Nucleon structure functions and longitudinal spin asymmetries

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The XIIth Quark Confinement and the Hadron Spectrum
Athens, GREECE
August 28 - September 04, 2016

Outline

Naive Quark Model

- **Internal Structure:** The knowledge of internal structure of nucleon in terms of quark and gluon degrees of freedom in QCD provide a basis for understanding more complex, strongly interacting matter.
- Knowledge has been rather limited because of confinement and it is still a big challenge to perform the calculations from the first principles of QCD.
- **Naive Quark Model** is able to provide a intuitive picture and successfully accounts for many of the low-energy properties of the hadrons in terms of the valence quarks.

Fundamental quantities

- **Structure:** Magnetic moments
  Dirac theory (1.0 μ_N) and experiment (2.5 μ_N).
  Proton is not an elementary Dirac particle but has an inner structure.
- **Size:** Spatial extension.
  Proton charge distribution given by charge radius r_p.
- **Shape:** Nonspherical charge distribution.
  Quadrupole moment of the transition $N \rightarrow \Delta$.
- Relation between the properties??

Quantum Chromodynamics (QCD): Present Theory of Strong Interactions

- At high energies, (α_s is small), QCD can be used perturbatively.
- At low energies, (α_s becomes large), one has to use other methods such as effective Lagrangian models to describe physics.
- Wide range of applications ranging from the dynamics and structure of hadrons and nuclei to the properties and phases of hadronic matter at the earliest stages of the universe.
- New experimental tools are continually being developed to probe the non-perturbative structure of the theory, for example the hard diffractive reactions, semi-inclusive reactions, deeply virtual Compton scattering etc..
- Many fundamental questions have not been resolved. The most challenging nonperturbative problem in QCD is to determine the structure and spectrum of hadrons in terms of their quark and gluon degrees of freedom.

Proton Spin Problem: The driving question

- 1988 European Muon Collaboration (Valence quarks carry 30% of proton spin)
- Naive Quark Model contradicts this results (Based on Pure valence description: proton = 2u + d)
  "Proton spin crisis"
- Confirmed by the measurements of polarized structure functions of proton in the deep inelastic scattering (DIS) experiments by SMC, E142-3 and HERMES experiments.
- Provides evidence that the valence quarks of proton carry only a small fraction of its spin suggesting that they should be surrounded by an indistinct sea of quark-antiquark pairs.

Flavor Structure

- 1991 NMC result: Asymmetric nucleon sea ($\bar{d} > \bar{u}$)
  Recently confirmed by E866 and HERMES
- Measured quark sea asymmetry established that the study of the structure of the nucleon is intrinsically a nonperturbative phenomena.
- Sum Rules
  - Bjorken Sum Rule: $\Delta_3 = \Delta u - \Delta d$
  - Ellis-Jaffe Sum Rule: $\Delta_8 = \Delta u + \Delta d - 2\Delta s$
    (Reduces to $\Delta_8 = \Delta\Sigma$ when $\Delta s = 0$)
  - Strange quark fraction: $f_s \simeq 0.10$
  - Gottfried Sum Rule: $I_G = \frac{1}{3} + \frac{2}{3}\int_0^1 [\bar{u}(x) - \bar{d}(x)]dx = 0.254 \pm 0.026$

Quark Sea

- Recently, a wide variety of accurately measured data have been accumulated for
  static properties of hadrons: masses, electromagnetic moments, charge radii etc.
  low energy dynamical properties: scattering lengths and decay rates etc.
- These lie in the nonperturbative range of QCD and require nonperturbative methods. The direct calculations of these quantities form the first principle of QCD are extremely difficult, because they.
- Flavor and spin structure of the nucleon is not limited to *u* and *d* quarks only. Nonperturbative effects explained only through the generation of "quark sea".
- Techniques such as lattice gauge theory, QCD sum rules, and a wide variety of models have been developed to study this extremely interesting energy regime.

