Predictions on the second-class current decays $\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau$

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Objectives

1. We reassess the $\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau$ decays, unseen in Nature so far.
2. To motivate near future B-factories e.g. Belle-II, their discovery.

Introduction

- $\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau$ decays belong to the so-called second-class current processes: parity conservation implies that these decays must proceed through the vector current, which has opposed G-parity to the $\pi^- \eta^{(\prime)}$ system.
- In the limit of exact isospin symmetry G-parity is exact and these processes are forbidden.
- Isospin is however an approximate symmetry, slightly broken both by $m_u \neq m_d$ (in QCD) and $q_u \neq q_d$ (in QED), which results in a sizable suppression of these decays.

Decay width

- Amplitude of the process

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{ud} \bar{u}(p_{\nu_\tau}) \gamma_\mu (1-\gamma_5) u(p_\tau) \langle \pi^- \eta^{(\prime)} | \bar{d} \gamma^\mu u | 0 \rangle \, .$$

  - Hadronic matrix element in terms of form factors

$$\langle \pi^- \eta^{(\prime)} | \bar{d} \gamma^\mu u | 0 \rangle = \left[(p_{\eta^{(\prime)}} - p_\pi)^\mu + \frac{\Delta_{\pi^- \eta^{(\prime)}}}{s} q^\mu \right] c^V_{\pi\eta^{(\prime)}} F_+^{\pi\eta^{(\prime)}}(s) + \frac{\Delta^{QCD}_{K^0 K^+}}{s} q^\mu c^S_{\pi^-\eta^{(\prime)}} F_0^{\pi^-\eta^{(\prime)}}(s) \, .$$

  - The finiteness of the matrix element at the origin imposes

$$F_+^{\pi^-\eta^{(\prime)}}(0) = -\frac{c^S_{\pi^-\eta^{(\prime)}} \Delta^{QCD}_{K^0 K^+}}{c^V_{\pi^-\eta^{(\prime)}} \Delta_{\pi^-\eta^{(\prime)}}} F_0^{\pi^-\eta^{(\prime)}}(0) \, , \quad \Delta_{PQ} = m_P^2 - m_Q^2 \, . \quad (1)$$

- Differential decay width as a function of the $\pi^- \eta^{(\prime)}$ invariant mass

$$\frac{d\Gamma\left(\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau\right)}{d\sqrt{s}} = \frac{G_F^2 M_\tau^3}{24\pi^3 s} S_{EW} |V_{ud} F_+^{\pi^-\eta^{(\prime)}}(0)|^2 \left(1 - \frac{s}{M_\tau^2}\right)^2$$

$$\left\{ \left(1 + \frac{2s}{M_\tau^2}\right) q^3_{\pi^-\eta^{(\prime)}}(s) |\widetilde{F}_+^{\pi^-\eta^{(\prime)}}(s)|^2 + \frac{3\Delta^2_{\pi^-\eta^{(\prime)}}}{4s} q_{\pi^-\eta^{(\prime)}}(s) |\widetilde{F}_0^{\pi^-\eta^{(\prime)}}(s)|^2 \right\} ,$$

with $q_{PQ}(s) = \frac{\sqrt{s^2 - 2s(m_P^2 + m_Q^2) + \Delta^2_{PQ}}}{2\sqrt{s}}$, and where $\widetilde{F}_{+,0}^{\pi^-\eta^{(\prime)}}(s) = \frac{F_{+,0}^{\pi^-\eta^{(\prime)}}(s)}{F_{+,0}^{\pi^-\eta^{(\prime)}}(0)}$, are the two form factors normalised to unity at the origin.

Form factors parameterizations

- Vector Form Factor
  - Chiral Perturbation Theory with Resonances [1]

we employ data from $\tau^- \to \pi^- \pi^0 \nu_\tau$

$$F_+^{\pi^-\eta}(s) = \varepsilon_{\pi\eta^{(\prime)}} \left[1 + \sum_V \frac{F_V G_V}{F^2} \frac{s}{M_V^2 - s} \right] = \varepsilon_{\pi\eta^{(\prime)}} F_+^{\pi^-\pi^0}(s) \, . \quad (2)$$

where $\varepsilon_{\pi\eta}$ and $\varepsilon_{\pi\eta'}$ accounts for the $\pi - \eta^{(\prime)}$ mixing.

