Pion and Kaon Properties from Dyson-Schwinger Eons

Peter C. Tandy
Dept of Physics
Kent State University USA
Pion and Kaon Properties from Dyson-Schwinger Eons

Peter C. Tandy
Dept of Physics
Kent State University USA
• DSE continuum approach to QCD

• Old work on pion and kaon properties and decays

• Parton distribution amplitudes and PDFs—mainly mesons as an example. DSE-model calculations with direct connection to QCD. Comparison to LQCD.

• Some applications to uv physics (Form Factors, HS behavior)

• PDFs including X. Ji’s space-like correlator approximation for LQCD—a model investigation.
The Pion (1947)
The Pion (1947)

- Cecil Powell (U. Bristol)
- Giuseppe Occhialini (Italy)
- Cesar Lattes (student, Brazil)
- Counting grains in emulsions
The Pion (1947)

\[\text{\LARGE u} \leftrightarrow \text{\LARGE d}\]
The Pion (1947)
The Pion (1947)

- Nobel Prize 1950 to Cecil Powell
- Now the simplest, lightest hadron—a test system for calibrating our npQCD understanding & tools
- Similar to the 1960-80s role of deuteron for nuclear structure/reaction tools...
Most common: Rainbow-ladder truncation of QCD’s eqns of motion. Approximation to full BSE kernel now starting to produce results……

Constrain modeling by preserving AV-Ward-Takahashi Id, V-WTI. [Color singlet] Naturally implements DCSB, conserved vector current, Goldstone Thm, PCAC…

RL truncation only good for ground state vector & pseudoscalar mesons, q-q\bar{q} descriptions of baryons with AV and S diquarks.

At the very least: DSE continuum QCD modeling suited for surveying the landscape quickly from large to small scales; finding out which underlying mechanisms are dominant. Applicable to all scales, high Q^2 form factors, etc. Do not expect ab initio final-precision QCD results, except in special cases. [pion, kaon.. ]

Unifying DSE treatment of light front quantities (PDFs, GPDs, DA) with other aspects of hadron structure: masses, decays, charge form factors, transition form factors…..

Pion & kaon q-q\bar{q} Bethe-Salpeter wavefn is very well known

\[
\text{AV} - \text{WTI} : m_q \to 0, P \to 0 \Rightarrow \Gamma_{\pi q \bar{q}}(k^2) = i\gamma_5 \frac{1}{4} \text{tr} \left[ S_0^{-1} - f_0 \right] + O(P)
\]

Ladder-Rainbow Model

Landau gauge only

\[ K_{\text{BSE}} \rightarrow -\gamma_\mu \frac{\alpha^a}{2} 4\pi \alpha_{\text{eff}}(q^2) \left( D^\text{free}_{\mu\nu}(q) \gamma^\nu \frac{\alpha^c}{2} \right) \]

1 true phen parameter

- \( K_{\text{BSE}} \rightarrow -\gamma_\mu \frac{\alpha^a}{2} 4\pi \alpha_{\text{eff}}(q^2) D^\text{free}_{\mu\nu}(q) \gamma^\nu \frac{\alpha^c}{2} \)
- \( \alpha_{\text{eff}}(q^2) \rightarrow \langle \bar{q}q \rangle_{\mu=1} \text{ GeV} = -(240 \text{ MeV})^3 \), incl vertex dressing
- \( \alpha_{\text{eff}}(q^2) \rightarrow \text{IR} \)
- \( \alpha_{\text{eff}}(q^2) \rightarrow \text{UV} \) \( \alpha_s^{1-\text{loop}}(q^2) \)

modern \( \pi, K \) qDSE-BSE strategy: Maris & Roberts, PRC56, 3369 (1997)

P. Maris & P.C. Tandy, PRC60, 055214 (1999)

\( M_\rho, M_\phi, M_{K^*} \) good to 5\%, \( f_\rho, f_\phi, f_{K^*} \) good to 10\%

[fit : \( m_\pi, m_K, f_\pi \), \( f_K(2\%) \)]

### Summary of light meson results

\( m_{u=d} = 5.5 \text{ MeV}, m_s = 125 \text{ MeV at } \mu = 1 \text{ GeV} \)

<table>
<thead>
<tr>
<th>Pseudoscalar (PM, Roberts, PRC56, 3369)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(-\langle qq\rangle^0_{\mu})</td>
<td>(0.236 GeV)(^3)</td>
<td>(0.241(^\dagger))(^3)</td>
</tr>
<tr>
<td>(m_\pi)</td>
<td>0.1385 GeV</td>
<td>0.138(^\dagger)</td>
</tr>
<tr>
<td>(f_\pi)</td>
<td>0.0924 GeV</td>
<td>0.093(^\dagger)</td>
</tr>
<tr>
<td>(m_K)</td>
<td>0.496 GeV</td>
<td>0.497(^\dagger)</td>
</tr>
<tr>
<td>(f_K)</td>
<td>0.113 GeV</td>
<td>0.109</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Charge radii (PM, Tandy, PRC62, 055204)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(r^2_\pi)</td>
<td>0.44 fm(^2)</td>
<td>0.45</td>
</tr>
<tr>
<td>(r^2_{K^+})</td>
<td>0.34 fm(^2)</td>
<td>0.38</td>
</tr>
<tr>
<td>(r^2_{K^0})</td>
<td>-0.054 fm(^2)</td>
<td>-0.086</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(\gamma\pi\gamma) transition (PM, Tandy, PRC65, 045211)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(g_{\pi\gamma})</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>(r^2_{\pi\gamma})</td>
<td>0.42 fm(^2)</td>
<td>0.41</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Weak (K_{l3}) decay (PM, Ji, PRD64, 014032)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(\lambda_+(e3))</td>
<td>0.028</td>
<td>0.027</td>
</tr>
<tr>
<td>(\Gamma(K_{e3}))</td>
<td>7.6 (\cdot 10^6) s(^{-1})</td>
<td>7.38</td>
</tr>
<tr>
<td>(\Gamma(K_{\mu3}))</td>
<td>5.2 (\cdot 10^6) s(^{-1})</td>
<td>4.90</td>
</tr>
</tbody>
</table>

### Vector mesons (PM, Tandy, PRC60, 055214)

<table>
<thead>
<tr>
<th></th>
<th>0.770 GeV</th>
<th>0.742</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_\rho/\omega)</td>
<td>0.216 GeV</td>
<td>0.207</td>
</tr>
<tr>
<td>(m_{K^*})</td>
<td>0.892 GeV</td>
<td>0.936</td>
</tr>
<tr>
<td>(f_{K^*})</td>
<td>0.225 GeV</td>
<td>0.241</td>
</tr>
<tr>
<td>(m_\phi)</td>
<td>1.020 GeV</td>
<td>1.072</td>
</tr>
<tr>
<td>(f_\phi)</td>
<td>0.236 GeV</td>
<td>0.259</td>
</tr>
</tbody>
</table>

### Strong decay (Jarecke, PM, Tandy, PRC67, 035202)

<table>
<thead>
<tr>
<th></th>
<th>6.02</th>
<th>5.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\delta_{\rho\pi\pi})</td>
<td>4.64</td>
<td>4.3</td>
</tr>
<tr>
<td>(\delta_{\phi\pi\pi})</td>
<td>4.60</td>
<td>4.1</td>
</tr>
</tbody>
</table>

### Radiative decay (PM, nucl-th/0112022)

<table>
<thead>
<tr>
<th></th>
<th>0.74</th>
<th>0.69</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\delta_{\rho\pi\gamma}/m_\rho)</td>
<td>2.31</td>
<td>2.07</td>
</tr>
<tr>
<td>(\delta_{\omega\pi\gamma}/m_\omega)</td>
<td>0.83</td>
<td>0.99</td>
</tr>
<tr>
<td>((\delta_{K^*\gamma}/m_K)^0)</td>
<td>1.28</td>
<td>1.19</td>
</tr>
</tbody>
</table>

### Scattering length (PM, Cotanch, PRD66, 116010)

<table>
<thead>
<tr>
<th></th>
<th>0.220</th>
<th>0.170</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_0)</td>
<td>0.044</td>
<td>0.045</td>
</tr>
<tr>
<td>(a_1)</td>
<td>0.038</td>
<td>0.036</td>
</tr>
</tbody>
</table>
Bridging a gap between continuum-QCD and ab initio predictions of hadron observables

Table 1
Row 1 - Computed values determined from the interaction tension in Eq. (23), quoted in GeV; and Row 2 - the difference: \( \varepsilon_\frac{L}{L_0} = \frac{\xi_L}{\xi_{L_0}} - 1 \). So as to represent the domain of constant ground-state physics, described in connection with Eq. (5), we list values obtained with bottom-up interactions using \( \omega = 0.5, 0.6 \) GeV.

