Exotic and excited states from functional approaches

Christian S. Fischer

Justus Liebig Universität Gießen

Eichmann, CF, Sanchis-Alepuz, submitted to PLB, [1607.05748]
Eichman, CF, Heupel, PLB, [1508.07178]

Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP in press [1606.09602]

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Baryons and Tetraquarks from Dyson-Schwinger equations

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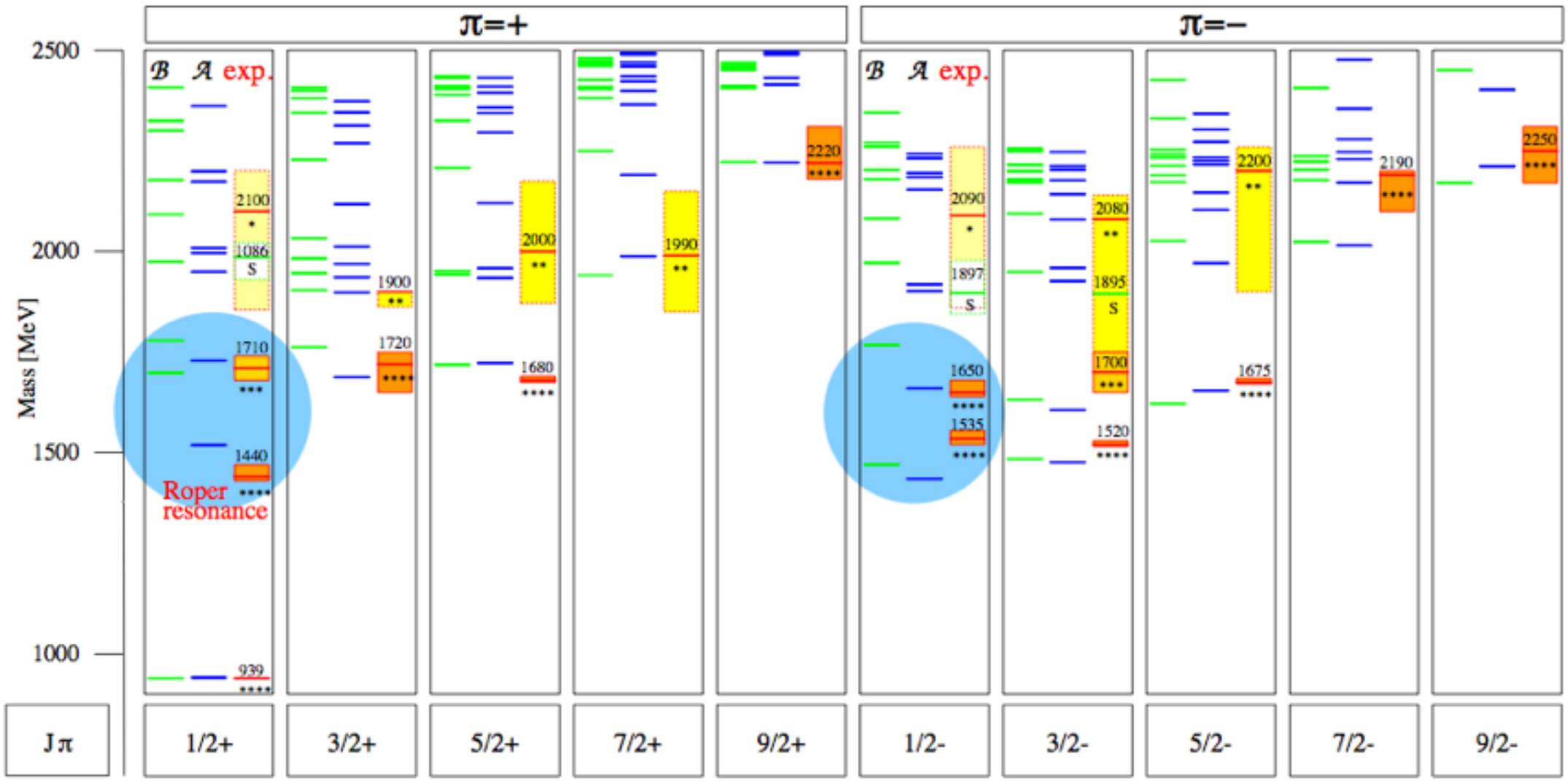
Baryons: Quark model

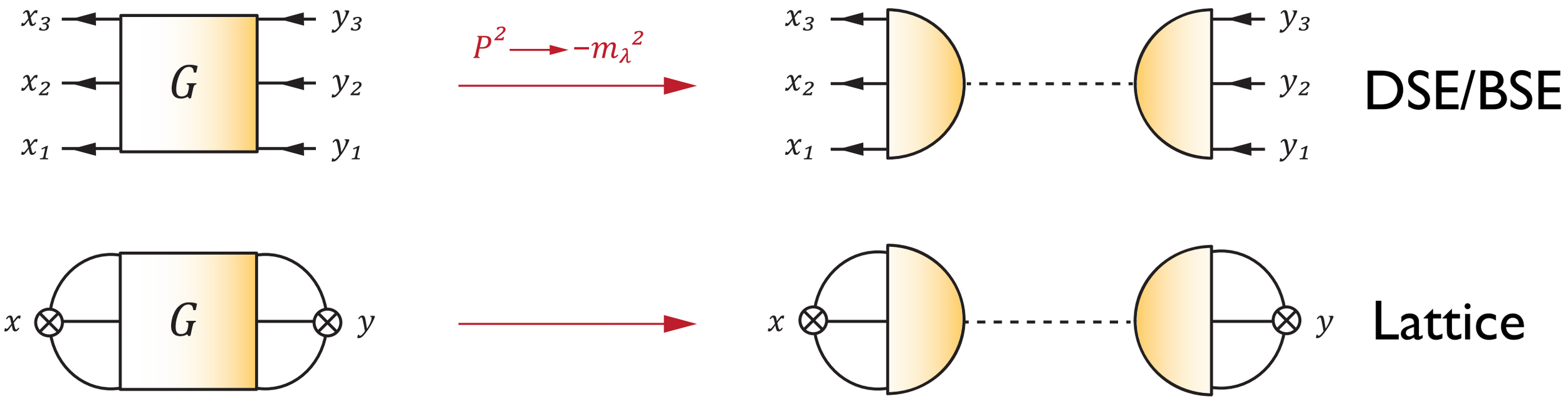


Loring, Metsch, Petry, EPJA 10 (2001) 395

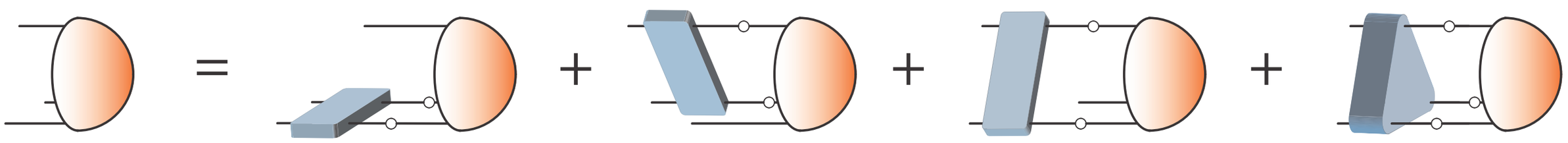
- 'missing resonances' - three-body vs. quark-diquark
- level ordering: $N\frac{1}{2}^{\pm}$ vs. $\Lambda\frac{1}{2}^{\pm}$

