Constraints on the NLO coefficients in $S=-1$ sector.

XII Quark Confinement and the Hadron Spectrum.


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Co-authors: Volodymyr Magas & Àngels Ramos

Since Perturbative QCD is inappropriate to describe low energy hadron interactions, an effective theory with hadrons as degrees of freedom which respects the symmetries of QCD is needed, namely **Chiral Perturbation Theory**.

But, actually, we are in $S=-1$ sector, where $\bar{K}N$ interaction at low energy is dominated by the presence of the $\Lambda(1405)$ resonance. ChPT is not applicable in such a region, consequently, we have to go further.

A nonperturbative resummation is mandatory [Unitary extension of Chiral Perturbation Theory (UChPT)].

This scheme allows the generation of bound-states and resonances dynamically and at the same time respects the symmetries of QCD, particularly (spontaneously broken) chiral symmetry.

The pioneering work -- *Kaiser, Siegel, Weise*, NP A594 (1995) 325
**INTRODUCTION**

UChPT as nonperturbative scheme to obtain scattering amplitude.

**Bethe-Salpeter equation:**

\[
T_{ij} = V_{ij} + V_{il}G_{l}V_{lj} + V_{il}G_{l}V_{lk}G_{k}V_{kj} + \ldots
\]

\[
T_{ij} = V_{ij} + V_{ij}G_{l}T_{lj}
\]

\[
T_{ij}(E; k_{i}, k_{j}) = V_{ij}(k_{i}, k_{j}) + \sum_{k} \int d^{3}q_{k}V_{ik}(k_{i}, q_{k})\tilde{G}_{k}(E; q_{k})T_{kj}(E; q_{k}, k_{j})
\]

On shell factorization of \(T_{kj}\) and \(V_{ik}\)

\[
T_{ij}(E) = V_{ij} + \sum_{k} V_{ik}G_{k}(E)T_{kj}(E), \quad T = (1 - VG)^{-1}V
\]

where \(G_{k}(E) = \int d^{3}q_{k}\tilde{G}_{k}(E; q_{k})\)

Coupled-channel algebraic equations system

In \(S=-1\) sector, \(i, j\) and \(k\) indexes run over these 10 channels:

\[K^{-}p, \bar{K}^{0}n, \pi^{0}\Lambda, \pi^{0}\Sigma^{0}, \pi^{+}\Sigma^{-}, \pi^{-}\Sigma^{+}, \eta\Lambda, \eta\Sigma^{0}, K^{+}\Xi^{-}, K^{0}\Xi^{0}\]
INTRODUCTION
UChPT as nonperturbative scheme to obtain scattering amplitude.

Loop function:

$$G_k = i \int \frac{d^4q}{(2\pi)^4} \frac{M_k}{E_k(q)} \frac{1}{\sqrt{s - q^0 - E_k(q)} + i\epsilon} q^2 - m_k^2 + i\epsilon$$

Adopting the dimensional regularization:

$$G_k = \frac{M_k}{16\pi^2} \left\{ a_k(\mu) + \ln \frac{M_k^2}{\mu^2} + \frac{m_k^2 - M_k^2}{2s} \ln \frac{m_k^2}{M_k^2} - 2i\pi \frac{q_k}{\sqrt{s}} \right\} + \frac{q_k}{\sqrt{s}} \ln \left( \frac{s^2 - \left( (M_k^2 - m_k^2) + 2q_k\sqrt{s} \right)^2}{s^2 - \left( (M_k^2 - m_k^2) - 2q_k\sqrt{s} \right)^2} \right)$$

subtraction constants for the dimensional regularization scale $\mu = 1GeV$ in all the $k$ channels.

With isospin symmetry

$$a_{K^-p} = a_{K^0n} = a_{KN}$$
$$a_{\pi^0\Lambda} = a_{\pi\Lambda}$$
$$a_{\pi^0\Sigma^0} = a_{\pi^+\Sigma^-} = a_{\pi^-\Sigma^+} = a_{\pi\Sigma}$$
$$a_{\eta\Lambda}$$
$$a_{\eta\Sigma^0} = a_{\eta\Sigma}$$
$$a_{K^+\Sigma^-} = a_{K^0\Xi^0} = a_{K\Xi}$$

6 PARAMETERS!
FORMALISM
Effective lagrangian up to LO
Weinberg-Tomozawa Term, WT

\[ \mathcal{L}_{MB}^{(1)} (B, U) = \langle B i \gamma^\mu \nabla_\mu B \rangle - M_B \langle BB \rangle + \frac{1}{2} D \langle B \gamma^\mu \gamma_5 \{u_\mu, B\} \rangle + \frac{1}{2} F \langle B \gamma^\mu \gamma_5 [u_\mu, B] \rangle \]

\[ V_{ij}^{WT} = -C_{ij} \frac{1}{4f^2} N_i N_j \left( \sqrt{s} - M_i - M_j \right) \]

Special attention is paid to \( K^- p \rightarrow K \Xi \) reactions:

- There is no direct contribution from these reactions at lowest order
  \[ C_{K^- p \rightarrow K^0 \Xi^0} = C_{K^- p \rightarrow K^+ \Xi^-} = 0 \]
- The rescattering terms due to the coupled channels are the only contribution to the scattering amplitude.