<table>
<thead>
<tr>
<th>( \xi_L )</th>
<th>( \frac{L}{L_0} )</th>
<th>( \frac{L_0}{L} )</th>
<th>( \frac{L_0}{L} )</th>
<th>( \frac{L}{L_0} )</th>
<th>( \frac{L}{L_0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.86</td>
<td>1.91</td>
<td>1.82</td>
<td>3.14</td>
<td>2.90</td>
<td></td>
</tr>
</tbody>
</table>
Modern Context for DSE Interaction Kernel


Bridging a gap between continuum-QCD and ab initio predictions of hadron observables

<table>
<thead>
<tr>
<th>$\xi_Z$</th>
<th>$\xi_\zeta$</th>
<th>$\tau_{DB}^{\omega=0.5}$</th>
<th>$\tau_{DB}^{\omega=0.5}$</th>
<th>$\tau_{RL}^{\omega=0.5}$</th>
<th>$\tau_{RL}^{\omega=0.5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.85</td>
<td>1.85</td>
<td>1.82</td>
<td>3.14</td>
<td>2.90</td>
<td></td>
</tr>
<tr>
<td>0.0</td>
<td>2.8%</td>
<td>-2.4%</td>
<td>68.5%</td>
<td>55.8%</td>
<td></td>
</tr>
</tbody>
</table>

\[ m_G(k^2) \quad m_G(0) \sim 0.38 \text{ GeV} \]

Bridging a gap between continuum-QCD and ab initio predictions of hadron observables


Exact Mass Relation for Flavor Non-Singlet PS Mesons

PCAC ⇒ \langle \bar{q}(x)q(y) (\partial_\mu J_{5\mu} = 2m_q J_5) \rangle ⇒ AV - WTI :

\[ -iP_\mu \Gamma_{5\mu}(k; P) = S^{-1}(k+P/2) \gamma_5 \frac{\tau}{2} + \gamma_5 \frac{\tau}{2} S^{-1}(k-P/2) - 2m_q \Gamma_5(k; P) \]

\[ \Gamma_\pi(k; P) \frac{f_\pi P_\mu}{P^2 + m_\pi^2} \]

\[ \Gamma_\pi(k; P) \frac{i \rho_\pi}{P^2 + m_\pi^2} \]

- \[ m_q = 0 : \quad S_0^{-1}(k) = i \kappa A_0(k^2) + B_0(k^2) \]

- \[ m_q = 0, P = 0 ⇒ GT_q : \quad \Gamma_\pi(k^2; 0) = i\gamma_5 \tau \frac{B_0(k^2)}{f_\pi^0} + \cdots \quad \text{ie, Goldstone Thm} \]

- \[ m_q \neq 0 : ⇒ f_\pi m_\pi^2 = 2m_q \rho_\pi(m_q) \quad \text{[for all } m_q, \text{ all ps mesons]} \]

- \[ \rho_{ps}(\mu) = -\langle 0 | \bar{q} \gamma_5 q | ps \rangle \quad \Rightarrow \frac{\Gamma_\pi}{\langle \bar{q}q \rangle f_\pi^0} + O(m_q) \quad \text{GMOR} \]

Pion $F(Q^2)$: Low $Q^2$

(P Maris & PCT, PRC 61, 045202 (2000))

(P. Maris & PCT, PRC 62, 0555204 (2000))

$r_{DSE}^\pi = 0.68$ fm \hspace{1cm} $r_{\pi}^{\text{expt}} = 0.663 \pm 0.006$ fm

\[ \Gamma_\pi = \sqrt{1 - \alpha^2} \Gamma_{q\bar{q}}^{RL} + \alpha \Gamma_{\pi q\bar{q}} \]

CPT: 18\% effect

\[ r_{ch}^2 = (1 - \alpha^2) r_{RL}^2 + \alpha^2 r_{\pi - lp}^2 \]

DSE-RL: $r_{RL}^2 = r_{ch}^2 \Rightarrow \alpha^2 = 18\%$
JLab data: G. Huber et al., PRC78, 045203 (2008)
Much More Work on Mesons and Baryons…

Results: beyond rainbow-ladder

Efficacy commensurate with other approaches

Light mesons: ground and excited states

- Sensitivity to interaction exasperated in light sector
- Deficiencies in many channels

[Kubrak, Fischer, RW EPJA 50 (2014) 126]
[Hitzer, Gomez-Rocha, Krassnigg arXiv 1508.07183]
Much More Work on Mesons and Baryons…

Light baryons: ground and excited states

Nucleon/Delta ground + excited states good

Expected deficiencies in diquarks/meson analogs

Flavour separated proton form factors

Primae facie, these experimental results are remarkable.
Parton Distribution Functions

Covariant formulation and calculation

\[ \int d^4q \ F(q^2, q \cdot P, q \cdot k, k^2) \]
Pion Valence PDF

Nguyen, Bashir, Roberts, PCT, PRC 83 062201 (2011); arXiv:1102.2448

Aicher, Schafer, Vogelsang, arXiv:1009.2481

soft gluon resummation

\[(1 - x)^{2+\gamma(Q^2)}\]

DSE, pQCD

prev PDF expt parm

\[(1 - x)^{1.5}\]

CQM, duality.. \((1 - x)^1\)

NJL (pt \(\pi\)) : \((1 - x)^0\)
Environmental Dependence of Valence $u(x)$


(valence is not isolated)
The Leading Order PDF

\[ q_f(x) = \frac{1}{4\pi} \int d\lambda \ e^{-ix P \cdot n\lambda} \left\langle \pi(P) | \bar{\psi}(\lambda n) \not\! \psi_f(0) | \pi(P) \right\rangle_c \]

RL DSE:

\[ \langle x^m \rangle^{RL}_v = \frac{-N_c}{2P \cdot n} \text{tr} \int_\ell \Gamma_\pi(\ell - \frac{P}{2}) \left[ \left( \frac{\ell \cdot n}{P \cdot n} \right)^m n \cdot \partial_\ell S(\ell) \right] \Gamma_\pi(\ell - \frac{P}{2}) S(\ell - P) \]

Method can easily exceed the Lattice – QCD practical limit: \( m = 3 \)
Fit numerical DSE-BSE solns to PTIRs (Nakanishi)

Use Nakanishi Repn (or PTIR) (1965)

\[ \mathcal{F}(q^2; q \cdot P) = \int_{-1}^{1} d\alpha \int_{0}^{\infty} d\Lambda \left\{ \frac{\rho_{\text{IR}}(\alpha; \Lambda)}{(q^2 + \alpha q \cdot P + \Lambda^2)^{m+n}} + \frac{\rho_{\text{UV}}(\alpha; \Lambda)}{(q^2 + \alpha q \cdot P + \Lambda^2)^n} \right\} \]

npQCD info is in the variables and constants that are not momenta
--- Wick rotation is trivial as in pert thy.

\[ \rho_{\text{IR}}(\alpha; \Lambda) \rightarrow \rho_1(\alpha) \delta(\Lambda - \Lambda_{\text{IR}1}) + \cdots 3 \]

Works for u-, d-, s-, c-, b-quarks.
Also for lattice-QCD propagators.