—> Eichmann, A2

Extracting spectra from correlators



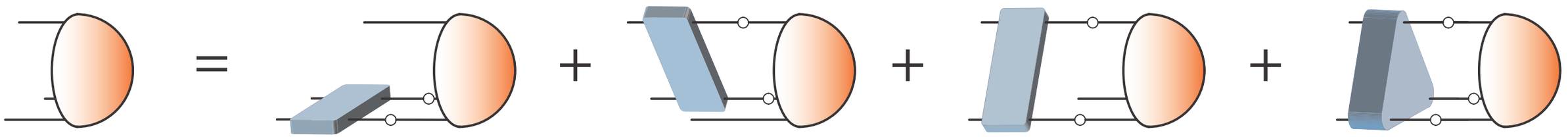
BSE for baryons (derived from equation of motion for G)



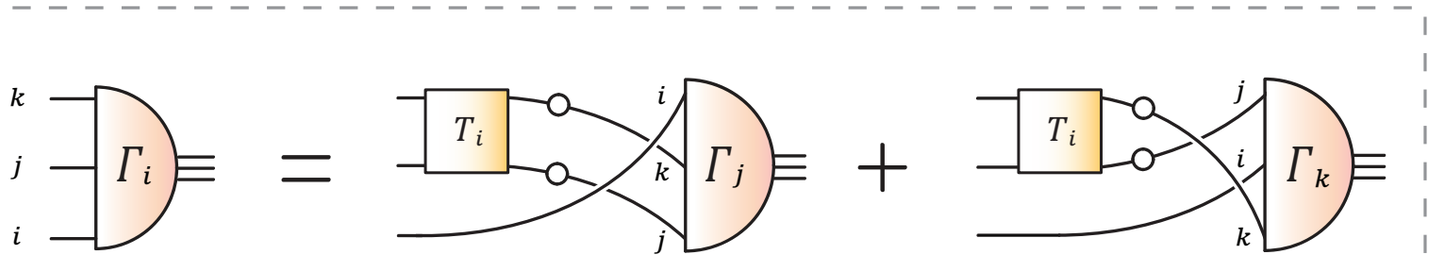
- exact equation for baryon ‘wave function’

Diquark-Quark approximation

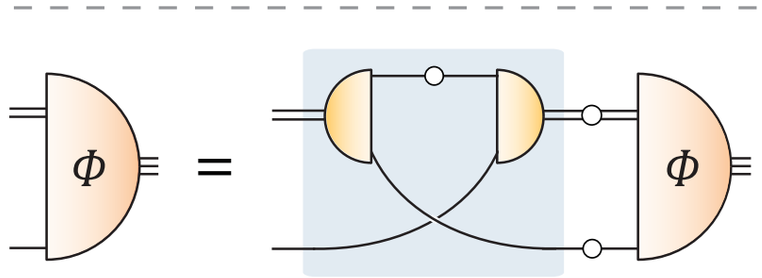
BSE for baryons (derived from equation of motion for G)



Faddeev equation (no three-body forces)



Diquark-quark



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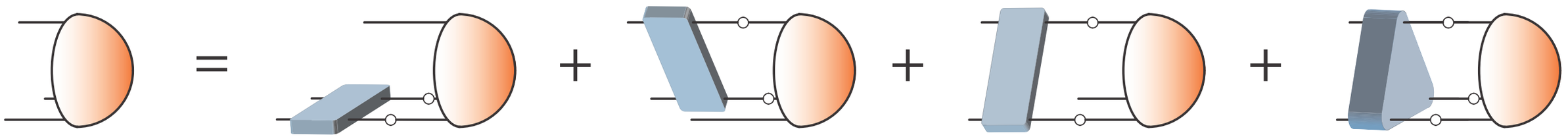

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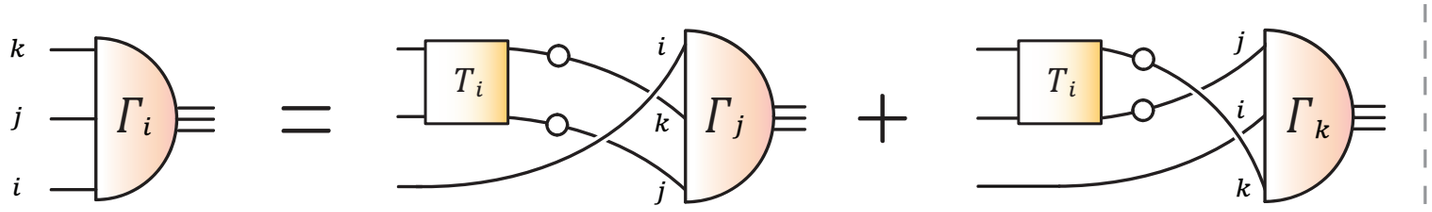

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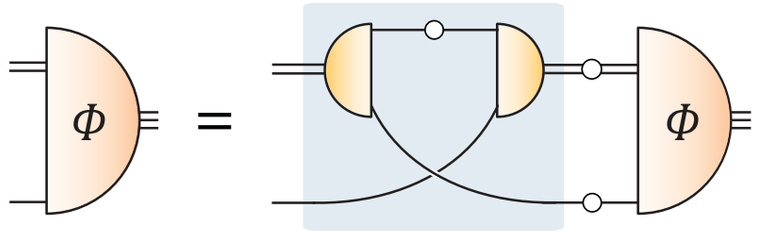

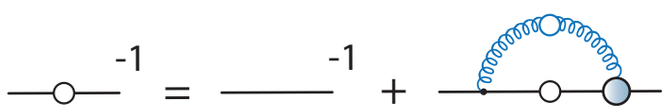

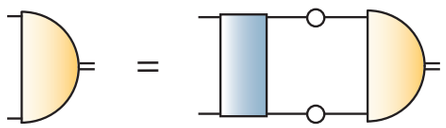

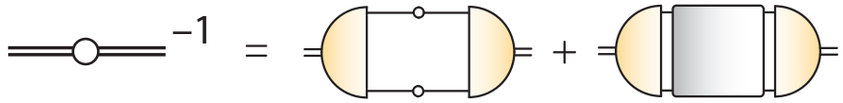

Diquark-Quark approximation

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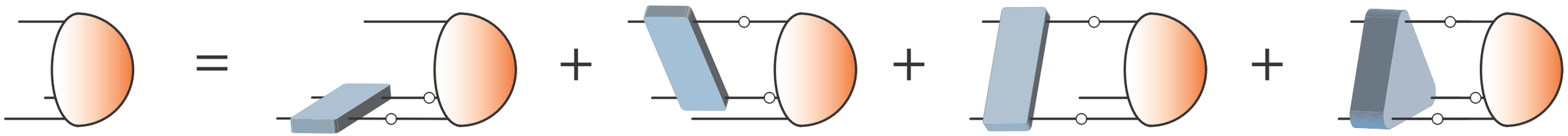

