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TMD Functions: Status and Prospects
Outline

❖ Factorization theorems with TMDs
❖ Evolution and OPE of TMDs
❖ The NNLO and N3LL’ new results
❖ Conclusions
DY, SIDIS, e+e- to 2 hadrons, TMD factorization

TMDs offer a unified description of very different experiments, run at different energy scales.

\[ q^2 = Q^2 \gg q_T^2 \]

Example: DY type experiments

SIDIS

e+e- to 2 hadrons

TMDs go beyond the usual parton distribution formalism. New factorization theorem! (Collins '11; Echevarría, Idilbi, S. '12 (EIS))

\[ q_T^2 \sim \Lambda_{QCD}^2 \]

\[ d\sigma \sim H(Q) \int d^2 b_T e^{-iq_T b_T} F(x; b_T) D(z; b_T) \]

Each mode has the same invariant mass, but different rapidity structure. Rapidity divergences need an appropriate regulator. Status: NNLO for all unpolarized TMDs.
A lot of TMD’s: Spin Fun

Quark Polarization

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Similar structures for

Gluons as initial states: in

Higgs production both \( f^g_1 \) and \( h^g_1 \)

TMDFF

\[
\tilde{D}^{[\gamma^+ \gamma_5]}_{h/f}(z, b_\perp, S_h; \zeta_D, \mu^2) = \tilde{D}_1 - \frac{\epsilon^{ij}_L b_{iL} S_{jL}}{(-ib_T)M_h} \tilde{D}^\perp_{1T}(1),
\]

\[
\tilde{D}^{[\gamma^- \gamma_5]}_{h/f}(z, b_\perp, S_h; \zeta_D, \mu^2) = \lambda \tilde{G}_{1L} + \frac{(b_{iL} \cdot S_{jL})}{(-ib_T)M_h} \tilde{G}^{(1)}_{1T},
\]

\[
\tilde{D}^{[\gamma^\pm \gamma_5]}_{h/f}(z, b_\perp, S_h; \zeta_D, \mu^2) = S_{h\perp} \tilde{H}_1 + \frac{\lambda b_{iL}}{(-ib_T)M_h} \tilde{H}^\perp_{1L}(1),
\]

\[
\tilde{D}^{[\gamma^\pm \gamma_5]}_{h/f}(z, b_\perp, S_h; \zeta_D, \mu^2) = \left(\frac{b_{iL} b^j_L + \frac{1}{2} b_T^2 g_{iL}^{(ij)}}{(-ib_T)^2 M^2_h}\right) \tilde{H}_1^{\perp(2)} - \frac{\epsilon^{ij}_L b_{iL} S_{jL}}{(-ib_T)M_h} \tilde{H}^\perp_{1T} - \frac{\epsilon^{ij}_L b_{iL} S_{jL}}{(-ib_T)M_h} \tilde{H}^\perp_{1T}(1)
\]

T-odd distributions

\( f^\perp_{1T, DIS} = -f^\perp_{1T, DY} \)

...but just one Soft Function!
TMD’s factorization and OPE: general outlook

Factorized hadronic tensor

\[ q^2 = Q^2 \gg q_T^2 \]

\[ \tilde{M} = \tilde{C}_{n/j}(b^2 \mu^2, Q^2 \mu^2) f_{j/h}(x_n; \mu^2) + O(b^2/B^2) \]

All coefficients are extracted matching effective field theories. During the matching the IR parts have to be regulated consistently above and below the matching scales.
DY, SIDIS, e+e- to 2 hadrons, TMDs status

A complete analysis of the TMDs requires:

- A complete knowledge of the perturbative structure of TMDs at NNLO: the perturbative knowledge should be maximally implemented in programs (Required to match the LHC and future colliders program).

- Consequently, a correct estimate of perturbative QCD errors and understanding of model dependence (see for instance: D’Alesio, Echevarría, Melis, S., JHEP 1411 (2014) 098 and arXiv:1510.0288)

The status of perturbative knowledge at NNLO is homogeneous (only for the unpolarized case!)

