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TMD Functions: Status and Prospects

Outline

- ❖ Factorization theorems with TMDs
- ❖ Evolution and OPE of TMDs
- ❖ The NNLO and N3LL' new results
- ❖ Conclusions

DY, SIDIS, e+e- to 2 hadrons, TMD factorization

TMDs offer a unified description of very different experiments, run at different energy scales

$$q^2 = Q^2 \gg q_T^2$$

TMD's go beyond the usual parton distribution formalism. **New factorization theorem!** (Collins '11; Echevarría, Idilbi, S. '12 (EIS))

$$q_T^2 \sim \Lambda_{QCD}^2$$

$$d\sigma \sim H(Q) \int d^2 b_T e^{-iq_T b_T} F(x; b_T) D(z; b_T)$$

Example: DY type experiments

SIDIS

e+e- to 2 hadrons



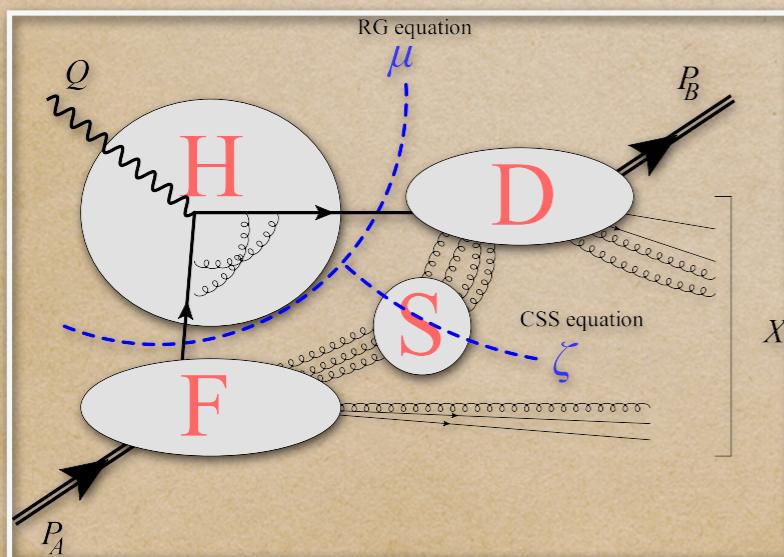
TMDPDFs



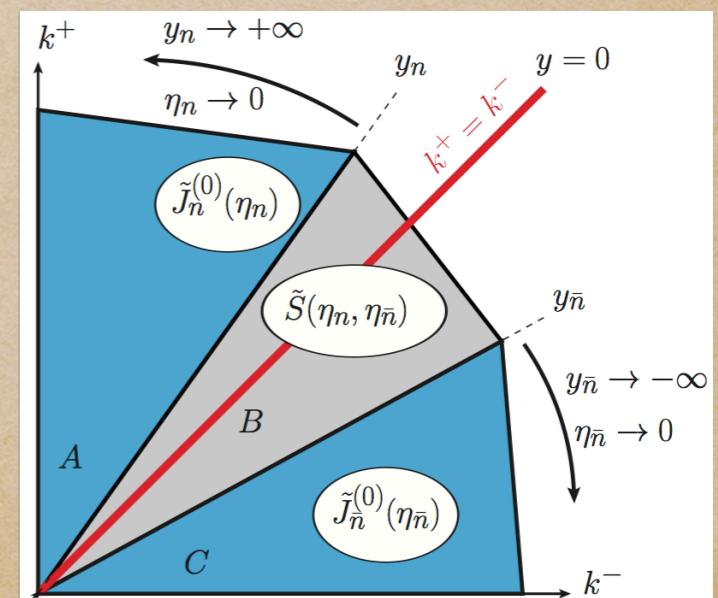
TMDPDF and TMDFF



TMDFFs



Each mode has the same invariant mass, but different rapidity structure.
Rapidity divergences need an appropriate regulator.
Status: NNLO for all unpolarized TMDs



A lot of TMD's: Spin Fun

Quark Polarization

TMDPDF

Nucleon
Polarization

	U	L	T
U	f_1		h_1^\perp
L		g_1	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp

$$\begin{aligned}\tilde{F}_{f/N}^{[\gamma^+]}(x, \mathbf{b}_\perp, S; \zeta_F, \mu^2) &= \tilde{f}_1 - \frac{\epsilon_\perp^{ij} \mathbf{b}_{\perp i} \mathbf{S}_{\perp j}}{ib_T M_N} \tilde{f}_{1T}^{\perp(1)}, \\ \tilde{F}_{f/N}^{[\gamma^+ \gamma_5]}(x, \mathbf{b}_\perp, S; \zeta_F, \mu^2) &= \lambda \tilde{g}_{1L} + \frac{(\mathbf{b}_\perp \cdot \mathbf{S}_\perp)}{ib_T M_N} \tilde{g}_{1T}^{(1)}, \\ \tilde{F}_{f/N}^{[i\sigma^i + \gamma_5]}(x, \mathbf{b}_\perp, S; \zeta_F, \mu^2) &= \mathbf{S}_\perp^i \tilde{h}_1 + \frac{\lambda \mathbf{b}_\perp^i}{ib_T M_N} \tilde{h}_{1L}^{\perp(1)} \\ &\quad - \frac{(\mathbf{b}_\perp^i \mathbf{b}_\perp^j + \frac{1}{2} b_T^2 g_\perp^{ij}) \mathbf{S}_{\perp j}}{(i)^2 b_T^2 M_N^2} \tilde{h}_{1T}^{\perp(2)} - \frac{\epsilon_\perp^{ij} \mathbf{b}_{\perp j}}{ib_T M_N} \tilde{h}_1^{\perp(1)}\end{aligned}$$

T-odd distributions

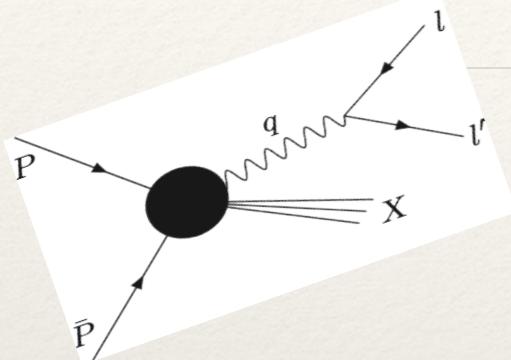
$$f_{1T,DIS}^\perp = -f_{1T,DY}^\perp$$

Similar structures for
Gluons as initial states: in
Higgs production both f_1^g and $h_1^{\perp g}$

