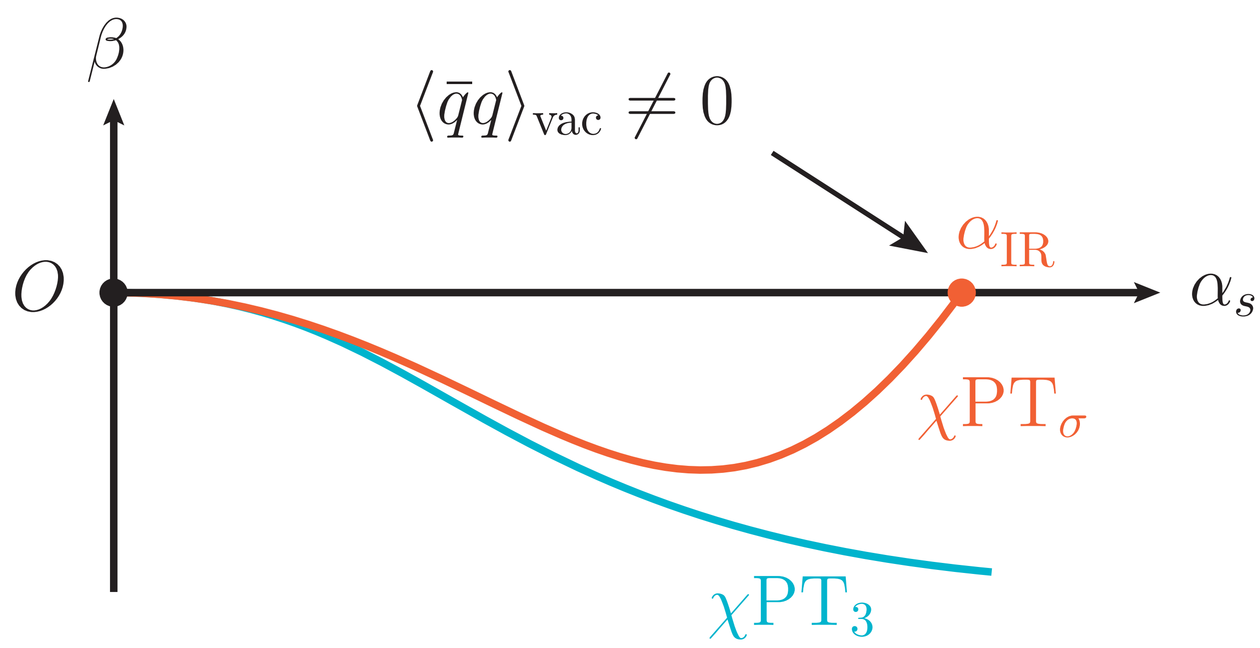
Chiral-Scale Perturbation Theory χPT_σ and the Renormalization Group



Based on work with R.J. Crewther (Adelaide)
[arXiv:1203.1321 & 1312.3319 & 1510.01322]

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0 | Motivation

1 Can we make sense of the lowest 0^{++} QCD resonance $f_0(500)$?

The mass and width have been precisely determined

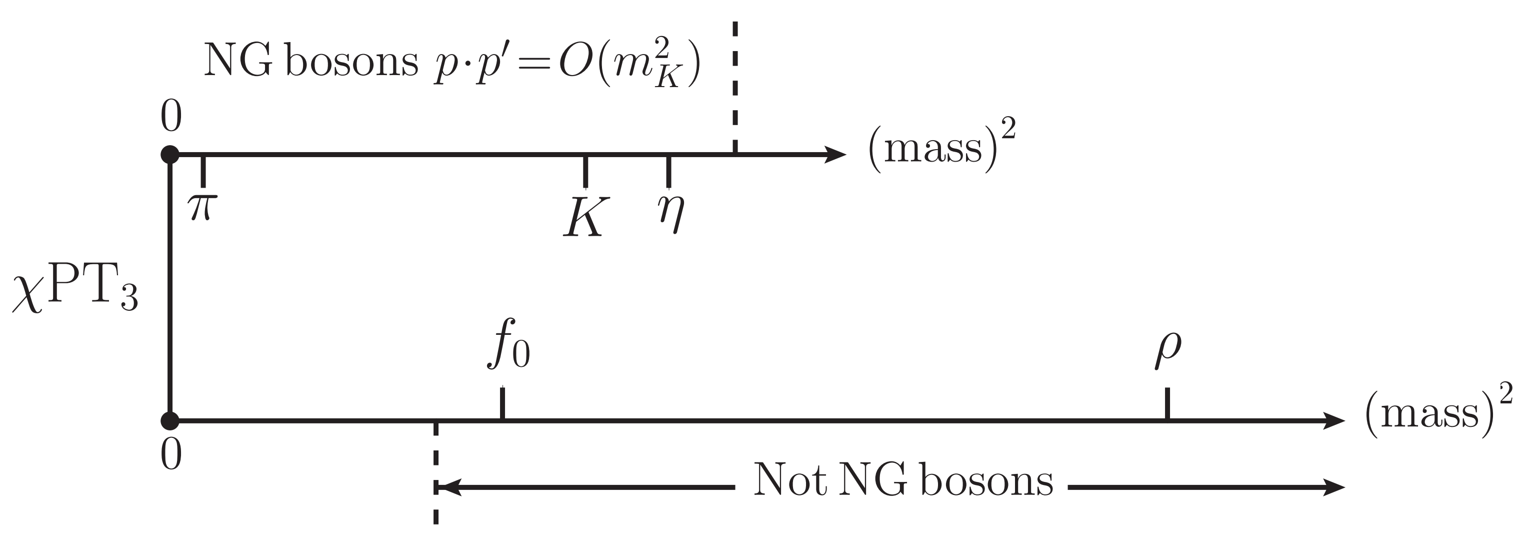
$$m_{f_0} = 441^{+16}_{-8}\ \mathrm{MeV} \qquad \Gamma_{f_0} = 544^{+18}_{-25}\ \mathrm{MeV}$$

