factor at Spacelike Momenta

L. Chang,¹ I.C. Cloët,² C.D. Roberts,² S.M. Schmidt,³ and P.C. Tandy⁴

UV-QCD is not Asymptotic QCD

$$Q^2 \gg \Lambda_{\text{QCD}}^2 : Q^2 F_\pi(Q^2) \rightarrow 16 \pi f_\pi^2 \alpha_s(Q^2) \omega_\phi^2(Q^2) + \mathcal{O}(1/Q^2)$$



Pion Electromagnetic Form Factor at Spacelike MomentaL. Chang,¹ I.C. Cloët,² C. D. Roberts,² S. M. Schmidt,³ and P. C. Tandy⁴

Jab data: G. Huber et al., PRC78, 045203 (2008)

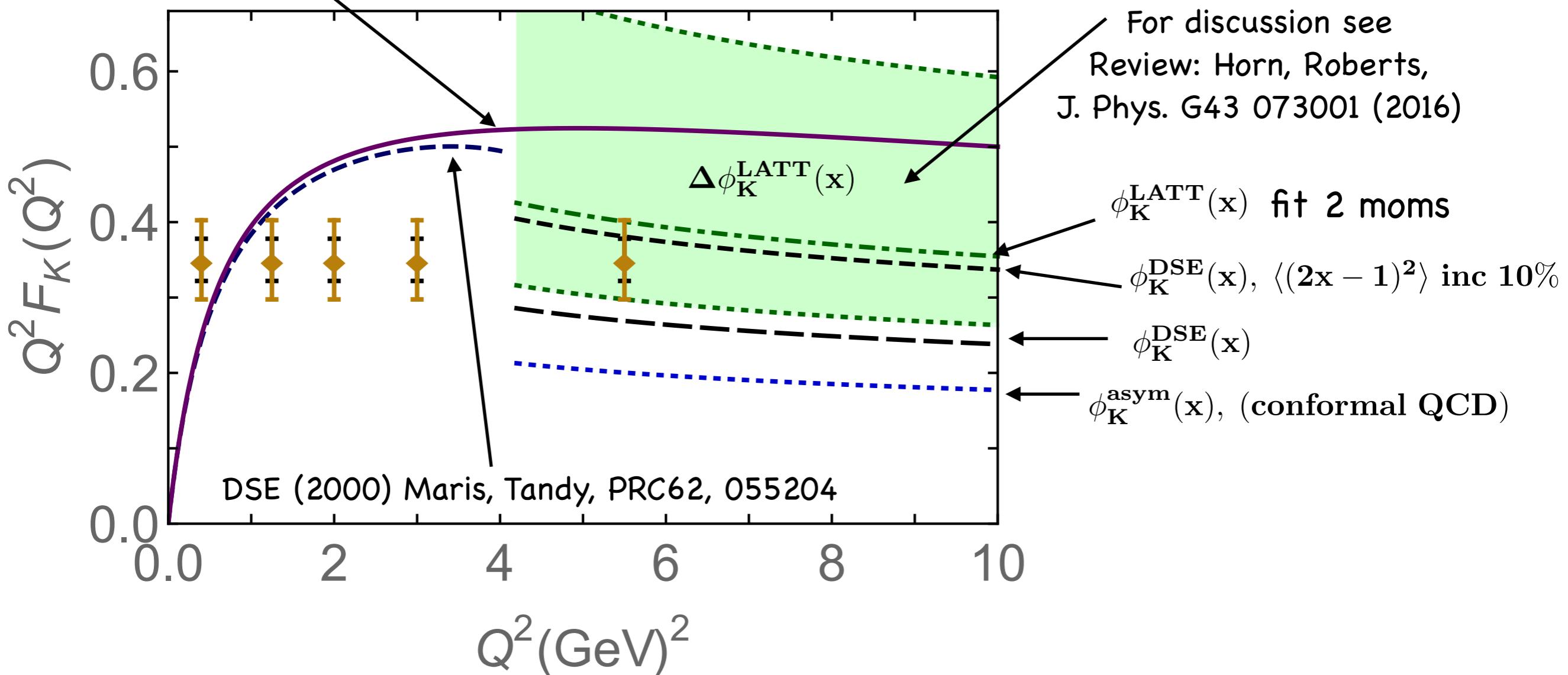
Kaon Elastic Form Factor

$$\text{HS/UV : } Q^2 F_K(Q^2) \rightarrow 16 \pi \alpha_s(Q^2) f_K^2 \omega_K^2(Q^2)$$

$$\omega_K^2 = e_u \omega_u^2 + e_{\bar{s}} \omega_{\bar{s}}^2 \rightarrow 1, Q^2 \rightarrow \infty$$

$$\omega_u = \frac{1}{3} \int_0^1 dx \frac{\phi_K(x)}{x} \neq \omega_{\bar{s}} = \frac{1}{3} \int_0^1 dx \frac{\phi_K(x)}{1-x}$$

DSE (2016) Gao, Chang, Liu, Roberts,
near completion



Hard Scattering Kaon Elastic Form Factor

$$\text{HS/UV} : Q^2 F_K(Q^2) \rightarrow 16\pi \alpha_s(Q^2) f_K^2 \omega_K^2(Q^2)$$

$$\omega_K^2 = e_u \omega_u^2 + e_{\bar{s}} \omega_{\bar{s}}^2 \quad \rightarrow 1, Q^2 \rightarrow \infty$$

$$\omega_u = \frac{1}{3} \int_0^1 dx \frac{\phi_K(x)}{x} \neq \omega_{\bar{s}} = \frac{1}{3} \int_0^1 dx \frac{\phi_K(x)}{1-x}$$

Expt ($s_U = 17.4 \text{ GeV}^2$ timelike) : $\frac{F_K(s_U)}{F_\pi(s_U)} = 0.92(5)$ K. Seth et al., PRL110, 022002 (2013)

$$\frac{f_K^2}{f_\pi^2} = 1.43$$

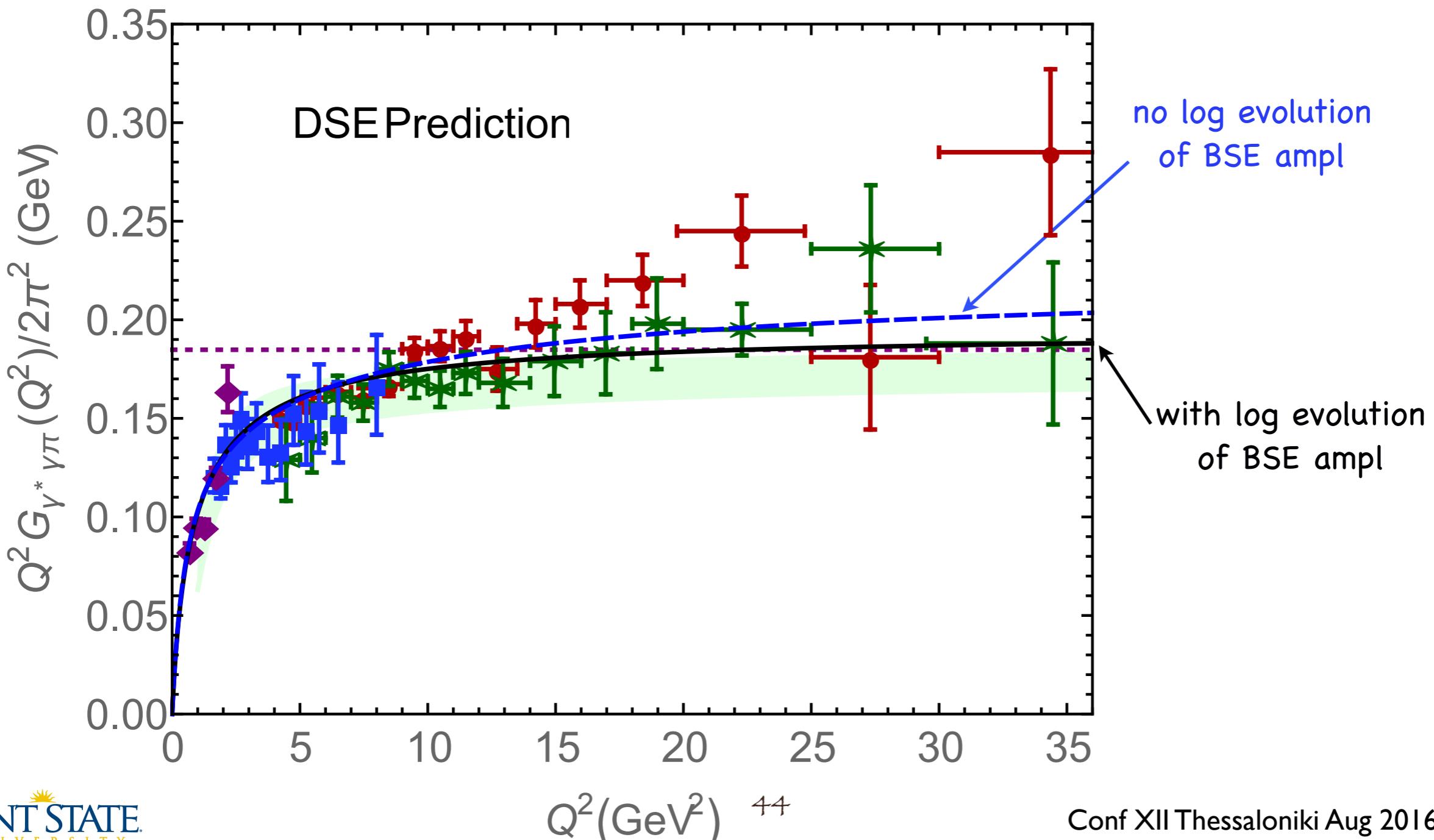
$$\frac{F_K(s_U)}{F_\pi(s_U)} = 1.16 (\phi^{\text{DSE}}); \quad 1.16 + 50\% (\phi^{\text{LQCD}})$$

C. Shi, L. Chang, C.D. Roberts, S.Schmidt, PCT, H-S. Zong, PLB738, 512 (2014)

Pion Transition Form Factor

K. Raya, L. Chang, A. Bashir, J.J.Cobos-Martinez, L.X. Gutierrez-Guerrero, C.D.Roberts, P.C.Tandy,
PRD93, 074017 (2016)

From unified treatment of DA, elastic FF, and transition FF



Summary

- **DSE approach** works extremely well for pion & kaon due to symmetry dominance.
- **Parton Distribution Amplitudes** (pion, kaon). DSE approach shows good contact with available lattice-QCD moments. Flavor symmetry breaking & dynamical chiral symmetry breaking evident and quantitative in the shapes.
- **Pion Transition & Elastic Form Factors** DSE TFF calculation for all Q^2 —agrees with Belle not BaBar. DSE eIFF—Connection with ultraviolet /hard scattering QCD reconciled. Identify that the ultraviolet partonic behavior is within reach of proposed JLab pion FF experiments.
- **Parton Distribution Functions** (pion). Qualitative behavior of empirical data fits reproduced by DSE $q\bar{q}$ + pion loop analysis.
- Time to declare we understand the pion and kaon in QCD ?
- X. Ji's **space-like correlator approach to PDFs**—a model investigation. Spurious anti-quark contributions seem unavoidable if $P_z < 2 \text{ GeV}$. For $x > 0.8$, need $P_z > 4 \text{ GeV}$ for confidence in the qualitative shape. Further work in progress.

The End

Collaborators

- Craig Roberts, Argonne National Lab, USA
- Adnan Bashir, University of Michoacan, Morelia, Mexico
- Ian Cloet, Argonne National Lab, USA
- Sixue Qin, Argonne National Lab, USA
- Hong-shi Zong, Nanjing Univ, China
- Lei Chang, Peking U, Argonne/Julich/Univ Adelaide, Australia
- Chao Shi, Nanjing Univ, [visiting Kent State U]
- Konstantin Khitrin, PhD student, Kent State Univ, USA
- Javier Cobos-Martinez, Univ of Sonora, Mexico

Lattice-QCD and DSE-based modeling

- Lattice: $\langle \mathcal{O} \rangle = \int D\bar{q}qG \mathcal{O}(\bar{q}, q, G) e^{-S[\bar{q}, q, G]}$
 - Euclidean metric, x-space, Monte-Carlo
 - Issues: lattice spacing and vol, sea and valence m_q , fermion Det
 - Large time limit \Rightarrow nearest hadronic mass pole
- EOMs (DSEs): $0 = \int D\bar{q}qG \frac{\delta}{\delta q(x)} e^{-S[\bar{q}, q, G] + (\bar{\eta}, q) + (\bar{q}, \eta) + (J, G)}$
 - Euclidean metric, p-space, continuum integral eqns
 - Issues: truncation and phenomenology—not full QCD
 - Analytic contin. \Rightarrow nearest hadronic mass pole
 - Can be quick to identify systematics, mechanisms, ···

Pion, Kaon...