TMDFF

$$\begin{aligned}\tilde{D}_{h/f}^{[\gamma^-]}(z, \mathbf{b}_\perp, S_h; \zeta_D, \mu^2) &= \tilde{D}_1 - \frac{\epsilon_\perp^{ij} \mathbf{b}_{\perp i} \mathbf{S}_{\mathbf{h} \perp j}}{(-ib_T) M_h} \tilde{D}_{1T}^{\perp(1)}, \\ \tilde{D}_{h/f}^{[\gamma^- \gamma_5]}(z, \mathbf{b}_\perp, S_h; \zeta_D, \mu^2) &= \lambda \tilde{G}_{1L} + \frac{(\mathbf{b}_\perp \cdot \mathbf{S}_{\mathbf{h} \perp})}{(-ib_T) M_h} \tilde{G}_{1T}^{(1)}, \\ \tilde{D}_{h/f}^{[i\sigma^i - \gamma_5]}(z, \mathbf{b}_\perp, S_h; \zeta_D, \mu^2) &= \mathbf{S}_{\mathbf{h} \perp}^i \tilde{H}_1 + \frac{\lambda \mathbf{b}_\perp^i}{(-ib_T) M_h} \tilde{H}_{1L}^{\perp(1)} \\ &\quad - \frac{(\mathbf{b}_\perp^i \mathbf{b}_\perp^j + \frac{1}{2} b_T^2 g_\perp^{ij}) \mathbf{S}_{\mathbf{h} \perp j}}{(-ib_T)^2 M_h^2} \tilde{H}_{1T}^{\perp(2)} - \frac{\epsilon_\perp^{ij} \mathbf{b}_{\perp j}}{(-ib_T) M_h} \tilde{H}_1^{\perp(1)}\end{aligned}$$

TMD's factorization and OPE: general outlook



Factorized hadronic tensor

$$q^2 = Q^2 \gg q_T^2$$

$Q=M$ =di-lepton invariant mass

Factorization

$$q_T^2 \sim \Lambda_{QCD}^2 \rightarrow \tilde{M} = H(Q^2/\mu^2) \tilde{F}_n(x_n, b; Q^2, \mu^2) \tilde{F}_{\bar{n}}(x_{\bar{n}}, b; Q^2, \mu^2)$$

OPE

$$q_T^2 \gg \Lambda_{QCD}^2 \rightarrow \tilde{F}_n(x_n, b; Q^2, \mu^2) = \tilde{C}_{n/j}(b^2 \mu^2, Q^2 \mu^2) f_{j/h}(x_n; \mu^2) + \mathcal{O}(b^2/B^2)$$

Very important

The factorization theorem predicts that each coefficient can be extracted on its own:
this checked at 2 loops (unpolarized case)

M.G. Echevarría, I.S., A. Vladimirov '15-'16

All coefficients are extracted matching effective field theories. During the matching the IR parts have to be regulated consistently above and below the matching scales

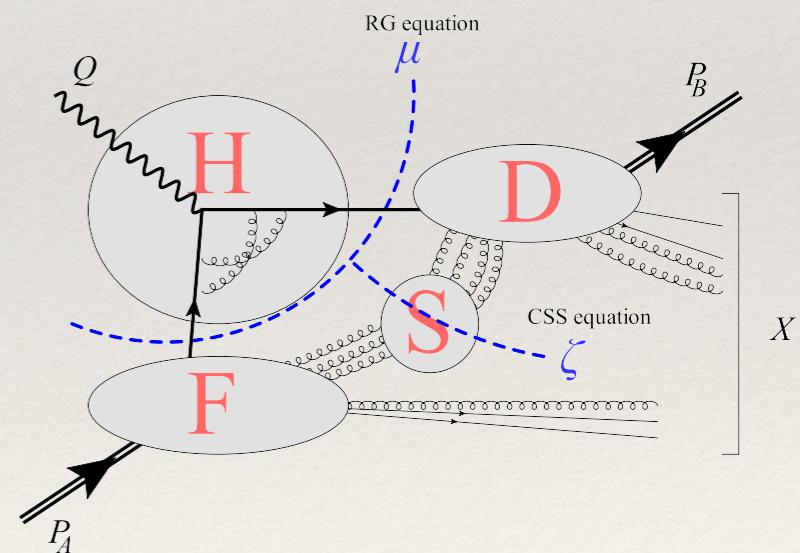
DY, SIDIS, e^+e^- to 2 hadrons, TMDs status

A complete analysis of the TMDs requires:

- ❖ A complete knowledge of the perturbative structure of TMDs at NNLO: the perturbative knowledge should be maximally implemented in programs (Required to match the LHC and future colliders program).
- ❖ Consequently, a correct estimate of perturbative QCD errors and understanding of model dependence (see for instance: D'Alesio, Echevarría, Melis, S., JHEP 1411 (2014) 098 and arXiv:1510.0288)

The status of perturbative knowledge at NNLO is homogeneous (only for the unpolarized case!)

M.G. Echevarría, I.S., A. Vladimirov, arXiv:1604.07869

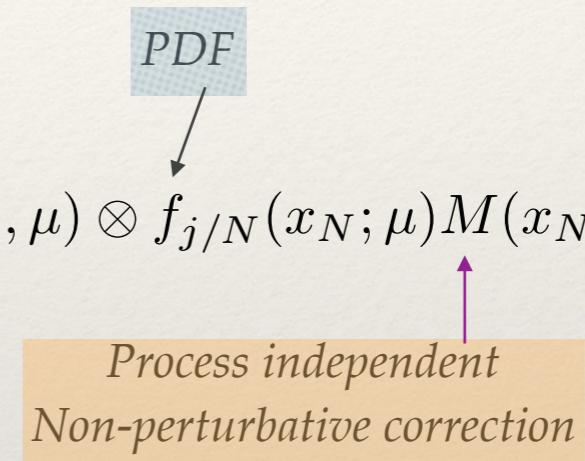


Construction of (un)polarized TMDPDFs

In the asymptotic limit (High Q, qT) of each TMDPDF

$$\tilde{F}_{q/N}(x, b_T; \zeta, \mu) = \left(\frac{\zeta}{C_\zeta \mu_b^2} \right)^{-D^R(b_T; \mu)} \sum_{j=q, \bar{q}, g} \tilde{C}_{q/j}(x_A, b_T; C_\zeta \mu_b^2, \mu) \otimes f_{j/N}(x_N; \mu) M(x_N, b_T, \zeta)$$

OPE to PDF, valid ONLY for $qT \gg \Lambda_{QCD}$



This construction formally recovers the perturbative limit.

2-loop matching of PDFs deduced from the calculation of the cross section [Firenze (Catani et al. 2008)], or products of TMDs [Zurich (Gehrmann. et al. 2012-2014)].