[Caprini, Colangelo & Leutwyler (06); García-Martin et al. (11)]

but what is the quark content: $q\bar{q}$, $q\bar{q}q\bar{q}$ ... (something else)? [Peláez (15)]

Related puzzles:

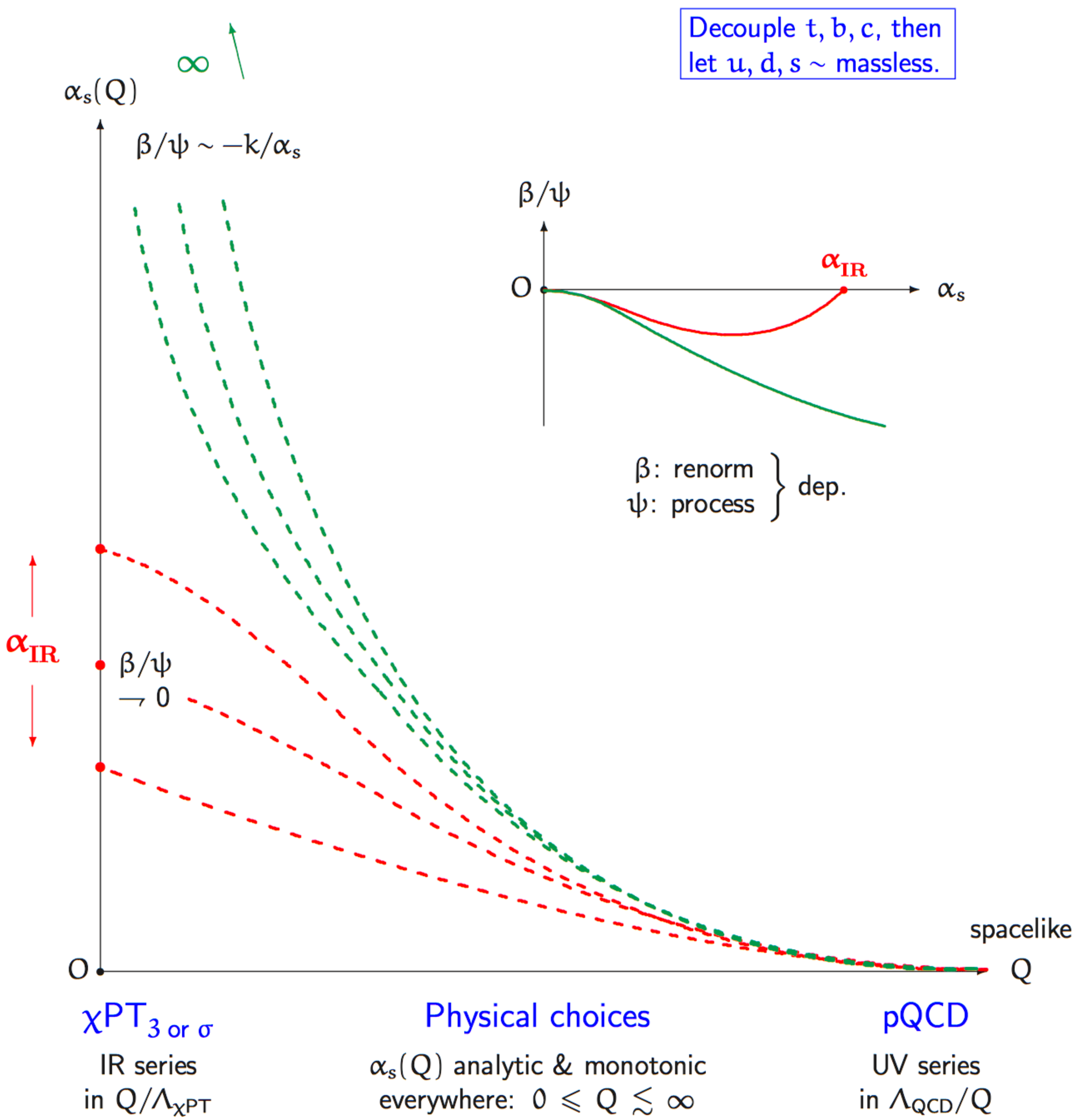
- $m_{f_0} \lesssim m_K \quad \Rightarrow \quad$ poor $\chi\mathrm{PT}_3$ convergence in 0^{++} channel [Meißner (91)]