Construction of (un)polarized TMDPDFs

In the asymptotic limit (High \( Q, q_T \)) of each TMDPDF

\[
\tilde{F}_{q/N}(x, b_T; \zeta, \mu) = \left( \frac{\zeta}{C_{\zeta} \mu_b^2} \right)^{-D_R(b_T; \mu)} \sum_{j=q,\bar{q},g} \tilde{C}_{q/j} \left( x_A, b_T; C_{\zeta} \mu_b^2, \mu \right) \otimes f_{j/N}(x_N; \mu) M(x_N, b_T, \zeta)
\]

\( OPE \) to PDF, valid ONLY for \( q_T >> \Lambda_{QCD} \)

This construction formally recovers the perturbative limit.

\[
\begin{align*}
\tilde{C}^{Q}_{g \leftarrow g} &= O(\alpha^0) \\
\tilde{C}^{Q}_{q \leftarrow q} &= O(\alpha_s^0) \\
\tilde{C}^{Q}_{g \leftarrow q} &= O(\alpha^1) \\
\tilde{C}^{Q}_{q \leftarrow g} &= O(\alpha_s^1) \\
\tilde{C}^{Q}_{q \leftarrow \bar{q}} &= O(\alpha_s^2) \\
\tilde{C}^{Q}_{q \leftarrow q'} &= O(\alpha_s^2)
\end{align*}
\]

Starting order

\[ \tilde{F}_{q/N}(x, b_T; \zeta, \mu) \]

Process independent
Non-perturbative correction

2-loop matching of PDFs deduced from the calculation of the cross section [Firenze (Catani et al. 2008)], or products of TMDs [Zurich (Gehrmann. et al. 2012-2014)].

M.G. Echevarría, I.S., A. Vladimirov: 1604.07869!

Direct application of the TMD formalism and agreement with previous works

Status: This formula predicts that one TMDPDF matches onto a sum of PDFs (FLAVOR MIXING).
Currently all analysis of low energy data have fully exploited this up to first order.
Construction of (un)polarized TMDFFs

In the asymptotic limit (High $Q$, $qT$) of each TMDFF

\[
\tilde{D}_{q/N}(z, b_T; \zeta, \mu) = \left( \frac{\zeta}{\mu_b} \right)^{-D(b, \mu)} \sum_j \int_z^1 \frac{d\tau}{\tau^{3-2\varepsilon}} \mathcal{C}^{Q}_{q \rightarrow j} \left( \frac{z}{\tau}, b_T; \mu_b, \mu \right) d_{j/N}(\tau, \mu) M_j(\tau, b_T, \zeta)
\]

OPE to FF, valid ONLY for $qT >> \Lambda_{QCD}$

This construction formally recovers the perturbative limit.

Starting order $O(\alpha_s^2)$?

\(\mathcal{O}(\alpha_s^0)\)
\(\mathcal{O}(\alpha_s^1)\)
\(\mathcal{O}(\alpha_s^2)\)

(Recent phenomenological work
Bacchetta et al., 2015, in e+e-)

**Status:** This formula predicts that one TMDFF matches onto a sum of FFs. (FLAVOR MIXING)
Currently all analysis of low energy data have fully exploited this up to first order
Factorization theorem basics: DIS case

\[ d\sigma \sim \int d^4xe^{iqx} \sum_X \langle h_1|J^\mu(x)|X, h_2\rangle \langle X, h_2|J^\nu(0)|h_1\rangle \]

\[ d\sigma \sim \int d^2b_Te^{-iqT}H(Q^2)\Phi_{h_1}(z_1, b_T)S(b_T)\Delta_{h_2}(z_2, b_T) + Y \]

❖ PDF, Soft Factor, FF have “rapidity divergences”
❖ Soft Factor mixes PDF and FF: no “real factorization”, however ..
❖ We have the splitting of rapidity singularities in the Soft Factor:

\[ S(b_T) = \sqrt{S(b_T, \zeta)}\sqrt{S(b_T, \zeta^{-1})} \]

\[ d\sigma \sim H(Q) \int d^2b_Te^{-iqT}F(x; b_T)D(z; b_T) \]