Next terms in hierarchy could play a relevant role in these channels!!!

\[ \mathcal{L}_{eff} (B, U) = \mathcal{L}_{MB}^{(1)} (B, U) \]
**FORMALISM**

Effective lagrangian up to LO Born Terms

\[ \mathcal{L}^{(1)}_{MB}(B, U) = \langle \bar{B} i \gamma^\mu \nabla_\mu B \rangle - M_B \langle \bar{B} B \rangle + \frac{1}{2} D \langle \bar{B} \gamma^\mu \gamma_5 \{ u_\mu, B \} \rangle + \frac{1}{2} F \langle \bar{B} \gamma^\mu \gamma_5 [ u_\mu, B ] \rangle \]

i) \[ V^D_{ij} = - \sum_{k=1}^{8} \frac{C^{(\text{Born})}_{i,k} C^{(\text{Born})}_{j,k}}{12 f^2} \mathcal{N}_i \mathcal{N}_j \frac{(\sqrt{s} - M_i)(\sqrt{s} - M_k)(\sqrt{s} - M_j)}{s - M_k^2} \]

ii) \[ V^C_{ij} = \sum_{k=1}^{8} \frac{C^{(\text{Born})}_{j,k,i} C^{(\text{Born})}_{i,k,j}}{12 f^2} \mathcal{N}_i \mathcal{N}_j \]

\[ \times \left[ \sqrt{s} + M_k - \frac{(M_i + M_k)(M_j + M_k)}{2 (M_i + E_i)(M_j + E_j)} (\sqrt{s} - M_k + M_i + M_j) \right. \]

\[ + \frac{(M_i + M_k)(M_j + M_k)}{4 q_i q_j} \left\{ \sqrt{s} + M_k - M_i - M_j \right\} \]

\[ - \frac{s + M_k^2 - m_i^2 - m_j^2 - 2E_i E_j}{2 (M_i + E_i)(M_j + E_j)} (\sqrt{s} - M_k + M_i + M_j) \}

\[ \times \ln \frac{s + M_k^2 - m_i^2 - m_j^2 - 2E_i E_j - 2 q_i q_j}{s + M_k^2 - m_i^2 - m_j^2 - 2E_i E_j + 2 q_i q_j} \]

\[ \mathcal{L}_{eff}(B, U) = \mathcal{L}^{(1)}_{MB}(B, U) \]
FORMALISM
Effective lagrangian up to NLO

\[ \mathcal{L}_{\text{MB}}^{(2)}(B, U) = b_D \langle B \chi_+ , B \rangle + b_F \langle \bar{B} \chi_+ , B \rangle + b_0 \langle \bar{B} B \rangle \chi_+ + d_1 \langle \bar{B} [u_\mu , [u^\mu , B]] \rangle \\
+ d_2 \langle \bar{B} [u_\mu , [u^\mu , B]] \rangle + d_3 \langle \bar{B} u_\mu \rangle u^\mu B + d_4 \langle \bar{B} B \rangle u^\mu u_\mu \]

The low energy coefficients are not fixed so we have to consider them as parameters of the model!

\[ V_{ij}^{\text{NLO}} = \frac{1}{f^2} N_i N_j \left[ D_{ij} - 2 \left( \omega_i \omega_j + \frac{q_i q_j}{3 (M_i + E_i) (M_j + E_j)} \right) \right] \]

\[ L_{K^- p \rightarrow K^0 \bar{\pi}^0} \neq 0 \quad L_{K^- p \rightarrow K^+ \bar{\pi}^-} \neq 0 \]

\[ \mathcal{L}_{\text{eff}}(B, U) = \mathcal{L}_{\text{MB}}^{(1)}(B, U) + \mathcal{L}_{\text{MB}}^{(2)}(B, U) \]
Finally:

\[ V_{ij} = V_{ij}^{WT} + V_{ij}^D + V_{ij}^C + V_{ij}^{NLO} \]

\[ T = (1 - VG)^{-1}V \]

**Fitting parameters:**

- Decay constant \( f \)
- Axial vector couplings \( D, F \)
- 7 coefficients of the NLO lagrangian terms \( b_0, b_D, b_F, d_1, d_2, d_3, d_4 \)
- 6 subtracting constants \( a_{KN}, a_{\pi\Lambda}, a_{\pi\Sigma}, a_{\eta\Lambda}, a_{\eta\Sigma}, a_{K\Xi} \)

\[ L_{eff} (B, U) = L_{MB}^{(1)} (B, U) + L_{MB}^{(2)} (B, U) \]
Assumption: the contribution of the Born diagrams would be very moderate.

This idea was reinforced by other works:

\[ V_{ij} = V_{ij}^{WT} + V_{ij}^D + V_{ij}^C + V_{ij}^{NLO} \]

\[ T = (1 - V_G)^{-1}V \]

\[ T_{ij}^{NLO} \]

\( K^- p \rightarrow K \Xi \) reactions could be very sensitive to the NLO corrections!!!

Fitting parameters:
\[ f, b_0, b_D, b_F, d_1, d_2, d_3, d_4, a_{KN}, a_{\pi \Lambda}, a_{\pi \Sigma}, a_{\eta \Lambda}, a_{\eta \Sigma}, a_{K \Xi} \]
RESULTS I
Results for $\bar{K}N \rightarrow K\Xi$
INCLUSION OF HYPERONIC RESONANCES
Motivation for including resonances

- Inclusion of high spin and high mass resonances allows us to study the accuracy and stability of the NLO parameters \( b_0, b_D, b_F, d_1, d_2, d_3, d_4 \).