- role of $f_0(500)$ in $\Delta I=1/2$ rule for non-leptonic kaon decays?

0 | Motivation

2 A variety of **non-perturbative** approaches indicate that α_s "freezes" in the IR to a constant value α_{IR} for **small** values of $N_f \leq 3$



[Diagram from RJ Crewther]

Main problems:

- scheme / definition dependence of α_s $\Rightarrow$ the existence of α_{IR} for small N_f is not entirely settled

  [For a catalog of existing results see: Deur, Brodsky & de Teramond (16)]

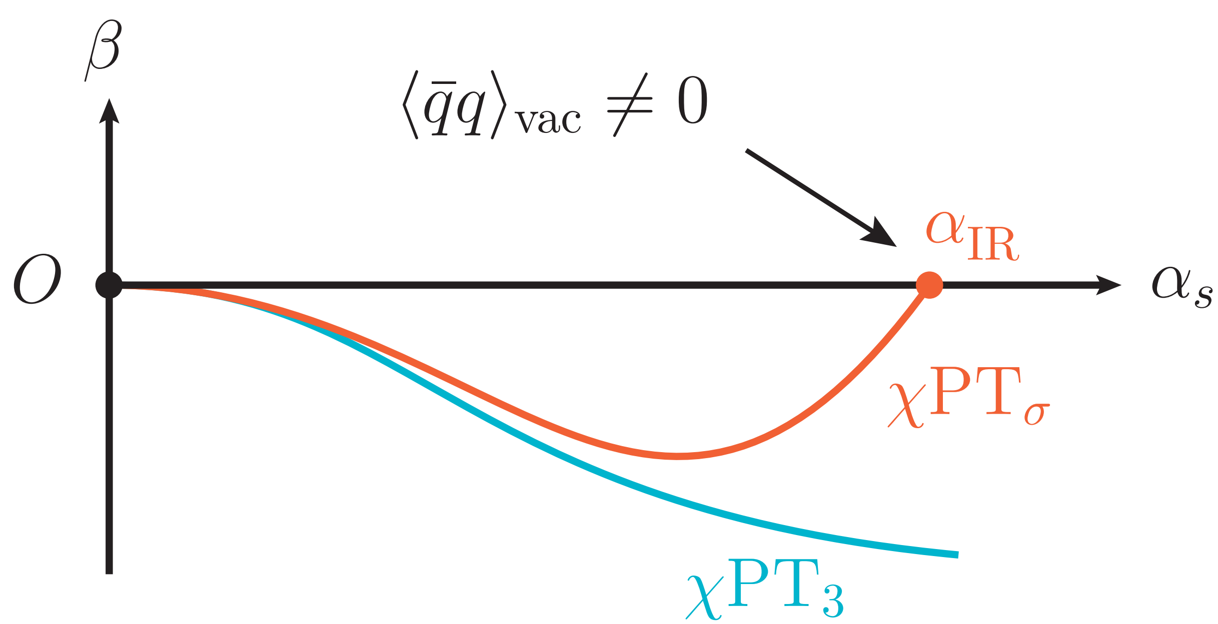
- when is α_{IR} **physical**?

Q: If α_{IR} exists, what are the implications for low energy physics, especially χPT?

1 | Chiral-scale perturbation theory

Decouple t,b,c $\Rightarrow$ relevance of **scale (or dilatation) invariance** determined by trace anomaly of $N_f = 3$ theory:

$$\partial^\mu \{x^\alpha \theta_{\alpha\mu}\} = \theta^\mu_\mu = \frac{\beta(\alpha_s)}{4\alpha_s} G^2 + (1+\gamma_m) \sum_{q=u,d,s} m_q \bar{q} q$$



IR fixed point: $\beta(\alpha_{\text{IR}}) = 0$

χPT_σ = asymptotic expansion in

$$\alpha_s \lesssim \alpha_{\text{IR}} \quad \text{and} \quad m_{u,d,s} \sim 0$$

about scale-**dependent** $|\text{vac}\rangle$

$\Rightarrow$ **9** NG bosons π, K, η, σ

In limit $\theta^\mu_\mu \to 0$ dilaton σ couples to masses in non-NG sector [Gell-Mann (62)]

$$\langle \text{vac}|\theta_{\mu\nu}|\sigma(q)\rangle = \tfrac{1}{3}(q_\mu q_\nu - q^2 g_{\mu\nu}) F_\sigma \quad \Rightarrow \quad \text{e.g. } F_\sigma g_{\sigma NN} = M_N$$