Each TMD is now free of rapidity singularity: nice renormalizable non-perturbative object
Operator definitions of TMDs

bare TMDPDF

\[ O^{\text{bare}}_q(x, b_T) = \frac{1}{2} \sum_X \int \frac{d\xi^-}{2\pi} e^{-ixp^+\xi^-} \left\{ T \left[ \bar{q}_i W_n^{T^+} \right]_a \left( \frac{\xi}{2} \right) |X\rangle \gamma_{ij}^+ \langle X | \bar{T} [W_n^{T^+} q_j]_a \left( -\frac{\xi}{2} \right) \right\} \]

bare TMDFF

\[ O^{\text{bare}}_q(z, b_T) = \frac{1}{4zN_c} \sum_X \int \frac{d\xi^-}{2\pi} e^{-ip^+\xi^- / z} \langle 0 | T [W_n^{T^+} q_j]_a \left( \frac{\xi}{2} \right) |X, \frac{\delta}{\delta J} \rangle \gamma_{ij}^+ \langle X, \frac{\delta}{\delta J} | \bar{T} [\bar{q}_i W_n^{T^+}]_a \left( -\frac{\xi}{2} \right) |0\rangle \]

**Formal definition of TMD operator**

Applying these operators to the hadron states we obtain unsubtracted TMDs

\[ \Phi_{q \rightarrow h}(x, b_T) = \langle h | O^{\text{bare}}_q(x, b_T) | h \rangle \]

\[ \Delta_{q \rightarrow h}(z, b_T) = \langle h | O^{\text{bare}}_q(z, b_T) | h \rangle \]

To define individual TMD we have to take into account rapidity divergences, UV divergences and overlap regions

\[ F_{q \rightarrow h}(x, b_T; \zeta, \mu) = \sqrt{S(b_T; \zeta)} \langle h | Z_q(\mu) O^{\text{bare}}_q(x, b_T) | h \rangle \bigg|_{\text{zero-bin}} \]

\[ D_{q \rightarrow h}(x, b_T; \zeta, \mu) = \sqrt{S(b_T; \zeta)} \langle h | Z_q(\mu) O^{\text{bare}}_q(x, b_T) | h \rangle \bigg|_{\text{zero-bin}} \]

- \( \mu \) is scale of UV renormalization.
- \( \zeta \) is scale of rapidity-divergences separation.
Operator definitions of TMDs

\[ O_q(x, b_T, \mu, \zeta) = Z_q(\zeta, \mu) R_q(\zeta, \mu) O_{q \text{ bare}}^q(x, b_T) \]

\[ Q_q(z, b_T, \mu, \zeta) = Z_q(\zeta, \mu) R_q(\zeta, \mu) Q_{q \text{ bare}}^q(z, b_T) \]

**Universal** (=process independent) subtraction constants

\[ R_q = \frac{\sqrt{S(b_T)}}{\text{zero - bin}} \]

Rapidity divergences subtraction

\[ Z_q \]

UV divergences subtraction

**Tilted WL’s [Collins]**

\[ R_q(\zeta, \mu) = \frac{\sqrt{S(b_T; +\infty, y_s)}}{\sqrt{S(b_T; +\infty, -\infty)S(b_T; y_s, -\infty)}} \]

\[ \zeta \sim m^2 e^{-2y_s} \]

**δ-regularization [EIS]**

zero-bin coincides with soft-factor

\[ R_q(\zeta, \mu) = \frac{1}{\sqrt{S(b_T; \alpha \delta^+, \delta^+)}} \]

\[ \zeta \sim \alpha Q^2 \]
Rapidity divergences and the Soft Function

\[ S(b_T) = \frac{1}{N_c} \langle 0 | [\infty_\eta, b_T, \infty_\eta] [\infty_\eta, 0, -\infty_\eta] | 0 \rangle, \quad [\gamma] \sim P \exp \left( -ig \int_\gamma A_\gamma \right) \]