M.G. Echevarría, I.S., A. Vladimirov: 1604.07869!
Direct application of the TMD formalism and agreement with previous works

$C_{g \leftarrow g}^Q = \mathcal{O}(\alpha^0)$ $\tilde{C}_{q \leftarrow q}^Q = \mathcal{O}(\alpha_s^0)$

$C_{g \leftarrow q}^Q = \mathcal{O}(\alpha^1)$ $\tilde{C}_{q \leftarrow g}^Q = \mathcal{O}(\alpha_s^1)$

$\tilde{C}_{q \leftarrow \bar{q}}^Q = \mathcal{O}(\alpha_s^2)$

$\tilde{C}_{q \leftarrow q'}^Q = \mathcal{O}(\alpha_s^2)$

Starting order

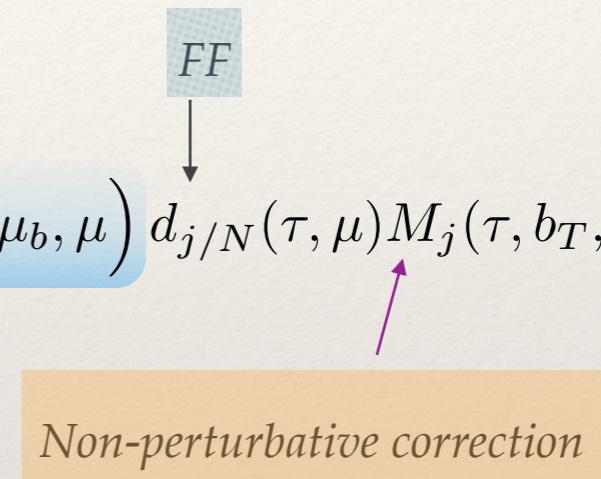
Status: This formula predicts that one TMDPDF matches onto a sum of PDFs (FLAVOR MIXING).
Currently all analysis of low energy data have fully exploited this up to first order

Construction of (un)polarized TMDFFs

In the asymptotic limit (High Q, qT) of each TMDFF

$$\tilde{D}_{q/N}(z, b_T; \zeta, \mu) = \left(\frac{\zeta}{\mu_b}\right)^{-D(b, \mu)} \sum_j \int_z^1 \frac{d\tau}{\tau^{3-2\varepsilon}} C_{q \rightarrow j}^Q \left(\frac{z}{\tau}, b_T; \mu_b, \mu\right) d_{j/N}(\tau, \mu) M_j(\tau, b_T, \zeta)$$

OPE to FF, valid ONLY for $qT \gg \Lambda_{QCD}$



Non-perturbative correction

This construction formally recovers the perturbative limit.

M.G. Echevarría, I.S., A.Vladimirov
arXiv:1509.06392 and 1604.07869

$$C_{g \rightarrow g}^Q = \mathcal{O}(\alpha^0)$$

$$C_{g \rightarrow q}^Q = \mathcal{O}(\alpha^1)$$

$$C_{q \rightarrow q}^Q = \mathcal{O}(\alpha_s^0)$$

$$C_{q \rightarrow g}^Q = \mathcal{O}(\alpha_s^1)$$

$$C_{q \rightarrow \bar{q}}^Q = \mathcal{O}(\alpha_s^2)$$

$$C_{q \rightarrow q'}^Q = \mathcal{O}(\alpha_s^2)$$

Starting order

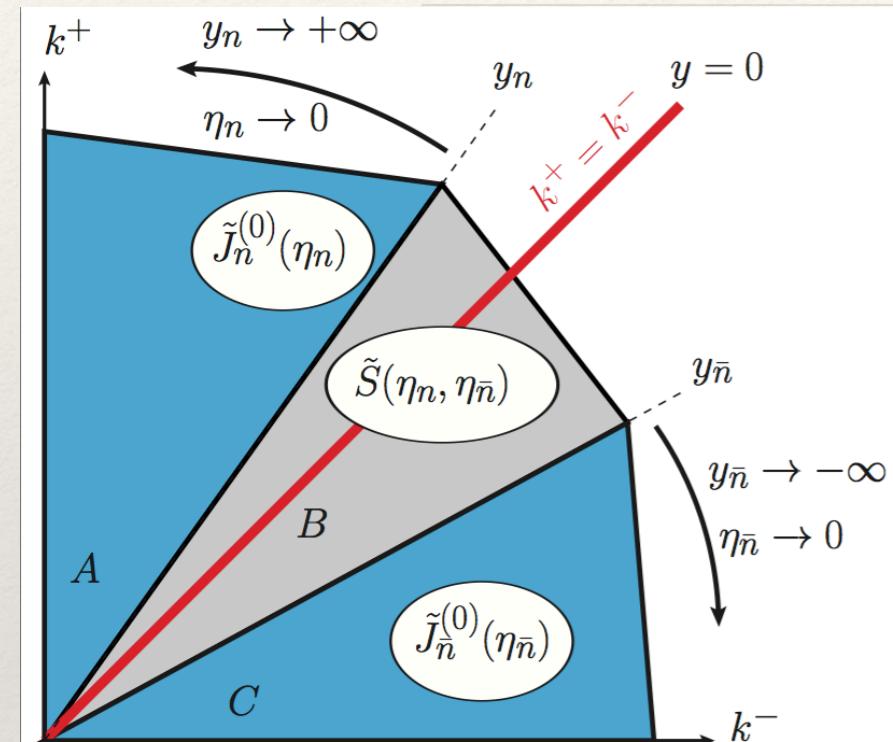
$\mathcal{O}(\alpha_s^2)?$

(Recent phenomenological work
Bacchetta et al. , 2015, in e+e-)

Status: This formula predicts that one TMDFF matches onto a sum of FFs. (FLAVOR MIXING)

Currently all analysis of low energy data have fully exploited this up to first order

Factorization theorem basics: DIS case



$$d\sigma \sim \int d^4x e^{iqx} \sum_X \langle h_1 | J^\mu(x) | X, h_2 \rangle \langle X, h_2 | J^\nu(0) | h_1 \rangle$$

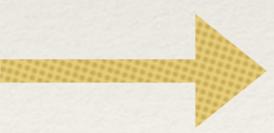
↓

$$d\sigma \sim \int d^2 b_T e^{-iq_T b_T} H(Q^2) \Phi_{h_1}(z_1, b_T) S(b_T) \Delta_{h_2}(z_2, b_T) + Y$$

TMDPDF Soft Factor TMDFF

- ❖ PDF, Soft Factor, FF have “rapidity divergences”
- ❖ Soft Factor mixes PDF and FF: no “real factorization”, however ..
- ❖ We have the **splitting of rapidity singularities in the Soft Factor:**

$$S(b_T) = \sqrt{S(b_T, \zeta)} \sqrt{S(b_T, \zeta^{-1})}$$



$$d\sigma \sim H(Q) \int d^2 b_T e^{-iq_T b_T} F(x; b_T) D(z; b_T)$$

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Each TMD is now free of rapidity singularity: nice renormalizable non-perturbative object