1 | Chiral-scale perturbation theory

Key results

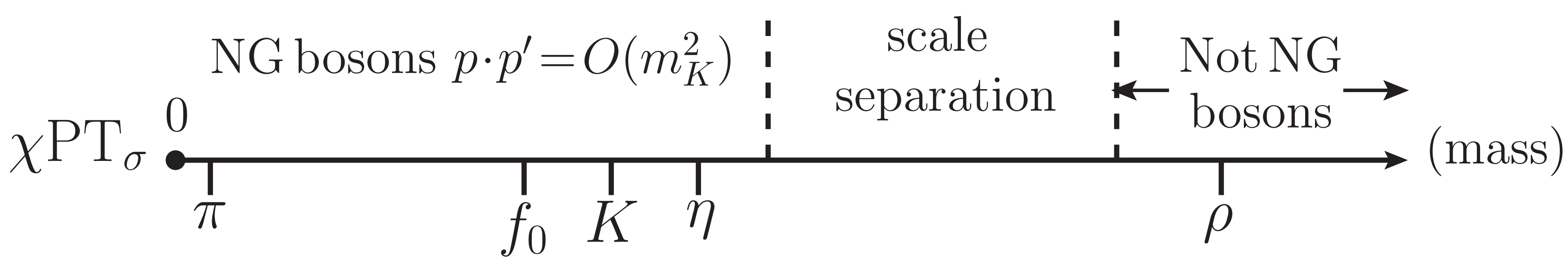
1. Associate dilaton σ with $f_0(500)$: the $O(m_K)$ mass understood due to $\sim m_s\bar{s}s$ term in θ^μ_μ while the width is explained by $O(p^2)$ relation

$$F_\sigma g_{\sigma\pi\pi} \simeq -m_\sigma^2$$

[Ellis (70); Crewther (70)]

NB. In chiral-scale limit $\theta^\mu_\mu \to 0$ get massless π, K, η and **stable** σ/f_0 (phase space = 0) $\Rightarrow$ OK to use local field σ in $\mathcal{L}_{\mathrm{eff}}$

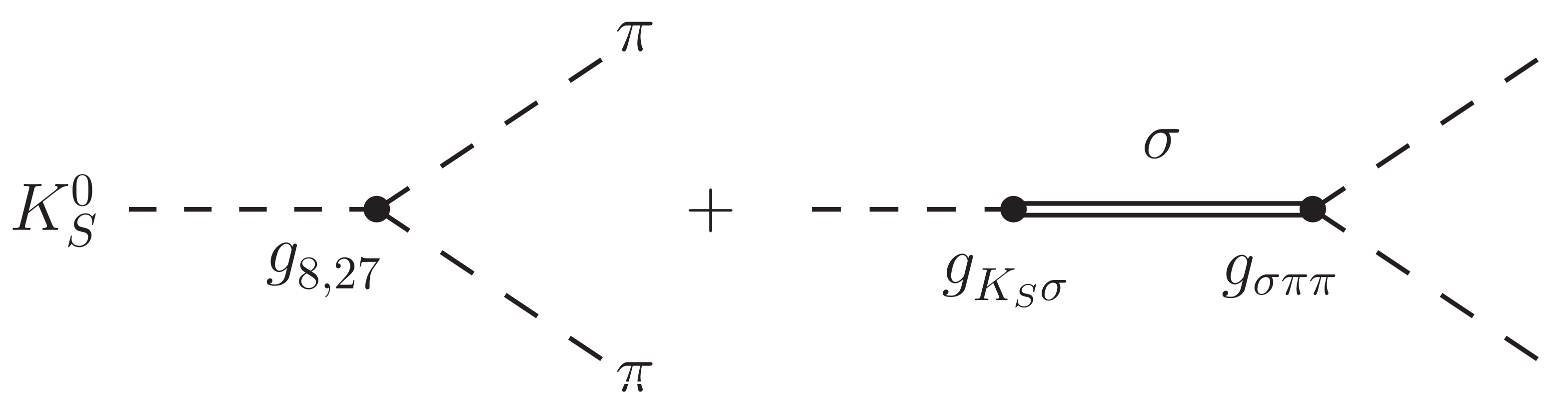
2. Promotion of $f_0(500)$ to NG sector $\Rightarrow$ scale separation restored



Expect improved convergence in 0^{++} channels ($O(p^4)$ analysis in progress)

1 | Chiral-scale perturbation theory

3 Lowest order $O(p^2)$ $\chi\mathrm{PT}_\sigma$ explains the ΔI=1/2 rule for kaon decays



4 Relation between **σγγ** coupling and electromagnetic trace anomaly

$$\tilde{\theta}^\mu_\mu = \theta^\mu_\mu + (R\alpha/6\pi)F^2$$

[Crewther (72); Chanowitz & Ellis (72)]