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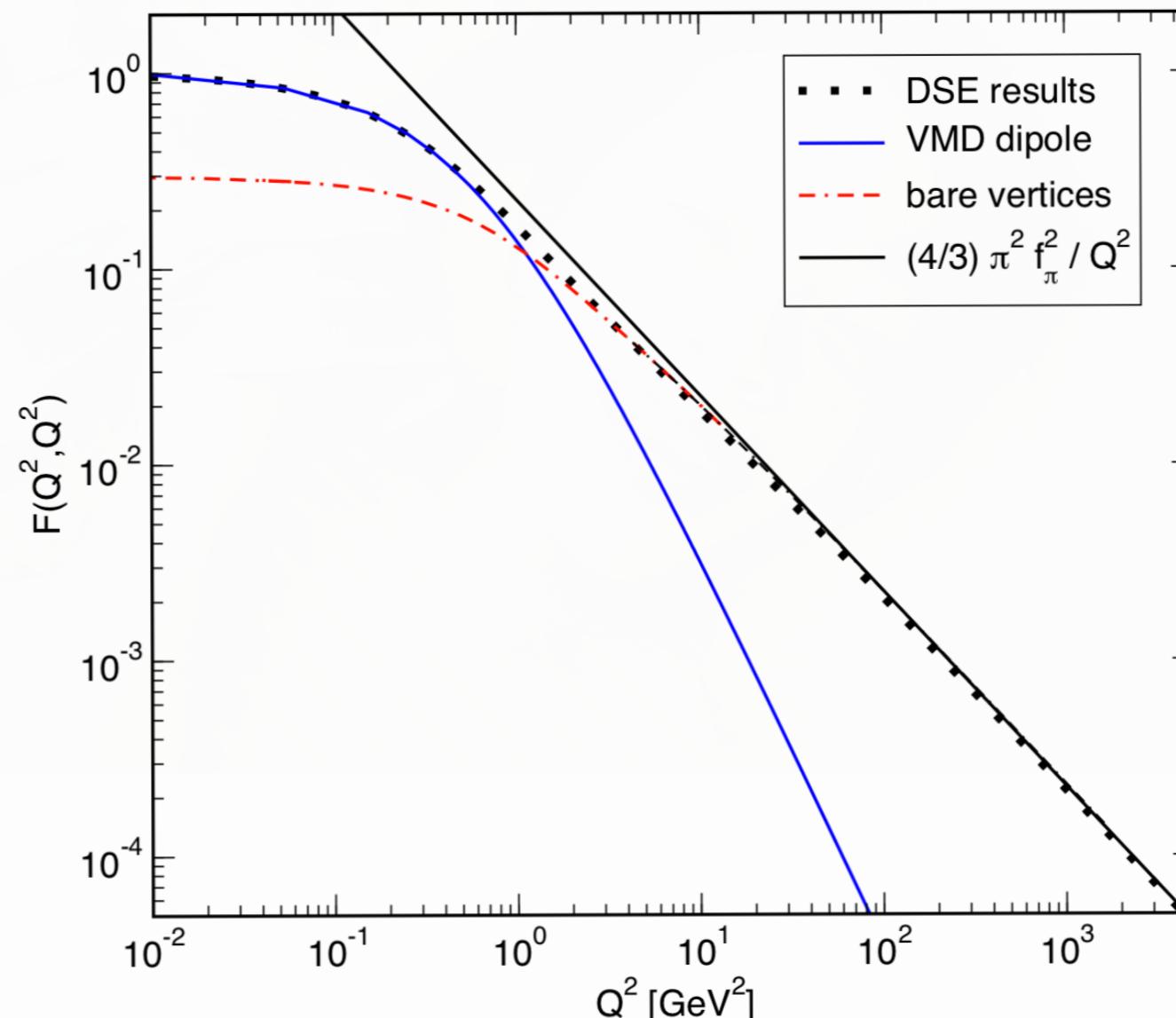
Expect: qualitatively new insight where other methods can't, eg high Q^2

Do not expect: final, precision-QCD results, except in special cases **Pion, Kaon...**

Where Asym FF Could be Calculated, its Power Law was Correct:-

$\gamma^* \pi \gamma^*$ Asymptotic Limit

Lepage and Brodsky, PRD22, 2157 (1980): LC-QCD/OPE \Rightarrow



Estimate 1-Pion Loop Contribution to Pion PDF

$$\pi^+ : \langle x^1 \rangle_\mu = \int_0^1 dx x \{ \mathbf{u} + \bar{\mathbf{u}}_{\text{sea}} + \bar{\mathbf{d}} + \mathbf{d}_{\text{sea}} + \mathbf{g}(x) \} \approx 2\langle x q_v(x) \rangle + 4\langle x q_{\text{sea}}(x) \rangle + \langle x g(x) \rangle = 1$$

$$\mathbf{u} = \mathbf{u}_v + \mathbf{u}_{\text{sea}}, \quad \bar{\mathbf{d}} = \bar{\mathbf{d}}_v + \bar{\mathbf{d}}_{\text{sea}} \quad \text{Empirical GRS/ASV} \Rightarrow \text{universal } q_v(x), q_{\text{sea}}(x) \text{ at } \mu = 0.630 \text{ GeV}$$

$$\Gamma_\pi = \sqrt{1 - \alpha^2} \Gamma_{q\bar{q}}^{\text{RL}} + \alpha \Gamma_{\pi q\bar{q}}$$

CPT: 18% effect

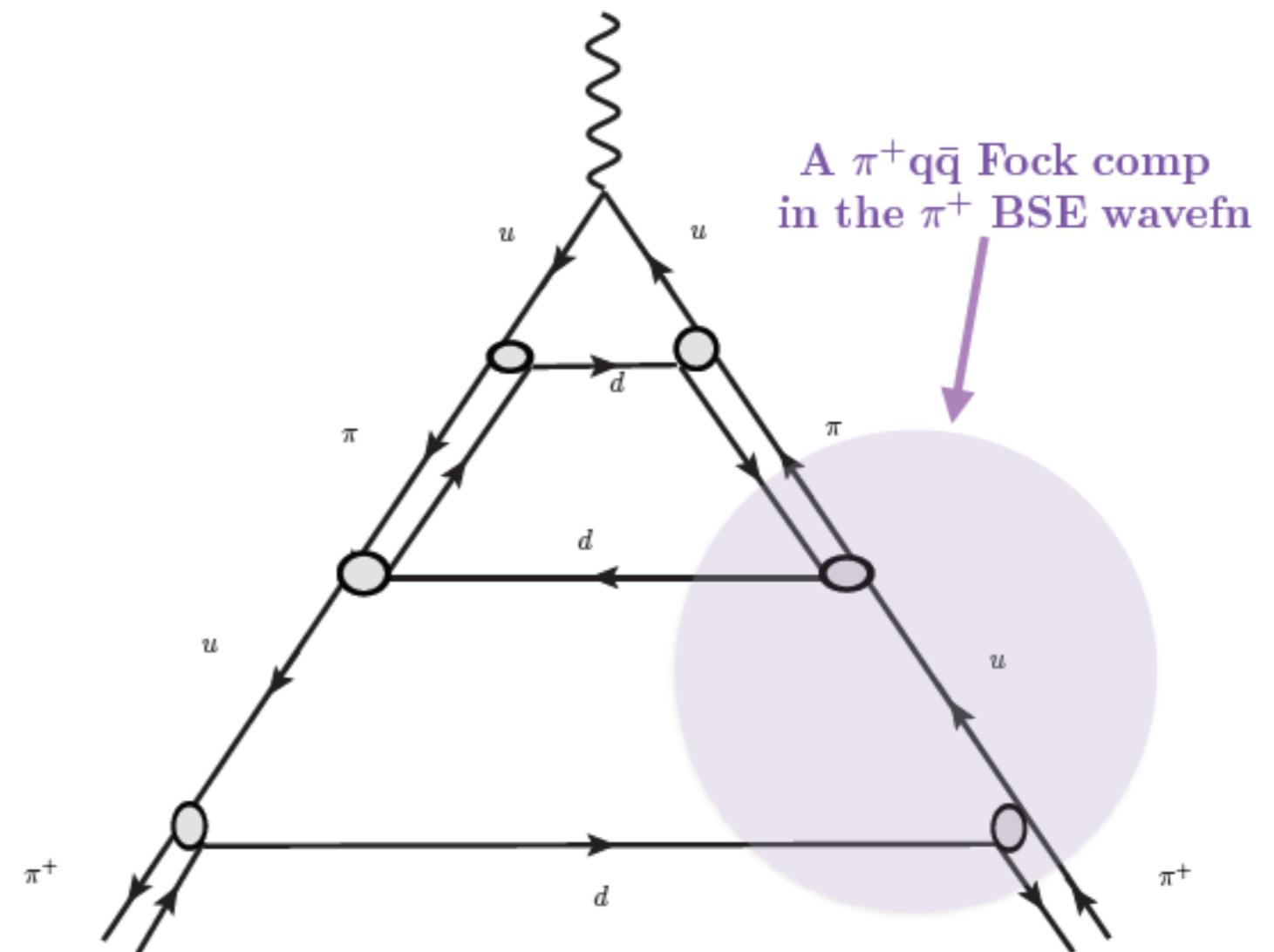
$$r_{\text{ch}}^2 = (1 - \alpha^2) r_{\text{RL}}^2 + \alpha^2 r_{\pi-\text{lp}}^2$$

$$\text{DSE-RL: } r_{\text{RL}}^2 = r_{\text{ch}}^2 \Rightarrow \alpha^2 = 18\%$$

PDF Consequence:

$$q_v(x) = (1 - \alpha^2) q^{\text{RL}}(x) + q_v^{\pi-\text{lp}}(x)$$

$$\text{with } \langle q_v^{\pi-\text{lp}}(x) \rangle = \alpha^2 = 0.18$$



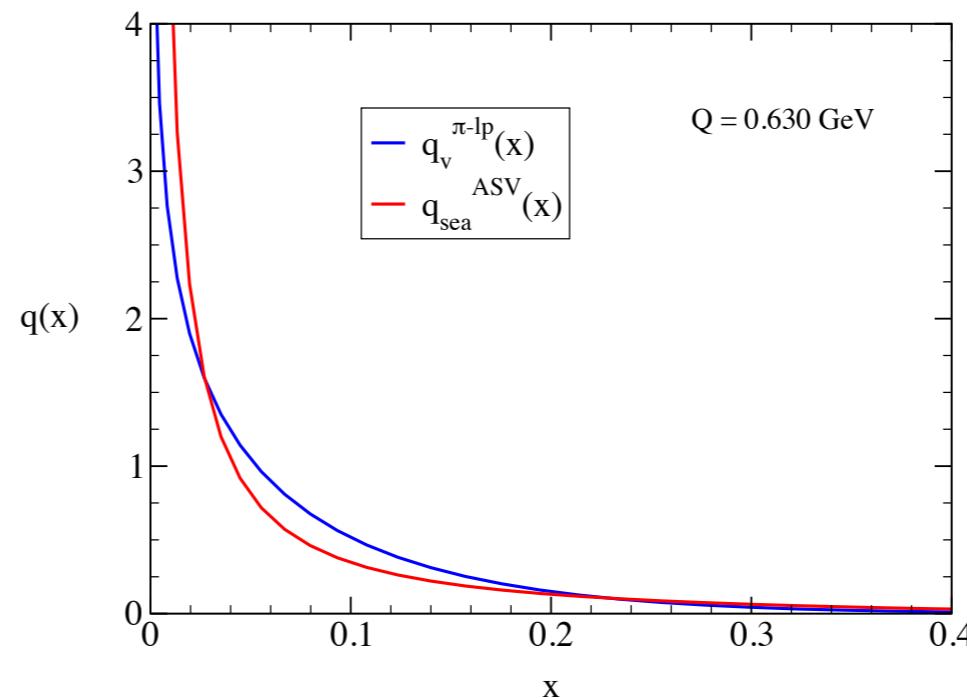
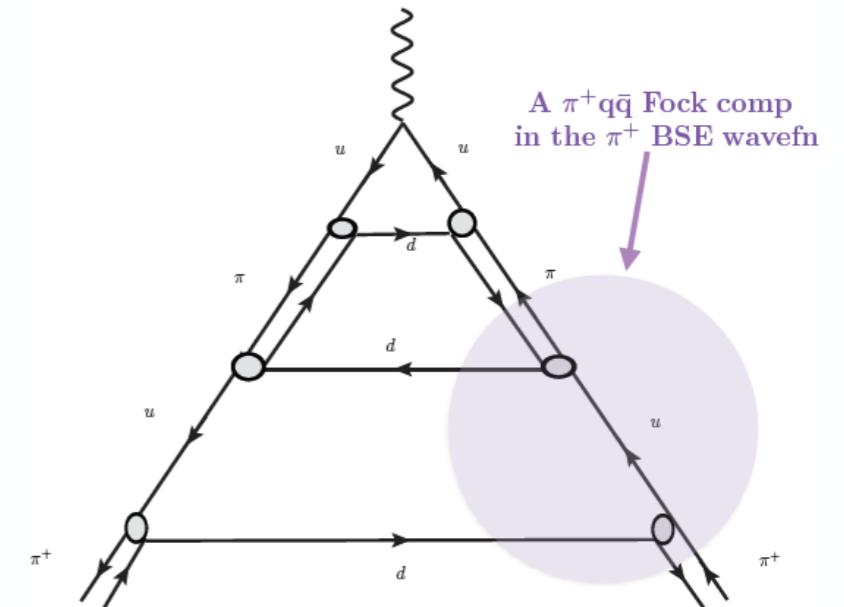
Convolution Model for $q(x)$ from virtual pi loop

$$q_v^{\pi-lp}(x) \sim \mathcal{P}_{\mathbf{q}/T}(x) = \int_x^1 \frac{dy}{|y|} \mathcal{P}_{\pi/T}(y) \mathcal{P}_{\mathbf{q}/\pi}\left(\frac{x}{y}\right),$$