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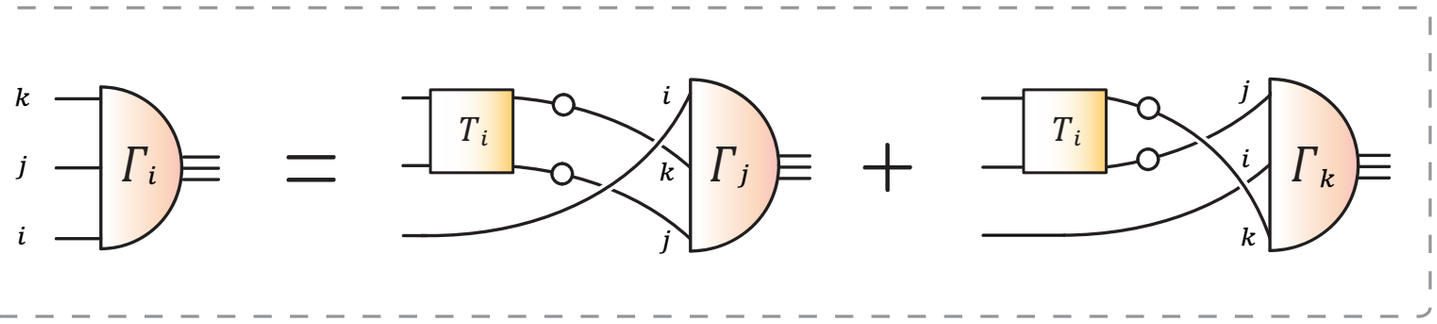
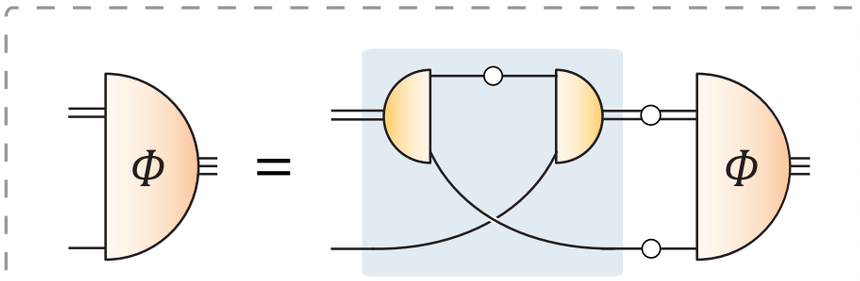
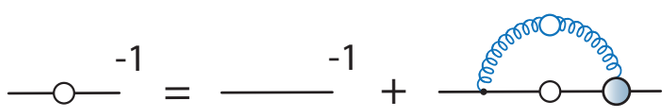
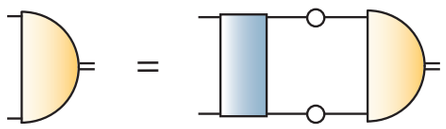
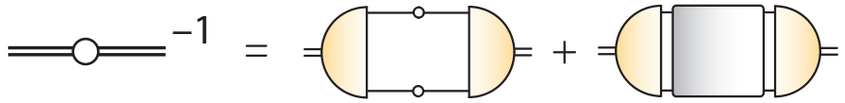


Diquark-quark

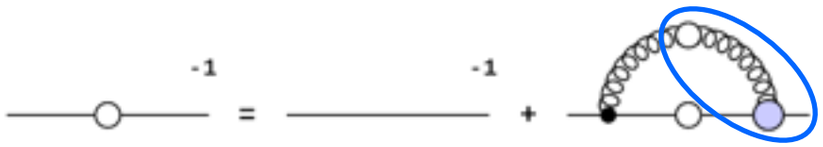








- Input: Non-perturbative quark, quark-gluon interaction (RL)



$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right) e^{-\eta^2\left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)$$

DSE/Faddeev landscape

	Quark-diquark			Three-quark		
	Contact interaction	QCD-based model	DSE (RL)	RL	bRL	bRL + 3q
N, Δ masses	√	√	√	√	√	...
N, Δ em. FFs	√	√	√	√		
$N \to \Delta\gamma$	√	√	√	...		
Roper	√	√		...		
$N \to N^*\gamma$	√	√		...		
$N^*(1535), \ldots$	...	...		...	...	
$N \to N^*\gamma$	...	...				
	Roberts et al	Oettel, Alkofer Roberts, Bloch Segovia et al.	Eichmann, Alkofer Nicmorus, Krassnigg	Eichmann, Alkofer Sanchis-Alepuz, CF	Sanchis-Alepuz, CF Williams	

Eichmann, N*-Workshop, Trento 2015

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Roper	√	√		...		
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$N^*(1535), \ldots$	...	...		...	...	
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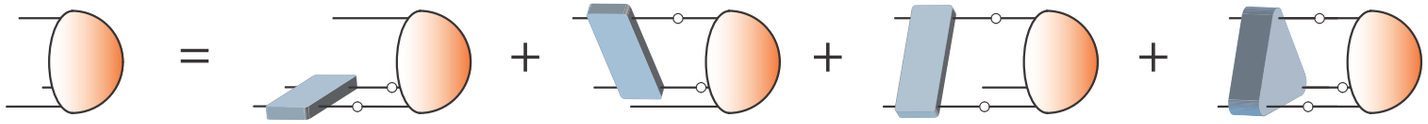
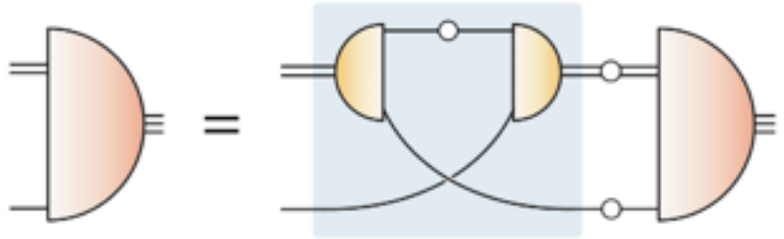
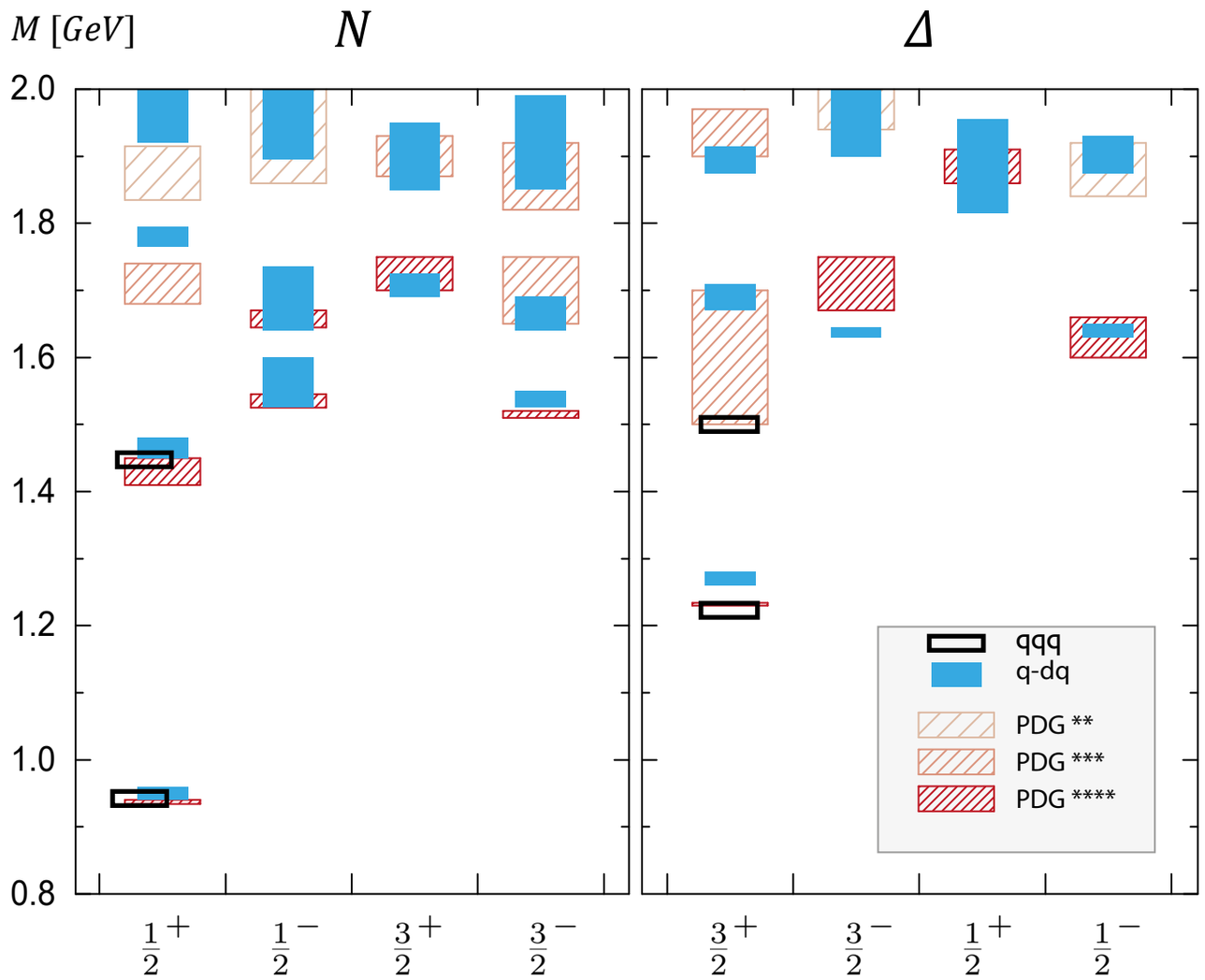
Eichmann, N*-Workshop, Trento 2015