Fit data on $\pi\pi \to \gamma\gamma$, $K_S \to \gamma\gamma$, and $\sigma \to \gamma\gamma$ $\Rightarrow$ bounds on asymptotic Drell-Yan ratio at $\alpha_s = \alpha_{\mathrm{IR}}$:

$$2.5 \lesssim R_{\mathrm{IR}} \lesssim 5$$

This talk: some formal aspects concerning **hidden** scale invariance & a proposal to test $\chi\mathrm{PT}_\sigma$ on the lattice

2 | The NG mechanism at an IR fixed point

The idea that particles can have mass in the scale invariant limit $\theta^\mu_\mu \to 0$ seems counter-intuitive, especially for gauge theories like QCD

Usually one considers fixed points in the **Wigner-Weyl** mode, as in e.g. the **conformal window**: two-loop $\beta = -b_1\alpha_s^2 - b_2\alpha_s^3$ $\Rightarrow$ IR fixed point

$$\alpha_{\text{CBZ}} = -b_2/b_1 \quad \text{if } b_2 < 0\,, \quad \text{i.e. for } 9 \leq N_f \leq 16$$

[Caswell (74); Banks & Zaks (82)]

But ... if IR fixed point is **non-perturbative** dimensional transmutation may cause $|\text{vac}\rangle$ to break scale invariance $\Rightarrow$ symmetry is **hidden**

General theory of the NG mode for scale and conformal invariance established long ago [Salam & Strathdee (69); Zumino (70); Carruthers (71)]

Let us apply these ideas to $N_f = 3$ QCD . . .

3 | Chiral and scale condensates

Recall **chiral** Ward identities for e.g. $J^a_{\mu 5} = \bar{q}\gamma_\mu\gamma_5 T^a q$ and its divergence:

$$\partial^\mu J^a_{\mu 5} = D^a_5 = 2mi\bar{q}\gamma_5 T^a q$$

Let $\mathcal{O}$ be any operator which is not chiral invariant and consider amplitude

$$\mathcal{A}^a_{\mu 5}(q) = \int d^4x\, e^{iq\cdot x}\, \mathrm{T}\big\langle \mathrm{vac}\big| J^a_{\mu 5}(x)\mathcal{O}(0)\big|\mathrm{vac}\big\rangle$$

Consider $q = 0$ limit of the chiral Ward identities

$$q^\mu \mathcal{A}^a_{\mu 5} = \text{commutators } + \int_x e^{iq\cdot x} D^a_5 \quad \text{insertion}$$

$$\Rightarrow \qquad 0 = \big\langle \mathrm{vac}\big|\delta^a_5\mathcal{O}\big|\mathrm{vac}\big\rangle + i\int d^4x\, \mathrm{T}\big\langle \mathrm{vac}\big| D^a_5(x)\mathcal{O}(0)\big|\mathrm{vac}\big\rangle$$

The term $\mathrm{T}\langle D_5\mathcal{O}\rangle$ does not vanish in limit $D^a_5 \to 0$ if $\exists\ q = 0$ pion pole

$$\Rightarrow \qquad \langle\delta\mathcal{O}\rangle \to iF_\pi\langle \mathrm{vac}|\mathcal{O}|\pi\rangle \neq 0\,, \quad m \to 0\,, \quad F_\pi \simeq 93\ \mathrm{MeV}$$

So e.g. $\langle\bar{q}q\rangle \neq 0$ **hides** the symmetry; $M_B \neq 0$ and mesons not P-doubled

3 | Chiral and scale condensates

For scale transformations, the relevant divergence is

$$\partial^\mu\{x^\alpha\theta_{\alpha\mu}\} = \theta^\mu_\mu = \frac{\beta(\alpha_s)}{4\alpha_s}G^2 + (1+\gamma_m(\alpha_s))\sum_{q=u,d,s} m_q\bar{q}q$$

so the q=0 Ward identities are the Callan-Symanzik equations

$$\left\{\mu\frac{\partial}{\partial\mu} + \beta(\alpha_s)\frac{\partial}{\partial\alpha_s} + \gamma_{\mathcal{O}}(\alpha_s) + (1+\gamma_m(\alpha_s))\sum_{u,d,s} m_q\frac{\partial}{\partial m_q}\right\}\langle\mathcal{O}\rangle_{\text{vac}} = 0$$

The terms $\beta\partial/\partial\alpha_s$ and $(1+\gamma_m)\sum_q m_q\partial/\partial m_q$ correspond to $q=0$ insertion of θ^μ_μ:

$$\left\{\mu\frac{\partial}{\partial\mu} + \gamma_{\mathcal{O}}(\alpha_s)\right\}\langle\text{vac}|\mathcal{O}(0)|\text{vac}\rangle = i\int d^4x\,\text{T}\langle\text{vac}|\theta^\mu_\mu(x)\mathcal{O}(0)|\text{vac}\rangle \quad (\star)$$

What happens to RH side of $(\star)$ if α_{IR} exists and limit $\theta^\mu_\mu \to 0$ is taken?