$T = \text{target} = \pi$ here

$\mathcal{P}_{\pi/T}(y)$ should strongly favor $y \leq \frac{m_\pi}{2M_q + m_\pi} \approx 0.2$,

$\mathcal{P}_{\mathbf{q}/\pi}\left(\frac{x}{y}\right)$ is self-consistently determined



Result is strongly constrained

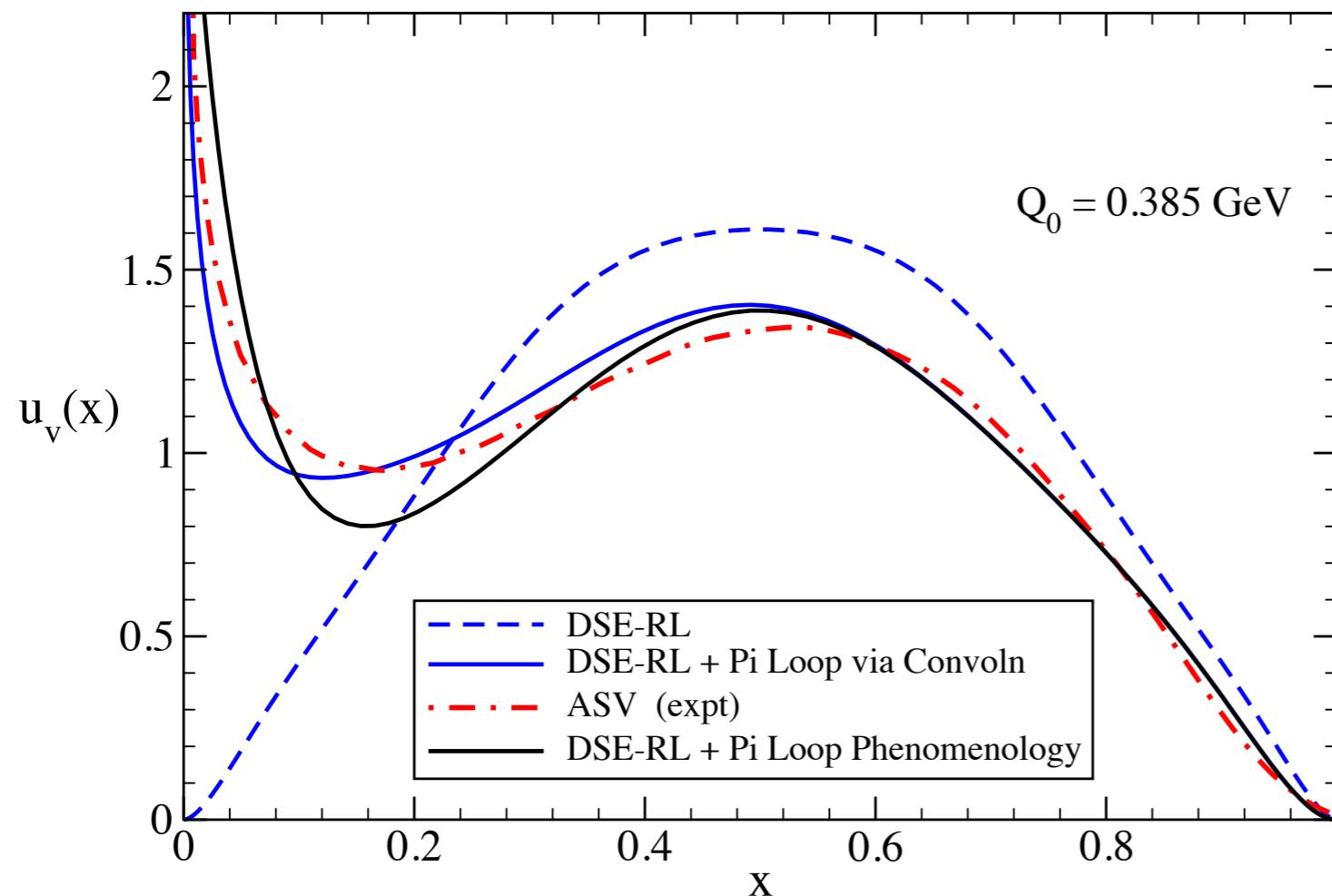
Analysis of Pion Parton Momentum Sum Rule

TABLE II: Momentum fraction sum rule from this work at scale $Q_0 = 0.630$ GeV corresponding to the ASV [13] compilation.

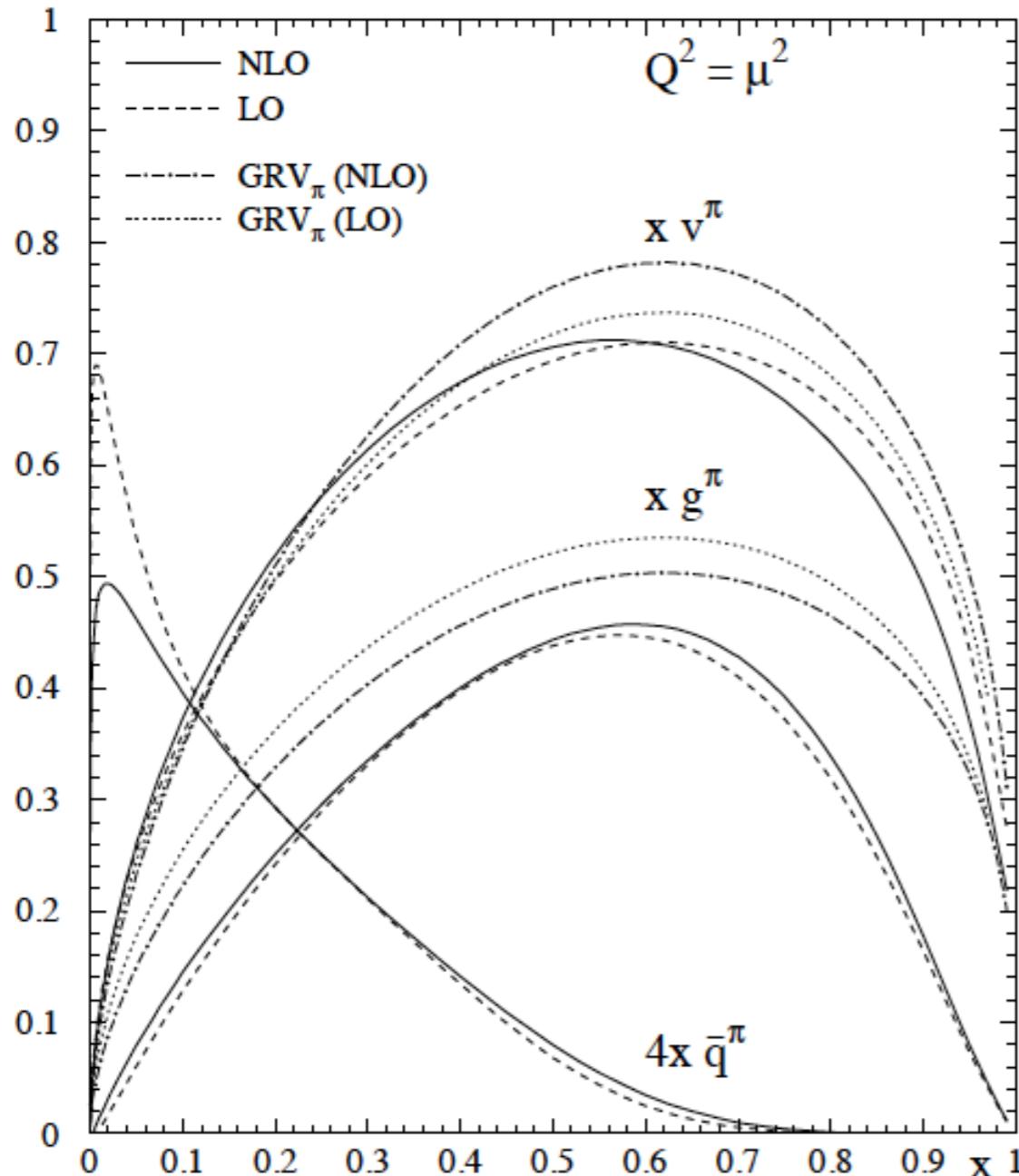
	$2 q_{\text{val}}^{\text{RL}}$	$2 q_{\text{val}}^{\text{DSE}}$	$4 q_{\text{sea}}^{\text{ASV}}$	gluon	Total
$\langle x \rangle_\pi$	0.770	0.649	0.0498	0.300	0.999

K. Khitrit, P. Tandy, in progress (2015)

Modern empirical expt parameterization:
Aicher, Shafer, Vogelsang, (ASV) PRL 105, 252003 (2010)



Pion PDFs—Expt “Data” Parameterizations



Eur. Phys. J. C 10, 313–317 (1999)
 Digital Object Identifier (DOI) 10.1007/s100529900124

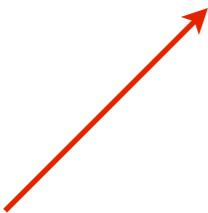
Pionic parton distributions revisited

M. Glück, E. Reya, I. Schienbein

Institut für Physik, Universität Dortmund, D-44221 Dortmund, Germany

Aicher, Schafer, Vogelsang,
 arXiv:1009.2481
 soft gluon resummation

Fig. 1. The valence and valence-like input distributions $xf^\pi(x, Q^2 = \mu^2)$ with $f = v, \bar{q}, g$ as compared to those of GRV _{π} [5]. Notice that GRV _{π} employs a vanishing SU(3)_{flavor} symmetric \bar{q}^π input at $\mu_{\text{LO}}^2 = 0.25$ GeV² and $\mu_{\text{NLO}}^2 = 0.3$ GeV² [5]. Our present SU(3)_{flavor} broken sea densities refer to a vanishing s^π input in (3), as for GRV _{π} [5]

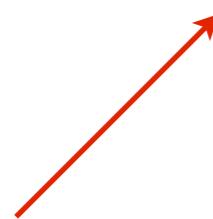


$$\Rightarrow \frac{\alpha_{\text{eff}}^{\text{RL}}(0)}{\pi} \approx 3$$

$\Rightarrow \frac{\alpha_{\text{eff}}^{\text{DB}}(0)}{\pi} \approx 1$, [with dressed vertex effects]

BSE kernel from ab initio gauge sector DSE
work now agrees satisfactorily with the
kernel from fitting data: Binosi, Chang,
Papavassiliou, Roberts, PLB742, 183 (2015)