Operator definitions of TMDs

Light cone Wilson lines

$\xi = (0, \xi^-, \xi_T)$

bare TMDPDF $O_q^{bare}(x, b_T) = \frac{1}{2} \sum_X \int \frac{d\xi^-}{2\pi} e^{-ixp^+ \xi^-} \left\{ T \left[\bar{q}_i \tilde{W}_n^T \right]_a \left(\frac{\xi}{2} \right) |X\rangle \gamma_{ij}^+ \langle X| \bar{T} \left[\tilde{W}_n^{T\dagger} q_j \right]_a \left(-\frac{\xi}{2} \right) \right\}$

bare TMDFF $\mathbb{O}_q^{bare}(z, b_T) = \frac{1}{4zN_c} \sum_X \int \frac{d\xi^-}{2\pi} e^{-ip^+ \xi^- / z} \langle 0 | T \left[\tilde{W}_n^{T\dagger} q_j \right]_a \left(\frac{\xi}{2} \right) |X, \frac{\delta}{\delta J}\rangle \gamma_{ij}^+ \langle X, \frac{\delta}{\delta J} | \bar{T} \left[\bar{q}_i \tilde{W}_n^T \right]_a \left(-\frac{\xi}{2} \right) |0\rangle$

Formal definition of TMD operator

Applying these operators to the hadron states we obtain **unsubtracted** TMDs

$$\begin{aligned} \Phi_{q \leftarrow h}(x, b_T) &= \langle h | O_q^{bare}(x, b_T) | h \rangle \\ \Delta_{q \rightarrow h}(z, b_T) &= \langle h | \mathbb{O}_q^{bare}(z, b_T) | h \rangle \end{aligned}$$

To define individual TMD we have to take into account rapidity divergences, UV divergences and overlap regions

$$\begin{aligned} F_{q \leftarrow h}(x, b_T; \zeta, \mu) &= \sqrt{S(b_T; \zeta)} \langle h | Z_q(\mu) O_q^{bare}(x, b_T) | h \rangle \Big|_{zero-bin} \\ D_{q \rightarrow h}(x, b_T; \zeta, \mu) &= \sqrt{S(b_T; \zeta)} \langle h | Z_q(\mu) \mathbb{O}_q^{bare}(x, b_T) | h \rangle \Big|_{zero-bin} \end{aligned}$$

- μ is scale of UV renormalization.
- ζ is scale of rapidity-divergences separation.

Operator definitions of TMDs

$$O_q(x, b_T, \mu, \zeta) = Z_q(\zeta, \mu) R_q(\zeta, \mu) O_q^{\text{bare}}(x, b_T)$$

$$\mathbb{O}_q(z, b_T, \mu, \zeta) = Z_q(\zeta, \mu) R_q(\zeta, \mu) \mathbb{O}_q^{\text{bare}}(z, b_T)$$

Universal (=process independent) subtraction constants

$$R_q = \frac{\sqrt{S(b_T)}}{\text{zero} - \text{bin}}$$

Rapidity divergences subtraction

$$Z_q$$

UV divergences subtraction

Tilted WL's [Collins]

$$R_q(\zeta, \mu) = \frac{\sqrt{S(b_T; +\infty, y_s)}}{\sqrt{S(b_T; +\infty, -\infty) S(b_T; y_s, -\infty)}}$$

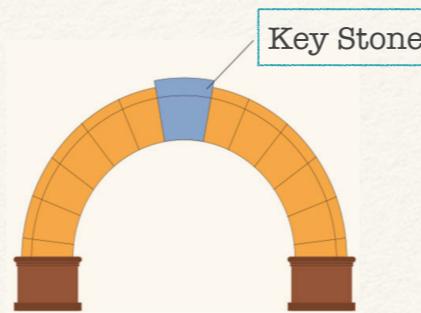
$$\zeta \sim m^2 e^{-2y_s}$$

δ -regularization [EIS]

zero-bin coincides with soft-factor

$$R_q(\zeta, \mu) = \frac{1}{\sqrt{S(b_T; \alpha \delta^+, \delta^+)}}$$

$$\zeta \sim \alpha Q^2$$



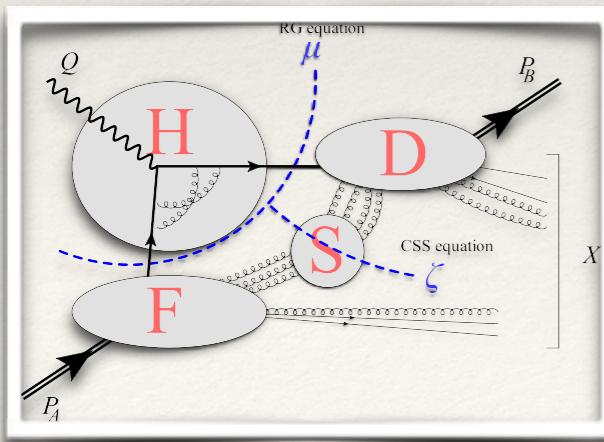
Rapidity divergences and the Soft Function

$$S(b_T) = \frac{1}{N_c} \langle 0 | [-\infty_n, b_T, \infty_{\bar{n}}] [\infty_{\bar{n}}, 0, -\infty_n] | 0 \rangle, \quad [\gamma] \sim P \exp \left(-ig \int_\gamma A_\gamma \right)$$

- ❖ The rapidity divergences arise in the limit $k^+ \rightarrow \infty, k^- \rightarrow 0, k^+ k^-$ fixed
- ❖ The Soft Function is undefined without a regulator for rapidity divergences
- ❖ The log of the Soft Function is linear in the rapidity regulator at all orders

$$\tilde{S}(\mathbf{L}_\mu, \mathbf{L}_{\sqrt{\delta^+ \delta^-}}) = \tilde{S}^{\frac{1}{2}}(\mathbf{L}_\mu, \mathbf{L}_{\delta^+}) \tilde{S}^{\frac{1}{2}}(\mathbf{L}_\mu, \mathbf{L}_{\delta^-})$$

- ❖ By definition the Soft Function is just 1 in the limit $b_T \rightarrow 0$



$$\begin{aligned} \mathbf{L}_X &\equiv \ln(X^2 b_T^2 e^{2\gamma_E}/4) \\ \mathbf{l}_\zeta &= \ln(\mu^2/\zeta) \end{aligned}$$