3 | Chiral and scale condensates

Wigner-Weyl (WW) mode

The standard procedure is to set all amplitudes involving $\int \theta^\mu_\mu$ to zero

$\Rightarrow$ no NG mechanism and scale invariance realised in **WW mode**

$$\left\{ \mu \frac{\partial}{\partial \mu} + \gamma_{\mathcal{O}}(\alpha_{\mathrm{WW}}) \right\} \langle \mathrm{vac} | \mathcal{O}(0) | \mathrm{vac} \rangle_{\mathrm{WW}} = 0$$

$\therefore$ theory at a WW fixed point α_{WW} is **manifestly** scale invariant

$\Rightarrow$ Green's functions scale according to power-laws $\sim \mu^{-\gamma_{\mathcal{O}}(\alpha_{\mathrm{WW}})}$

$\Rightarrow$ No mass gap and dimensional transmutation does not occur. In particular, fermions cannot condense at α_{WW}. Must assume that fermion condensation is consequence of **explicit** scale symmetry breaking, e.g. in walking gauge theories [Appelquist et al (97)]

3 | Chiral and scale condensates

Nambu-Goldstone (NG) mode

$$\left\{\mu\frac{\partial}{\partial\mu}+\gamma_{\mathcal{O}}(\alpha_s)\right\}\langle\mathrm{vac}|\mathcal{O}(0)|\mathrm{vac}\rangle = i\int d^4x\,\mathrm{T}\langle\mathrm{vac}\big|\theta^\mu_\mu(x)\mathcal{O}(0)\big|\mathrm{vac}\rangle \quad (\star)$$

In this case the RH side of $(\star)$ **does not vanish** at α_{IR} as $\theta^\mu_\mu \to 0$

Occurs if $\exists$ dilaton σ where $\langle\mathrm{vac}|\theta^\mu_\mu|\sigma\rangle = -m_\sigma^2 F_\sigma$, so key result follows:

$$\left\{\mu\frac{\partial}{\partial\mu}+\gamma_{\mathcal{O}}(\alpha_s)\right\}\langle\mathrm{vac}|\mathcal{O}(0)|\mathrm{vac}\rangle_{\mathrm{NG}} \to -F_\sigma\langle\sigma(q=0)|\,\mathcal{O}(0)\,|\mathrm{vac}\rangle\,, \quad \theta^\mu_\mu\to 0$$

Then $\langle\mathcal{O}\rangle_{\mathrm{vac}}$ is a **scale condensate** (e.g. $\langle\bar{q}q\rangle_{\mathrm{vac}}\neq 0$) and the vacuum breaks scale invariance.

NB.
- all particles except π, K, η, σ can **remain massive** at α_{IR}
- Green's functions do not exhibit power-law behaviour
- $|\mathrm{vac}\rangle$ carries μ dependence of theory and "knows" about dimensional transmutation; driven by strength of interactions in H

4 | Chiral-scale effective Lagrangian

If $\alpha_{\rm IR}$ exists in $N_f = 3$ QCD, it is **necessary** to extend $\chi\mathrm{PT}_3$ to account for non-linearly realised scale invariance [Salam et al (69) & (70); Ellis (70)]

In **physical region** $0 < \alpha_s < \alpha_{\rm IR}$ consider the combined limit

$$m_{u,d,s} \sim 0 \qquad \text{and} \qquad \alpha_s \lesssim \alpha_{\rm IR}$$

Effective chiral-scale Lagrangian composed of terms with differing scale dimension

$$\mathcal{L}_{\chi\mathrm{PT}_\sigma} = \mathcal{L}_{\rm inv}^{d=4} + \mathcal{L}_{\rm anom}^{d>4} + \mathcal{L}_{\rm mass}^{d<4}$$

Operator dimensions satisfy

$$d_{\rm inv} = 4 \quad \text{and} \quad 1 \leq 3 - \gamma_m(\alpha_{\rm IR}) < 4$$ [Wilson (69)]

Callan-Symanzik equations for QCD amplitudes implies that at LO

$$\beta(\alpha_s) \simeq \beta'(\alpha_{\rm IR})(\alpha_s - \alpha_{\rm IR}) \quad \Rightarrow \quad d_{\rm anom} = 4 + \beta' > 4$$

4 | Chiral-scale effective Lagrangian

Want $O(p^2)$ expression for $\mathcal{L}_{\chi\mathrm{PT}_\sigma} = \mathcal{L}_{\mathrm{inv}}^{d=4} + \mathcal{L}_{\mathrm{anom}}^{d>4} + \mathcal{L}_{\mathrm{mass}}^{d<4}$