Modern Context for Rainbow-Ladder Kernel



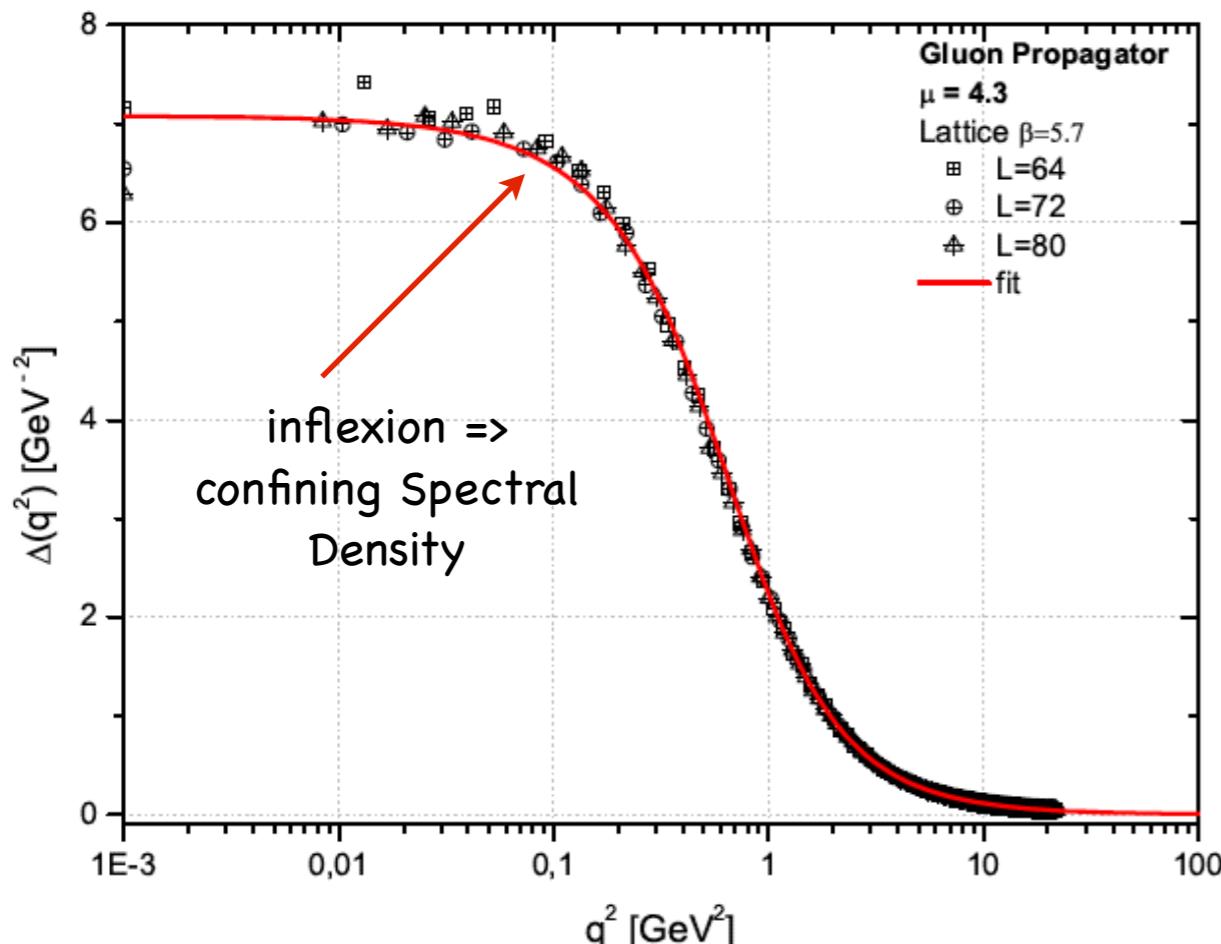
Landau gauge, lattice – QCD gluon propagator,
I.L.Bogolubsky *etal.*, PosLAT2007, 290 (2007)

$$\Rightarrow \frac{\alpha_{\text{eff}}^{\text{RL}}(0)}{\pi} \approx 3$$

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Papavassiliou, Roberts, PLB742, 183 (2015)

Modern Context for Rainbow-Ladder Kernel



Landau gauge, lattice – QCD gluon propagator,
 I.L.Bogolubsky *et al.*, PoS(LAT2007), 290 (2007)

Identified enough strength for physical DCSB
 $\Rightarrow m_G(k^2)$ $m_G(0) \sim 0.38 \text{ GeV}$

$$K_{\text{BSE}}^{\text{RL}} = \frac{4\pi\hat{\alpha}_{\text{eff}}(q^2)}{m_G^2(q^2) + q^2} \Rightarrow \frac{\alpha_{\text{eff}}^{\text{RL}}(0)}{\pi} \approx 3$$

$$\Rightarrow \frac{\alpha_{\text{eff}}^{\text{DB}}(0)}{\pi} \approx 1, \text{ [with dressed vertex effects]}$$

BSE kernel from ab initio gauge sector DSE
 work now agrees satisfactorily with the
 kernel from fitting data: Binosi, Chang,
 Papavassiliou, Roberts, PLB742, 183 (2015)

Other Meson Distribution Amplitudes

Table 1: Meson PDA moments obtained using numerical simulations of lattice-regularised QCD with $N_f = 2 + 1$ domain-wall fermions and nonperturbative renormalisation of lattice operators [29]: linear extrapolation to physical pion mass, $\overline{\text{MS}}$ -scheme at $\zeta = 2 \text{ GeV}$, two lattice volumes. The first error is statistical, the second represents an estimate of systematic errors, including those from the s -quark mass, discretisation and renormalisation.

meson	$\langle (x - \bar{x})^n \rangle$	$16^3 \times 32$	$24^3 \times 64$
π	n=2	0.25(1)(2)	0.28(1)(2)
ρ_{\parallel}	n=2	0.25(2)(2)	0.27(1)(2)
ϕ	n=2	0.25(2)(2)	0.25(2)(1)
K	n=1	0.035(2)(2)	0.036(1)(2)
K_{\parallel}^*	n=1	0.037(1)(2)	0.043(2)(3)
K	n=2	0.25(1)(2)	0.26(1)(2)
K_{\parallel}^*	n=2	0.25(1)(2)	0.25(2)(2)

$$\varphi(x) = x^\alpha (1-x)^\beta / B(\alpha, \beta).$$

$$16^3 \times 32: \quad \alpha_{us} = 0.56^{+0.21}_{-0.18}, \beta_{us} = 0.45^{+0.19}_{-0.16},$$

$$24^3 \times 64: \quad \alpha_{us} = 0.48^{+0.19}_{-0.16}, \beta_{us} = 0.38^{+0.17}_{-0.15}.$$

DAs of light quark mesons look
much the same--with small
flavor breaking

DSE analysis of LQCD moments:
Segovia, Chang, Cloet, Roberts, Schmidt, Zong
PLB731, 13, (2014)

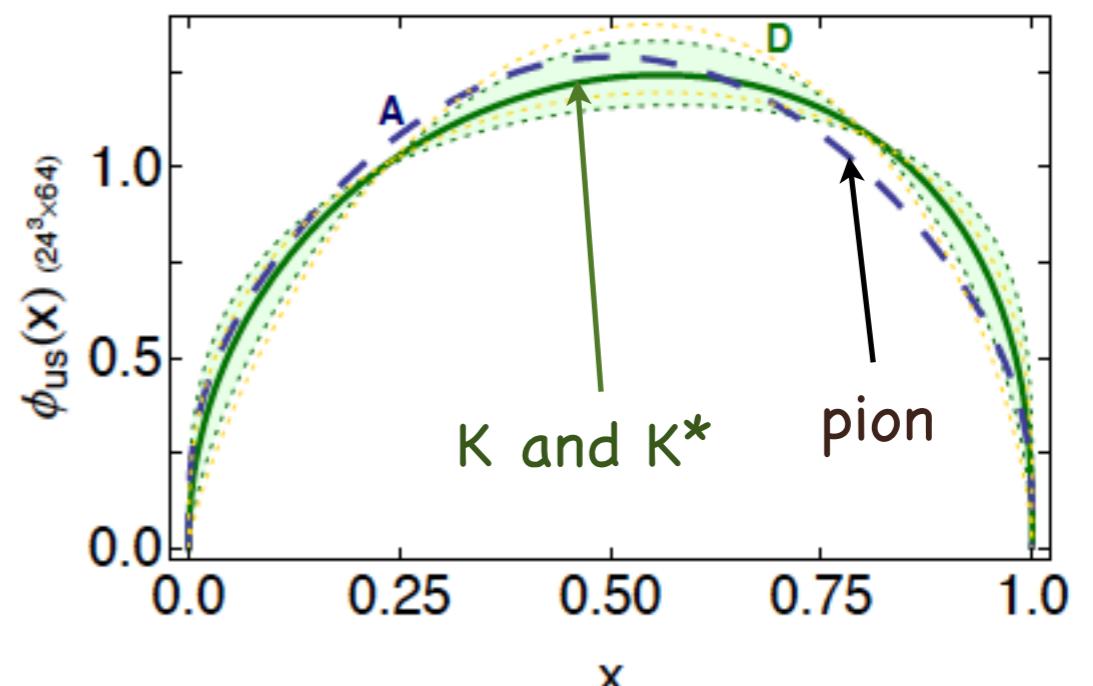


Figure 2: Solid curve and associated error band (shaded region labelled "D"): PDA in Eq. (15), describing $u\bar{s}$ pseudoscalar and vector mesons, reconstructed using Eq. (8) and obtained from the $24^3 \times 64$ -lattice configurations. The result obtained from the $16^3 \times 32$ -lattice moments in Table 1 is not materially different. The dashed curve "A" is the DSE prediction for the pion's PDA in Eq. (13).