Light baryon spectrum







Eichmann, CF, Sanchis-Alepuz, 1607.05748
Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP in press [1606.09602]

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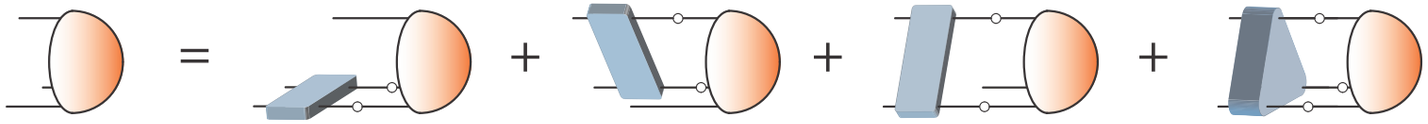
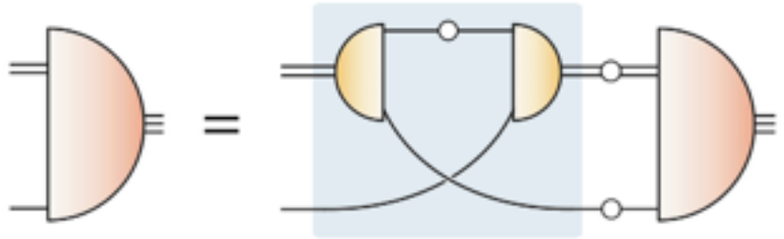
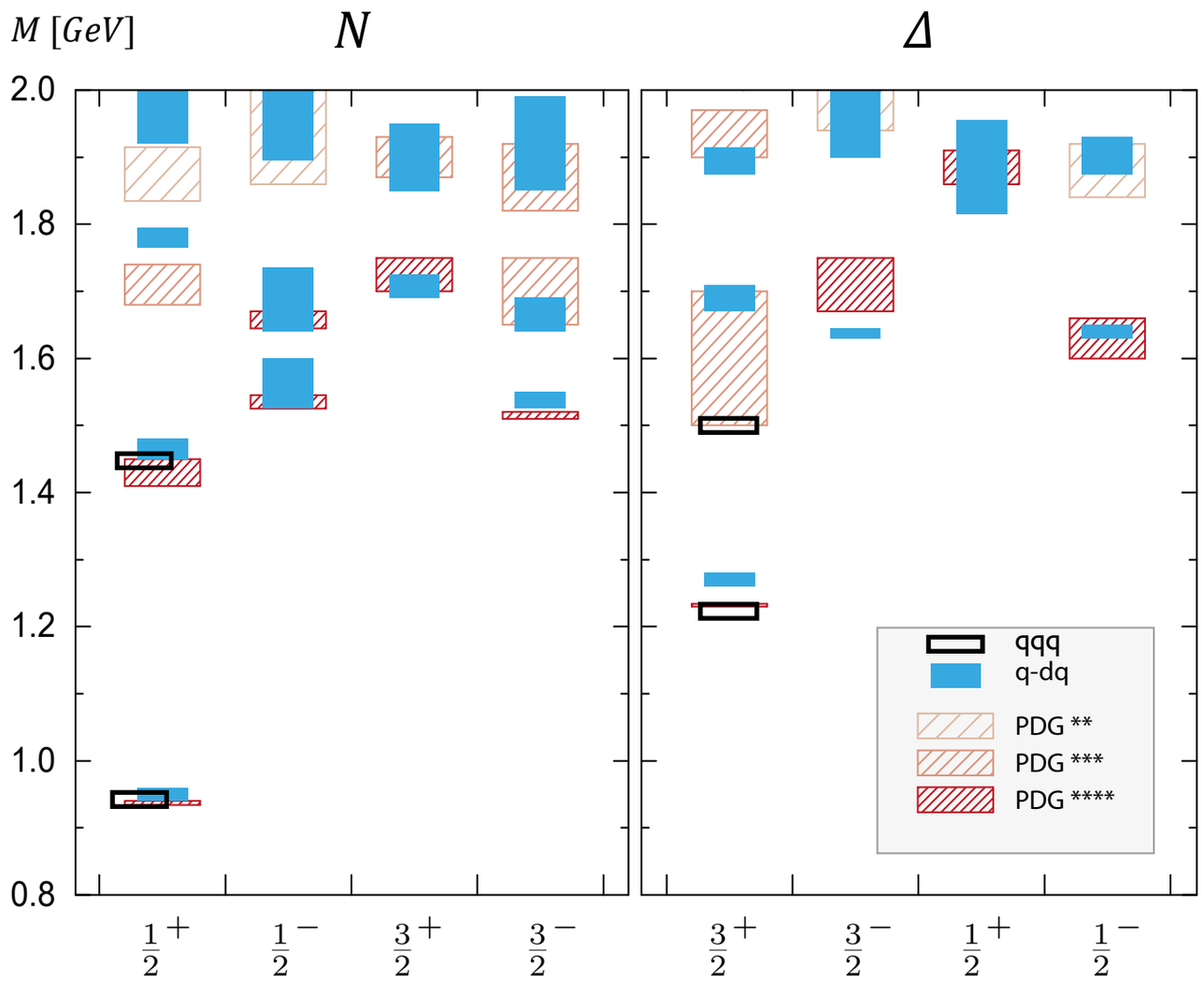