Use covariant field e^{σ/F_σ} with scale dimension 1 for

$$\sigma \to \sigma - \tfrac{1}{4} F_\sigma \log \left| \det(\partial x'/\partial x) \right|$$

$\Rightarrow$ use powers of e^{σ/F_σ} to adjust dimensions of operators like

$$\mathcal{K}\left[U, U^\dagger\right] = \tfrac{1}{4} F_\pi^2 \mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) \qquad \mathcal{K}_\sigma = \tfrac{1}{2} \partial_\mu \sigma \partial^\mu \sigma$$

Then strong interactions at lowest order given by

$$\begin{aligned}
\mathcal{L}_{\mathrm{inv}}^{d=4} &= \left\{ c_1 \mathcal{K} + c_2 \mathcal{K}_\sigma + c_3 e^{2\sigma/F_\sigma} \right\} e^{2\sigma/F_\sigma} \\
\mathcal{L}_{\mathrm{anom}}^{d>4} &= \left\{ (1 - c_1)\mathcal{K} + (1 - c_2)\mathcal{K}_\sigma + c_4 e^{2\sigma/F_\sigma} \right\} e^{(2+\beta')\sigma/F_\sigma} \\
\mathcal{L}_{\mathrm{mass}}^{d<4} &= \mathrm{Tr}(M U^\dagger + U M^\dagger) e^{(3-\gamma_m)\sigma/F_\sigma}
\end{aligned}$$

New set of low energy constants $c_{1,\ldots,4}$, $\beta'(\alpha_{\mathrm{IR}})$, $\gamma_m(\alpha_{\mathrm{IR}})$

4 | Chiral-scale effective Lagrangian

Can we constrain the low energy constants?

$$\mathcal{L}_{\text{inv}}^{d=4} = \left\{c_1\mathcal{K} + c_2\mathcal{K}_\sigma + c_3 e^{2\sigma/F_\sigma}\right\}e^{2\sigma/F_\sigma}$$
$$\mathcal{L}_{\text{anom}}^{d>4} = \left\{(1-c_1)\mathcal{K} + (1-c_2)\mathcal{K}_\sigma + c_4 e^{2\sigma/F_\sigma}\right\}e^{(2+\beta')\sigma/F_\sigma}$$
$$\mathcal{L}_{\text{mass}}^{d<4} = \text{Tr}(MU^\dagger + UM^\dagger)e^{(3-\gamma_m)\sigma/F_\sigma}$$

For consistency, c_3 and c_4 are $O(M)$. Vacuum stability (no tadpoles) req:

$$4c_3 + (4+\beta')c_4 = -(3-\gamma_m)F_\pi^2\left(m_K^2 + \tfrac{1}{2}m_\pi^2\right)$$

Recently, we found a **simplification**: for

$$\theta_\mu^\mu\big|_{\text{eff}} = \sum_d (d-4)\mathcal{L}_d \;=\; :\beta'\mathcal{L}_{\text{anom}}^{d>4} - (1+\gamma_m)\mathcal{L}_{\text{mass}}^{d<4}:$$

to vanish in chiral-scale limit, c_1 and c_2 are both $1 + O(M)$

5 | Lowest order results

Mass formula

$$m_\sigma^2 F_\sigma^2 = F_\pi^2(m_K^2 + \tfrac{1}{2}m_\pi^2)(3-\gamma_m)(1+\gamma_m) - \beta'(4+\beta')c_4$$

Factor $d(d-4)$ in each term: $d_{\mathrm{mass}} = 3 - \gamma_m$ and $d_{\mathrm{anom}} = 4 + \beta'$

Sign of c_4 not determined by theory: depends on m_K/m_σ :

$$\Rightarrow \begin{cases} c_4 > 0 \text{ unless } d_{\mathrm{mass}} \text{ close to 0 or 4} \\ \text{for (say) } d_{\mathrm{mass}} = 2 \text{ and } \beta'=1\text{: } c_4 \simeq \tfrac{3}{4} m_K^2 F_\pi^2 \end{cases}$$

Width

$$\mathcal{L}_{\sigma\pi\pi} = \{2|\partial\pi|^2 - (3-\gamma_m)m_\pi^2|\pi|^2\}\sigma/(2F_\sigma)$$

$\Rightarrow$ small effect on $\pi\pi$ scattering in $SU(2)\times SU(2)$ limit with $\partial = O(m_\pi)$

Vertex for on shell amplitude $g_{\sigma\pi\pi} = -\big(m_\sigma^2 + (1-\gamma_m)m_\pi^2\big)/F_\sigma$