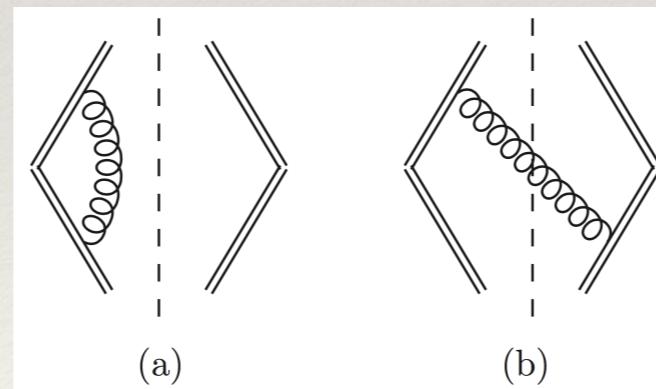
One loop Soft Function

Origin of divergences: A.Idilbi, M.G. Echevarria, I.S. arXiv:1310.8541, Int.J.Mod.Phys.Conf.Ser. 25 (2014) 1460005

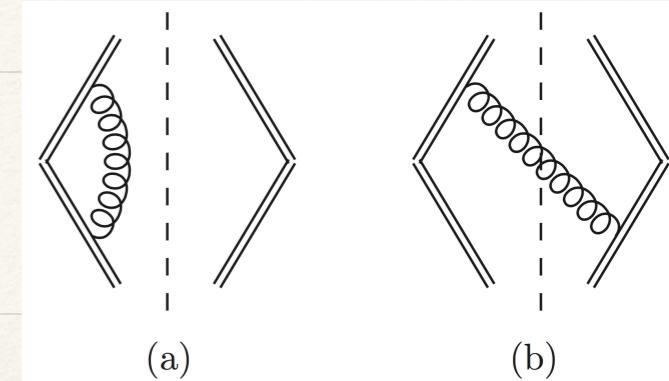
$$S_1^v = -2ig_s^2 C_F \delta^{(2)}(\vec{k}_{s\perp}) \mu^{2\varepsilon} \int \frac{d^d k}{(2\pi)^d} \frac{1}{[k^+ - i\delta^+][k^- + i\delta^-][k^2 - \lambda^2 + i0]} + h.c.$$

$$S_1^r = 4\pi g_s^2 C_F \mu^{2\varepsilon} \int d^d \mathbf{k}_s e^{i\mathbf{k}_s \cdot \mathbf{b}} \int \frac{d^d k}{(2\pi)^d} \frac{\delta^{(2)}(\mathbf{k} + \mathbf{k}_s) \delta(k^2 - \lambda^2) \theta(k^+)}{(k^+ + i\delta^+)(k^- - i\delta^-)} + h.c.$$

\cancel{d} : UV-Dim.Reg.
 λ : IR div.
 δ^\pm : rapidity div.



One loop Soft Function



Origin of divergences: A.Idilbi, M.G. Echevarria, I.S. arXiv:1310.8541, Int.J.Mod.Phys.Conf.Ser. 25 (2014) 1460005

$$\tilde{S}_1^v = \frac{\alpha_s C_F}{2\pi} \left[\frac{-2}{\varepsilon_{\text{UV}}^2} + \frac{2}{\varepsilon_{\text{UV}}} \ln \frac{\delta^+ \delta^-}{\mu^2} + \ln^2 \frac{\lambda^2}{\mu^2} - 2 \ln \frac{\lambda^2}{\mu^2} \ln \frac{\delta^+ \delta^-}{\mu^2} + \frac{\pi^2}{6} \right]$$

ε_{UV} : Dim. Reg.
 λ : IR div.

$$\tilde{S}_1^r = \frac{\alpha_s C_F}{2\pi} \left[L_\perp^2 + 2L_\perp \ln \frac{\delta^+ \delta^-}{\mu^2} + 2 \ln \frac{\lambda^2}{\mu^2} \ln \frac{\delta^+ \delta^-}{\mu^2} - \ln^2 \frac{\lambda^2}{\mu^2} \right]$$

δ^\pm : rapidity div.

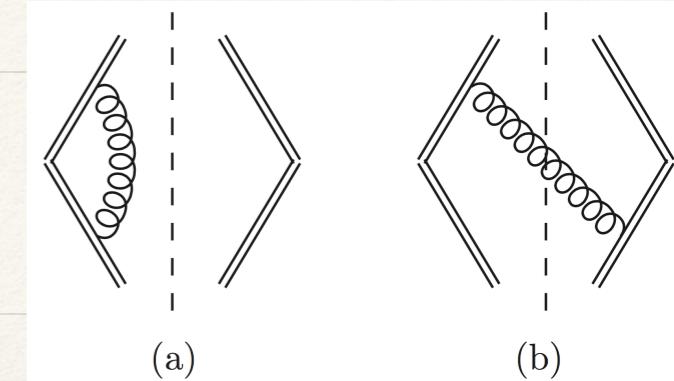
$$L_\perp = \ln \frac{\mu^2 b_T^2 e^{2\gamma_E}}{4}$$

Summing up only UV and rapidity divergences survives

$$\tilde{S}_1 = \frac{\alpha_s C_F}{2\pi} \left[\frac{-2}{\varepsilon_{\text{UV}}^2} + \frac{2}{\varepsilon_{\text{UV}}} \ln \frac{\delta^+ \delta^-}{\mu^2} + L_\perp^2 + 2L_\perp \ln \frac{\delta^+ \delta^-}{\mu^2} + \frac{\pi^2}{6} \right]$$

To compute a NNLO calculation we need this result at all orders in ε

One loop Soft Function



Modified delta-regularization:

$$P \exp \left[-ig \int_0^\infty d\sigma A_\pm(\sigma n) \right] \rightarrow P \exp \left[-ig \int_0^\infty d\sigma A_\pm(\sigma n) e^{-\delta^\pm |\sigma|} \right]$$

$$S_v^{[1]} = -\frac{2g^2}{(4\pi)^{\frac{d}{2}}} \delta^{-\epsilon} \Gamma^2(\epsilon) \Gamma(1-\epsilon) + h.c.$$

Mixed divergences cancel at all orders

$$S_r^{[1]} = \frac{2g^2}{(4\pi)^{\frac{d}{2}}} \left[\delta^{-\epsilon} \Gamma^2(\epsilon) \Gamma(1-\epsilon) - B^\epsilon \Gamma(-\epsilon) (L_+ - \psi(-\epsilon) - \gamma_E) \right] + h.c.$$