- It also allows the production of angular dependent scattering amplitudes; and hence, a better reproduction of the differential cross sections.

<table>
<thead>
<tr>
<th>Resonance</th>
<th>( I(J^P) )</th>
<th>Mass (MeV)</th>
<th>( \Gamma ) (MeV)</th>
<th>( \Gamma_{K\Xi}/\Gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Lambda(1890) )</td>
<td>0 ( \frac{3}{2}^+ )</td>
<td>1850 - 1910</td>
<td>60 - 200</td>
<td></td>
</tr>
<tr>
<td>( \Lambda(2100) )</td>
<td>0 ( \frac{7}{2}^- )</td>
<td>2090 - 2110</td>
<td>100 - 250</td>
<td>&lt; 3%</td>
</tr>
<tr>
<td>( \Lambda(2110) )</td>
<td>0 ( \frac{3}{2}^+ )</td>
<td>2090 - 2140</td>
<td>150 - 250</td>
<td></td>
</tr>
<tr>
<td>( \Lambda(2350) )</td>
<td>0 ( \frac{9}{2}^+ )</td>
<td>2340 - 2370</td>
<td>100 - 250</td>
<td></td>
</tr>
<tr>
<td>( \Sigma(1915) )</td>
<td>1 ( \frac{3}{2}^+ )</td>
<td>1900 - 1935</td>
<td>80 - 160</td>
<td></td>
</tr>
<tr>
<td>( \Sigma(1940) )</td>
<td>1 ( \frac{3}{2}^- )</td>
<td>1900 - 1950</td>
<td>150 - 300</td>
<td></td>
</tr>
<tr>
<td>( \Sigma(2030) )</td>
<td>1 ( \frac{7}{2}^+ )</td>
<td>2025 - 2040</td>
<td>150 - 200</td>
<td>&lt; 2%</td>
</tr>
<tr>
<td>( \Sigma(2250) )</td>
<td>1 (?)</td>
<td>2210 - 2280</td>
<td>60 - 150</td>
<td></td>
</tr>
</tbody>
</table>

In Sharov, Korotkikh, Lansko, EPJA 47 (2011) 109, a phenomenological model was suggested in which several combinations of resonances were tested concluding that \( \Sigma(2030) \) and \( \Sigma(2250) \) were the most relevant.
The total scattering amplitude for the $\bar{K}N \rightarrow K\Xi$ reaction taking into account the unitarized chiral contributions up to NLO plus the phenomenological contributions from the resonances reads:

$$T_{ij,s,s'}^{\text{tot}} = T_{ij,s,s'}^{\text{NLO}} + T_{s,s'}^{5/2^-} + T_{s,s'}^{7/2^+}$$

Fitting parameters.

- Decay constant $f$
- Subtracting constants $a_{\bar{K}N}$, $a_{\pi\Lambda}$, $a_{\pi\Sigma}$, $a_{\eta\Lambda}$, $a_{\eta\Sigma}$, $a_{K\Xi}$
- Coefficients of the NLO lagrangian terms $b_0, b_D, b_F, d_1, d_2, d_3, d_4$
- Masses and width of the resonances $M_{Y_5/2}$, $M_{Y_{7/2}}$, $\Gamma_{5/2}$, $\Gamma_{7/2}$
  Not free at all, their values are constrained according to PDG summary
- Cutoff parameters from the form factor $\Lambda_{5/2}$, $\Lambda_{7/2}$
- Product of the coupling constants (one for each vertex) for both resonances $g_{\bar{\Xi}Y_{5/2}K} \cdot g_{N_{Y_{5/2}\bar{\Xi}}}$, $g_{\bar{\Xi}Y_{7/2}K} \cdot g_{N_{Y_{7/2}\bar{\Xi}}}$
RESULTS I
Results for $\bar{K}N \rightarrow K\Xi$ including $\Sigma(2030)$, $\Sigma(2250)$ resonances

<table>
<thead>
<tr>
<th></th>
<th>$\gamma$</th>
<th>$R_n$</th>
<th>$R_c$</th>
<th>$a_p(K^-p \rightarrow K^-p)$</th>
<th>$\Delta E_{1s}$</th>
<th>$\Gamma_{1s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NLO*</td>
<td>2.37</td>
<td>0.189</td>
<td>0.664</td>
<td>$-0.69 + 10.86$</td>
<td>300</td>
<td>570</td>
</tr>
<tr>
<td>NLO+RES</td>
<td>2.39</td>
<td>0.187</td>
<td>0.668</td>
<td>$-0.66 + 10.84$</td>
<td>286</td>
<td>562</td>
</tr>
<tr>
<td>Exp.</td>
<td>2.36</td>
<td>0.189</td>
<td>0.664</td>
<td>$-0.66 + 10.81$</td>
<td>283</td>
<td>541</td>
</tr>
<tr>
<td></td>
<td>$\pm 0.04$</td>
<td>$\pm 0.015$</td>
<td>$\pm 0.011$</td>
<td>$(\pm 0.07) + i(\pm 0.15)$</td>
<td>$\pm 36$</td>
<td>$\pm 92$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>NLO*</th>
<th>NLO+RES</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_0$ (GeV$^{-1}$)</td>
<td>$-1.158 \pm 0.021$</td>
<td>$-0.907 \pm 0.004$</td>
</tr>
<tr>
<td>$b_D$ (GeV$^{-1}$)</td>
<td>$0.082 \pm 0.050$</td>
<td>$-0.151 \pm 0.008$</td>
</tr>
<tr>
<td>$b_F$ (GeV$^{-1}$)</td>
<td>$0.294 \pm 0.149$</td>
<td>$0.535 \pm 0.047$</td>
</tr>
<tr>
<td>$d_1$ (GeV$^{-1}$)</td>
<td>$-0.071 \pm 0.069$</td>
<td>$-0.055 \pm 0.055$</td>
</tr>
<tr>
<td>$d_2$ (GeV$^{-1}$)</td>
<td>$0.634 \pm 0.023$</td>
<td>$0.383 \pm 0.014$</td>
</tr>
<tr>
<td>$d_3$ (GeV$^{-1}$)</td>
<td>$2.819 \pm 0.058$</td>
<td>$2.180 \pm 0.011$</td>
</tr>
<tr>
<td>$d_4$ (GeV$^{-1}$)</td>
<td>$-2.036 \pm 0.035$</td>
<td>$-1.429 \pm 0.006$</td>
</tr>
</tbody>
</table>