EG: \[ q_{A}(x) = i N_c \text{tr} \int \frac{dk^+ dk^- d^2k_{\perp}}{(2\pi)^4} \delta(k^+ - xP^+) \text{tr}[\Gamma_\pi S(i\gamma^+) S \Gamma_\pi S] \]
Pion Distribution Amplitude (leading twist)

\[
f_\pi \phi_\pi(x) = \int \frac{d\lambda}{2\pi} \ e^{-ixP.n\lambda} \langle 0 | \bar{q}(0) \gamma_5 \not{q} \ (\lambda n) \ | \pi(P) \rangle
\]

\[
f_\pi \langle x^m \rangle_\phi = \frac{Z_2 N_c}{P \cdot n} \ tr \int_k \left( \frac{k \cdot n}{P \cdot n} \right)^m \gamma_5 \not{n} \ [S(k) \Gamma_\pi (k - \frac{P}{2}; P) \ S(k - P)]
\]

\( \mu = 2 \text{ GeV} \)

Broadening of PDA is an expression of DCSB ---long sought after in LF QFT

DSE RL

DSE beyond RL

\( \phi^{\text{asym}}(x) = \phi_\pi(x; \mu \to \infty) \)
Pion Distribution Amplitude (leading twist)

\[ f_\pi \phi_\pi(x) = \int \frac{d\lambda}{2\pi} e^{-ixP \cdot n \lambda} \langle 0 | \bar{q}(0) \gamma_5 \not{n} q(\lambda n) | \pi(P) \rangle \]

\[ f_\pi \langle x^m \rangle_\phi = \frac{Z_2 N_c}{P \cdot n} \text{tr} \int \frac{\langle k \cdot n \rangle^m}{\langle P \cdot n \rangle} \gamma_5 \not{n} \left[ S(k) \Gamma_\pi (k - \frac{P}{2}; P) S(k - P) \right] \]

\[ \mu = 2 \text{ GeV} \]

Imaging Dynamical Chiral-Symmetry Breaking: Pion Wave Function on the Light Front

Broadening of PDA is an expression of DCSB
---long sought after in LF QFT
Pion Distribution Amplitude

ERBL (~1980):
\[ \phi_\pi(x; \mu) = 6x(1-x) \left\{ 1 + \sum_{n=2,4...} a_n(\mu) C_n^{3/2}(2x - 1) \right\} \]

\[ a_n(\mu) = a_n(\mu_0) \left[ \frac{\alpha_s(\mu_0)}{\alpha_s(\mu)} \right] \gamma_n^{(0)} / \beta_0 \]

Evolution to higher scales is EXTREMELY SLOW
Not much change up to LHC energy

Conformal limit: \( a_n(\mu \to \infty) = 0 \)

Efficient representation of DSE results:

\[ \phi_\pi(x; \mu) = N_\alpha x^\alpha (1-x)^\alpha \left\{ 1 + \sum_{n=2}^\infty \tilde{a}_n(\mu) C_n^{\alpha+1/2}(2x - 1) \right\} \]

\[ \phi_K(x; \mu) = N_\alpha x^\alpha (1-x)^\alpha \left\{ 1 + \sum_{n=2,4...} \tilde{a}_n(\mu) C_n^{\alpha+1/2}(2x - 1) \right\} \]

\[ + \ N_\beta x^\beta (1-x)^\beta \left\{ \sum_{n=1,3...} \tilde{a}_n(\mu) C_n^{\beta+1/2}(2x - 1) \right\} \]
Low Order Truncation of ERBL-Gegenbauer Expn of PDA

\[
\phi_\pi(x; \mu) = 6x(1 - x) \left\{ 1 + \sum_{n=2,4\ldots} a_n(\mu) C_{n}^{3/2}(2x - 1) \right\}
\]

DSE soln

\[
\{(0, 1.), (2, 0.233104), (4, 0.112135), (6, 0.0683202), (8, 0.0469145), (10, 0.0346469), (12, 0.0268732), (14, 0.0215933), (16, 0.0178199), (18, 0.0150159), (20, 0.0128672), (22, 0.0111788), (24, 0.00982438), (26, 0.00871886), (28, 0.00780296), (30, 0.00703438), (32, 0.0063823), (34, 0.00582279), (36, 0.00534272), (38, 0.00493277), (40, 0.00447911)\}\]

10%

\[\mu = 2 \text{ GeV}\]

\[\phi_\pi(x; \mu) = C_{n}^{3/2}(2x - 1) \text{ projection}\]

\[
\phi_\pi^\text{QCDSR}(x = 1/2; \mu = 2) = 1.2 \pm 0.3
\]

A double-humped PDA is almost ruled out by V. Braun, I. Filyanov, Z. Phys. C44, 157 (1989)
One Lattice-QCD Moment Almost Determines Pion DA

\[ \phi_{\pi}^{LQCD}(x; \mu = 2) = N x^\alpha (1 - x)^\alpha \]
\[ \alpha = 0.35 + 0.32 - 0.24 \]

\[ \langle (2x - 1)^2 \rangle_{\mu=2}^{LQCD} = 0.27 \pm 0.04 \]

V. Braun et al., PRD74, 074501 (2006)

DSE beyond RL

\[ \phi_{\pi}^{\text{asym}}(x) = \phi_{\pi}(x; \mu \to \infty) \]

\[ \mu = 2 \text{ GeV} \]

DSE RL

Lattice-QCD

Conf XII Thessaloniki Aug 2016
\[ \langle (2x - 1)^2 \rangle_{\mu=2 \text{ GeV}}^{\text{LQCD}} = 0.2361(41)(39) \]

V. Braun et al., arXiv:1503.03656 [hep-lat]

DSE prediction: 0.251
Kaon Distribution Amplitude

Size of SU(2) x SU(3) spin-flavor symmetry-breaking?

that, as strong interaction bound states whose decay is mediated only by the weak interaction, so that they have a relatively long lifetime, kaons have been instrumental in establishing the foundation and properties of the Standard Model; notably, the physics of CP violation. In this connection the nonleptonic decays of B mesons are crucial because, e.g., the transitions $B^\pm \rightarrow (\pi K)^\pm$ and $B^\pm \rightarrow \pi^\pm\pi^0$ provide access to the imaginary part of the CKM matrix element $V_{ub}$: $\gamma = \text{Arg}(V_{ub}^*)$ [4]. Factorisation theorems have been derived and are applicable to such decays [5]. However, the formulae involve a certain class of so-called “non-factorisable” corrections because the parton distribution amplitudes (PDAs) of strange mesons are not symmetric with respect to quark and antiquark momenta. Therefore, any derived estimate of $\gamma$ is only as accurate as the evaluation of both the difference between $K$ and $\pi$ PDAs and also their respective differences from the asymptotic distribution, $\varphi_{asy}(u) = 6u(1-u)$. Amplitudes of twist-two and -three are involved. With this motivation, we focus on the twist-two amplitudes herein.

Kaon Distribution Amplitude


\[ \mu = 2 \text{ GeV} \]

\[ \varphi_K \]

\[ u \]

DSE-DB

DSE-RL

skewness implies only 14% flavor symm breaking due to DCSB

\[ \left\{ \frac{m_s - m_u}{m_s + m_u} \right\} 2 \text{ GeV} \sim 66\% \]

Kaon DA Moments

Table 1

Moments \( u_\Delta = 2u - 1 \) of the \( K \)-meson PDA computed using Eqs. (11) and (12), compared with selected results obtained elsewhere: Refs. [40,41], lattice-QCD; Ref. [10], analysis of lattice-QCD results in Ref. [41]; Refs. [42–46], compilation of results from QCD sum rules; and Ref. [47], holographic soft-wall Ansatz for the kaon's light-front wave function. We also list values obtained with \( \varphi = \varphi_{\text{asy}} \), Eq. (14), and \( \varphi = \varphi_{\text{ms}} \), Eq. (16), because they represent lower and upper bounds, respectively, for concave distribution amplitudes.