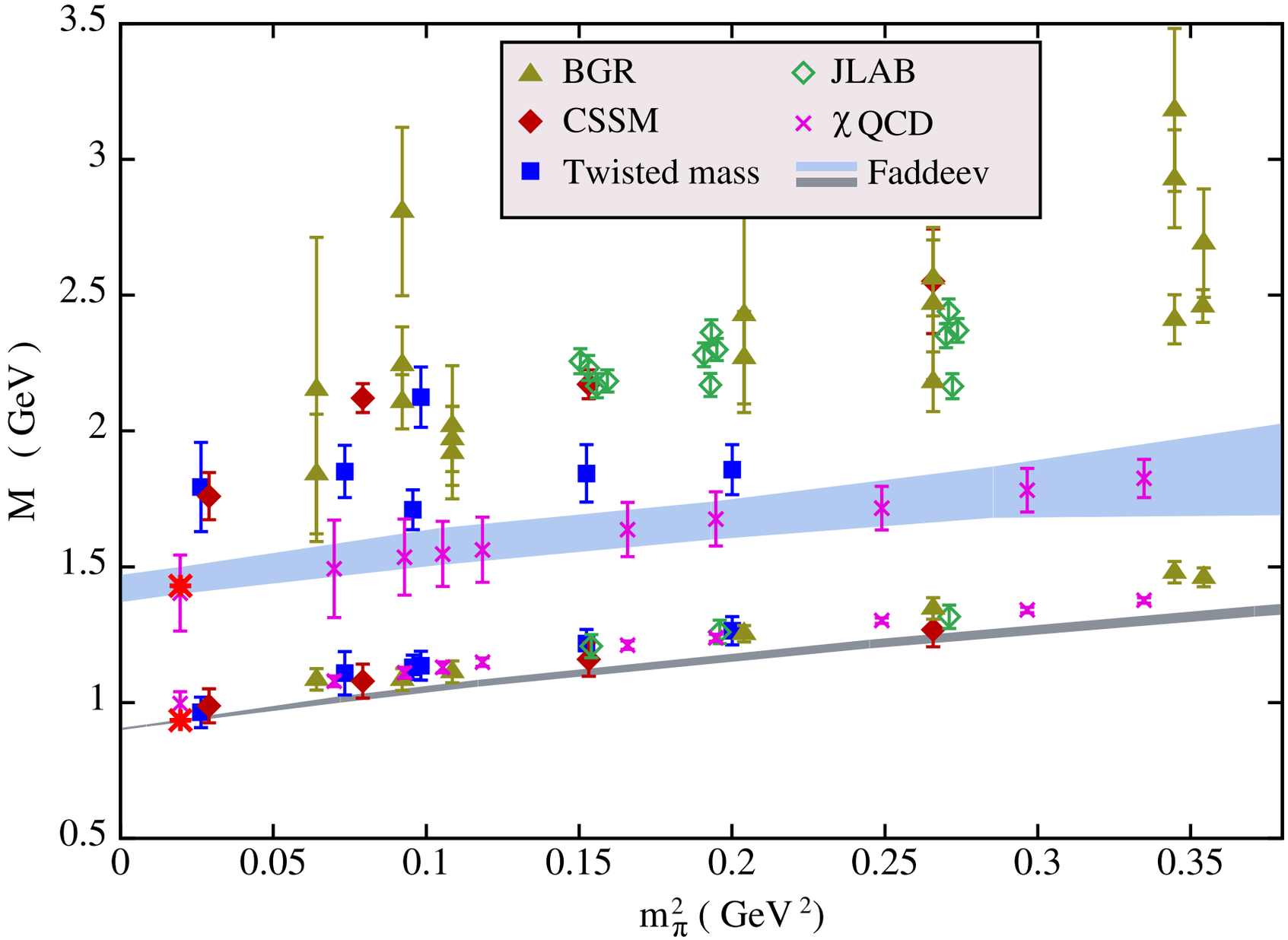




Eichmann, CF, Sanchis-Alepuz, 1607.05748
Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP in press [1606.09602]

- Three-body and diquark-quark approach agree qualitatively
- Spectrum in one-to-one agreement with experiment
- Correct level ordering (wo. coupled channel effects) !

Mass evolution



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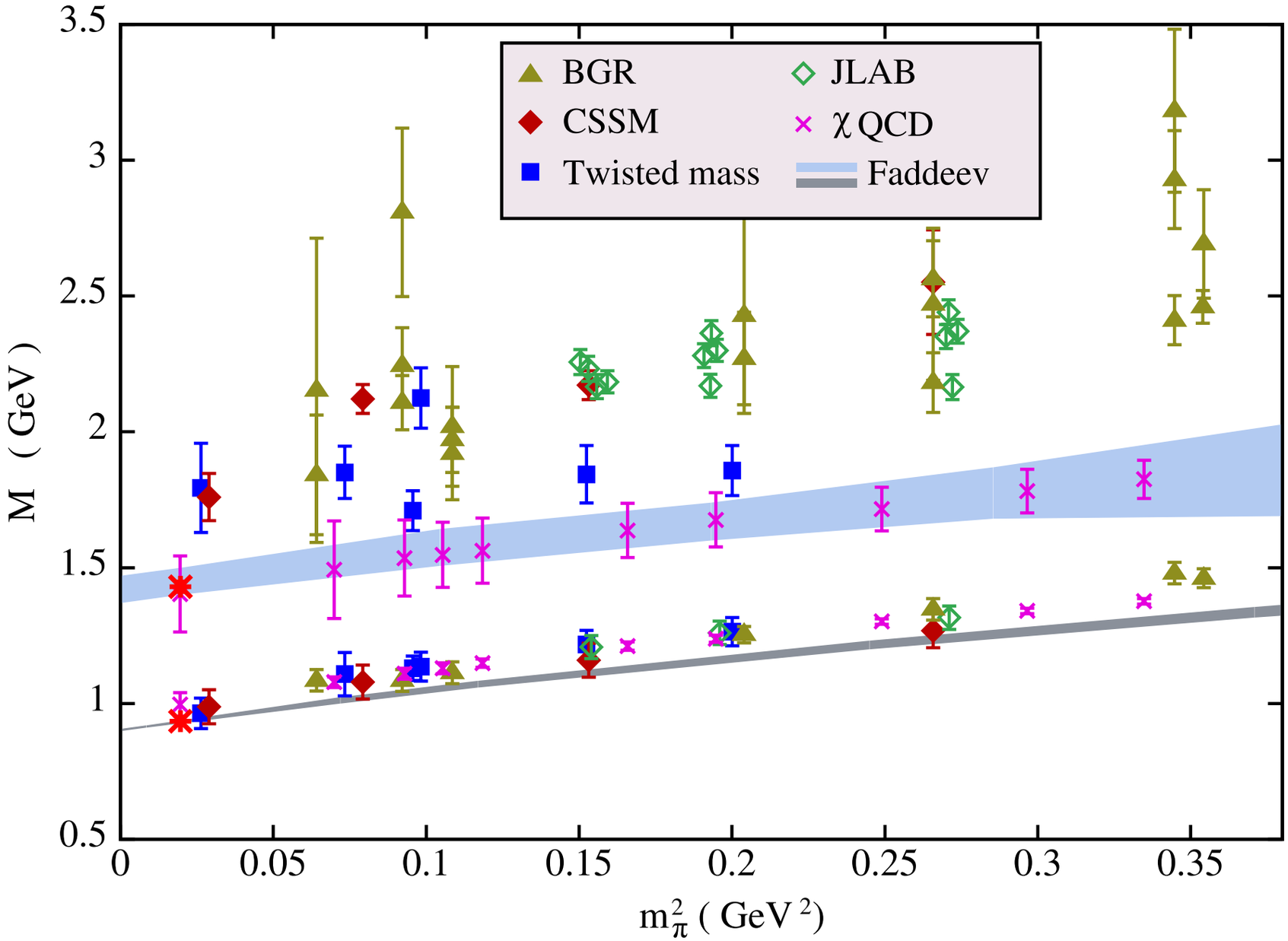
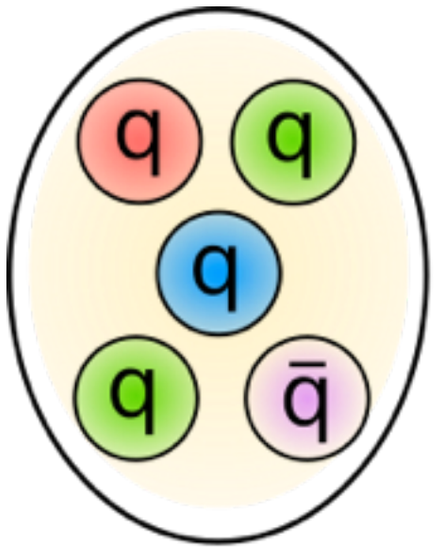