INCLUSION OF BORN TERMS

\[ V_{ij} = V_{ij}^{WT} + V_{ij}^{D} + V_{ij}^{C} + V_{ij}^{NLO} \quad \Rightarrow \quad T = (1 - V G)^{-1} V \quad \Rightarrow \quad T_{ij}^{NLO} \]

What if we include Born diagrams???
They might be relevant in \( K^- p \rightarrow K \Xi \) reactions.

A new fit which includes the Born contributions was performed.
New parametrization was obtained for:
\( f, b_0, b_D, b_F, d_1, d_2, d_3, d_4, a_{\bar{K}N}, a_{\pi\Lambda}, a_{\eta\Lambda}, a_{\eta\Sigma}, a_{K\Xi}, D, F \)

The contribution of Born terms is at the same order as the NLO one!!!
The goodness of the fits is almost equal. Where does the difference in the physical interpretation lie???
INCLUSION OF BORN TERMS
Comparison between Models in isospin basis decomposition

New scenarios consisting of processes which filter isospin could provide more constraints in order to get more reliable values of NLO coefficients.

• \( \Lambda_b \to J/\psi \eta \Lambda, J/\psi K \Xi \) decayment, pure \( I = 0 \) process.

• J-Lab proposal for the secondary \( K_L \) beam for the reaction \( K^- n \to K^0 \Xi^- \), pure \( I = 1 \) process.

• Inclusion of the experimental data from \( \eta \Lambda, \eta \Sigma^0 \) channels in the fitting procedure, pure \( I = 0 \) and \( I = 1 \) processes respectively.
  Until then the scattering data used in the fits come from:
  \( K^- p \to K^- p, K^0 n, \pi^0 \Lambda, \pi^0 \Sigma^0, \pi^+ \Sigma^-, \pi^- \Sigma^+, K^+ \Xi^-, K^0 \Xi^0 \)
INCLUSION OF BORN TERMS (work in progress)

Considering $K^- p \rightarrow \eta \Lambda, \eta \Sigma^0$ scattering data in addition in the fit
INCLUSION OF BORN TERMS (work in progress)
Considering $K^- p \rightarrow \eta \Lambda, \eta \Sigma^0$ scattering data in addition in the fit

<table>
<thead>
<tr>
<th></th>
<th>WT+NLO+Born</th>
<th>WT+NLO+Born ($\eta$ chan)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{KN}$ ($10^{-3}$)</td>
<td>1.77 ± 2.38</td>
<td>1.27 ± 0.12</td>
</tr>
<tr>
<td>$a_{\pi\Lambda}$ ($10^{-3}$)</td>
<td>55.2 ± 13.5</td>
<td>-6.1 ± 12.9</td>
</tr>
<tr>
<td>$a_{\pi\Sigma}$ ($10^{-3}$)</td>
<td>2.33 ± 3.17</td>
<td>0.68 ± 1.43</td>
</tr>
<tr>
<td>$a_{\eta\Lambda}$ ($10^{-3}$)</td>
<td>8.00 ± 5.04</td>
<td>-0.67 ± 1.06</td>
</tr>
<tr>
<td>$a_{\eta\Sigma}$ ($10^{-3}$)</td>
<td>6.5 ± 20.6</td>
<td>8.00 ± 3.26</td>
</tr>
<tr>
<td>$a_{K\Xi}$ ($10^{-3}$)</td>
<td>-9.04 ± 3.63</td>
<td>-2.51 ± 0.99</td>
</tr>
<tr>
<td>$f/f_\pi$</td>
<td>1.21 ± 0.03</td>
<td>1.20 ± 0.03</td>
</tr>
<tr>
<td>$b_0$ ($GeV^{-1}$)</td>
<td>-0.70 ± 0.23</td>
<td>0.13 ± 0.04</td>
</tr>
<tr>
<td>$b_D$ ($GeV^{-1}$)</td>
<td>0.31 ± 0.20</td>
<td>0.12 ± 0.01</td>
</tr>
<tr>
<td>$b_F$ ($GeV^{-1}$)</td>
<td>0.65 ± 0.41</td>
<td>0.21 ± 0.02</td>
</tr>
<tr>
<td>$d_1$ ($GeV^{-1}$)</td>
<td>0.17 ± 0.26</td>
<td>0.15 ± 0.03</td>
</tr>
<tr>
<td>$d_2$ ($GeV^{-1}$)</td>
<td>0.17 ± 0.11</td>
<td>0.13 ± 0.03</td>
</tr>
<tr>
<td>$d_3$ ($GeV^{-1}$)</td>
<td>0.37 ± 0.16</td>
<td>0.30 ± 0.02</td>
</tr>
<tr>
<td>$d_4$ ($GeV^{-1}$)</td>
<td>0.01 ± 0.09</td>
<td>0.25 ± 0.03</td>
</tr>
<tr>
<td>$D$</td>
<td>0.90 ± 0.10</td>
<td>0.70 ± 0.16</td>
</tr>
<tr>
<td>$F$</td>
<td>0.40 ± 0.08</td>
<td>0.51 ± 0.11</td>
</tr>
<tr>
<td>$\chi^2_{d.o.f.}$</td>
<td>0.73</td>
<td>1.14</td>
</tr>
</tbody>
</table>
INCLUSION OF BORN TERMS (work in progress)