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
\langle u_\Delta^m \rangle & m = 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
\text{RL} & 0.11 & 0.24 & 0.064 & 0.12 & 0.045 & 0.076 \\
\text{DB} & 0.040 & 0.23 & 0.021 & 0.11 & 0.013 & 0.063 \\
\text{Lattice-QCD:} & & & & & & \\
[40] & 0.027(2) & 0.26(2) & & & & \\
[41] & 0.036(2) & 0.26(2) & & & & \\
[10] & 0.036(2) & 0.26(2) & 0.020(2) & 0.13(2) & 0.014(2) & 0.085(15) \\
\text{QCD Sum Rules:} & & & & & & \\
[42–46] & 0.035(8) & & & & & \\
[47] & 0.04(2) & 0.24(1) & & & & \\
\varphi = \varphi_{\text{ms}} & 0.33 & 0.33 & 0.2 & 0.2 & 0.14 & 0.14 \\
\varphi = \varphi_{\text{asy}} & 0 & 0.2 & 0 & 0.086 & 0 & 0.048 \\
\hline
\end{array}
\]

Shi Chao, L. Chang, C.D. Roberts, P.C. Tandy, PLB738, 512 (2014)
\[ \mu = 2 \text{ GeV} \]

### Table 1

Moments \((u_{\Delta} = 2u - 1)\) of the \(K\)-meson PDA computed using Eqs. (11) and (12), compared with selected results obtained elsewhere: Refs. [40,41], lattice-QCD; Ref. [10], analysis of lattice-QCD results in Ref. [41]; Refs. [42–46], compilation of results from QCD sum rules; and Ref. [47], holographic soft-wall Ansatz for the kaon's light-front wave function. We also list values obtained with \(\varphi = \varphi^{\text{asy}}\), Eq. (14), and \(\varphi = \varphi_{\text{ms}}\), Eq. (16), because they represent lower and upper bounds, respectively, for concave distribution amplitudes.

<table>
<thead>
<tr>
<th>(u_{\Delta}^m)</th>
<th>(m = 1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RL</td>
<td>0.11</td>
<td>0.24</td>
<td>0.064</td>
<td>0.12</td>
<td>0.045</td>
<td>0.076</td>
</tr>
<tr>
<td>DB</td>
<td><strong>0.040</strong></td>
<td>0.23</td>
<td><strong>0.021</strong></td>
<td>0.11</td>
<td><strong>0.013</strong></td>
<td>0.063</td>
</tr>
<tr>
<td>[40]</td>
<td>0.027(2)</td>
<td>0.26(2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[41]</td>
<td>0.036(2)</td>
<td>0.26(2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[10]</td>
<td>0.036(2)</td>
<td>0.26(2)</td>
<td>0.20(2)</td>
<td>0.13(2)</td>
<td>0.014(2)</td>
<td>0.085(15)</td>
</tr>
<tr>
<td>[42–46]</td>
<td>0.035(8)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[47]</td>
<td>0.04(2)</td>
<td>0.24(1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(\varphi = \varphi_{\text{ms}}\) \[ \begin{array}{cccccc} 
0.33 & 0.33 & 0.2 & 0.2 & 0.14 & 0.14 \\
0 & 0.2 & 0 & 0.086 & 0 & 0.048 
\end{array} \]
$\mu = 2 \text{ GeV}$

1st Excited State Pion & Kaon DAs

Bo-Lin, L. Chang, F. Gao, C.D.Roberts, S.M.Schmidt, H-S., Zong,
Spacelike Correlator Approximation for PDFs
To help lattice-QCD be more applicable to hadron PDFs and GPDs than just the first 3 moments?

Parton Physics on a Euclidean Lattice

Xiangdong Ji\textsuperscript{1,2}

Standard light-cone correlator, leading twist: \( x = k \cdot n / P \cdot n = k^+ / P^+ \in [0, 1] \)

\[
q_f(x) = \frac{1}{4\pi} \int d\lambda \ e^{-ixP \cdot n} \lambda \langle\pi(P)| \bar{\psi}_f(\lambda n) \gamma \psi_f(0) |\pi(P)\rangle_c
\]

\( n^2 = 0 ; \quad z^- = \lambda n ; \quad z^+ = 0 = z_\perp \)

Ji: Take large \( Pz \) limit of frame-dependent equal-time correlator: \( x = kz / Pz \in [-\infty, +\infty] \)

\[
\tilde{q}_f(x; Pz) = \frac{1}{4\pi} \int dz \ e^{-ixPz} \ z \langle\pi(P)| \bar{\psi}_f(z) \gamma z \psi_f(0) |\pi(P)\rangle_c
\]

\( \rightarrow q_f(x) \text{ as } Pz \rightarrow \infty \quad \text{How fast?} \)
Quark Distribution

§ Back to the continuum

\[ q(x, \mu) = \bar{q}(x, \mu, P_z) + \mathcal{O}(M_N^2/P_z^2) + \mathcal{O}(\alpha_s) + \mathcal{O}(\Lambda_{QCD}^2/P_z^2) \]

Finite \( P_z \to \infty P_z' \) perturbative matching

\[ \bar{q}(x, \mu, P_z) = \int_{-1}^{1} \frac{dy}{|y|} Z\left(\frac{x, \mu}{y, P_z}\right) q(y, \mu) \]

\[ Z(x, \mu/P_z) = \delta(x - 1) + \frac{\alpha_s}{2\pi} Z^{(1)}(x, \mu/P_z) \]

Non-singlet case only

X. Xiong, X. Ji, J. Zhang,
1310.7471 [hep-ph]; Y. Zhao, this workshop

FIG. 1: One loop corrections to quasi quark distribution.
\[ \int \frac{dz}{4\pi} e^{-izk_z} \left( P \left\langle \psi(z) \gamma_z \exp\left( -ig \int_0^Z dz' A_z(z') \right) \psi(0) \right| P \right) \]

\[ P_z \in \{1, 2, 3\} \frac{2\pi}{L} \]

Distribution gets sharper as \( P_z \) increases
Artifacts due to finite \( P_z \) on the lattice

Improvement?
Work out leading-\( P_z \) corrections
Simple model for pion PDF & Quasi-PDF

\[ S(k) = \frac{1}{i k^+ + M}, \quad M = 0.4 \text{ GeV} \]

\[ \Gamma_\pi(q, P) = \gamma_5 N_\pi \int_{-1}^{1} d\alpha \frac{\rho(\alpha)}{q^2 + \alpha q \cdot P + \Lambda^2}, \quad \rho(\alpha) = \text{even} \]

Euclidean to Minkowski:

Evaluate \( q(x) \) directly using Cauchy Residue Thm for \( \int_{-\infty}^{\infty} dk^- \)

\[ q_A(x) = i N_c \text{ tr} \int \frac{dk^+ dk^- d^2k_\perp}{(2\pi)^4} \delta(k^+ - x P^+) \text{ tr}[\Gamma_\pi S (i \gamma^+) S \Gamma_\pi S] \]

Evaluate \( \tilde{q}(x; P_z) \) directly using Cauchy Residue Thm for \( \int_{-\infty}^{\infty} dk^0 \)

\[ \tilde{q}_A(x) = i N_c \text{ tr} \int \frac{dk^0 dk_z d^2k_\perp}{(2\pi)^4} \delta(k_z - x P_z) \text{ tr}[\Gamma_\pi S (i \gamma^z) S \Gamma_\pi S] \]
DIVERSION——A full DSE calculation of the true pion valence PDF

TABLE II: Momentum fraction sum rule from this work at scale $Q_0 = 0.630$ GeV corresponding to the ASV [13] compilation.