Pion Cloud Mechanism

- Quark sea is believed to originate from process such as virtual pion production.
- It is suggested that in the deep inelastic lepton-nucleon scattering, the lepton probe also scatters off the pion cloud surrounding the target proton. The $\pi^+(\bar{d}u)$ cloud, dominant in the process $p \rightarrow \pi^+ n$, leads to an excess of $\bar{d}$ sea.
- However, this effect should be significantly reduced by the emissions such as $p \rightarrow \Delta^{++} + \pi^-$ with $\pi^-(\bar{u}d)$ cloud. Therefore, the pion cloud idea is not able to explain the significant $\bar{d} > \bar{u}$ asymmetry.
- This approach can be improved upon by adopting a mechanism which operates in the *interior* of the hadron.

Chiral Symmetry Breaking

- The dynamics of light quarks (u, d, and s) and gluons can be described by the QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} + i\bar{\psi}_R \not{D} \psi_R + i\bar{\psi}_L \not{D} \psi_L - \bar{\psi}_R M \psi_L - \bar{\psi}_L M \psi_R \,,$$

  $G^a_{\mu\nu}$ is the gluonic gauge field strength tensor, D^μ is the gauge-covariant derivative, M is the quark mass matrix and ψ_L and ψ_R are the left and right handed quark fields

- Mass terms change sign as $\psi_R \to \psi_R$ and $\psi_L \to -\psi_L$ under the chiral transformation ($\psi \to \gamma^5 \psi$), the Lagrangian no longer remains invariant. If neglected, the Lagrangian will have global chiral symmetry of the $SU(3)_L \times SU(3)_R$ group. Hadrons do not display parity doublets $\to$ the chiral symmetry is believed to be spontaneously broken around a scale of 1 GeV as

$$SU(3)_L \times SU(3)_R \to SU(3)_{L+R} \,.$$

- As a consequence, there exists a set of massless particles, referred to as the Goldstone bosons (GBs), which are identified with the observed (π, K, η mesons).
- Within the region of QCD confinement scale ($\Lambda_{QCD} \simeq 0.1 - 0.3$ GeV) and the chiral symmetry breaking scale $\Lambda_{\chi SB}$, the constituent quarks, the octet of GBs (π, K, η mesons), and the *weakly* interacting gluons are the appropriate degrees of freedom.
- The effective interaction Lagrangian in this region can be expressed as

$$\mathcal{L}_{\rm int} = \bar{\psi}(i\not{D} + \not{V})\psi + ig_A\bar{\psi}\not{A}\gamma^5\psi + \cdots ,$$

where g_A is the axial-vector coupling constant. The gluonic degrees of freedom can be neglected owing to small effect in the effective quark model at low energy scale. The vector and axial-vector currents V_μ and A_μ are defined as

$$\begin{pmatrix} V_\mu \\ A_\mu \end{pmatrix} = \frac{1}{2}(\xi^\dagger\partial_\mu\xi \pm \xi\partial_\mu\xi^\dagger),$$

where $\xi = \exp(2i\Phi/f_\pi)$, f_π is the pseudoscalar pion decay constant ($\simeq$ 93 MeV).

- The field Φ describes the dynamics of GBs as

$$\Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \beta\frac{\eta}{\sqrt{6}} & \pi^+ & \alpha K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \beta\frac{\eta}{\sqrt{6}} & \alpha K^0 \\ \alpha K^- & \alpha \bar{K}^0 & -\beta\frac{2\eta}{\sqrt{6}} \end{pmatrix} .$$

Expanding V_μ and A_μ in the powers of Φ/f_π, we get

$$V_\mu = 0 + O\left((\Phi/f_\pi)^2\right) ,$$
$$A_\mu = \frac{i}{f_\pi}\partial_\mu\Phi + O\left((\Phi/f_\pi)^2\right) .$$

- The effective interaction Lagrangian between GBs and quarks from in the leading order can now be expressed as

$$\mathcal{L}_{\rm int} = -\frac{g_A}{f_\pi}\bar{\psi}\partial_\mu\Phi\gamma^\mu\gamma^5\psi ,$$

which using the Dirac equation $(i\gamma^\mu\partial_\mu - m_q)q = 0$ can be reduced to

$$\mathcal{L}_{\rm int} \approx i\sum_{q=u,d,s}\frac{m_q + m_{q'}}{f_\pi}\bar{q}'\Phi\gamma^5 q = i\sum_{q=u,d,s} c_8\bar{q}'\Phi\gamma^5 q .$$