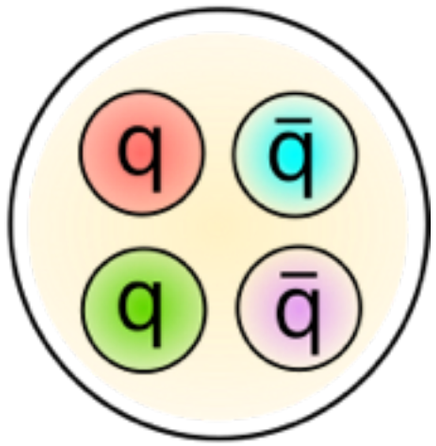
Eichmann, CF, Sanchis-Alepuz, 1607.05748
Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP in press [1606.09602]

- Mass evolution as expected for three-body state…

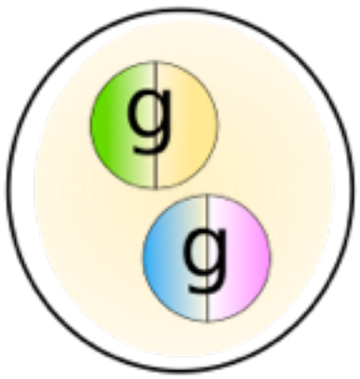
Tetraquarks in the light meson sector



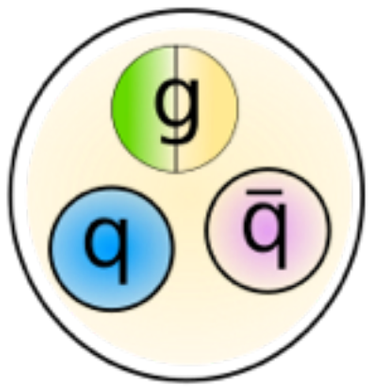
Pentaquark



Tetraquark

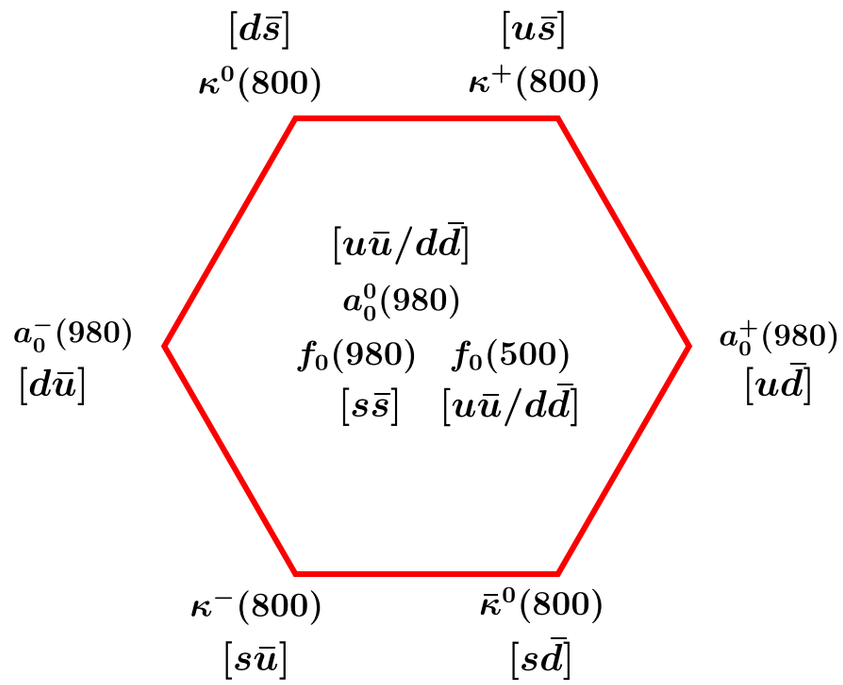


Glueball



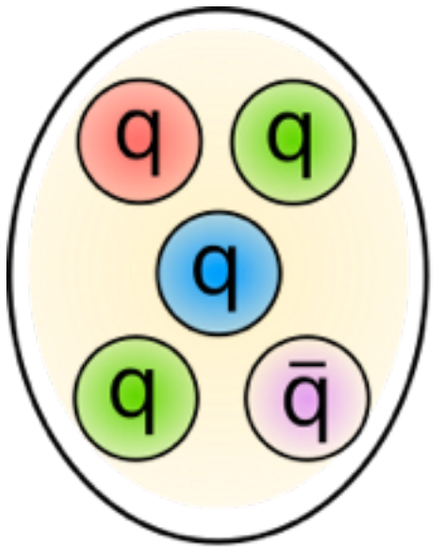
Hybrid

Light meson sector: scalars!



$$f_0(980) \to \pi\pi, K\bar{K}$$
$$a_0(980) \to \pi\eta, K\bar{K}$$

Tetraquarks in the light meson sector



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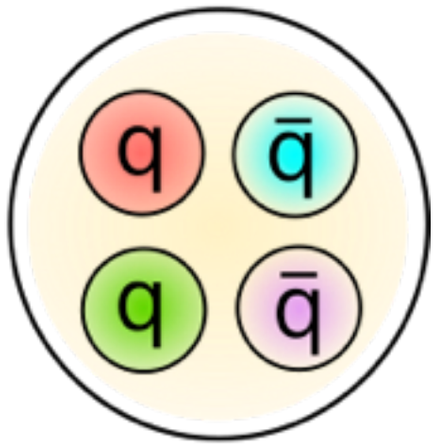