<table>
<thead>
<tr>
<th>$q_{val}$</th>
<th>$q_{val}^{DSE}$</th>
<th>$q_{sea}^{ASV}$</th>
<th>gluon</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(x)_\pi$</td>
<td>0.770</td>
<td>0.649</td>
<td>0.0498</td>
<td>0.300</td>
</tr>
</tbody>
</table>


Modern empirical expt parameterization:
Aicher, Shafer, Vogelsang, (ASV) PRL 105, 252003 (2010)

$Q_0 = 0.385$ GeV
Back to: Spacelike Correlator Approximation for PDFs

Model-exact PDF & Quasi-PDF @ Pz=10 GeV
Pz Dependence of quasi-pdf of valence model pion

valence π toy model, quasi-pdf moments

rel err 10^0 10^1 10^2 10^3
Pz 10^-3 10^-2 10^-1 10^0 10^1 10^2 10^3

Conf XII Thessaloniki Aug 2016
\[ \langle x^m \rangle \text{ for toy model pion at } P_z = 3 \text{ GeV} \]
Pz Dependence of quasi-pdf of u-ubar “pion”

\[ q_\pi(x; P_z) \]

- \( P_z = 0.5 \)
- \( P_z = 1 \)
- \( P_z = 2 \)
- \( P_z = 3 \)
- \( P_z = 5 \)
- True pdf (\( P_z = \infty \))

---I.Cloet, Lei Chang, PCT, in progress (2015)----
Applications:-

eg: Form Factors
The Pion Charge Form Factor: Transition from npQCD to pQCD

\[ F_\pi(Q^2 = uv) = \int_0^1 dx \int_0^1 dy \phi_\pi^*(x; Q) [T_H(x, y; Q^2)] \phi_\pi(y; Q) + \text{NLO/higher twist} \]


\[ Q^2 \gg \Lambda_{\text{QCD}}^2 : Q^2 F_\pi(Q^2) \to 16 \pi f_\pi^2 \alpha_s(Q^2) \omega_\phi^2(Q^2) + O(1/Q^2) \]

at \( Q^2 \sim 3 - 4 \text{ GeV}^2 \), \( \Rightarrow 0.1 \)

JLab expt, Theory \( \Rightarrow 0.45 \)

But, recent DSE theory \( \Rightarrow \phi_\pi(x; \mu = 2 \text{ GeV}) \Rightarrow \omega_\phi^2 = 3.3 \)

Pion Electromagnetic Form Factor at Spacelike Momenta

L. Chang, 1 I. C. Cloët, 2 C. D. Roberts, 2 S. M. Schmidt, 3 and P. C. Tandy 4

PRL 111, 141802 (2013)
UV-QCD is not Asymptotic QCD

\[ Q^2 \gg \Lambda_{QCD}^2 : \quad Q^2 F_\pi(Q^2) \rightarrow 16 \pi f_\pi^2 \alpha_s(Q^2) \omega_\phi^2(Q^2) + O(1/Q^2) \]
Pion Electromagnetic Form Factor at Spacelike Momenta

L. Chang, I. C. Cloët, C. D. Roberts, S. M. Schmidt, and P. C. Tandy

JLab data: G. Huber et al., PRC78, 045203 (2008)

Conf XII Thessaloniki Aug 2016
Kaon Elastic Form Factor

\[ Q^2 F_K(Q^2) \to 16 \pi \alpha_s(Q^2) f_K^2 \omega_K^2(Q^2) \]

\[ \omega_K^2 = e_u \omega_u^2 + e_s \omega_s^2 \to 1, Q^2 \to \infty \]

\[ \omega_u = \frac{1}{3} \int_0^1 dx \frac{\phi_K(x)}{x} \neq \omega_s = \frac{1}{3} \int_0^1 dx \frac{\phi_K(x)}{1-x} \]


\[ \Delta \phi_{\text{LAT}}^\text{LATT}(x) \]

DSE (2016) Gao, Chang, Liu, Roberts, near completion


\[ \phi_{\text{LAT}}^\text{LATT}(x) \text{ fit 2 moms} \]

\[ \phi_{\text{DSE}}^\text{DSE}(x), \langle (2x - 1)^2 \rangle \text{ inc 10\%} \]

\[ \phi_{\text{K}}^\text{asym}(x), \text{(conformal QCD)} \]
Hard Scattering Kaon Elastic Form Factor

\[ Q^2 F_K(Q^2) \rightarrow 16 \pi g_s(Q^2) f_K^2 \omega_K^2(Q^2) \]

\[ \omega_K^2 = e_u \omega_u^2 + e_s \omega_s^2 \rightarrow 1, Q^2 \rightarrow \infty \]

\[ \omega_u = \frac{1}{3} \int_0^1 dx \frac{\phi_K(x)}{x} \neq \omega_s = \frac{1}{3} \int_0^1 dx \frac{\phi_K(x)}{1-x} \]

Expt \((s_U = 17.4 \text{ GeV}^2 \text{ timelike})\):

\[ \frac{F_K(s_U)}{F_\pi(s_U)} = 0.92(5) \]

K. Seth et al., PRL110, 022002 (2013)

\[ \frac{f_K^2}{f_\pi^2} = 1.43 \]

\[ \frac{F_K(s_U)}{F_\pi(s_U)} = 1.16 (\phi^{\text{DSE}}); \quad 1.16 + 50\% (\phi^{\text{LQCD}}) \]

Pion Transition Form Factor

From unified treatment of DA, elastic FF, and transition FF

Summary

• DSE approach works extremely well for pion & kaon due to symmetry dominance.

• Parton Distribution Amplitudes (pion, kaon). DSE approach shows good contact with available lattice-QCD moments. Flavor symmetry breaking & dynamical chiral symmetry breaking evident and quantitative in the shapes.

• Pion Transition & Elastic Form Factors DSE TFF calculation for all $Q^2$—agrees with Belle not BaBar. DSE elFF—Connection with ultraviolet/hard scattering QCD reconciled. Identify that the ultraviolet partonic behavior is within reach of proposed JLab pion FF experiments.

• Parton Distribution Functions (pion). Qualitative behavior of empirical data fits reproduced by DSE q-qbar + pion loop analysis.

• Time to declare we understand the pion and kaon in QCD?

• X. Ji’s space-like correlator approach to PDFs—a model investigation. Spurious anti-quark contributions seem unavoidable if $P_z < 2$ GeV. For $x > 0.8$, need $P_z > 4$ GeV for confidence in the qualitative shape. Further work in progress.
The End
Collaborators

- Craig Roberts, Argonne National Lab, USA
- Adnan Bashir, University of Michoacan, Morelia, Mexico
- Ian Cloet, Argonne National Lab, USA
- Sixue Qin, Argonne National Lab, USA
- Hong-shi Zong, Nanjing Univ, China
- Lei Chang, Peking U, Argonne/Julich/Univ Adelaide, Australia
- Chao Shi, Nanjing Univ, [visiting Kent State U]
- Konstantin Khitrin, PhD student, Kent State Univ, USA
- Javier Cobos-Martinez, Univ of Sonora, Mexico


**Lattice-QCD and DSE-based modeling**

- **Lattice:**
  \[ \langle O \rangle = \int D\bar{q}qG \ O(\bar{q}, q, G) \ e^{-S[\bar{q}, q, G]} \]
  - Euclidean metric, x-space, Monte-Carlo
  - Issues: lattice spacing and vol, sea and valence \( m_q \), fermion Det
  - **Large time limit \( \Rightarrow \) nearest hadronic mass pole**

- **EOMs (DSEs):**
  \[ 0 = \int D\bar{q}qG \ \delta \frac{S[\bar{q}, q, G]}{\delta q(x)} e^{-S[\bar{q}, q, G]+(\bar{\eta}, q)+(\bar{q}, \eta)+(J, G)} \]
  - Euclidean metric, p-space, continuum integral eqns
  - Issues: truncation and phenomenology—not full QCD
  - **Analytic contin. \( \Rightarrow \) nearest hadronic mass pole**
  - Can be quick to identify systematics, mechanisms, \( \cdots \)

---

Pion, Kaon...
Lattice-QCD and DSE-based modeling

- Lattice: \( \langle \mathcal{O} \rangle = \int D\bar{q}qG \mathcal{O}(\bar{q}, q, G) e^{-S[\bar{q}, q, G]} \)
  - Euclidean metric, x-space, Monte-Carlo
  - Issues: lattice spacing and vol, sea and valence \( m_q \), fermion Det
  - Large time limit \( \Rightarrow \) nearest hadronic mass pole

- EOMs (DSEs): \( 0 = \int D\bar{q}qG \frac{\delta}{\delta q(x)} e^{-S[\bar{q}, q, G] + (\bar{q}, q) + (\bar{q}, \eta) + (J, G)} \)
  - Euclidean metric, p-space, continuum integral eqns
  - Issues: truncation and phenomenology—not full QCD
  - Analytic contin. \( \Rightarrow \) nearest hadronic mass pole
  - Can be quick to identify systematics, mechanisms, \( \cdots \)