Considering $K^- p \to \eta \Lambda, \eta \Sigma^0$ scattering data in addition in the fit

Prediction for $K^- n \to K^0 \Xi^-$ reaction

(pure $I = 1$ process!!!)
• Chiral Perturbation Theory with unitarization in coupled channels is a very powerful technique to describe low energy hadron dynamics.

• The $\bar{K}N \rightarrow KE$ channels are very sensitive to the NLO terms of the lagrangian as well as to the Born terms, so they provide more reliable values of the NLO parameters.

• Experimental data from processes which filter isospin have been shown to be very helpful to reproduce properly the whole meson-baryon channels of the $S=-1$ sector and to constrain the fitting parameters.

• Addition of resonant terms in the scattering amplitude could play a significant role in the $\bar{K}N \rightarrow KE, \eta \Lambda$ reactions giving a significantly better agreement with experimental data and making the NLO coefficients gain notable accuracy.
THANK YOU

KEEP CALM AND WAIT FOR THE NEXT FIT
INCLUSION OF HYPERONIC RESONANCES

\( \bar{K}N \rightarrow Y \rightarrow K\Xi \)

\[ Y = \Sigma(2030), \Sigma(2250) \]

Finally, the scattering amplitudes related to the resonances can be obtained in the following way:

For \( \Sigma(2030) \), \( J^P = \frac{7^+}{2}, T^{7/2^+} \):

\[ T^{7/2^+}(s',s) = \frac{g_{\Xi Y_{5/2} K N Y_{5/2} K}}{m_K^6} u_{\Xi}(p') \frac{k_{\beta_1} k_{\beta_2} \Lambda_{\alpha_1\alpha_2} k_{\alpha_1} k_{\alpha_2}}{\not{q} - \not{M}_{7/2} + i \Gamma_{7/2}/2} u_{\Xi}^\dagger(p) \exp\left(-\frac{\not{q}^2}{\Lambda_{7/2}^2}\right) \exp\left(-\frac{\not{q}^2}{\Lambda_{7/2}^2}\right) \]

For \( \Sigma(2250) \), \( J^P = \frac{5^-}{2}, T^{5/2^-} \):

\[ T^{5/2^-}(s',s) = \frac{g_{\Xi Y_{5/2} K N Y_{5/2} K}}{m_K^6} u_{\Xi}(p') \frac{k_{\beta_1} k_{\beta_2} \Lambda_{\alpha_1\alpha_2} k_{\alpha_1} k_{\alpha_2}}{\not{q} - \not{M}_{7/2} + i \Gamma_{7/2}/2} u_{\Xi}^\dagger(p) \exp\left(-\frac{\not{q}^2}{\Lambda_{7/2}^2}\right) \exp\left(-\frac{\not{q}^2}{\Lambda_{7/2}^2}\right) \]
RESULTS I
Results for $\bar{K}N \rightarrow K\Xi$ including $\Sigma(2030), \Sigma(2250)$ resonances

\[
\begin{array}{cccccc}
\gamma & R_n & R_c & \alpha_p(K^- p \rightarrow K^- p) & \Delta E_{1s} & \Gamma_{1s} \\
\hline
\text{NLO}^* & 2.37 & 0.189 & 0.664 & -0.69 + i 0.86 & 300 & 570 \\
\text{WT+RES} & 2.37 & 0.193 & 0.667 & -0.73 + i 0.81 & 307 & 528 \\
\text{NLO+RES} & 2.39 & 0.187 & 0.668 & -0.66 + i 0.84 & 286 & 562 \\
\text{Exp.} & 2.36 & 0.189 & 0.664 & -0.66 + i 0.81 & 283 & 541 \\
& & & & \pm 0.04 & \pm 0.015 & \pm 0.011 \\
& & & & (\pm 0.07) + i (\pm 0.15) & \pm 36 & \pm 92 \\
\end{array}
\]