- $c_8 \left(= \frac{m_q + m_{q'}}{f_\pi}\right)$ is the coupling constant for octet of GBs and m_q ($m_{q'}$) is the quark mass parameter. The Lagrangian of the quark-GB interaction, suppressing all the space-time structure to the lowest order, can now be expressed as

$$\mathcal{L}_{\rm int} = c_8 \bar{\psi} \Phi \psi \, .$$

- The QCD Lagrangian is also invariant under the axial $U(1)$ symmetry, which would imply the existence of ninth GB. This breaking symmetry picks the η' as the ninth GB.
- The effective Lagrangian describing interaction between quarks and a nonet of GBs, consisting of octet and a singlet, can now be expressed as

$$\mathcal{L}_{\rm int} = c_8 \bar{\psi} \Phi \psi + c_1 \bar{\psi} \frac{\eta'}{\sqrt{3}} \psi = c_8 \bar{\psi} \left(\Phi + \zeta \frac{\eta'}{\sqrt{3}} I \right) \psi = c_8 \bar{\psi} \left(\Phi' \right) \psi \, ,$$

where $\zeta = c_1/c_8$, c_1 is the coupling constant for the singlet GB and I is the 3×3 identity matrix.

Chiral Constituent Quark Model

- χCQM initiated by Weinberg and developed by Manohar and Georgi to explain the successes of NQM.
- The fluctuation process describing the effective Lagrangian is

$$q^{\pm} \rightarrow \mathrm{GB} + q^{'\mp} \rightarrow (q\bar{q}^{'}) + q^{'\mp},$$

where $q\bar{q}^{'} + q^{'}$ constitute the sea quarks.

- Incorporates *confinement* and *chiral symmetry breaking.*
- "Justifies" the idea of constituent quarks and scope of the model extended in the context of **"proton spin crisis"**

χCQM

- The GB field can be expressed in terms of the GBs and their transition probabilities as

$$\Phi' = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \beta \frac{\eta}{\sqrt{6}} + \zeta \frac{\eta'}{\sqrt{3}} & \pi^+ & \alpha K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \beta \frac{\eta}{\sqrt{6}} + \zeta \frac{\eta'}{\sqrt{3}} & \alpha K^o \\ \alpha K^- & \alpha \bar{K}^0 & -\beta \frac{2\eta}{\sqrt{6}} + \zeta \frac{\eta'}{\sqrt{3}} \end{pmatrix}.$$

- The transition probability of chiral fluctuation $u(d) \rightarrow d(u) + \pi^{+(-)}$ 'a' is introduced by considering nondegenerate quark masses $M_s > M_{u,d}$. In terms of a, the probabilities of transitions of $u(d) \rightarrow s + K^{+(0)}$, $u(d,s) \rightarrow u(d,s) + \eta$, and $u(d,s) \rightarrow u(d,s) + \eta'$ are given as $\alpha^2 a$, $\beta^2 a$ and $\zeta^2 a$ respectively.
- The parameters α and β are introduced by considering nondegenerate GB masses $M_K, M_\eta > M_\pi$ and the parameter ζ is introduced by considering $M_{\eta'} > M_K, M_\eta$. The hierarchy for the probabilities, which scale as $\frac{1}{M_q^2}$, can be obtained as

$$a > a\alpha^2 \geq a\beta^2 > a\zeta^2.$$

Successes of χCQM

- "Proton Spin Problem"
- Magnetic moments of octet and decuplet baryons including their transitions and the Coleman-Glashow sum rule.
- Hyperon β decay parameters including the axial-vector coupling parameters F and D.
- Magnetic moments of octet baryon resonancesas well as Λ resonances.
- Charge radii and quadrupole moment of the baryons.
- Small intrinsic charm content in the nucleon spin in the SU(4) χCQM and the magnetic moment and charge radii of charm baryons including their radiative decays