**Expect:** qualitatively new insight where other methods can’t, eg high \( Q^2 \)

**Do not expect:** final, precision-QCD results, except in special cases Pion, Kaon...
Where Asym FF Could be Calculated, its Power Law was Correct:

\[ \gamma^* \pi \gamma^* \text{ Asymptotic Limit} \]

Lepage and Brodsky, PRD22, 2157 (1980): LC-QCD/OPE ⇒

\[ F(Q^2, Q^2) \]

\[ Q^2 [\text{GeV}^2] \]
Estimate 1-Pion Loop Contribution to Pion PDF

\[ \pi^+ : \langle x^1 \rangle_{\mu} = \int_0^1 dx \, x \left\{ u + \bar{u}_{\text{sea}} + \bar{d} + d_{\text{sea}} + g(x) \right\} \approx 2 \langle x \, q_v(x) \rangle + 4 \langle x \, q_{\text{sea}}(x) \rangle + \langle x \, g(x) \rangle = 1 \]

\[ u = u_v + u_{\text{sea}}, \quad \bar{d} = \bar{d}_v + \bar{d}_{\text{sea}} \]

Empirical GRS/ASV ⇒ universal \( q_v(x), q_{\text{sea}}(x) \) at \( \mu = 0.630 \, \text{GeV} \)

\[ \Gamma_\pi = \sqrt{1 - \alpha^2} \, \Gamma_{q\bar{q}}^{\text{RL}} + \alpha \, \Gamma_{\pi q\bar{q}} \]

CPT: 18% effect

\[ r_{\text{ch}}^2 = (1 - \alpha^2) \, r_{\text{RL}}^2 + \alpha^2 \, r_{\pi - \text{lp}}^2 \]

DSE-RL: \( r_{\text{RL}}^2 = r_{\text{ch}}^2 \Rightarrow \alpha^2 = 18\% \)

PDF Consequence:

\[ q_v(x) = (1 - \alpha^2) \, q^{\text{RL}}(x) + q_v^{\pi - \text{lp}}(x) \]

with \( \langle q_v^{\pi - \text{lp}}(x) \rangle = \alpha^2 = 0.18 \)
Convolution Model for $q(x)$ from virtual $\pi$ loop

$$q^{\pi lp}_v(x) \sim \mathcal{P}_{q/T}(x) = \int_x^1 \frac{dy}{|y|} \mathcal{P}_{\pi/T}(y) \mathcal{P}_{q/\pi}(\frac{x}{y}) ,$$

$$T = \text{target} = \pi \text{ here}$$

$\mathcal{P}_{\pi/T}(y)$ should strongly favor $y \leq \frac{m_\pi}{2M_q + m_\pi} \approx 0.2$ ,

$\mathcal{P}_{q/\pi}(\frac{x}{y})$ is self – consistently determined

Result is strongly constrained
Analysis of Pion Parton Momentum Sum Rule

TABLE II: Momentum fraction sum rule from this work at scale $Q_0 = 0.630$ GeV corresponding to the ASV [13] compilation.

<table>
<thead>
<tr>
<th>$(x)_\pi$</th>
<th>$q_{val}^{RL}$</th>
<th>$q_{val}^{DSE}$</th>
<th>$q_{sea}^{ASV}$</th>
<th>$q_{gluon}$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.770</td>
<td>0.649</td>
<td>0.0498</td>
<td>0.300</td>
<td>0.999</td>
<td></td>
</tr>
</tbody>
</table>

Modern empirical expt parameterization:
Aicher, Shafer, Vogelsang, (ASV) PRL 105, 252003 (2010)


Conf XII Thessaloniki Aug 2016
Fig. 1. The valence and valence–like input distributions $xF^\pi(x, Q^2 = \mu^2)$ with $f = v, q, g$ as compared to those of GRV$\pi$ [5]. Notice that GRV$\pi$ employs a vanishing SU(3)$_{flavor}$ symmetric $\bar{q}^\pi$ input at $\mu_{LO}^2 = 0.25$ GeV$^2$ and $\mu_{NLO}^2 = 0.3$ GeV$^2$ [5]. Our present SU(3)$_{flavor}$ broken sea densities refer to a vanishing $s^\pi$ input in (3), as for GRV$\pi$ [5].

Digital Object Identifier (DOI) 10.1007/s100529900124

Pionic parton distributions revisited

M. Glück, E. Reya, I. Schienbein
Institut für Physik, Universität Dortmund, D-44221 Dortmund, Germany

Aicher, Schafer, Vogelsang,
arXiv:1009.2481
soft gluon resummation
$\Rightarrow \frac{\alpha_{\text{eff}}^{\text{RL}}(0)}{\pi} \approx 3$

$\Rightarrow \frac{\alpha_{\text{eff}}^{\text{DB}}(0)}{\pi} \approx 1$, [with dressed vertex effects]

BSE kernel from ab initio gauge sector DSE work now agrees satisfactorily with the kernel from fitting data: Binosi, Chang, Papavassiliou, Roberts, PLB742, 183 (2015)

\[ \Rightarrow \frac{\alpha_{\text{eff}}^{\text{RL}}(0)}{\pi} \approx 3 \]

\[ \Rightarrow \frac{\alpha_{\text{eff}}^{\text{DB}}(0)}{\pi} \approx 1, \text{ [with dressed vertex effects]} \]

BSE kernel from ab initio gauge sector DSE work now agrees satisfactorily with the kernel from fitting data: Binosi, Chang, Papavassiliou, Roberts, PLB 742, 183 (2015)
Modern Context for Rainbow-Ladder Kernel

\[ K_{\text{BSE}}^{\text{RL}} = \frac{4\pi \hat{\alpha}_{\text{eff}}(q^2)}{m_G^2(q^2) + q^2} \Rightarrow \frac{\alpha_{\text{eff}}^{\text{RL}}(0)}{\pi} \approx 3 \]

\Rightarrow \frac{\alpha_{\text{eff}}^{\text{DB}}(0)}{\pi} \approx 1 \quad \text{[with dressed vertex effects]} \]


Identified enough strength for physical DCSB

\[ \Rightarrow m_G(k^2) \quad m_G(0) \sim 0.38 \text{ GeV} \]

BSE kernel from ab initio gauge sector DSE work now agrees satisfactorily with the kernel from fitting data: Binosi, Chang, Papavassiliou, Roberts, PLB742, 183 (2015)
Other Meson Distribution Amplitudes

Table 1: Meson PDA moments obtained using numerical simulations of lattice-regularised QCD with $N_f = 2 + 1$ domain-wall fermions and nonperturbative renormalisation of lattice operators [29]: linear extrapolation to physical pion mass, MS-scheme at $\zeta = 2$ GeV, two lattice volumes. The first error is statistical, the second represents an estimate of systematic errors, including those from the $s$-quark mass, discretisation and renormalisation.

<table>
<thead>
<tr>
<th>meson</th>
<th>$\langle(x-\bar{x})^n\rangle$</th>
<th>$16^3 \times 32$</th>
<th>$24^3 \times 64$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>n=2</td>
<td>0.25(1)(2)</td>
<td>0.28(1)(2)</td>
</tr>
<tr>
<td>$\rho_\parallel$</td>
<td>n=2</td>
<td>0.25(2)(2)</td>
<td>0.27(1)(2)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>n=2</td>
<td>0.25(2)(2)</td>
<td>0.25(2)(1)</td>
</tr>
<tr>
<td>$K$</td>
<td>n=1</td>
<td>0.035(2)(2)</td>
<td>0.036(1)(2)</td>
</tr>
<tr>
<td>$K^*$</td>
<td>n=1</td>
<td>0.037(1)(2)</td>
<td>0.043(2)(3)</td>
</tr>
<tr>
<td>$K$</td>
<td>n=2</td>
<td>0.25(1)(2)</td>
<td>0.26(1)(2)</td>
</tr>
<tr>
<td>$K^*$</td>
<td>n=2</td>
<td>0.25(1)(2)</td>
<td>0.25(2)(2)</td>
</tr>
</tbody>
</table>


![Graph showing $\varphi(x)$ vs. $x$ with annotations for $K$ and $K^*$ pion]  

$\varphi(x) = x^\alpha (1-x)^\beta / B(\alpha, \beta)$. 