- The rapidity divergences arise in the limit \( k^+ \to \infty, \; k^- \to 0, \; k^+ k^- \) fixed
- The Soft Function is undefined without a regulator for rapidity divergences
- The log of the Soft Function is linear in the rapidity regulator at all orders
  \[ \tilde{S}(L_\mu, L_{\sqrt{\delta^+ \delta^-}}) = \tilde{S}^{\frac{1}{2}}(L_\mu, L_{\delta^+}) \tilde{S}^{\frac{1}{2}}(L_\mu, L_{\delta^-}) \]
- By definition the Soft Function is just 1 in the limit \( b_T \to 0 \)

\[ L_X \equiv \ln(X^2 b_T^2 e^{-2\gamma E} / 4) \]
\[ l_\zeta = \ln(\mu^2 / \zeta) \]
One loop Soft Function


\[ S^v_1 = -2i g_s^2 C_F \delta^{(2)}(\vec{k}_{s\perp}) \mu^{2\varepsilon} \int \frac{d^d k}{(2\pi)^d} \frac{1}{[k^+ - i\delta^+][k^- + i\delta^-][k^2 - \lambda^2 + i0]} + h.c. \]

\[ S^T_1 = 4\pi g_s^2 C_F \mu^{2\varepsilon} \int d^d k_s e^{ik_s b} \int \frac{d^d k}{(2\pi)^d} \frac{\delta^{(2)}(k + k_s)\delta(k^2 - \lambda^2)\theta(k^+)}{(k^+ + i\delta^+)(k^- - i\delta^-)} + h.c. \]

\( \delta : \) UV-Dim.Reg.
\( \lambda : \) IR div.
\( \delta^{\pm} : \) rapidity div.

(a) (b)
One loop Soft Function


\[ \tilde{S}_1^v = \frac{\alpha_s C_F}{2\pi} \left[ \frac{-2}{\varepsilon_{UV}^2} + \frac{2}{\varepsilon_{UV}} \ln \frac{\delta^+ \delta^-}{\mu^2} + \ln^2 \frac{\lambda^2}{\mu^2} - 2\ln \frac{\lambda^2}{\mu^2} \ln \frac{\delta^+ \delta^-}{\mu^2} + \frac{\pi^2}{6} \right] \]

\[ \tilde{S}_1^r = \frac{\alpha_s C_F}{2\pi} \left[ L_\perp^2 + 2L_\perp \ln \frac{\delta^+ \delta^-}{\mu^2} + 2\ln \frac{\lambda^2}{\mu^2} \ln \frac{\delta^+ \delta^-}{\mu^2} - \ln^2 \frac{\lambda^2}{\mu^2} \right] \]

Summing up only UV and rapidity divergences survives:

\[ \tilde{S}_1 = \frac{\alpha_s C_F}{2\pi} \left[ \frac{-2}{\varepsilon_{UV}^2} + \frac{2}{\varepsilon_{UV}} \ln \frac{\delta^+ \delta^-}{\mu^2} + L_\perp^2 + 2L_\perp \ln \frac{\delta^+ \delta^-}{\mu^2} + \frac{\pi^2}{6} \right] \]

\( \varepsilon_{UV} \): Dim. Reg.
\( \lambda \): IR div.
\( \delta^\pm \): rapidity div.

\( L_\perp = \ln \frac{\mu^2 b_T^{-2} e^{2\gamma_E}}{4} \)

To compute a NNLO calculation we need this result at all orders in \( \varepsilon \)
One loop Soft Function

Modified delta-regularization:

\[ P \exp \left[ -ig \int_0^\infty d\sigma A_\pm (\sigma n) \right] \rightarrow P \exp \left[ -ig \int_0^\infty d\sigma A_\pm (\sigma n) e^{-\delta^\pm |\sigma|} \right] \]

\[ S_v^{[1]} = -\frac{2g^2}{(4\pi)^d/2} \delta^{-\epsilon} \Gamma^2(\epsilon) \Gamma(1-\epsilon) + h.c. \]

\[ S_r^{[1]} = \frac{2g^2}{(4\pi)^d/2} \left[ \delta^{-\epsilon} \Gamma^2(\epsilon) \Gamma(1-\epsilon) - B^\epsilon \Gamma(-\epsilon) \left( L_+ - \psi(-\epsilon) - \gamma_E \right) \right] + h.c. \]