Motivation

- The polarized deep inelastic lepton-nucleon scattering have determined the unpolarized and polarized structure functions of the nucleon through the measurement of the longitudinal spin asymmetries with the target spin being parallel and antiparallel to the longitudinally polarized beam.
- Major surprise was found in the flavor structure when the famous DIS experiments by the New Muon Collaboration (NMC) in 1991 established the sea quark asymmetry of the unpolarized quarks by measuring the violation of the Gottfried sum rule $(\int_0^1 [\bar{d}(x) - \bar{u}(x)]dx)$.
- Confirmed by two independent experiments in various $0 \leq x \leq 1$ ranges. Fermilab E866 experiments, measuring a large sea quark asymmetry ratio $\bar{d}/\bar{u}$ as well as $\bar{d} - \bar{u} \neq 0$. Drell-Yan cross section ratios of the NA51 experiments. HERMES presented sea quark asymmetry $\frac{\bar{d}-\bar{u}}{u-d}$.

Motivation

- The information on the strange sea is obtained from the neutrino-induced DIS experiments as well as through the charm production with dimuon events in the final states of the experiments CDHS, CCFR, CHARMII, NOMAD, NuTeV and CHORUS.
- Neutrino-induced DIS experiments emphasized that the valence quark distributions dominate for $x > 0.3$ and it is a relatively clean region to test the valence structure of the nucleon as well as to estimate the structure functions and related quantities, whereas the sea quarks dominate for the $x < 0.3$. Renewed considerable interest in the sea quark flavor structure as well as asymmetries and they point out the need for additional refined data.
- Ongoing Drell-Yan experiment at Fermilab and a proposed experiment at J-PARC facility are working towards extending the kinematic coverage and improving the accuracy of the sea quark asymmetry.

Purpose

- To determine the spin independent structure functions $F_1^{p,n}(x)$ and $F_2^{p,n}(x)$, spin dependent structure functions $g_1^{p,n}(x)$.
- The p and n longitudinal spin asymmetries $A_1^p(x)$ and $A_1^n(x)$ come from the difference in cross sections in scattering of a polarized lepton from a polarized proton where the leptons are scattered with the same and unlike helicity as that of the proton.
- To compute the ratio of polarized to unpolarized quark distribution functions for up and down quarks in the p and n $\frac{\Delta u^p(x)}{u^p(x)}$, $\frac{\Delta d^p(x)}{d^p(x)}$, $\frac{\Delta u^n(x)}{u^n(x)}$, and $\frac{\Delta d^n(x)}{d^n(x)}$.

- The unpolarized distribution function of the quark (antiquark) $q_i(x)$ ($\bar{q}_i(x)$) is described as the probability of the i^{th} quark (antiquark) carrying a fraction x of the nucleon's momentum. It can be calculated from the scalar matrix element

$$\langle N|q\bar{q}|N\rangle,$$

where $|N\rangle$ is the nucleon wavefunction. The operator $q\bar{q}$ is defined in terms of the number $n_{q(\bar{q})}$ of $q(\bar{q})$ quarks with electric charge $e_q(e_{\bar{q}})$. We have

$$q\bar{q} = \sum_{q=u,d,s} (n_q q + n_{\bar{q}}\bar{q}) \quad = \quad n_u u + n_{\bar{u}}\bar{u} + n_d d + n_{\bar{d}}\bar{d} + n_s s + n_{\bar{s}}\bar{s}\,.$$

- The polarized distribution function of the i^{th} quark $\Delta q_i(x)$ is defined as

$$\Delta q_i(x) = q_i^{\uparrow}(x) - q_i^{\downarrow}(x),$$

where $q_i^{\uparrow}(x)$ ($q_i^{\downarrow}(x)$) is the probability that the i^{th} quark spin is aligned parallel or antiparallel to the nucleon spin. The polarized distribution function of the quarks can be calculated from the axial vector matrix element of the nucleon

$$\langle N | q^{\uparrow} q^{\downarrow} | N \rangle \, .$$

Here $\mathcal{N} = q^{\uparrow} q^{\downarrow}$ is the number operator defined in terms of the number $n_{q^{\uparrow}(q^{\downarrow})}$ of $q^{\uparrow}(q^{\downarrow})$ quarks. We have