Summing up only UV and rapidity divergences survive:

$$S^{[1]} = -4B^\epsilon \Gamma(-\epsilon) (L_0 - \psi(-\epsilon) - \gamma_E)$$

The trivial limit ($b_T = 0$) easily recovered

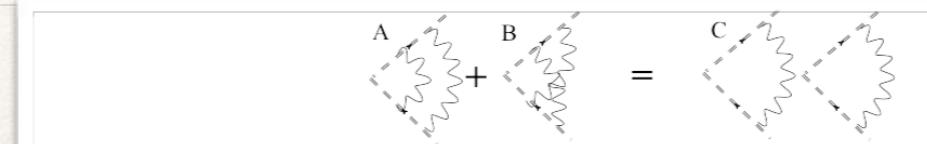
$$\delta = \pm \delta^+ \delta^-, \quad B = \frac{b_T^2}{4}$$

$$L_0 = \ln \left(\frac{B|\delta|}{e^{-2\gamma_E}} \right)$$

Two loops Soft Function

Full result in Echevarría, S., Vladimirov arXiv:1511.05590

Modified delta-regularization preserves abelian exponentiation



$$\begin{array}{c} p \quad k \quad l \\ \swarrow \quad \searrow \quad \swarrow \end{array} = \frac{1}{(p^+ + i\delta)(p^+ + k^+ + 2i\delta)(p^+ + k^+ + l^+ + 3i\delta)}$$

δ -regularization preserving exponentiation

The regularization should be implemented on the level of operator

$$P \exp \left[-ig \int_0^\infty d\sigma A_\pm(\sigma n) \right] \rightarrow P \exp \left[-ig \int_0^\infty d\sigma A_\pm(\sigma n) e^{-\delta^\pm |\sigma|} \right]$$

Then exponentiation is exact

$$\text{Diag}_A + \text{Diag}_B = \frac{\text{Diag}_C^2}{2}$$

The structure of each diagram recalls the one-loop result

$$\text{Diagram} = \mu^{2\epsilon} \left(A_1 \delta^{-2\epsilon} + A_2 \delta^{-\epsilon} B^\epsilon + A_3 B^{2\epsilon} \right)$$

$A_1, A_2 = 0$ in the sum of all diagrams

A_3 in the sum of all diagrams is linear in rapidity logs

We can omit calculation of virtual diagrams

The Soft Function is linear in rapidity logs at all orders in ϵ

Two loops Soft Function

The D-function which governs the TMD evolution kernel is the finite part of

$$D = \frac{1}{2} \frac{d \ln \tilde{S}}{d \mathbf{l}_\delta} |_{finite}$$

First direct calculation of the D- function at two loops

$$D = \sum_n \sum_{k=0}^n a_s^n \mathbf{L}_\mu^k d^{(n,k)}$$

NEW! 3-Loop results from Soft function of: Y. Li, H. X. Zu arXiv:1604.01404

$$d^{(2,2)} = \frac{\Gamma^{(0)} \beta_0}{4} = C_F \left(\frac{11}{3} C_A - \frac{4}{3} T_R N_f \right)$$

$$d^{(2,1)} = \frac{\Gamma^{(1)}}{2} = 2C_F \left(\left(\frac{67}{9} - \frac{\pi^2}{3} \right) C_A - \frac{20}{9} T_R N_f \right)$$

$$d^{(2,0)} = C_F \left(\left(\frac{404}{27} - \frac{14\zeta_3}{17} \right) C_A - \frac{112}{27} T_R N_f \right).$$

First deduced by Becher, Neubert '10

Two loops Soft Function

The D-function which governs the TMD evolution kernel is the finite part of

$$D = \frac{1}{2} \frac{d \ln \tilde{S}(b_T)}{d \ln \delta} |_{finite}$$

Comparing quark and gluon soft functions, the difference is just the Casimir scaling

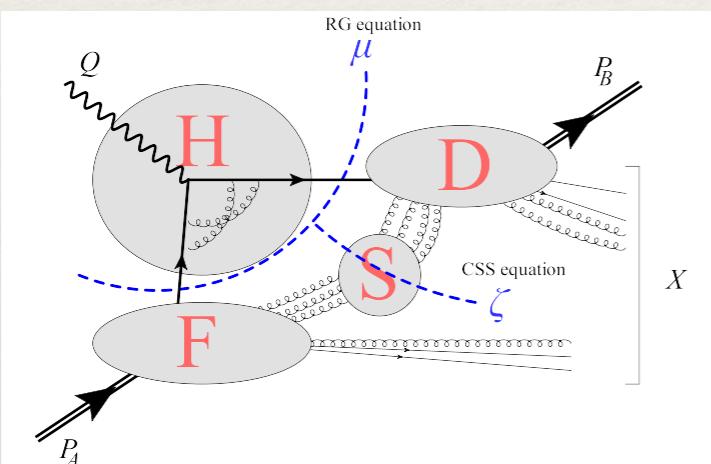
The Casimir scaling of D's is valid at all order in perturbation theory

$$\frac{D}{C_F} = \frac{D_g}{C_A}$$

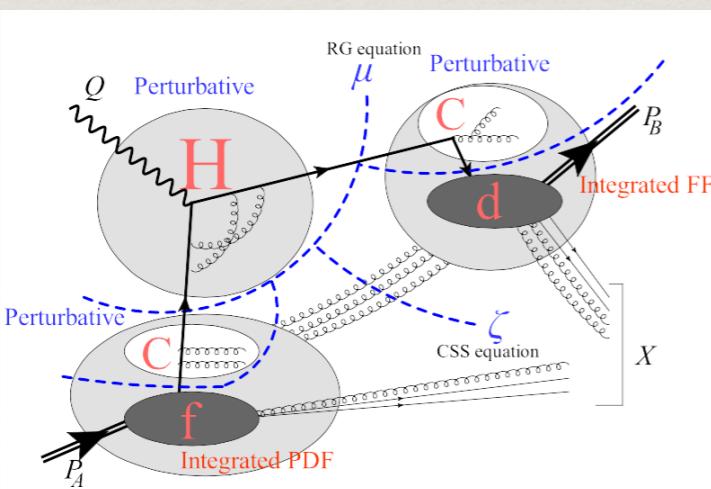
TMDFF at NNLO

❖ The Universal Soft function (Spin independent)

The unsubtracted TMDs



$$\tilde{D}_{i \rightarrow h}(z, \mathbf{L}_\mu, \mathbf{l}_{\zeta_D}) = \tilde{\Delta}_{i \rightarrow h}^{(0)}(z, \mathbf{L}_\mu, \mathbf{l}_{\zeta_D}, \mathbf{L}_{\delta^+}) \tilde{S}^{\frac{1}{2}}(\mathbf{L}_\mu, \mathbf{L}_{\delta^+})$$



$$\tilde{D}_{i \rightarrow h}(z, \mathbf{L}_\mu, \mathbf{l}_\zeta) = \int_z^1 \frac{d\tau}{\tau^{3-2\epsilon}} C_{i \rightarrow j}\left(\frac{z}{\tau}, \mathbf{L}_\mu, \mathbf{l}_\zeta\right) d_{j \rightarrow h}(\tau, \mu)$$