Summing up only UV and rapidity divergences survive:

\[ S^{[1]} = -4B^\epsilon \Gamma(-\epsilon) \left( L_0 - \psi(-\epsilon) - \gamma_E \right) \]

\[ \delta = \pm \delta^+ \delta^-, \quad B = \frac{b_T^2}{4} \]

\[ L_0 = \ln \left( \frac{B |\delta|}{e^{-2\gamma_E}} \right) \]

Mixed divergences cancel at all orders

The trivial limit (\( b_T = 0 \)) easily recovered
Two loops Soft Function

Modified delta-regularization preserves abelian exponentiation

The structure of each diagram recalls the one-loop result

\[
\text{Diagram} = \mu^{2\epsilon} \left( A_1 \delta^{-2\epsilon} + A_2 \delta^{-\epsilon} B^\epsilon + A_3 B^{2\epsilon} \right)
\]

\( A_1, A_2 = 0 \) in the sum of all diagrams
\( A_3 \) in the sum of all diagrams is linear in rapidity logs

We can omit calculation of virtual diagrams

The Soft Function is linear in rapidity logs at all orders in

Full result in Echevarría, S., Vladimirov  arXiv:1511.05590
Two loops Soft Function

The D-function which governs the TMD evolution kernel is the finite part of

\[ D = \sum_{n} \sum_{k=0}^{n} \alpha_s^n L^k \mu \delta^{(n,k)} \]

\[ D = \frac{1}{2} \frac{d \ln \tilde{S}}{d \ln \delta} \bigg|_{finite} \]

First direct calculation of the D-function at two loops


\[ d^{(2,2)} = \frac{\Gamma^{(0)} \beta_0}{4} = C_F \left( \frac{11}{3} C_A - \frac{4}{3} T_R N_f \right) \]

\[ d^{(2,1)} = \frac{\Gamma^{(1)}}{2} = 2C_F \left( \left( \frac{67}{9} - \frac{\pi^2}{3} \right) C_A - \frac{20}{9} T_R N_f \right) \]

\[ d^{(2,0)} = C_F \left( \left( \frac{404}{27} - \frac{14 \zeta_3}{17} \right) C_A - \frac{112}{27} T_R N_f \right). \]

First deduced by Becher, Neubert ‘10
Two loops Soft Function

The D-function which governs the TMD evolution kernel is the finite part of

\[ D = \frac{1}{2} \frac{d \ln \tilde{S}(b_T)}{d \ln \delta} \bigg|_{finite} \]

Comparing quark and gluon soft functions, the difference is just the Casimir scaling

The Casimir scaling of D’s is valid at all order in perturbation theory

\[ \frac{D}{C_F} = \frac{D_g}{C_A} \]
TMDFF at NNLO

The Universal Soft function (Spin independent)

The unsubstracted TMDs

\[ \tilde{D}_{i \rightarrow h}(z, L_\mu, l_{\zeta D}) = \tilde{\Delta}^{(0)}_{i \rightarrow h}(z, L_\mu, l_{\zeta D}, L_{\delta^+}) \hat{S}^{1/2}(L_\mu, L_{\delta^+}) \]

\[ \tilde{D}_{i \rightarrow h}(z, L_\mu, l_{\zeta}) = \int_z^1 \frac{d\tau}{\tau^{3-2\epsilon}} C_{i \rightarrow j} \left( \frac{z}{\tau}, L_\mu, l_{\zeta} \right) d_{j \rightarrow h}(\tau, \mu) \]
TMDFF expansion