### RESULTS

**Fitting parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>NLO*</th>
<th>WT+RES</th>
<th>NLO+RES</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{KN}$ $(10^{-3})$</td>
<td>6.799 ± 0.701</td>
<td>−1.965 ± 2.219</td>
<td>6.157 ± 0.090</td>
</tr>
<tr>
<td>$a_{\pi\Lambda}$ $(10^{-3})$</td>
<td>50.93 ± 9.18</td>
<td>−188.2 ± 131.7</td>
<td>59.10 ± 3.01</td>
</tr>
<tr>
<td>$a_{\pi\Sigma}$ $(10^{-3})$</td>
<td>−3.167 ± 1.978</td>
<td>0.228 ± 2.949</td>
<td>−1.172 ± 0.296</td>
</tr>
<tr>
<td>$a_{\eta\Lambda}$ $(10^{-3})$</td>
<td>−15.16 ± 12.32</td>
<td>1.608 ± 2.603</td>
<td>−6.987 ± 0.381</td>
</tr>
<tr>
<td>$a_{\eta\Sigma}$ $(10^{-3})$</td>
<td>−5.325 ± 0.111</td>
<td>208.9 ± 151.1</td>
<td>−5.791 ± 0.034</td>
</tr>
<tr>
<td>$a_{K\Xi}$ $(10^{-3})$</td>
<td>31.00 ± 9.441</td>
<td>43.04 ± 25.84</td>
<td>32.60 ± 11.65</td>
</tr>
<tr>
<td>$f/f_{\pi}$</td>
<td>1.197 ± 0.011</td>
<td>1.203 ± 0.023</td>
<td>1.193 ± 0.003</td>
</tr>
<tr>
<td>$b_0$ $(\text{GeV}^{-1})$</td>
<td>−1.158 ± 0.021</td>
<td>−0.907 ± 0.004</td>
<td></td>
</tr>
<tr>
<td>$b_D$ $(\text{GeV}^{-1})$</td>
<td>0.082 ± 0.050</td>
<td>−0.151 ± 0.008</td>
<td></td>
</tr>
<tr>
<td>$b_F$ $(\text{GeV}^{-1})$</td>
<td>0.294 ± 0.149</td>
<td>0.535 ± 0.047</td>
<td></td>
</tr>
<tr>
<td>$d_1$ $(\text{GeV}^{-1})$</td>
<td>−0.071 ± 0.069</td>
<td>−0.055 ± 0.055</td>
<td></td>
</tr>
<tr>
<td>$d_2$ $(\text{GeV}^{-1})$</td>
<td>0.634 ± 0.023</td>
<td>0.383 ± 0.014</td>
<td></td>
</tr>
<tr>
<td>$d_3$ $(\text{GeV}^{-1})$</td>
<td>2.819 ± 0.058</td>
<td>2.180 ± 0.011</td>
<td></td>
</tr>
<tr>
<td>$d_4$ $(\text{GeV}^{-1})$</td>
<td>−2.036 ± 0.035</td>
<td>−1.429 ± 0.006</td>
<td></td>
</tr>
<tr>
<td>$g_{\Xi_{5/2}} g_{\Xi_{4/2}}$</td>
<td>−5.42 ± 15.96</td>
<td>8.82 ± 5.72</td>
<td></td>
</tr>
<tr>
<td>$g_{\Xi_{7/2}} g_{\Xi_{4/2}}$</td>
<td>−0.61 ± 14.12</td>
<td>0.06 ± 0.20</td>
<td></td>
</tr>
<tr>
<td>$\Lambda_{5/2}$ $(\text{MeV})$</td>
<td>576.7 ± 275.2</td>
<td>522.7 ± 43.8</td>
<td></td>
</tr>
<tr>
<td>$\Lambda_{7/2}$ $(\text{MeV})$</td>
<td>623.7 ± 287.5</td>
<td>999.0 ± 288.0</td>
<td></td>
</tr>
<tr>
<td>$M_{\Xi_{5/2}}$ $(\text{MeV})$</td>
<td>2210.0 ± 39.8</td>
<td>2278.8 ± 67.4</td>
<td></td>
</tr>
<tr>
<td>$M_{\Xi_{7/2}}$ $(\text{MeV})$</td>
<td>2025.0 ± 9.4</td>
<td>2040.0 ± 9.4</td>
<td></td>
</tr>
<tr>
<td>$\Gamma_{5/2}$ $(\text{MeV})$</td>
<td>150.0 ± 71.3</td>
<td>150.0 ± 54.4</td>
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</tr>
<tr>
<td>$\Gamma_{7/2}$ $(\text{MeV})$</td>
<td>200.0 ± 44.6</td>
<td>200.0 ± 32.3</td>
<td></td>
</tr>
</tbody>
</table>

| $\chi^2_{d.o.f.}$                                                        | 1.48   | 2.26           | 1.05           |
INCLUSION OF BORN TERMS
Comparison between Models in isospin basis decomposition
INCLUSION OF BORN TERMS (work in progress)
Considering $K^- p \rightarrow \eta \Lambda, \eta \Sigma^0$ scattering data in addition in the fit

Comparison between Models in isospin basis decomposition
INCLUSION OF BORN TERMS (work in progress)

Considering $K^- p \rightarrow \eta \Lambda, \eta \Sigma^0$ scattering data in addition in the fit
### FORMALISM
Effective lagrangian up to NLO