$16^3 \times 32$: $\alpha_{us} = 0.56_{-0.21}^{+0.31}$, $\beta_{us} = 0.45_{-0.19}^{+0.17}$. 

$24^3 \times 64$: $\alpha_{us} = 0.48_{-0.16}^{+0.19}$, $\beta_{us} = 0.38_{-0.15}^{+0.17}$.

DAs of light quark mesons look much the same--with small flavor breaking.
Pion Form Factor: Running q Mass Fn Effect

\[ Q^2 \cdot F_\pi(Q^2) \ [\text{GeV}^2] \]

With dynamical \( M_q(p^2) \)

With constituent \( M_q \)

60% reduction

JLab data: G. Huber et al., PRC78, 045203 (2008)
Mommy, Daddy, where does mass come from?

Dynamical Breaking of Chiral Symmetry
Transition from constituent to parton quark

\[ 4 \text{ GeV}^2 = Q^2 \leftarrow \frac{Q}{2} \Rightarrow Q^2 = 8 \text{ GeV}^2 \]
Many Moments via Feyn PTIR--Easy

• DCSB: A large u/d quark constituent mass is generated from almost nothing for the same reason & and by the same mechanism that makes the pion almost massless!

• DCSB causes the shape of the pion DA to be significantly broader than the asymptotic-QCD DA at accessible scales for hadron physics, and a new analysis technique shows that lattice-QCD moments say the same thing. [DCSB identified in a LF-defined quantity.]

• The scale running of distribution amplitudes is exceedingly SLOW---even at LHC scales asymptotic-QCD for DAs and form factors they influence there are persistent sizeable npQCD effects and DCSB in the hadron states.

• The elastic form factor of the pion makes a transition from non-perturbative/constituent quark behavior to partonic perturbative behavior for $Q^2$ at 6-8 GeV^2 and the relevant extension of the Brodsky-LePage uv-QCD leading formula is just 15% below the recent DSE calculation there.

• The new DSE approach is applicable to form factors for all spacelike $Q^2$.

• DSE-QCD can now be applied to light-front-defined bound state properties as a fn of momentum fraction x. Meson DAs and PDFs work out well, nucleon PDFs and GPDs await...
Deep Inelastic Lepton Scattering

- PDFs: $u_\pi(x)$, $u_K(x)$, $s_K(x)$
- Drell-Yan data exists
- Pion and Kaon/Pion Ratio
- Employ LR DSE model
- Bjorken limit fixes quark $k^+$
- Covariant formulation: $\int d^4q \ F(q^2, q \cdot P, q \cdot k, k^2)$
- Evolve from model scale via LO DGLAP
\[ F_\pi(Q^2) = (1 - \alpha^2)F_{\pi \pi}^{RL}(Q^2) + \alpha^2 F_{\pi \pi}^{lp}(Q^2) \]

\[ F_\pi(Q^2) = (1 - \alpha^2)(1 - \frac{Q^2 r_{RL}^2}{6} + \cdots) + \alpha^2 (1 - \frac{Q^2 r_{\pi - lp}^2}{6} + \cdots) \]

\[ F_\pi(Q^2) = (1 - \frac{Q^2 r_{TOT}^2}{6} + \cdots), \quad r_{ch}^2 = (1 - \alpha^2)r_{RL}^2 + [\alpha^2 r_{\pi - lp}^2] \]
The Future?

• Excited meson & baryons states, especially exotics & hybrids

• PDFs and GPDS for nucleons and pions

• Continue to enhance understanding of EM form factors of baryons

• Focus on observables where LQCD has difficulty, FFS, GPDs, chem potl > 0

• Parton DAs for nucleons

• Will LQCD be able to obtain the x-dependence of PDFs, GPDs, rather than 2-3 moments?

• Direct solution of BSE and Faddeev eqn for excited mesons and baryons? J/Psi tower of states? It looks possible to directly solve the meson BSE to obtain the essential features of the Nakanishi “spectral function”\[ \rho(\alpha; \Lambda) \].
Previous DSE Limited Result 2000

Pion Form Factor: Broad Picture

\[ F_\pi(Q^2) \]

CERN '80s
JLab 2001, 6, 8

DSE-BSE 2000
DSE-BSE 2013

(BL) UV QCD: \( \phi_\pi^{1\text{loop}}(x) \)

(BL) Asym QCD: \( \phi_\pi^{\text{asym}}(x) \)

Ubatuba May 2014
Pion Transition Form Factor

$\gamma^* \pi^0 \rightarrow \gamma$ Transition

$F(Q^2)$

- CLEO
- CELLO
- DSE-QCD calc, Maris-PCT 2002
- VMD $\rho$ monopole
- BL QCD-inspired monopole

Ubatuba May 2014
Analysis of a quenched lattice QCD dressed quark propagator


Analysis of full-QCD and quenched-QCD lattice propagators


Dyson–Schwinger Equations and their Application to Hadronic Physics

CRAIG D. ROBERTS* and ANTHONY G. WILLIAMS†

* Physics Division, Argonne National Laboratory, Argonne, IL 60439-4843, U.S.A.
† Department of Physics and Mathematical Physics, University of Adelaide, SA 5005, Australia

Analysis of a quenched lattice QCD dressed quark propagator


Helicity Distribution

§ Exploratory study

\[ \int \frac{dz}{4\pi} e^{-izk_z} \left| \begin{array}{c} P \\ \bar{\psi}(z) \gamma_z \gamma_5 \exp \left( -ig \int_0^z dz' A_z(z') \right) \psi(0) \end{array} \right| P \]

Uncorrected bare lattice results

Huey-Wen Lin — Light Cone 2014
Typical Hadron PDF $q(x)$: a sketch for pion.
Gen axial vertex \[ \Gamma_{5\mu}(k; P) = \gamma_5 \left\{ \gamma_\mu F_R(k; P) + \not{k} k_\mu G_R(k; P) - \sigma_{\mu\nu} k_\nu H_R(k; P) \right\} \]
\[ + \tilde{\Gamma}_{5\mu}(k; P) + \Gamma_\pi(k; P) \frac{2f_\pi P_\mu}{P^2 + m_\pi^2} \]

Gen pion BS ampl \[ \Gamma_\pi(k; P) = \gamma_5 \left\{ iE_\pi(k; P) + P F_\pi(k; P) + \not{k} k.P G_\pi(k; P) + \sigma : k P H_\pi(k; P) \right\} \]

chiral+soft pi limits of AV-WTI give: DCSB

\[ S(k) = \frac{1}{i \not{k} A(k^2) + B(k^2)} \]

\[ f_\pi E_\pi(k; P = 0) = B(k^2) \]
\[ F_R(k; 0) + 2f_\pi F_\pi(k; 0) = A(k^2) \]
\[ G_R(k; 0) + 2f_\pi G_\pi(k; 0) = 2A'(k^2) \]
\[ H_R(k; 0) + 2f_\pi H_\pi(k; 0) = 0 \]

\[ \Rightarrow g_A^q(F) \approx 0.81 \text{ (RL)} + 6\% \text{ (BRL)} \]
\[ g_A^N \]
Roberts, Chang, Schmidt, 2012
Quark Level “Goldberger-Treiman” Relations


Gen axial vertex

\[ \Gamma_{5\mu}(k; P) = \gamma_5 \left\{ \gamma_\mu F_R(k; P) + \not{k} k_\mu G_R(k; P) - \sigma_{\mu\nu} k_\nu H_R(k; P) \right\} \]

\[ + \tilde{\Gamma}_{5\mu}(k; P) + \Gamma_\pi(k; P) \frac{2f_\pi P_\mu}{P^2 + m_\pi^2} \]