Perturbative expansion:

\[ \tilde{C}_{j \rightarrow f}^{[0]} = \delta(1 - z) \]

\[ \tilde{C}_{i \rightarrow j}^{[1]} = \tilde{D}_{i \rightarrow j}^{[1]} - \frac{d_{i \rightarrow j}^{[1]}}{z^{2 - 2\epsilon}} \]

\[ \tilde{C}_{i \rightarrow f}^{[2]} = \tilde{D}_{i \rightarrow f}^{[2]} - \tilde{C}_{i \rightarrow k}^{[1]} \otimes \frac{d_{k \rightarrow f}^{[1]}}{z^{2 - 2\epsilon}} - \frac{d_{i \rightarrow f}^{[2]}}{z^{2 - 2\epsilon}} \]

Missing piece

To recover the SF from the soft limit of FF (zero-bin correspondence):

\[ \frac{1}{(k_1^+ - i0)(k_2^+ - i0) \ldots (k_n^+ - i0)} \rightarrow \frac{1}{(k_1^+ - i\delta^+/z)(k_2^+ - 2i\delta^+/z) \ldots (k_n^+ - ni\delta^+/z)} \]
RGE for TMD’s

\[
\mu^2 \frac{d}{d\mu^2} \tilde{D}_{i \rightarrow h} = \frac{1}{2} \gamma_D^i \tilde{D}_{i \rightarrow h}
\]
\[
\gamma_D = \Gamma_{cusp}^i \zeta - \gamma_V^i
\]
\[
\zeta \frac{d}{d\zeta} \tilde{D}_{i \rightarrow h} = -\mathcal{D}^i \tilde{D}_{i \rightarrow h}, \quad 2\mu^2 \frac{d}{d\mu^2} \mathcal{D}^i = \Gamma_{cusp}^i
\]

RGE for TMD fragmentation

\[l_X \equiv \ln(\mu^2/X)\]

RGE for coefficients

\[
\zeta \frac{d}{d\zeta} \tilde{C}_{i \rightarrow j} = -\mathcal{D}^i \tilde{C}_{i \rightarrow j}
\]
\[
\mu^2 \frac{d}{d\mu^2} \tilde{C}_{i \rightarrow j} = \tilde{C}_{i \rightarrow k} \otimes \mathcal{K}_{k \rightarrow j}
\]
\[
\mathcal{K}_{k \rightarrow j}(z) = \frac{\delta_{kj}}{2} (\Gamma_{cusp}^i \zeta - \gamma_V^i) \delta(\bar{z}) - \frac{\mathcal{P}_{k \rightarrow j}(z)}{z^2}
\]

Re-writing of the coefficient

\[
\tilde{C}_{i \rightarrow j} = \exp \left[ -\mathcal{D}^i \mathbf{L} \sqrt{\zeta} \right] \tilde{C}_{i \rightarrow j}
\]
Structure of the result

Log structure of the result

\[ \tilde{C}_{i,j}^{[n]} = \sum_{k=0}^{2n} \tilde{C}_{i,j}^{(n;k)} L_k^\mu \]

Recursion relation

\[ (k+1) \tilde{C}_{i \rightarrow j}^{(n;k+1)} = \sum_{r=1}^{n} \frac{\Gamma_{\text{cusp}}^{[r]}}{2} \tilde{C}_{i \rightarrow j}^{(n-r;k-1)} \]
\[ - \frac{\gamma_i^{[r]}}{2} - 2(n-r)\beta_i^{[r]} \tilde{C}_{i \rightarrow j}^{(n-r;k)} - \tilde{C}_{i \rightarrow k}^{(n-r;k)} \otimes \frac{\mathcal{P}_{k \rightarrow j}^{[r]}}{z^2} \]

The original piece of the 2-loop coefficient is

\[ \tilde{C}_{q \rightarrow q}^{(2;0)}(z) = C_F^2 Q_F(z) + C_F C_A Q_A(z) + T_R N_f Q_N(z) \]
\[ \tilde{C}_{q \rightarrow \bar{q}}^{(2;0)}(z) = C_F \left( C_F - \frac{C_A}{2} \right) Q_{q \bar{q}}(z) \]
Sample of the result

\[ \tilde{C}_{ij}^{[n]} = \sum_{k=0}^{2n} \tilde{C}_{ij}^{(n;k)} L^k_{\mu} \]