Pion Form Factor: Running q Mass Fn Effect

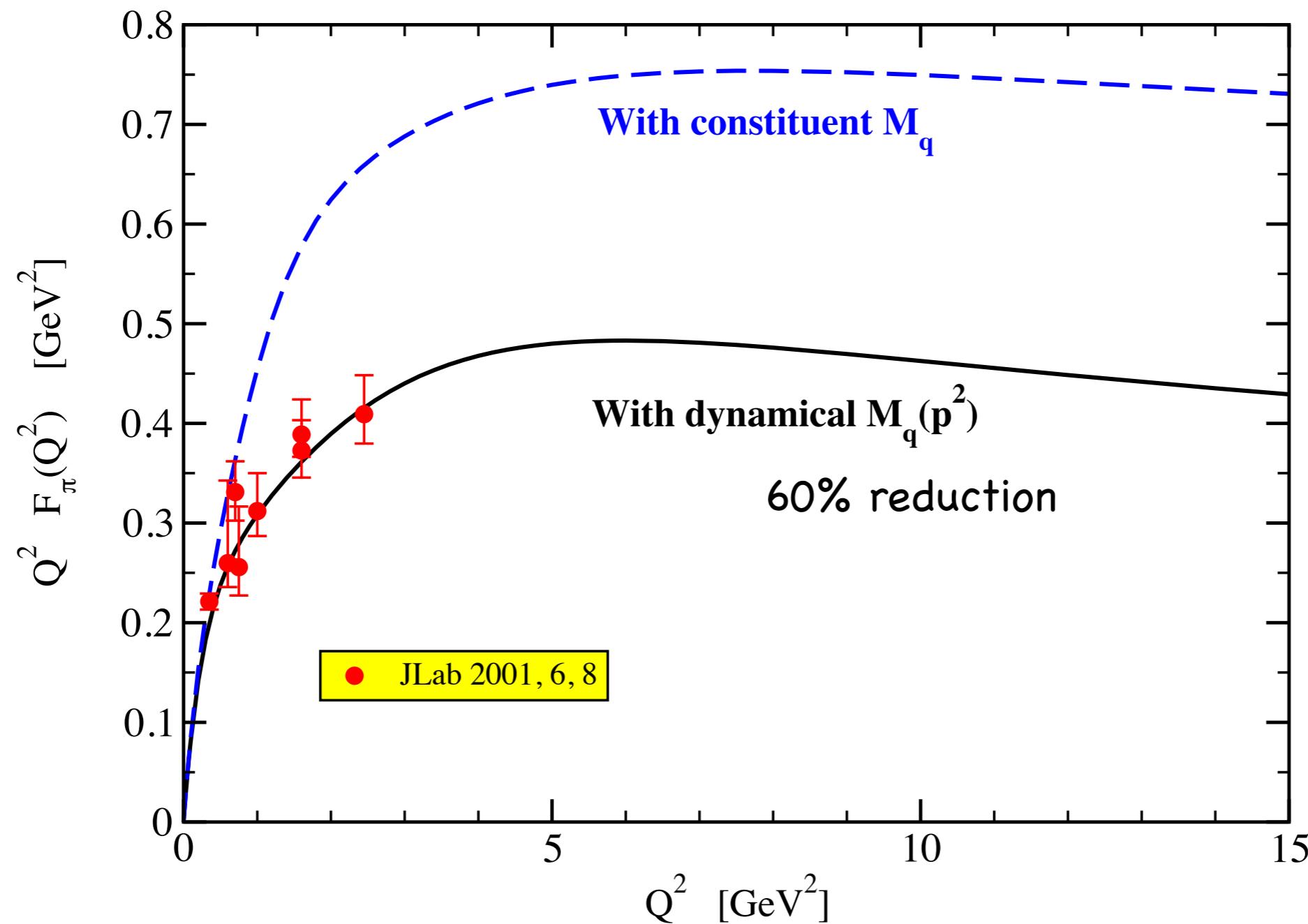


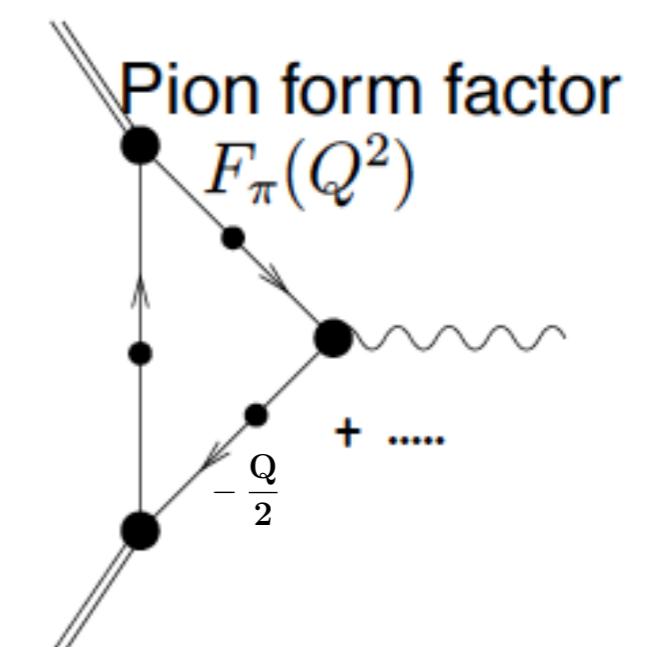
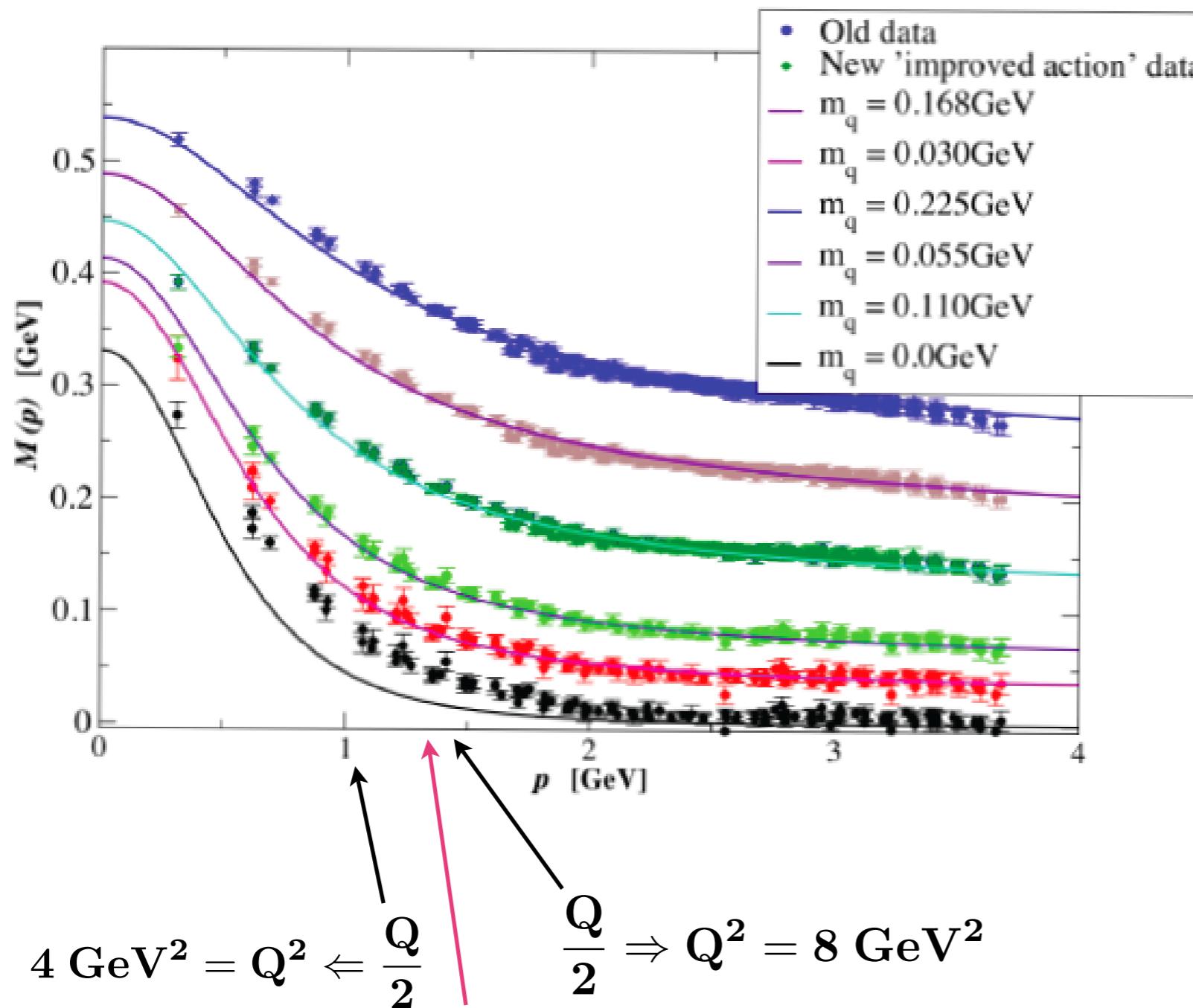


Illustration by Sandbox Studio, Chicago with Corinne Mucha

Mommy, Daddy, where does mass come from?

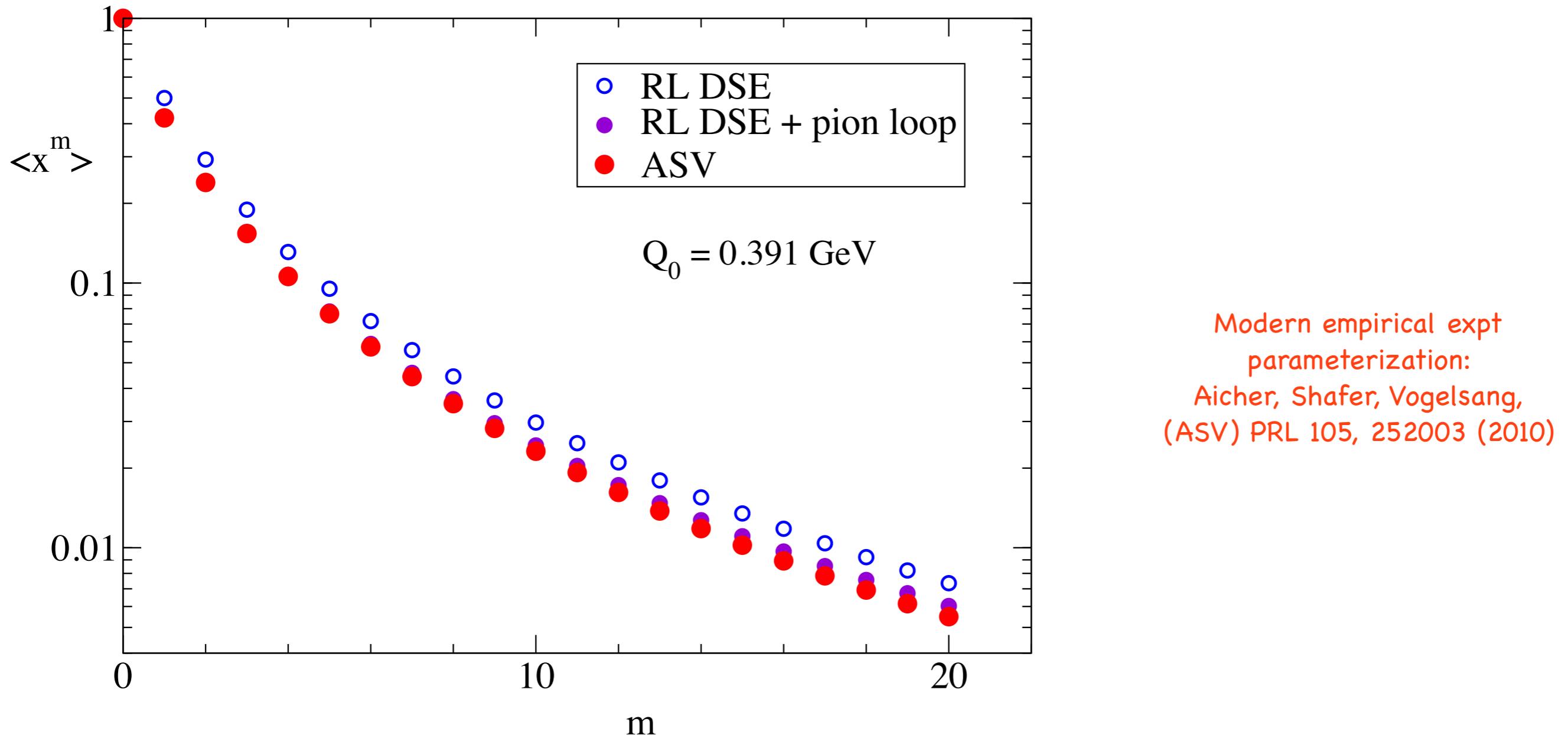
Dynamical Breaking of Chiral Symmetry

Transition from constituent to parton quark



JLab 12 GeV

Many Moments via Feyn PTIR--Easy

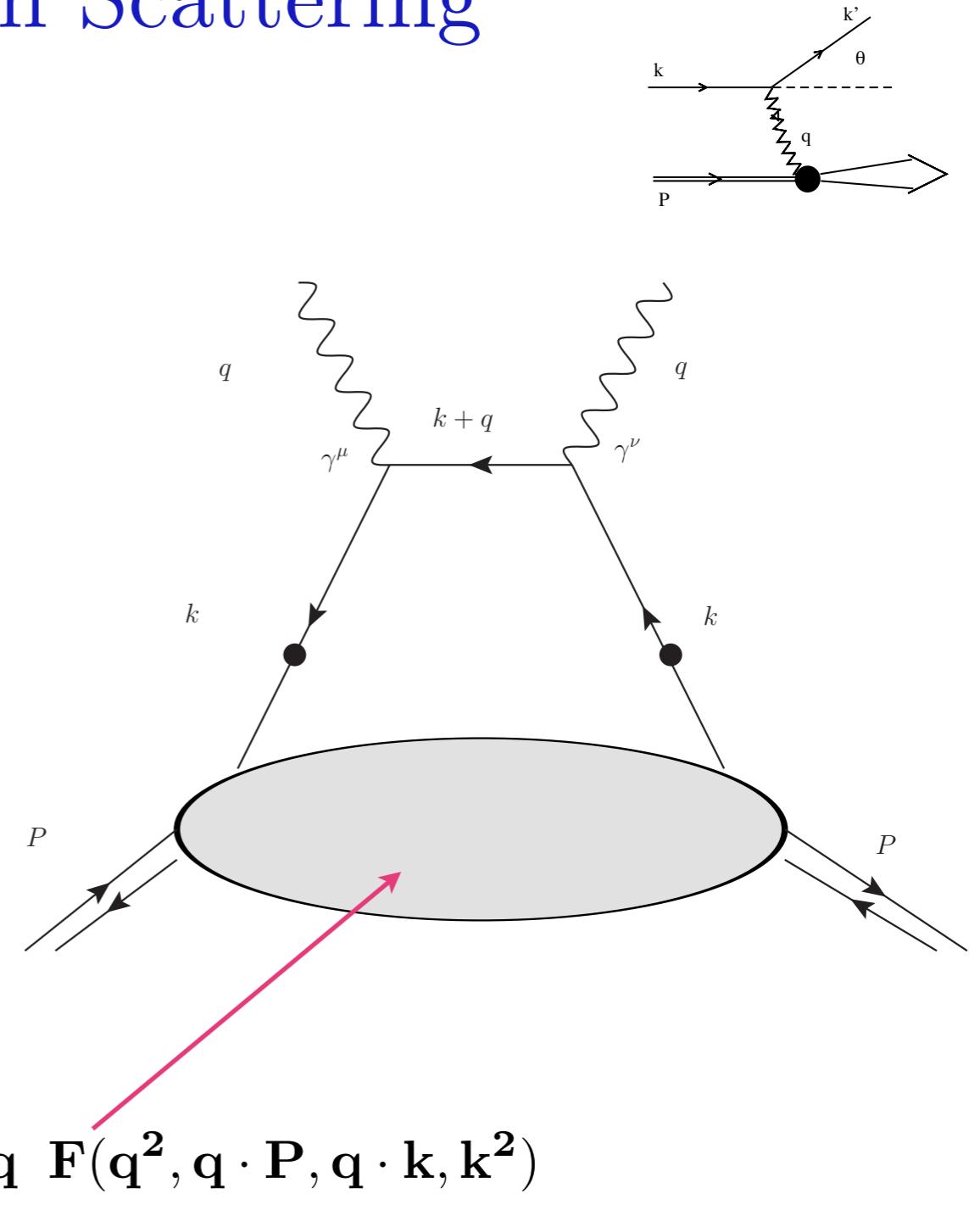


Summary

- DCSB: A large u/d quark constituent mass is generated from almost nothing for the same reason & and by the same mechanism that makes the pion almost massless!
- DCSB causes the shape of the pion DA to be significantly broader than the asymptotic-QCD DA at accessible scales for hadron physics, and a new analysis technique shows that lattice-QCD moments say the same thing. [DCSB identified in a LF-defined quantity.]
- The scale running of distribution amplitudes is exceedingly SLOW---even at LHC scales asymptotic-QCD for DAs and form factors they influence there are persistent sizeable npQCD effects and DCSB in the hadron states.
- The elastic form factor of the pion makes a transition from non-perturbative/constituent quark behavior to partonic perturbative behavior for Q^2 at 6-8 GeV 2 and the relevant extension of the Brodsky-LePage uv-QCD leading formula is just 15% below the recent DSE calculation there.
- The new DSE approach is applicable to form factors for all spacelike Q^2 .
- DSE-QCD can now be applied to light-front-defined bound state properties as a fn of momentum fraction x. Meson DAs and PDFs work out well, nucleon PDFs and GPDs await...