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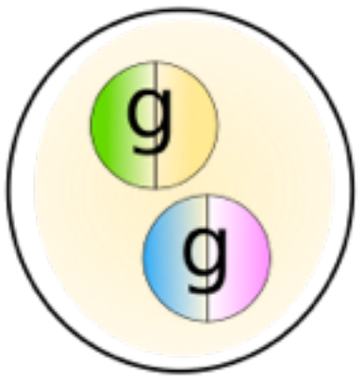

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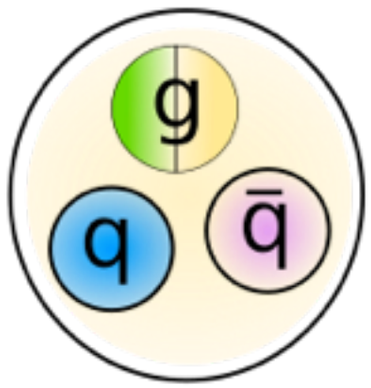

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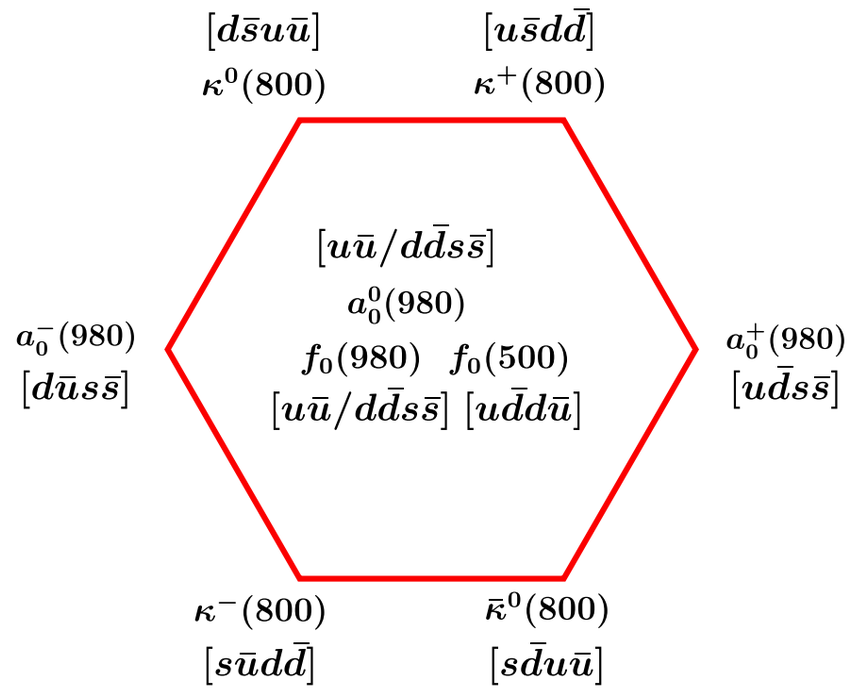
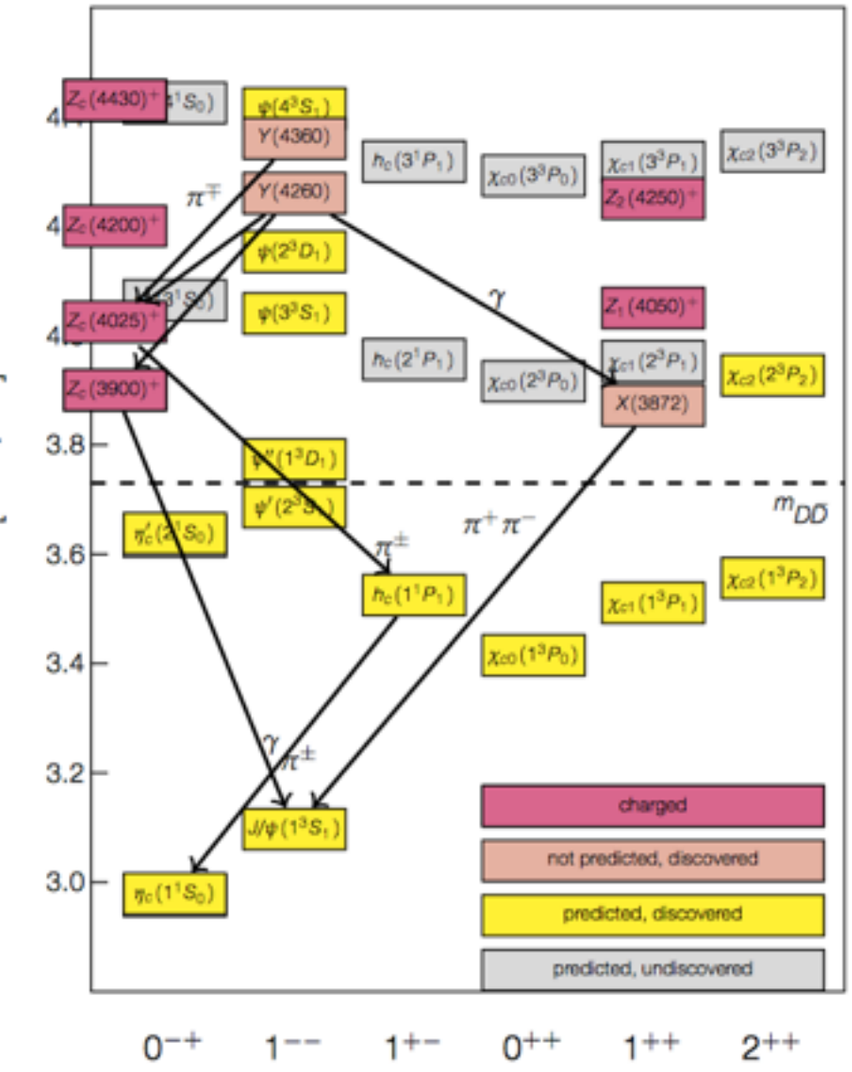


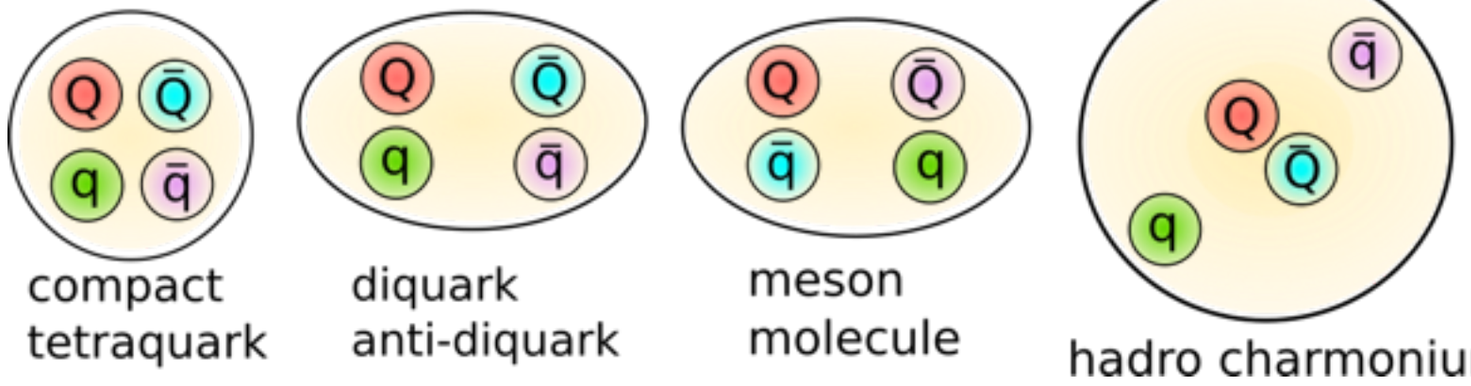
$$f_0(980) \to \pi\pi, K\bar{K}$$
$$a_0(980) \to \pi\eta, K\bar{K}$$