<table>
<thead>
<tr>
<th></th>
<th>$K^{-}p$</th>
<th>$K^{0}n$</th>
<th>$\pi^{0}\Lambda$</th>
<th>$\pi^{0}\Sigma^{0}$</th>
<th>$\eta\Lambda$</th>
<th>$\eta\Sigma^{0}$</th>
<th>$\pi^{+}\Sigma^{-}$</th>
<th>$\pi^{-}\Sigma^{+}$</th>
<th>$K^{+}\Xi^{-}$</th>
<th>$K^{0}\Xi^{0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^{-}p$</td>
<td>$4\left(b_{0} + b_{D}\right)m_{K}^{2}$</td>
<td>$2\left(b_{D} + b_{F}\right)m_{K}^{2}$</td>
<td>$\frac{-\left(b_{D} + b_{F}\right)m_{K}^{2}}{2\sqrt{3}}$</td>
<td>$\frac{\left(b_{D} - b_{F}\right)m_{K}^{2}}{2\sqrt{3}}$</td>
<td>$0$</td>
<td>$0$</td>
<td>$\left(b_{D} - b_{F}\right)m_{K}^{2}$</td>
<td>$0$</td>
<td>$0$</td>
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</tr>
<tr>
<td>$K^{0}n$</td>
<td>$4\left(b_{0} + b_{D}\right)m_{K}^{2}$</td>
<td>$\frac{-\left(b_{D} + b_{F}\right)m_{K}^{2}}{2\sqrt{3}}$</td>
<td>$\frac{\left(b_{D} - b_{F}\right)m_{K}^{2}}{2\sqrt{3}}$</td>
<td>$0$</td>
<td>$0$</td>
<td>$\left(b_{D} - b_{F}\right)m_{K}^{2}$</td>
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<td>$0$</td>
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</table>

$D_{ij}$

<table>
<thead>
<tr>
<th></th>
<th>$K^{-}p$</th>
<th>$K^{0}n$</th>
<th>$\pi^{0}\Lambda$</th>
<th>$\pi^{0}\Sigma^{0}$</th>
<th>$\eta\Lambda$</th>
<th>$\eta\Sigma^{0}$</th>
<th>$\pi^{+}\Sigma^{-}$</th>
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<th>$K^{+}\Xi^{-}$</th>
<th>$K^{0}\Xi^{0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^{-}p$</td>
<td>$2d_{2} + d_{3}$</td>
<td>$d_{1} + d_{2} + d_{3}$</td>
<td>$\sqrt{3}(d_{1} + d_{2})$</td>
<td>$-d_{1} - d_{2} + 2d_{3}$</td>
<td>$d_{1} - 3d_{2} + 2d_{3}$</td>
<td>$\frac{d_{1} - 3d_{2} + 2d_{3}}{2\sqrt{3}}$</td>
<td>$-2d_{2} + d_{3}$</td>
<td>$-d_{1} + d_{2} + d_{3}$</td>
<td>$-4d_{2} + 2d_{3}$</td>
<td>$-2d_{2} + d_{3}$</td>
</tr>
</tbody>
</table>

$L_{ij}$

<table>
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<tr>
<th></th>
<th>$K^{-}p$</th>
<th>$K^{0}n$</th>
<th>$\pi^{0}\Lambda$</th>
<th>$\pi^{0}\Sigma^{0}$</th>
<th>$\eta\Lambda$</th>
<th>$\eta\Sigma^{0}$</th>
<th>$\pi^{+}\Sigma^{-}$</th>
<th>$\pi^{-}\Sigma^{+}$</th>
<th>$K^{+}\Xi^{-}$</th>
<th>$K^{0}\Xi^{0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^{-}p$</td>
<td>$2d_{2} + d_{3} + 2d_{4}$</td>
<td>$d_{1} + d_{2} + d_{3}$</td>
<td>$\sqrt{3}(d_{1} + d_{2})$</td>
<td>$-d_{1} - d_{2} + 2d_{3}$</td>
<td>$d_{1} - 3d_{2} + 2d_{3}$</td>
<td>$\frac{d_{1} - 3d_{2} + 2d_{3}}{2\sqrt{3}}$</td>
<td>$-2d_{2} + d_{3}$</td>
<td>$-d_{1} + d_{2} + d_{3}$</td>
<td>$-4d_{2} + 2d_{3}$</td>
<td>$-2d_{2} + d_{3}$</td>
</tr>
</tbody>
</table>
What happens if a third resonance is added?
For instance \( \Lambda(1890) \), as it was done in B. C. Jackson, Y. Oh, H. Haberzettl and K. Nakayama, arXiv: 1503.00845 [nucl-th].
Results for $\bar{K}N \rightarrow K\Xi$
Results for $\bar{K}N \rightarrow KE$
Results for $\bar{K}N \rightarrow K\Sigma$ including $\Sigma(2030), \Sigma(2250)$ resonances