Deep Inelastic Lepton Scattering

- PDFs: $u_\pi(x)$, $u_K(x)$, $s_K(x)$
- Drell-Yan data exists
- Pion and Kaon/Pion Ratio
- Employ LR DSE model
- Bjorken limit fixes quark k^+
- Covariant formulation: $\int d^4q \ F(q^2, q \cdot P, q \cdot k, k^2)$
- Evolve from model scale via LO DGLAP

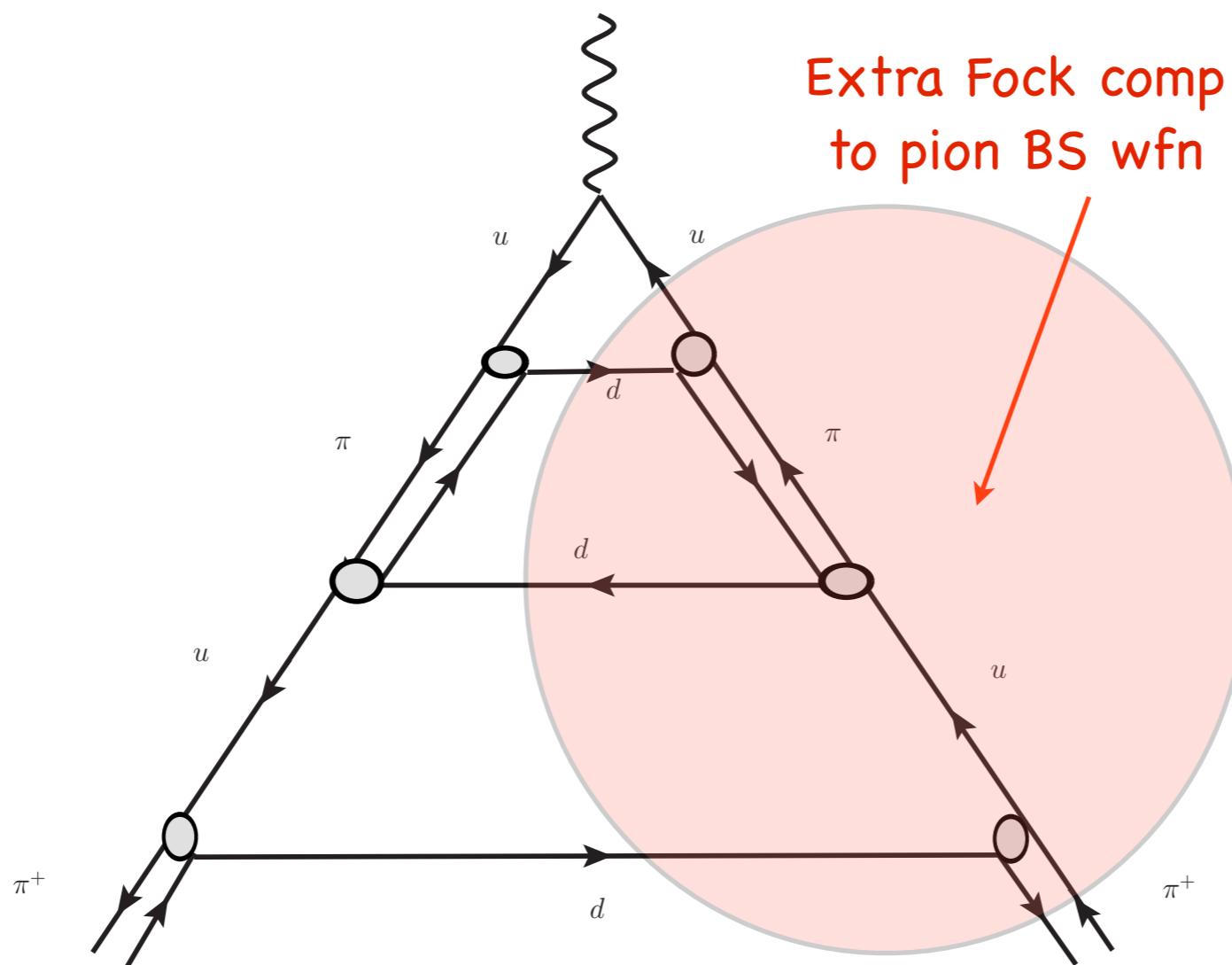


Pion Loop in Pion Charge Form Factor

$$F_\pi(Q^2) = (1 - \alpha^2) F_\pi^{\text{RL}}(Q^2) + \alpha^2 F_\pi^{\pi-\text{lp}}(Q^2)$$

$$F_\pi(Q^2) = (1 - \alpha^2) \left(1 - \frac{Q^2 r_{\text{RL}}^2}{6} + \dots \right) + \alpha^2 \left(1 - \frac{Q^2 r_{\pi-\text{lp}}^2}{6} + \dots \right)$$

$$F_\pi(Q^2) = \left(1 - \frac{Q^2 r_{\text{TOT}}^2}{6} + \dots \right), \quad r_{\text{ch}}^2 = (1 - \alpha^2) r_{\text{RL}}^2 + [\alpha^2 r_{\pi-\text{lp}}^2]$$

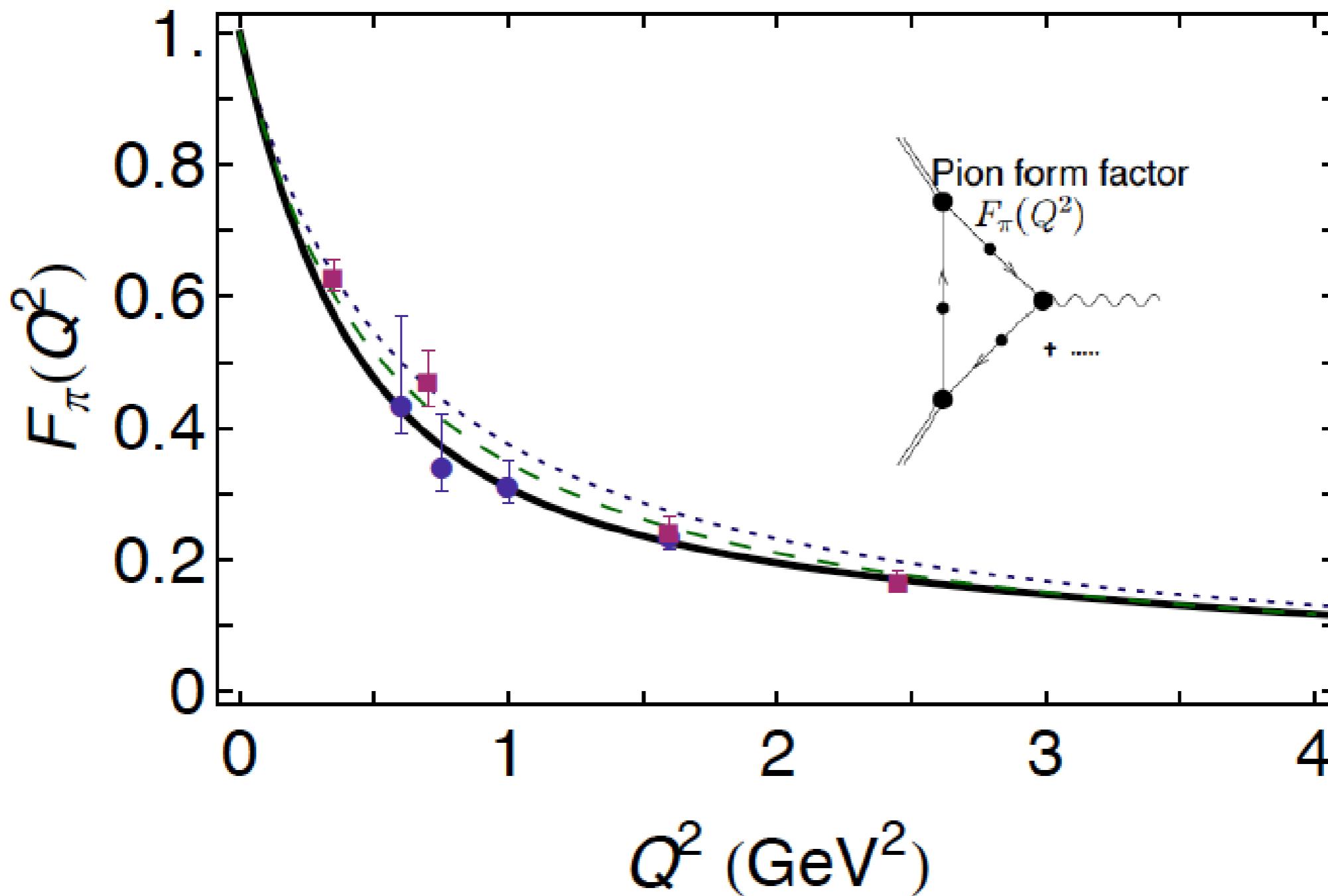


The Future?

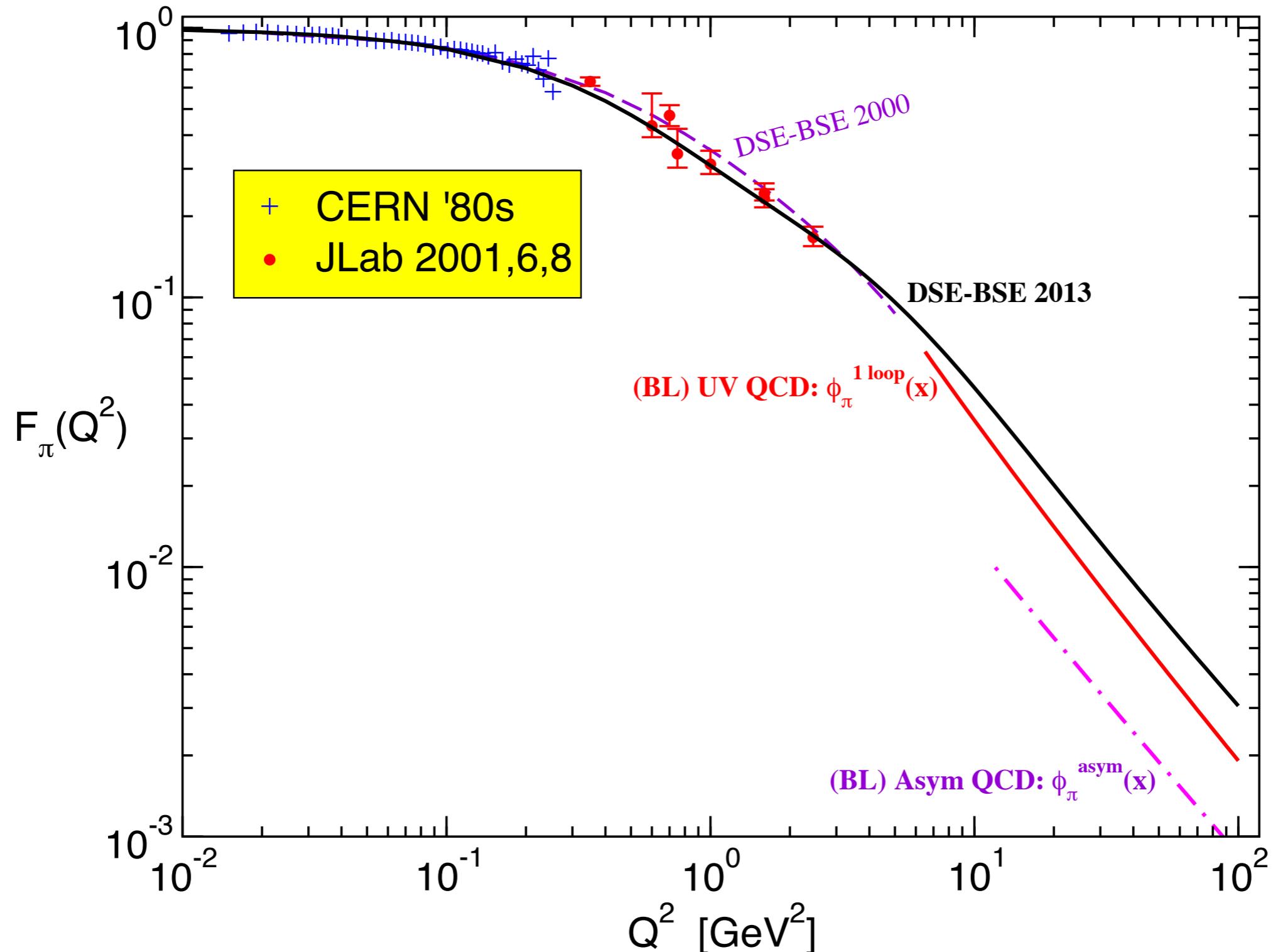
- Excited meson & baryons states, especially exotics & hybrids
- PDFs and GPDS for nucleons and pions
- Continue to enhance understanding of EM form factors of baryons
- Focus on observables where LQCD has difficulty, FFS, GPDs, chem potl > 0
- Parton DAs for nucleons
- Will LQCD be able to obtain the x-dependence of PDFs, GPDs, rather than 2-3 moments?
- Direct solution of BSE and Faddeev eqn for excited mesons and baryons? J/
Psi tower of states? It looks possible to directly solve the meson BSE to
obtain the essential features of the Nakanishi “spectral function” .