Fit to data on $f_0(500)$ width $\Rightarrow$ $|F_\sigma| \simeq 50$ MeV

6 | The ΔI=1/2 rule for kaon decays

Let $\mathcal{J}_{ij} = (U\partial_\mu U^\dagger)_{ij}$ and adjust dimensions of $\chi\mathrm{PT}_3$ operators:

$$Q_8 = \mathcal{J}_{13}\mathcal{J}_{21} - \mathcal{J}_{23}\mathcal{J}_{11}$$
$$Q_{27} = \mathcal{J}_{13}\mathcal{J}_{21} + \tfrac{3}{2}\mathcal{J}_{23}\mathcal{J}_{11}$$

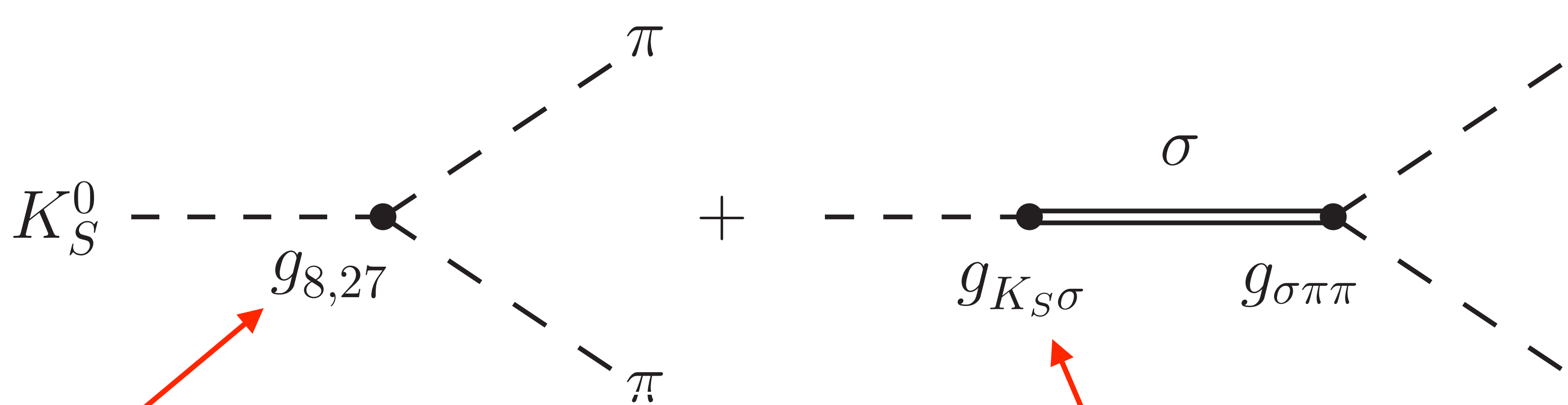
and

$$Q_{mw} = \mathrm{Tr}(\lambda_6 - i\lambda_7)\big(g_M M U^\dagger + \bar{g}_M U M^\dagger\big)$$

Vacuum alignment of the resulting effective Lagrangian [Crewther (86)]

$$\mathcal{L}_w = g_8 Q_8 e^{(2-\gamma_8)\sigma/F_\sigma} + g_{27} Q_{27} e^{(2-\gamma_{27})\sigma/F_\sigma} + Q_{mw} e^{(3-\gamma_{mw})\sigma/F_\sigma} + \text{ h.c.}$$

$\Rightarrow$ $K_S\sigma$ vertex from mismatch of mass and weak anomalous dimensions



g_8 and g_{27} allowed to have similar magnitude

Magnitude of $g_{K_S\sigma}$ needed is consistent with $K_S \to \gamma\gamma$ and $\gamma\gamma \to \pi\pi$

7 | Testing χPT_σ on the lattice

The signal on the lattice for an IR fixed point in NG mode is **freezing** of the running coupling **outside the conformal window** [e.g. Horseley et al. (13)]

Scheme/definition dependence of α_s $\Rightarrow$ hard to test $\chi\mathrm{PT}_\sigma$ conclusively

Instead, revive lattice proposal to measure $K \to \pi$ **on shell** [Crewther (86)] and consider low-energy theorems for $K \to \pi\pi$:

1. Vacuum alignment $\Rightarrow$ $\quad K \to |\mathrm{vac}\rangle = O(p^4)$ (still true in $\chi\mathrm{PT}_\sigma$)

   $\therefore$ try lattice determination for $\langle\pi|\mathcal{L}_{\mathrm{eff}}|K\rangle$ with both K and π on shell

2. In $\chi\mathrm{PT}_\sigma$ we have the following low-energy theorem

$$\{K \to \pi\pi\} = \{K \to \pi \text{ on shell}\} + \{\sigma/f_0 \text{ pole}\} + O(p^4)$$

3. The on shell value of $\langle\pi|\mathcal{L}_{\mathrm{eff}}|K\rangle$ is not affected by σ/f_0 pole

   $\Rightarrow$ a lattice determination of $K \to \pi$ measures $|g_8/g_{27}|$ directly which is what the ΔI=1/2 puzzle has always been about