$$q^{\uparrow} q^{\downarrow} = \sum_{q=u,d,s} (n_{q^{\uparrow}} q^{\uparrow} + n_{q^{\downarrow}} q^{\downarrow}) = n_{u^{\uparrow}} u^{\uparrow} + n_{u^{\downarrow}} u^{\downarrow} + n_{d^{\uparrow}} d^{\uparrow} + n_{d^{\downarrow}} d^{\downarrow} + n_{s^{\uparrow}} s^{\uparrow} + n_{s^{\downarrow}} s^{\downarrow}$$

with the coefficients of the $q^{\uparrow\downarrow}$ giving the number of $q^{\uparrow\downarrow}$ quarks.

- The spin independent structure functions of the nucleon can be further defined in terms of the unpolarized distribution functions of the quarks as

$$F_1^N(x) = \frac{1}{2}\sum_{u,d,s} e_i^2 (q_i(x) + \bar{q}_i(x)) \,,$$
$$F_2^N(x) = 2xF_1^N(x) \,.$$

- The spin dependent structure function of the nucleon can similarly be defined in terms of the polarized distribution function of the quarks as

$$g_1^N(x) = \frac{1}{2}\sum_{u,d,s} e_i^2 \Delta q_i(x) \,.$$

- The proton and neutron longitudinal spin asymmetries are given by

$$A_1^p(x) = \frac{4\Delta u^p(x) + \Delta d^p(x)}{4u^p(x) + d^p(x)}, \quad A_1^n(x) = \frac{4\Delta u^n(x) + \Delta d^n(x)}{4u^n(x) + d^n(x)}.$$

- The explicit ratio of polarized to unpolarized quark distribution functions for up and down quarks in the proton and neutron as

$$\begin{aligned}
\frac{\Delta u^p(x)}{u^p(x)} &= \frac{4}{15}A_1^p(x)\left(4 + \frac{d^p(x)}{u^p(x)}\right) - \frac{1}{15}A_1^n(x)\left(1 + 4\frac{d^p(x)}{u^p(x)}\right), \\
\frac{\Delta d^p(x)}{d^p(x)} &= \frac{4}{15}A_1^n(x)\left(4 + \frac{u^p(x)}{d^p(x)}\right) - \frac{1}{15}A_1^p(x)\left(1 + 4\frac{u^p(x)}{d^p(x)}\right), \\
\frac{\Delta u^n(x)}{u^n(x)} &= \frac{4}{15}A_1^p(x)\left(1 + 4\frac{d^n(x)}{u^n(x)}\right) - \frac{1}{15}A_1^n(x)\left(4 + \frac{d^p(x)}{u^p(x)}\right), \\
\frac{\Delta d^n(x)}{d^n(x)} &= \frac{4}{15}A_1^n(x)\left(1 + 4\frac{u^n(x)}{d^n(x)}\right) - \frac{1}{15}A_1^p(x)\left(4 + \frac{u^n(x)}{d^n(x)}\right).
\end{aligned}$$

Application Potential of the model

The present calculations suggest few important points

- Decomposition of various measurable quantities into the contributions from valence and sea components.
- Contribution of strange quarks in the nucleon which do not appear explicitly in most quark model descriptions of the nucleon and the role played by non-valence flavors in understanding the nucleon internal structure.

Long term

Understanding the spin and flavor structure of the proton will help to resolve the most challenging problems facing subatomic physics which include

- What happens to the spin in the transition between current and constituent quarks in the low energy QCD?
- How can we distinguish between the *current quarks* and the *constituent quarks*?
- How is the spin of the proton built out from the intrinsic spin and orbital angular momentum of its quark and gluonic constituents?
- What is the role played by non-valence flavors in understanding the nucleon internal structure?

Conclusions

- A small but non-zero value of SU(3) symmetry breaking within the dynamics of χCQM, suggests an important role for non-valence quark masses in the nonperturbative regime of QCD.
- Chiral symmetry breaking is the key to understand the contribution of the sea quarks in the nonperturbative regime of QCD.
- At leading order, the model envisages constituent quarks, the Goldstone bosons (π, K, η mesons) as appropriate degrees of freedom.

Thank You