<table>
<thead>
<tr>
<th></th>
<th>$\gamma$</th>
<th>$R_n$</th>
<th>$R_e$</th>
<th>$a_p(K^-p \rightarrow K^-p)$</th>
<th>$\Delta E_{1s}$</th>
<th>$\Gamma_{1s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NLO*</td>
<td>2.37</td>
<td>0.189</td>
<td>0.664</td>
<td>$-0.69 + i 0.86$</td>
<td>300</td>
<td>570</td>
</tr>
<tr>
<td>WT+RES</td>
<td>2.37</td>
<td>0.193</td>
<td>0.667</td>
<td>$-0.73 + i 0.81$</td>
<td>307</td>
<td>528</td>
</tr>
<tr>
<td>NLO+RES</td>
<td>2.39</td>
<td>0.187</td>
<td>0.668</td>
<td>$-0.66 + i 0.84$</td>
<td>286</td>
<td>562</td>
</tr>
<tr>
<td>Exp.</td>
<td>2.36</td>
<td>0.189</td>
<td>0.664</td>
<td>$-0.66 + i 0.81$</td>
<td>283</td>
<td>541</td>
</tr>
<tr>
<td></td>
<td>$\pm 0.04$</td>
<td>$\pm 0.015$</td>
<td>$\pm 0.011$</td>
<td>$(\pm 0.07) + i (\pm 0.15)$</td>
<td>$\pm 36$</td>
<td>$\pm 92$</td>
</tr>
</tbody>
</table>
Differential cross section of the $\bar{K}N \rightarrow K^0\Xi^0$
Differential cross section of the $\bar{K}N \rightarrow K^+\Xi^-$
RESULTS I

Experimental data VS. the NLO model.

In Sharov, Korotkikh, Lanskoy, EPJA 47 (2011) 109, a phenomenological model was suggested in which several combinations of resonances were tested concluding that \( \Sigma(2030) \) and \( \Sigma(2250) \) were...
INCLUSION OF HYPERONIC RESONANCES IN $\bar{K}N \rightarrow KE$

\[
\Delta_{\alpha_1 \alpha_2} (\frac{5}{2}) = \frac{1}{2} \left( \theta_{\alpha_1}^{\beta_1} \theta_{\alpha_2}^{\beta_2} + \theta_{\alpha_1}^{\beta_2} \theta_{\alpha_2}^{\beta_1} \right) - \frac{1}{2} \theta_{\alpha_1 \alpha_2} \theta_{\beta_1 \beta_2} - \frac{1}{10} \left( \overline{\gamma} \alpha_1 \overline{\gamma} \beta_1 \theta_{\alpha_2}^{\beta_2} + \overline{\gamma} \alpha_1 \overline{\gamma} \beta_2 \theta_{\alpha_2}^{\beta_1} + \overline{\gamma} \alpha_2 \overline{\gamma} \beta_1 \theta_{\alpha_1}^{\beta_2} + \overline{\gamma} \alpha_2 \overline{\gamma} \beta_2 \theta_{\alpha_1}^{\beta_1} \right)
\]

\[
\theta_{\mu}^{\nu} = g_{\mu}^{\nu} - \frac{q_{\mu} q^{\nu}}{M_f^2}
\]

\[
\overline{\gamma}_{\mu} = \gamma_{\mu} - \frac{q_{\mu} \phi}{M_f^2}
\]

\[
\Delta_{\alpha_1 \alpha_2 \alpha_3} (\frac{7}{2}) = \frac{1}{36} \sum_{F(\alpha)F(\beta)} \left( \theta_{\alpha_1}^{\beta_1} \theta_{\alpha_2}^{\beta_2} \theta_{\alpha_3}^{\beta_3} - \frac{3}{7} \theta_{\alpha_1}^{\beta_1} \theta_{\alpha_2 \alpha_3} \theta_{\beta_2 \beta_3} - \frac{3}{7} \overline{\gamma} \alpha_1 \overline{\gamma} \beta_1 \theta_{\alpha_2}^{\beta_2} \theta_{\alpha_3}^{\beta_3} + \frac{3}{35} \overline{\gamma} \alpha_1 \overline{\gamma} \beta_1 \theta_{\alpha_2 \alpha_3} \theta_{\beta_2 \beta_3} \right)
\]
Taking into account the scattering amplitude given by LS equations for a NLO Chiral Lagrangian and the phenomenological contributions from the resonances, the total scattering amplitude for the $\bar{K}N \rightarrow K\Xi$ reaction should be written as:

$$T_{ij,s,s'}^{tot} = T_{ij,s,s'}^{LS} + T_{s,s'}^{5/2^-} + T_{s,s'}^{7/2^+}$$

Being aware of isospin symmetry, the coupling constants for each channel have to integrate this fact in its value.

$$|K^+\Xi^-> = \frac{1}{\sqrt{2}} (|K\Xi⟩_{I=1} + |K\Xi⟩_{I=0})$$

Σ(2030), Σ(2250) both have $I=1$

$$|K^0\Xi^0⟩ = \frac{1}{\sqrt{2}} (|K\Xi⟩_{I=1} - |K\Xi⟩_{I=0})$$

Or in a equivalent manner:

- $K^-p \rightarrow K^+\Xi^-$

$$T_{s,s'}^{tot} = T_{s,s'}^{LS} - T_{s,s'}^{5/2^-} - T_{s,s'}^{7/2^+}$$

- $K^-p \rightarrow K^0\Xi^0$

$$T_{s,s'}^{tot} = T_{s,s'}^{LS} + T_{s,s'}^{5/2^-} + T_{s,s'}^{7/2^+}$$
On going work ...

In order to improve results, the model could be developed taking into account:

- Born (direct and cross) diagrams (fine tuning)

\[ \mathcal{L}_{MB}^{(YUKAWA)}(B, U) = \frac{1}{2} D\langle \bar{B} \gamma^\mu \gamma_5 \{u_\mu, B\} \rangle + \frac{1}{2} F\langle \bar{B} \gamma^\mu \gamma_5 [u_\mu, B] \rangle \]