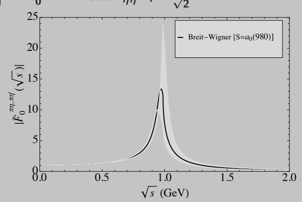
- Scalar Form Factor
  - Chiral Perturbation Theory with Resonances [1]

$$F_0^{\pi^-\eta^{(\prime)}}(s) = c_0^{\pi^-\eta^{(\prime)}} \frac{M_S^2 + \Delta_{\pi^-\eta^{(\prime)}}}{M_S^2 - s - iM_S\Gamma_S(s)} \, , \quad c_0^{\pi^-\eta} = \cos\theta_{\eta\eta'} - \sqrt{2}\sin\theta_{\eta\eta'} \, , \; c_0^{\pi^-\eta'} = \cos\theta_{\eta\eta'} + \frac{\sin\theta_{\eta\eta'}}{\sqrt{2}} \, , \quad (3)$$

From Eqs. (1,2,3) we find

$\varepsilon_{\pi\eta} = (9.8 \pm 0.3) \cdot 10^{-3}$

$\varepsilon_{\pi\eta'} = (2.5 \pm 1.5) \cdot 10^{-4}$



  - Dispersion Relations
    - Elastic unitarity: Omnès equation

$$F_0^{\pi^-\eta^{(\prime)}}(s) = P(s) \exp\left[\frac{s - s_0}{\pi} \int_{s_{th}}^{\infty} ds' \frac{\delta_{1,0}^{\pi^-\eta^{(\prime)}}(s')}{(s' - s_0)(s' - s - i\varepsilon)} \right] ,$$

where $\delta_{1,0}^{\pi^-\eta^{(\prime)}}(s) = \arctan \frac{\mathrm{Im}t_{1,0}^{\pi^-\eta^{(\prime)}}(s)}{\mathrm{Re}t_{1,0}^{\pi^-\eta^{(\prime)}}(s)}$ with $t^{\pi^-\eta^{(\prime)}}(s)$ from Ref. [2].

    - Inelastic cuts: $\pi^-\eta$ coupled to $\pi^-\eta'$ and/or K^-K^0 and viceversa [1].

Results: Branching ratio predictions

- $\tau^- \to \pi^- \eta \nu_\tau$

Scalar Form factor	Contribution (10^{-5})	Total (10^{-5})
ChPT (1 resonance)	$0.72^{+0.46}_{-0.22}$	0.98(51)
ChPT (2 resonances)	$0.48^{+0.29}_{-0.14}$	0.74(32)
Elastic approximation (Omnès)	$0.10^{+0.02}_{-0.03}$	0.36(4)
$\pi^-\eta$ coupled to $\pi^-\eta'$	0.15(9)	0.41(9)
$\pi^-\eta$ coupled to K^-K^0	1.86(11)	2.12(11)
$\pi^-\eta$ coupled to $\pi^-\eta'$ and K^-K^0	1.41(9)	1.67(9)

Table 1: Total branching ratio includes the vector, $\mathbf{0.26(2) \cdot 10^{-5}}$, and the scalar contributions.

- $\tau^- \to \pi^- \eta' \nu_\tau$

Scalar Form factor	Contribution
ChPT (1 resonance)	$[2 \cdot 10^{-11}, 7 \cdot 10^{-10}]$
ChPT (2 resonances)	$[5 \cdot 10^{-11}, 2 \cdot 10^{-9}]$
Elastic approximation (Omnès)	$[2 \cdot 10^{-9}, 4 \cdot 10^{-8}]$
$\pi^-\eta$ coupled to $\pi^-\eta'$	$[2 \cdot 10^{-7}, 2 \cdot 10^{-6}]$
$\pi^-\eta$ coupled to K^-K^0	$[3 \cdot 10^{-7}, 3 \cdot 10^{-6}]$
$\pi^-\eta$ coupled to $\pi^-\eta'$ and K^-K^0	$[1 \cdot 10^{-7}, 1 \cdot 10^{-6}]$

Table 2: Scalar contribution to the branching ratio. The (subleading) vector gives $\mathbf{[0.3, 6] \cdot 10^{-11}}$.

Results: Invariant mass distribution

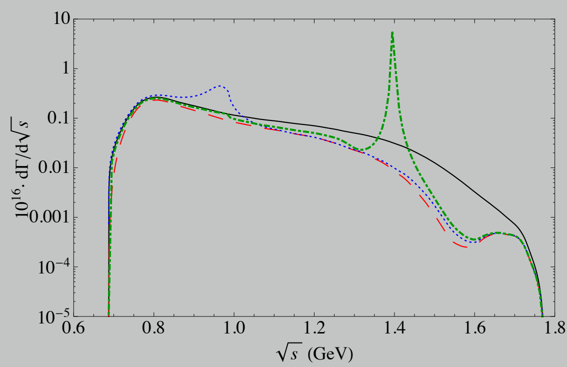


Figure 1: $\tau^- \to \pi^- \eta \nu_\tau$ distribution. We show the vector contribution (red-dashed curve) and the full distribution as obtained by employing: the scalar form factor in its elastic version (black solid curve), the three coupled-channels analysis (green dot-dashed curve) and ChPT with two resonances (blue dotted curve).

Conclusions

- We focus on the Standard Model prediction of $\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau$ decays.
- We have addressed the description of the participant vector and scalar form factors based on Chiral Perturbation Theory with resonances supplemented by dispersion relations.
- According to our results, the discovery of second-class currents might be possible at Belle-II thanks to the increased luminosity with respect to its predecessors.

References

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2 Z. H. Guo and J. A. Oller, Phys. Rev. D 84, 034005 (2011).

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