Gen pion BS ampl

\[ \Gamma_\pi(k; P) = \gamma_5 \left\{ iE_\pi(k; P) + P F_\pi(k; P) + \not{k} k \cdot P G_\pi(k; P) + \sigma : k P H_\pi(k; P) \right\} \]

chiral+soft pi limits of AV-WTI give: DCSB

\[ S(k) = \frac{1}{i \not{k} A(k^2) + B(k^2)} \]

\[ f_\pi E_\pi(k; P = 0) = B(k^2) \]

\[ F_R(k; 0) + 2f_\pi F_\pi(k; 0) = A(k^2) \quad \Rightarrow g_A^q(F) \approx 0.81 \text{ (RL)} + 6\% \text{ (BRL)} \]

\[ G_R(k; 0) + 2f_\pi G_\pi(k; 0) = 2A'(k^2) \]

\[ H_R(k; 0) + 2f_\pi H_\pi(k; 0) = 0 \]
Quark Level “Goldberger-Treiman” Relations


Gen axial vertex
\[ \Gamma_{5\mu}(k; P) = \gamma_5 \left\{ \gamma_\mu F_R(k; P) + \kappa k_\mu G_R(k; P) - \sigma_{\mu\nu} k_\nu H_R(k; P) \right\} \]
\[ + \tilde{\Gamma}_{5\mu}(k; P) + \Gamma_\pi(k; P) \frac{2f_\pi P_\mu}{P^2 + m_\pi^2} \]

Gen pion BS ampl
\[ \Gamma_\pi(k; P) = \gamma_5 \left\{ iE_\pi(k; P) + P F_\pi(k; P) + \kappa k \cdot P G_\pi(k; P) + \sigma \cdot k P H_\pi(k; P) \right\} \]

Chiral+soft pi limits of AV-WTI give:

DCSB
\[ S(k) = \frac{1}{i \kappa A(k^2) + B(k^2)} \]

- \[ f_\pi E_\pi(k; P = 0) = B(k^2) \]

\[ F_R(k; 0) + 2f_\pi F_\pi(k; 0) = A(k^2) \]
\[ \Rightarrow g_A^q(F) \approx 0.81 \ (RL) + 6\% \ (BRL) \]
\[ g_A^N \quad \text{Roberts, Chang, Schmidt, 2012} \]

\[ G_R(k; 0) + 2f_\pi G_\pi(k; 0) = 2A'(k^2) \]

\[ H_R(k; 0) + 2f_\pi H_\pi(k; 0) = 0 \]
Quark Level "Goldberger-Treiman" Relations


Gen axial vertex \[ \Gamma_{5\mu}(k; P) = \gamma_5 \left\{ \gamma_\mu F_R(k; P) + i k^\mu G_R(k; P) - \sigma_{\mu\nu} k^\nu H_R(k; P) \right\} \]

Gen pion BS ampl \[ \Gamma_{\pi}(k; P) = \gamma_5 \left\{ i E_{\pi}(k; P) + P F_{\pi}(k; P) + i k^\mu P G_{\pi}(k; P) + \sigma : k P H_{\pi}(k; P) \right\} \]

chiral+soft pi limits of AV-WTI give: DCSB

\[ S(k) = \frac{1}{i k^\mu A(k^2) + B(k^2)} \]

- \[ f_\pi E_{\pi}(k; P = 0) = B(k^2) \]

- \[ F_R(k; 0) + 2f_\pi F_{\pi}(k; 0) = A(k^2) \quad \Rightarrow g_A^q(F) \approx 0.81 \text{ (RL)} + 6\% \text{(BRL)} \]

- \[ G_R(k; 0) + 2f_\pi G_{\pi}(k; 0) = 2A'(k^2) \]

- \[ H_R(k; 0) + 2f_\pi H_{\pi}(k; 0) = 0 \]

Roberts, Chang, Schmidt, 2012
Quark Level “Goldberger-Treiman” Relations


Gen axial vertex
\[ \Gamma_{5\mu}(k; P) = \gamma_5 \left\{ \gamma_\mu F_R(k; P) + \not{k} k_\mu G_R(k; P) - \sigma_{\mu\nu} k_\nu H_R(k; P) \right\} + \tilde{\Gamma}_{5\mu}(k; P) + \Gamma_\pi(k; P) \frac{2f_\pi P_\mu}{P^2 + m_\pi^2} \]

Gen pion BS ampl
\[ \Gamma_\pi(k; P) = \gamma_5 \left\{ iE_\pi(k; P) + P F_\pi(k; P) + \not{k} k \cdot P G_\pi(k; P) + \sigma : k P H_\pi(k; P) \right\} \]

chiral+soft pi limits of AV-WTI give: DCSB

\[ S(k) = \frac{1}{\frac{1}{i} \frac{k}{k} A(k^2) + B(k^2)} \]

- \[ f_\pi E_\pi(k; P = 0) = B(k^2) \]

- \[ F_R(k; 0) + 2f_\pi F_\pi(k; 0) = A(k^2) \]

\[ G_R(k; 0) + 2f_\pi G_\pi(k; 0) = 2A'(k^2) \]

\[ H_R(k; 0) + 2f_\pi H_\pi(k; 0) = 0 \]

\[ g_A^q(F) \approx 0.81 \text{ (RL)} + 6\% \text{ (BRL)} \]

\[ g_A^N \] Roberts, Chang, Schmidt, 2012
Quark Level “Goldberger-Treiman” Relations


Gen axial vertex
\[ \Gamma_{5\mu}(k; P) = \gamma_5 \left\{ \gamma_\mu \, F_R(k; P) + k_\mu \, G_R(k; P) - \sigma_{\mu\nu} k_\nu \, H_R(k; P) \right\} \]
\[ + \tilde{\Gamma}_{5\mu}(k; P) + \Gamma_\pi(k; P) \frac{2f_\pi \, P_\mu}{P^2 + m_\pi^2} \]

Gen pion BS ampl
\[ \Gamma_\pi(k; P) = \gamma_5 \left\{ iE_\pi(k; P) + P \, F_\pi(k; P) + k_\mu \, P \, G_\pi(k; P) + \sigma : k \, P \, H_\pi(k; P) \right\} \]

chiral+soft pi limits of AV-WTI give: DCSB

\[ S(k) = \frac{1}{i \, k \, A(k^2) + B(k^2)} \]

- \[ f_\pi \, E_\pi(k; P = 0) = B(k^2) \]
- \[ F_R(k; 0) + 2f_\pi \, F_\pi(k; 0) = A(k^2) \]
\[ \Rightarrow g_A^q(F) \approx 0.81 \, (RL) + 6\% \, (BRL) \]
\[ g_A^N \quad \text{Roberts, Chang, Schmidt, 2012} \]
- \[ G_R(k; 0) + 2f_\pi \, G_\pi(k; 0) = 2A'(k^2) \]
- \[ H_R(k; 0) + 2f_\pi \, H_\pi(k; 0) = 0 \]
Quark Level “Goldberger-Treiman” Relations


\[ \Gamma_{5\mu}(k; P) = \gamma_5 \left\{ \gamma_\mu F_R(k; P) + \not{k} k_\mu G_R(k; P) - \sigma_{\mu\nu} k_\nu H_R(k; P) \right\} \]

\[ + \bar{\Gamma}_{5\mu}(k; P) + \Gamma_\pi(k; P) \frac{2f_\pi P_\mu \not{P}}{P^2 + m_\pi^2} \]

Gen pion BS ampl: \[ \Gamma_\pi(k; P) = \gamma_5 \left\{ iE_\pi(k; P) + \not{P} F_\pi(k; P) + \not{k} k \cdot P G_\pi(k; P) + \sigma \cdot k P H_\pi(k; P) \right\} \]

chiral+soft pi limits of AV-WTI give: \[ S(k) = \frac{1}{i \not{k} A(k^2) + B(k^2)} \]

- \[ f_\pi E_\pi(k; P = 0) = B(k^2) \]
- \[ F_R(k; 0) + 2f_\pi F_\pi(k; 0) = A(k^2) \]
- \[ G_R(k; 0) + 2f_\pi G_\pi(k; 0) = 2A'(k^2) \]
- \[ H_R(k; 0) + 2f_\pi H_\pi(k; 0) = 0 \]

Dibaryon Chiral Symmetry Breaking (DCSB)

\[ g_A^q(F) \approx 0.81 \text{(RL)} + 6\% \text{(BRL)} \]

\[ g_A^N \text{ Roberts, Chang, Schmidt, 2012} \]