TMDFF expansion

Perturbative expansion:

$$\tilde{C}_{j \rightarrow f}^{[0]} = \delta(1 - z)$$

$$\tilde{C}_{i \rightarrow j}^{[1]} = \tilde{D}_{i \rightarrow j}^{[1]} - \frac{d_{i \rightarrow j}^{[1]}}{z^{2-2\epsilon}}$$

$$\tilde{C}_{i \rightarrow f}^{[2]} = \boxed{\tilde{D}_{i \rightarrow f}^{[2]}} - \tilde{C}_{i \rightarrow k}^{[1]} \otimes \frac{d_{k \rightarrow f}^{[1]}}{z^{2-2\epsilon}} - \frac{d_{i \rightarrow f}^{[2]}}{z^{2-2\epsilon}}$$

Missing piece

To recover the SF from the soft limit of FF (zero-bin correspondence) :

$$\frac{1}{(k_1^+ - i0)(k_2^+ - i0) \dots (k_n^+ - i0)} \rightarrow \frac{1}{(k_1^+ - i\delta^+/\textcolor{red}{z})(k_2^+ - 2i\delta^+/\textcolor{red}{z}) \dots (k_n^+ - ni\delta^+/\textcolor{red}{z})}$$

RGE for TMD's

$$\mu^2 \frac{d}{d\mu^2} \tilde{D}_{i \rightarrow h} = \frac{1}{2} \gamma_D^i \tilde{D}_{i \rightarrow h}$$

$$\gamma_D = \Gamma_{cusp}^i \mathbf{l}_\zeta - \gamma_V^i$$

$$\zeta \frac{d}{d\zeta} \tilde{D}_{i \rightarrow h} = -\mathcal{D}^i \tilde{D}_{i \rightarrow h}, \quad 2\mu^2 \frac{d}{d\mu^2} \mathcal{D}^i = \Gamma_{cusp}^i$$

RGE for TMD fragmentation

$$\mathbf{l}_X \equiv \ln(\mu^2/X)$$

RGE for coefficients

$$\begin{aligned} \zeta \frac{d}{d\zeta} \tilde{C}_{i \rightarrow j} &= -\mathcal{D}^i \tilde{C}_{i \rightarrow j} \\ \mu^2 \frac{d}{d\mu^2} \tilde{C}_{i \rightarrow j} &= \tilde{C}_{i \rightarrow k} \otimes \mathcal{K}_{k \rightarrow j}^i \\ \mathcal{K}_{k \rightarrow j}^i(z) &= \frac{\delta_{kj}}{2} (\Gamma_{cusp}^i \mathbf{l}_\zeta - \gamma_V^i) \delta(\bar{z}) - \frac{\mathcal{P}_{k \rightarrow j}(z)}{z^2}. \end{aligned}$$

Re-writing of the coefficient

$$\tilde{C}_{i \rightarrow j} = \exp \left[-\mathcal{D}^i \mathbf{L}_{\sqrt{\zeta}} \right] \tilde{C}_{i \rightarrow j}$$

Structure of the result

Log structure of the result

$$\tilde{\mathcal{C}}_{ij}^{[n]} = \sum_{k=0}^{2n} \tilde{\mathcal{C}}_{ij}^{(n;k)} \mathbf{L}_\mu^k$$

Recursion relation

$$(k+1)\tilde{\mathcal{C}}_{i \rightarrow j}^{(n;k+1)} = \sum_{r=1}^n \frac{\Gamma_{cusp}^{[r]}}{2} \tilde{\mathcal{C}}_{i \rightarrow j}^{(n-r;k-1)} - \frac{\gamma_V^{i[r]} - 2(n-r)\beta^{[r]}}{2} \tilde{\mathcal{C}}_{i \rightarrow j}^{(n-r;k)} - \tilde{\mathcal{C}}_{i \rightarrow k}^{(n-r;k)} \otimes \frac{\mathcal{P}_{k \rightarrow j}^{[r]}}{z^2}$$

The original piece of
the 2-loop coefficient is

$$\begin{aligned}\tilde{\mathcal{C}}_{q \rightarrow q}^{(2;0)}(z) &= C_F^2 Q_F(z) + C_F C_A Q_A(z) + T_R N_f Q_N(z) \\ \tilde{\mathcal{C}}_{q \rightarrow \bar{q}}^{(2;0)}(z) &= C_F \left(C_F - \frac{C_A}{2} \right) Q_{q\bar{q}}(z)\end{aligned}$$

Sample of the result

Log structure of the result

$$\tilde{\mathcal{C}}_{ij}^{[n]} = \sum_{k=0}^{2n} \tilde{\mathcal{C}}_{ij}^{(n;k)} \mathbf{L}_\mu^k$$

$$\tilde{\mathcal{C}}_{q \rightarrow q}^{(2;0)}(z) = C_F^2 Q_F(z) + C_F C_A Q_A(z) + T_R N_f Q_N(z),$$

$$\tilde{\mathcal{C}}_{q \rightarrow \bar{q}}^{(2;0)}(z) = C_F \left(C_F - \frac{C_A}{2} \right) Q_{q\bar{q}}(z).$$

$$Q_N(z) = \frac{1}{z^2} \left[\left(\frac{2}{3} \ln^2 z - \frac{20}{3} \ln z + \frac{112}{27} \right) p(z) - \frac{16}{3} \bar{z} \ln z - \frac{4}{3} \bar{z} \right]_+ \\ + \delta(\bar{z}) \left(-\frac{2717}{162} + \frac{25\pi^2}{9} + \frac{52}{9} \zeta_3 \right)$$

Conclusions

- ❖ The comprehension of transverse momentum spectra for all energy ranges requires the correct inclusion of non-perturbative effects: TMDs.
- ❖ The check of the universality of TMD evolution requires the same degree of precision for TMDPDF and TMDFF: **NNLO**.
- ❖ The perturbative part of TMDs should be used at highest available order to control the perturbative series (NNLL only achieved in a limited set of TMDs): At present the TMD evolution kernel is known at **N3LO!** Casimir scaling of the D function established at all orders in perturbation theory.)
- ❖ The control of perturbative error is fundamental to understand the nature of non-perturbative effects.
- ❖ We have completed the calculation of the universal Soft Function and the matching of the ALL unpolarized TMDFF onto FF at NNLO using the EIS formulation. (TMDPDF onto PDF also checked.) We can perform a complete N3LL' analysis (only 4-loop cusp is missing, but one can use Padé estimations)
- ❖ The Soft Function can be used for the evaluation of the matching of all (un)polarized TMDs. One can reach in principle the same perturbative precision
- ❖ The implementation of all this information in an accepted TMD which includes also non-perturbative QCD effect is still work in progress (both theoretically and experimentally)

Thanks!!!