Log structure of the result

\[ \tilde{C}_{q\rightarrow q}^{(2;0)}(z) = C_F^2 Q_F(z) + C_F C_A Q_A(z) + T_R N_f Q_N(z), \]
\[ \tilde{C}_{q\rightarrow \bar{q}}^{(2;0)}(z) = C_F \left( C_F - \frac{C_A}{2} \right) Q_{q\bar{q}}(z). \]

\[ Q_N(z) = \frac{1}{z^2} \left[ \left( \frac{2}{3} \ln^2 z - \frac{20}{3} \ln z + \frac{112}{27} \right) p(z) - \frac{16}{3} \bar{z} \ln \bar{z} - \frac{4}{3} \bar{z} \right] + \]
\[ + \delta(\bar{z}) \left( -\frac{2717}{162} + \frac{25\pi^2}{9} + \frac{52}{9} \zeta_3 \right) \]

Full result in arXiv: Echevarría, S., Vladimirov 1509.06392, 1604.07869
Conclusions

❖ The comprehension of transverse momentum spectra for all energy ranges requires the correct inclusion of non-perturbative effects: TMDs.

❖ The check of the universality of TMD evolution requires the same degree of precision for TMDPDF and TMDFF: NNLO.

❖ The perturbative part of TMDs should be used at highest available order to control the perturbative series (NNLL only achieved in a limited set of TMDs): At present the TMD evolution kernel is known at N3LO! Casimir scaling of the D function established at all orders in perturbation theory.)

❖ The control of perturbative error is fundamental to understand the nature of non-perturbative effects.

❖ We have completed the calculation of the universal Soft Function and the matching of the ALL unpolarized TMDFF onto FF at NNLO using the EIS formulation. (TMDPDF onto PDF also checked.) We can perform a complete N3LL’ analysis (only 4-loop cusp is missing, but one can use Padé estimations)

❖ The Soft Function can be used for the evaluation of the matching of all (un)polarized TMDs. One can reach in principle the same perturbative precision

❖ The implementation of all this information in an accepted TMD which includes also non-perturbative QCD effect is still work in progress (both theoretically and experimentally)

Thanks!!!
TMDF expansion

1-loop

\[ D_{q/q}^{[1]} = \tilde{\Delta}_{q/q}^{[1]} - \tilde{S}_+^{[1]} - Z_2^{[1]} + Z_D^{[1]} \]

Rapidity divergences cancel here!

2-loops

\[ \tilde{D}_{i/f}^{[2]} = (\tilde{\Delta}_{i/f}^{[2]} - \tilde{S}_{+}^{[1]} \tilde{\Delta}_{i/f}^{[1]} - \tilde{S}_{+}^{[2]} \delta_{i/f} + \frac{3\tilde{S}_{+}^{[1]} \tilde{S}_{+}^{[1]}}{2} \delta_{i/f} + \left( Z_D^{[1]} - Z_2^{[1]} \right) \left( \tilde{\Delta}_{i/f}^{[1]} - \frac{\tilde{S}_{+}^{[1]} \delta_{i/f}}{2} \right) + \left( Z_D^{[2]} - Z_2^{[2]} - Z_2^{[1]} Z_D^{[1]} + Z_2^{[1]} Z_2^{[1]} \right) \delta_{i/f}. \]
An example: Z-production at LHC

D\textsuperscript{Y}Res results: $q_T$ spectrum of Z boson at the LHC

G. Ferrera talk at REF2015

Main differences
D\textsuperscript{Y}Res includes full 2-loop matching (NNLO) onto PDFs, but no low energy DY infos. Minimal Subtraction.

D' Alesio et al 2014, partially includes the 2-loop matching (NNLL) and full low energy DY infos (2 parameters)

A TMD approach has the potential to drastically reduce errors!
We can include QCD infos at different energies and experiments

Caveat: THE COMPLETE NNLO PERTURBATIVE QCD INFORMATION MUST BE INCLUDED