Previous DSE Limited Result 2000

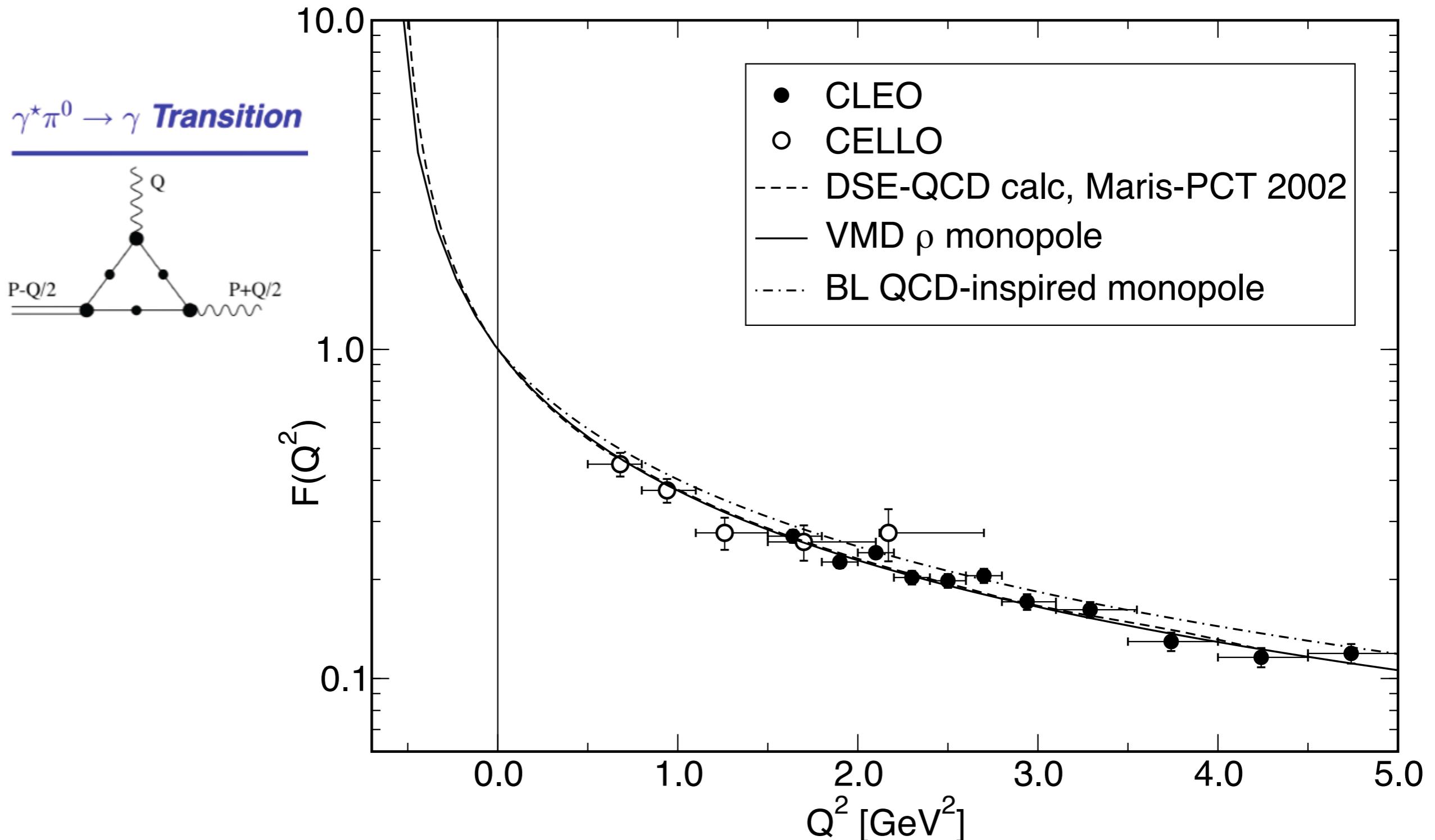
P. Maris and P.C. Tandy, PRC62, 055204, (2000)



Pion Form Factor: Broad Picture



Pion Transition Form Factor



Hadron Physics & QCD



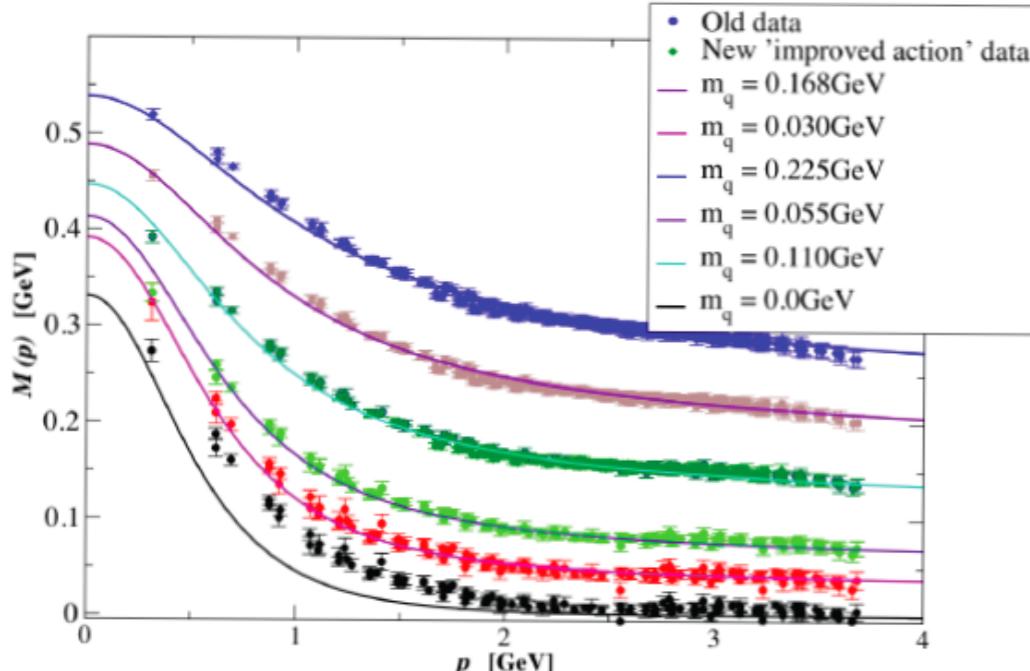
Prog. Part. Nucl. Phys., Vol. 33, pp. 477–575, 1994
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 0146-6410/94 \$26.00

Analysis of a quenched lattice QCD dressed quark propagator

M.S. Bhagwat, M.A. Pichowsky (Kent State U.),

C.D. Roberts (Argonne, PHY), P.C. Tandy (Kent State U.). Apr 2003. 9 pp.

Published in *Phys. Rev. C* 68 (2003) 015203 e-Print: [nucl-th/0304003](https://arxiv.org/abs/hep-th/0304003) | PDF



[13] P.O. Bowman, U.M. Heller, and A.G. Williams, *Phys. Rev. D* 66, 014505 (2002).

[14] P.O. Bowman, U.M. Heller, D.B. Leinweber, and A.G. Williams, [hep-lat/0209129](https://arxiv.org/abs/hep-lat/0209129).

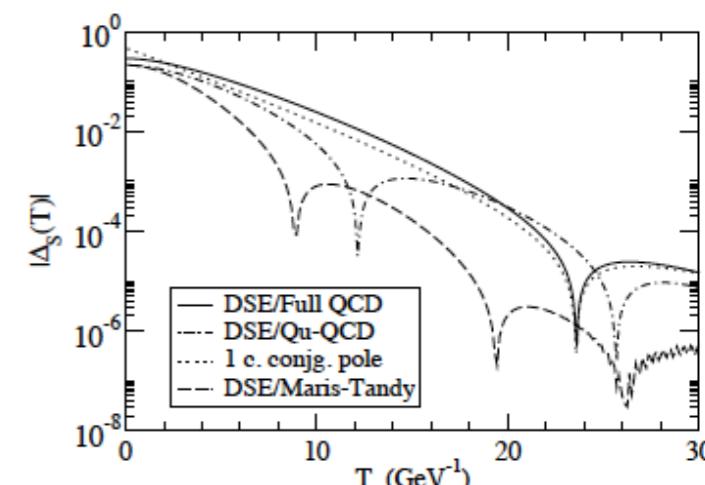
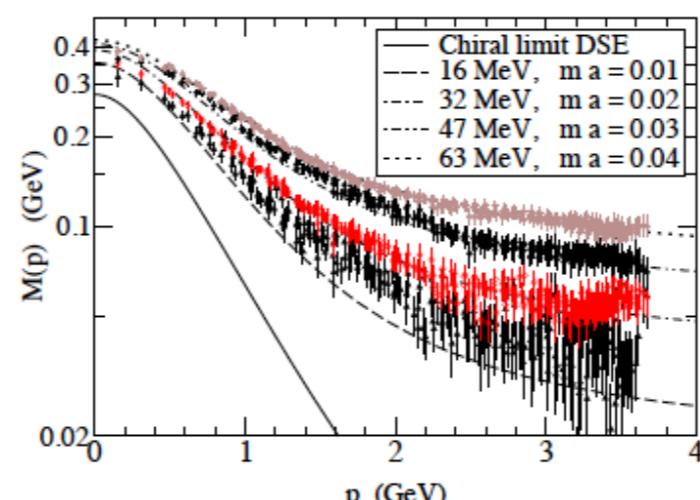
Dyson–Schwinger Equations and their Application to Hadronic Physics

CRAIG D. ROBERTS* and ANTHONY G. WILLIAMS†,‡

**Physics Division, Argonne National Laboratory, Argonne, IL 60439-4843, U.S.A.*

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‡*Department of Physics and the Supercomputer Computations Research Institute, Florida State University, Tallahassee, FL 32306, U.S.A.*



Analysis of full-QCD and quenched-QCD lattice propagators

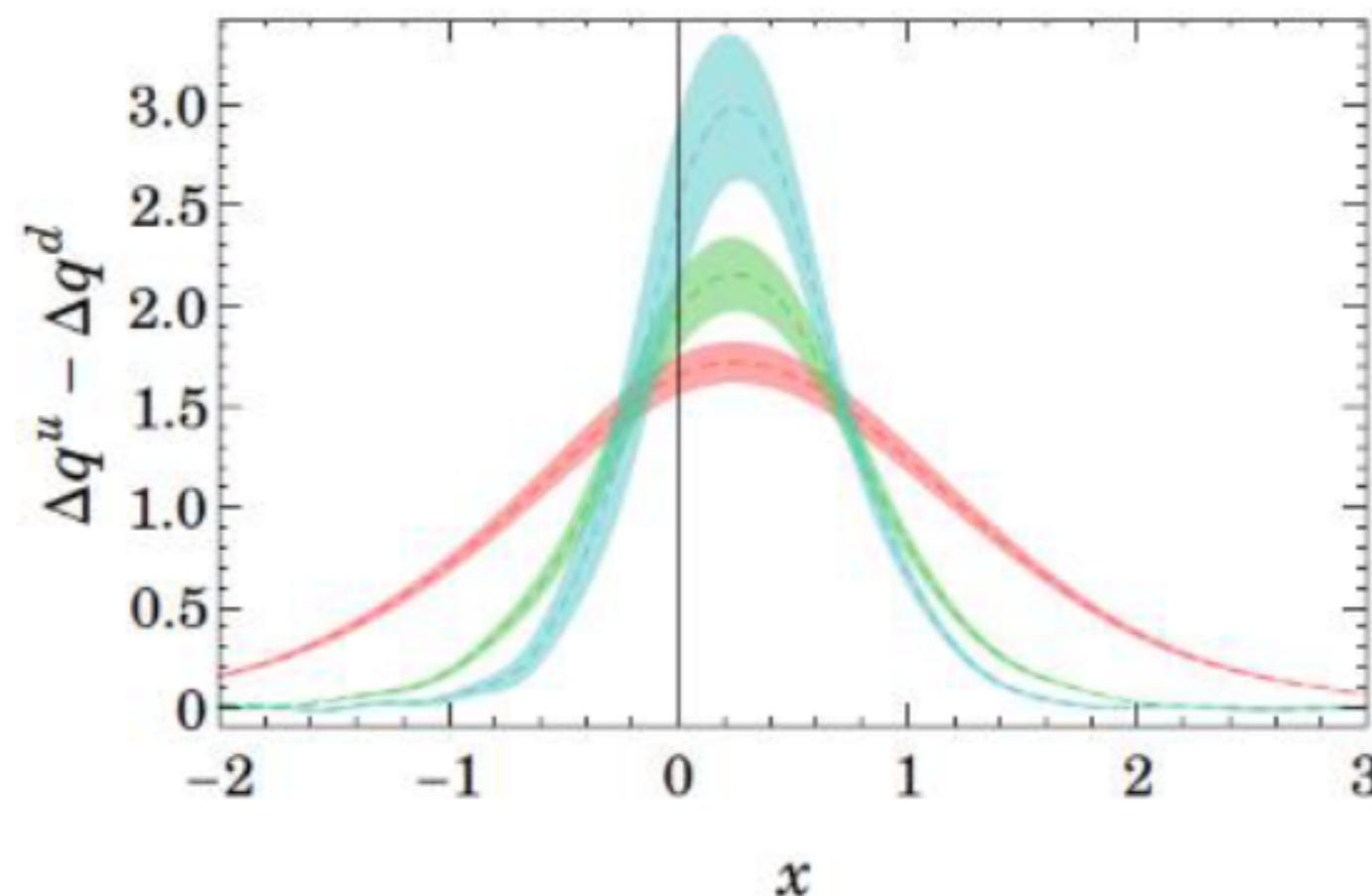
M.S. Bhagwat, P.C. Tandy (Kent State U.). Jan 2006. 3 pp.