TMDFF expansion

1-loop

$$D_{q/q}^{[1]} = \tilde{\Delta}_{q/q}^{[1]} - \tilde{S}_+^{[1]} - Z_2^{[1]} + Z_D^{[1]}$$

Rapidity divergences cancel here!

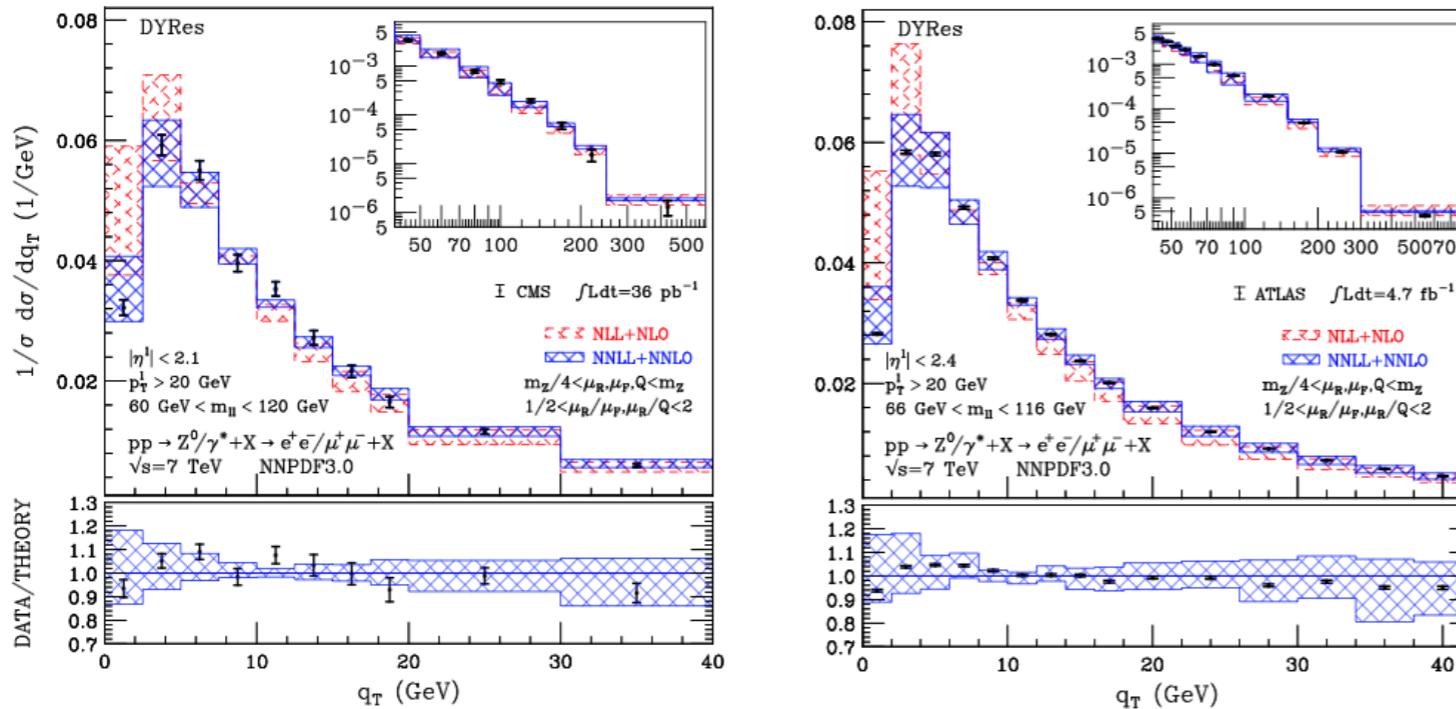
$$\begin{aligned} \text{2-loops } \tilde{D}_{i/f}^{[2]} &= \underbrace{\tilde{\Delta}_{i/f}^{[2]} - \tilde{S}_+^{[1]} \tilde{\Delta}_{i/f}^{[1]}}_{\text{rap.div.free}} - \tilde{S}_+^{[2]} \delta_{if} + \frac{3\tilde{S}_+^{[1]} \tilde{S}_+^{[1]}}{2} \delta_{if} \\ &\quad + \left(Z_D^{[1]} - Z_2^{[1]} \right) \left(\tilde{\Delta}_{i/f}^{[1]} - \frac{\tilde{S}_+^{[1]} \delta_{if}}{2} \right) \\ &\quad + \left(Z_D^{[2]} - Z_2^{[2]} - Z_2^{[1]} Z_D^{[1]} + Z_2^{[1]} Z_2^{[1]} \right) \delta_{if} . \end{aligned}$$

$\sim \delta(1 - z)$

Pure UV

An example: Z-production at LHC

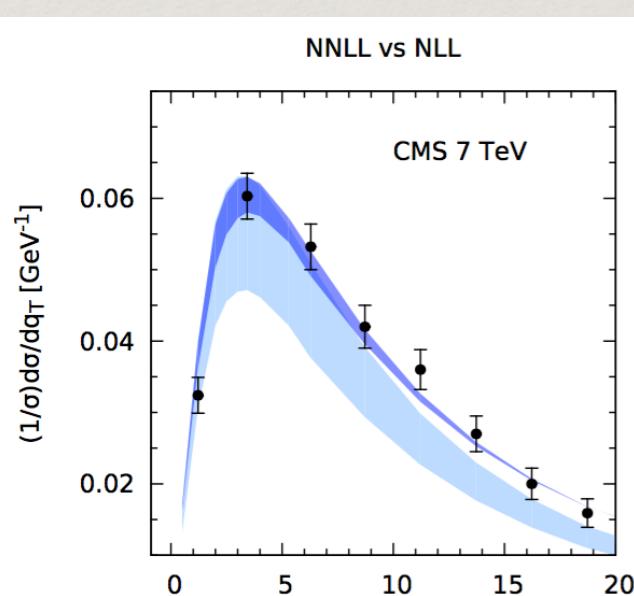
DYRes results: q_T spectrum of Z boson at the LHC G.Ferrera talk at REF2015



Main differences

DYRes includes full 2-loop matching (NNLO) onto PDFs, but no low energy DY infos.
Minimal Subtraction.

D'Alesio et al 2014, partially includes the 2-loop matching (NNLL) and full low energy DY infos (2 parameters)



D' Alesio et al. 2014

A TMD approach has the potential to drastically reduce errors!
We can include QCD infos at different energies and experiments

Caveat: THE COMPLETE NNLO PERTURBATIVE QCD INFORMATION MUST BE INCLUDED