AIP Conf. Proc. 842 (2006) 225–227 (PANIC05) e-Print: [nucl-th/0601020](https://arxiv.org/abs/hep-th/0601020) | PDF

Helicity Distribution

§ Exploratory study

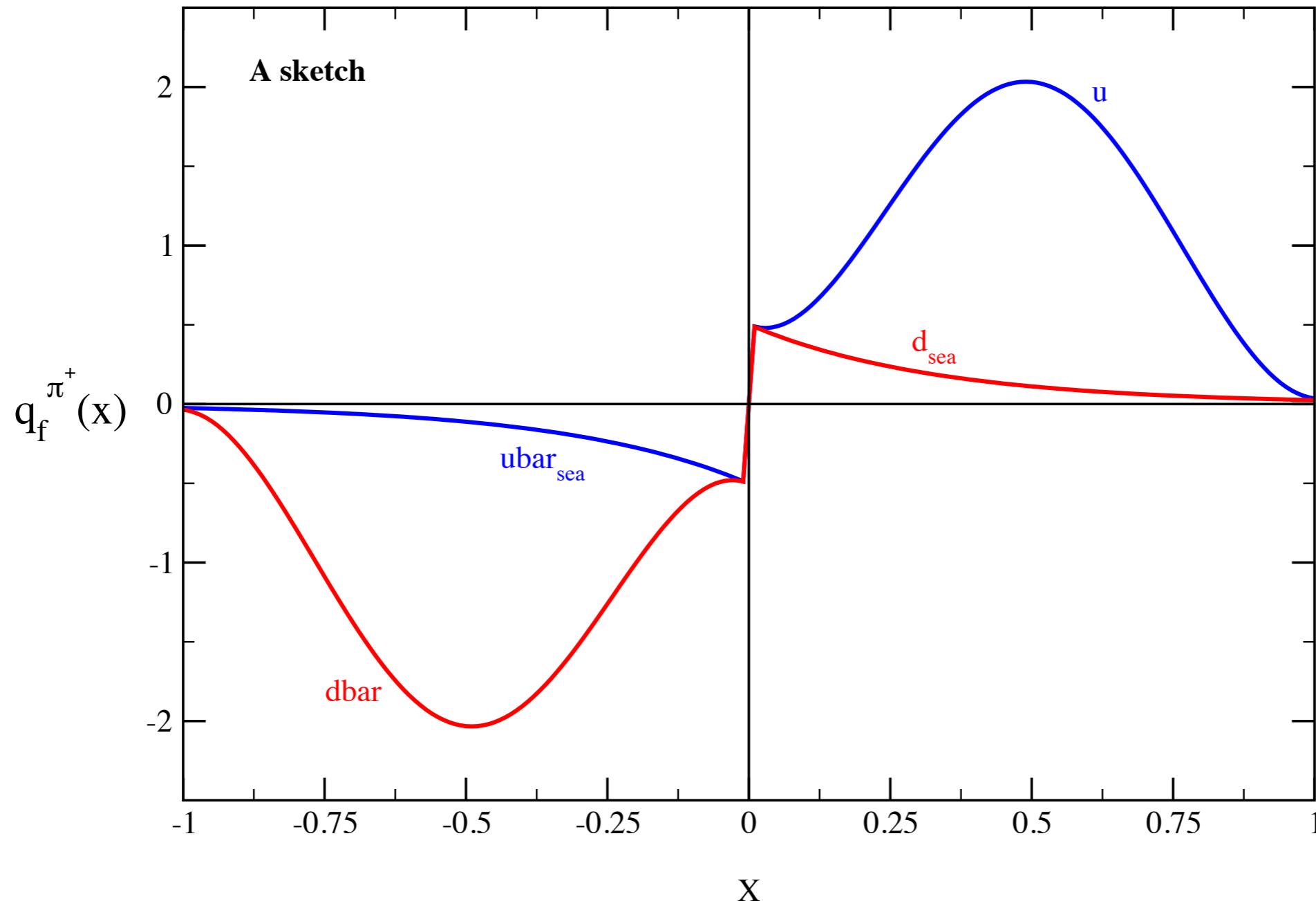
$$\int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \right| \bar{\psi}(z) \gamma_z \gamma_5 \exp\left(-ig \int_0^z dz' A_z(z')\right) \psi(0) \left| P \right\rangle$$



Uncorrected bare
lattice results



Typical Hadron PDF $q(x)$: a sketch for pion



Gen axial vertex $\Gamma_{5\mu}(\mathbf{k}; \mathbf{P}) = \gamma_5 \left\{ \gamma_\mu \mathbf{F}_R(\mathbf{k}; \mathbf{P}) + \not{k} k_\mu \mathbf{G}_R(\mathbf{k}; \mathbf{P}) - \sigma_{\mu\nu} k_\nu \mathbf{H}_R(\mathbf{k}; \mathbf{P}) \right\}$

$$+ \tilde{\Gamma}_{5\mu}(\mathbf{k}; \mathbf{P}) + \Gamma_\pi(\mathbf{k}; \mathbf{P}) \frac{2f_\pi P_\mu}{\mathbf{P}^2 + \mathbf{m}_\pi^2}$$

Gen pion BS ampl $\Gamma_\pi(\mathbf{k}; \mathbf{P}) = \gamma_5 \left\{ i\mathbf{E}_\pi(\mathbf{k}; \mathbf{P}) + \not{P} \mathbf{F}_\pi(\mathbf{k}; \mathbf{P}) + \not{k} \mathbf{k} \cdot \mathbf{P} \mathbf{G}_\pi(\mathbf{k}; \mathbf{P}) + \sigma : \mathbf{k} \mathbf{P} \mathbf{H}_\pi(\mathbf{k}; \mathbf{P}) \right\}$

chiral+soft pi limits of AV-WTI give: DCSB

$$\mathbf{S}(\mathbf{k}) = \frac{1}{i \not{k} \mathbf{A}(\mathbf{k}^2) + \mathbf{B}(\mathbf{k}^2)}$$

$$f_\pi \mathbf{E}_\pi(\mathbf{k}; \mathbf{P} = \mathbf{0}) = \mathbf{B}(\mathbf{k}^2)$$

$$\mathbf{F}_R(\mathbf{k}; \mathbf{0}) + 2f_\pi \mathbf{F}_\pi(\mathbf{k}; \mathbf{0}) = \mathbf{A}(\mathbf{k}^2) \Rightarrow g_A^q(F) \approx 0.81 \text{ (RL)} + 6\% \text{ (BRL)}$$

g_A^N Roberts, Chang, Schmidt, 2012

$$\mathbf{G}_R(\mathbf{k}; \mathbf{0}) + 2f_\pi \mathbf{G}_\pi(\mathbf{k}; \mathbf{0}) = 2\mathbf{A}'(\mathbf{k}^2)$$

$$\mathbf{H}_R(\mathbf{k}; \mathbf{0}) + 2f_\pi \mathbf{H}_\pi(\mathbf{k}; \mathbf{0}) = \mathbf{0}$$

Quark Level “Goldberger-Treiman” Relations

Maris, Roberts, PCT, Phys. Lett. B420, 267(1998) — an exact result in QCD

Gen axial vertex $\Gamma_{5\mu}(\mathbf{k}; \mathbf{P}) = \gamma_5 \left\{ \gamma_\mu \mathbf{F}_R(\mathbf{k}; \mathbf{P}) + \not{k} k_\mu \mathbf{G}_R(\mathbf{k}; \mathbf{P}) - \sigma_{\mu\nu} k_\nu \mathbf{H}_R(\mathbf{k}; \mathbf{P}) \right\}$

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g_A^N Roberts, Chang, Schmidt, 2012

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$$+ \tilde{\Gamma}_{5\mu}(\mathbf{k}; \mathbf{P}) + \Gamma_\pi(\mathbf{k}; \mathbf{P}) \frac{2f_\pi \mathbf{P}_\mu}{\mathbf{P}^2 + m_\pi^2}$$

Gen pion BS ampl $\Gamma_\pi(\mathbf{k}; \mathbf{P}) = \gamma_5 \left\{ i\mathbf{E}_\pi(\mathbf{k}; \mathbf{P}) + \not{P} \mathbf{F}_\pi(\mathbf{k}; \mathbf{P}) + \not{k} \mathbf{k} \cdot \mathbf{P} \mathbf{G}_\pi(\mathbf{k}; \mathbf{P}) + \sigma : \mathbf{k} \mathbf{P} \mathbf{H}_\pi(\mathbf{k}; \mathbf{P}) \right\}$

chiral+soft pi limits of AV-WTI give: DCSB

$$\mathbf{S}(\mathbf{k}) = \frac{1}{i \not{k} \mathbf{A}(\mathbf{k}^2) + \mathbf{B}(\mathbf{k}^2)}$$

- $f_\pi \mathbf{E}_\pi(\mathbf{k}; \mathbf{P} = \mathbf{0}) = \mathbf{B}(\mathbf{k}^2)$

$$\mathbf{F}_R(\mathbf{k}; \mathbf{0}) + 2f_\pi \mathbf{F}_\pi(\mathbf{k}; \mathbf{0}) = \mathbf{A}(\mathbf{k}^2) \Rightarrow g_A^q(F) \approx 0.81 \text{ (RL)} + 6\% \text{ (BRL)}$$

g_A^N Roberts, Chang, Schmidt, 2012

$$\mathbf{G}_R(\mathbf{k}; \mathbf{0}) + 2f_\pi \mathbf{G}_\pi(\mathbf{k}; \mathbf{0}) = 2\mathbf{A}'(\mathbf{k}^2)$$

$$\mathbf{H}_R(\mathbf{k}; \mathbf{0}) + 2f_\pi \mathbf{H}_\pi(\mathbf{k}; \mathbf{0}) = \mathbf{0}$$

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Maris, Roberts, PCT, Phys. Lett. B420, 267(1998) —— an exact result in QCD

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$$+ \tilde{\Gamma}_{5\mu}(k; P) + \Gamma_\pi(k; P) \frac{2f_\pi P_\mu}{P^2 + m_\pi^2}$$

Gen pion BS ampl $\Gamma_\pi(k; P) = \gamma_5 \left\{ iE_\pi(k; P) + \not{P} F_\pi(k; P) + \not{k} k \cdot P G_\pi(k; P) + \sigma : k P H_\pi(k; P) \right\}$

chiral+soft pi limits of AV-WTI give: DCSB

- $f_\pi E_\pi(k; P=0) = B(k^2)$

$$S(k) = \frac{1}{i \not{k} A(k^2) + B(k^2)}$$

$$F_R(k; 0) + 2f_\pi F_\pi(k; 0) = A(k^2) \Rightarrow g_A^q(F) \approx 0.81 \text{ (RL)} + 6\% \text{ (BRL)}$$

g_A^N Roberts, Chang, Schmidt, 2012

$$G_R(k; 0) + 2f_\pi G_\pi(k; 0) = 2A'(k^2)$$

$$H_R(k; 0) + 2f_\pi H_\pi(k; 0) = 0$$

Quark Level “Goldberger-Treiman” Relations

Maris, Roberts, PCT, Phys. Lett. B420, 267(1998) —— an exact result in QCD

Gen axial vertex $\Gamma_{5\mu}(k; P) = \gamma_5 \left\{ \gamma_\mu F_R(k; P) + \not{k} k_\mu G_R(k; P) - \sigma_{\mu\nu} k_\nu H_R(k; P) \right\}$

$$+ \tilde{\Gamma}_{5\mu}(k; P) + \Gamma_\pi(k; P) \frac{2f_\pi P_\mu}{P^2 + m_\pi^2}$$

Gen pion BS ampl $\Gamma_\pi(k; P) = \gamma_5 \left\{ iE_\pi(k; P) + \not{P} F_\pi(k; P) + \not{k} k \cdot P G_\pi(k; P) + \sigma : k P H_\pi(k; P) \right\}$

chiral+soft pi limits of AV-WTI give: DCSB

- $f_\pi E_\pi(k; P=0) = B(k^2)$

$$S(k) = \frac{1}{i \not{k} A(k^2) + B(k^2)}$$

- $F_R(k; 0) + 2f_\pi F_\pi(k; 0) = A(k^2)$

$$\Rightarrow g_A^q(F) \approx 0.81 \text{ (RL)} + 6\% \text{ (BRL)}$$

g_A^N Roberts, Chang, Schmidt, 2012

$$G_R(k; 0) + 2f_\pi G_\pi(k; 0) = 2A'(k^2)$$

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