Tetraquark candidates in charmonium region



Wolfgang Gradl, BESIII, St Goar 2015

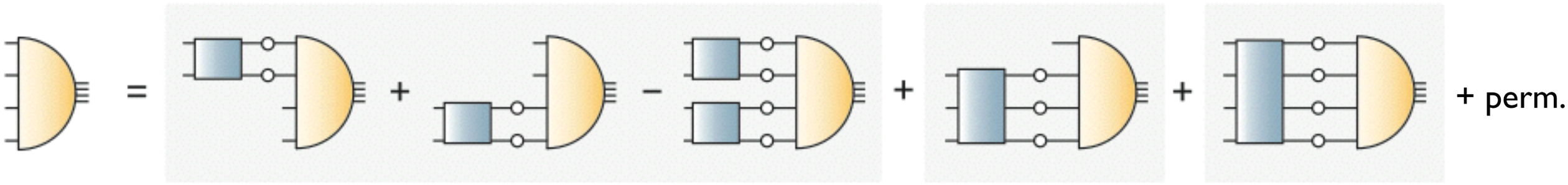
Internal structure ??



Related to details of underlying QCD forces between quarks

Tetraquarks from the four-body interaction

Exact equation:



Two-body interactions

Three- and four-body interactions

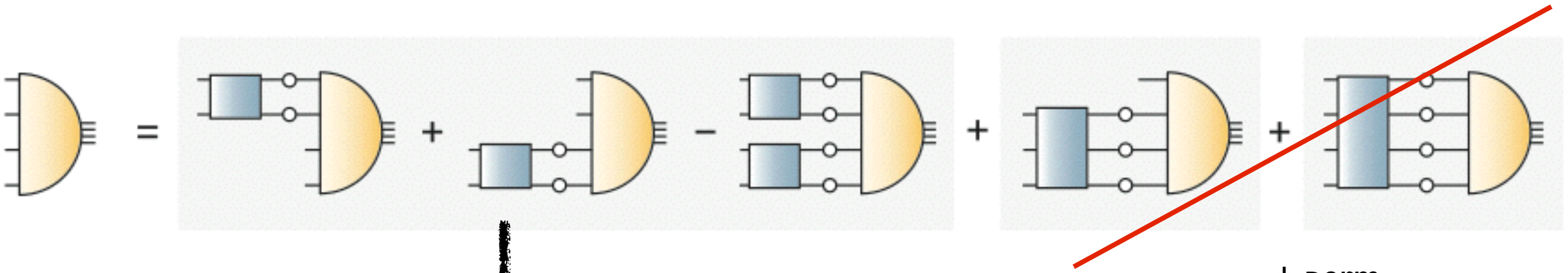
Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)

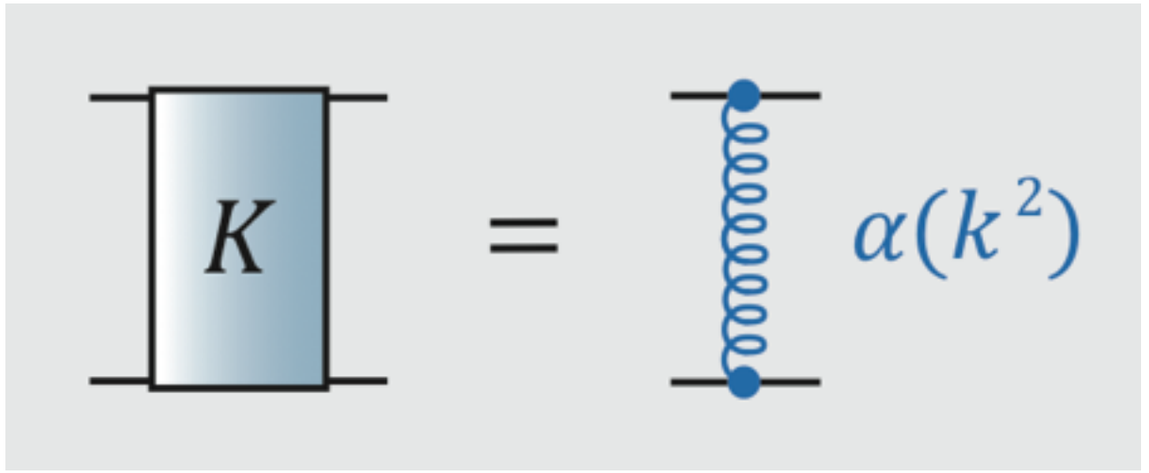
Heupel, Eichman, CF, PLB 718 (2012) 545-549

Eichman, CF, Heupel, PLB 753 (2016) 282-287

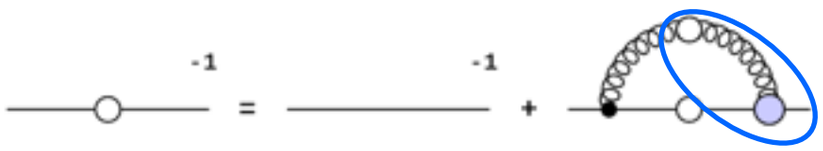
- Basic idea:
  solve four-body equation without any assumption on internal clustering
- Key elements: quark propagator and interaction kernels

Solving the four-body equation





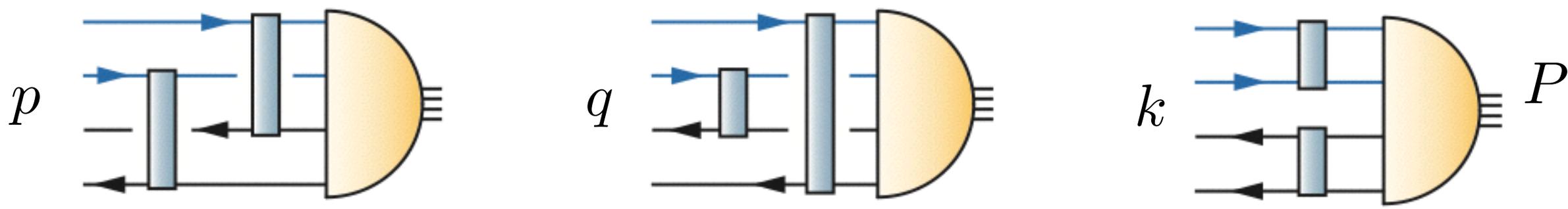
- Input: Non-perturbative quark, quark-gluon interaction



$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right) e^{-\eta^2\left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)$$

Structure of the amplitude

Scalar tetraquark:



$$\Gamma(P,p,q,k) = \sum_i f_i(s_1,...,s_9) \times \tau_i(P,p,q,k) \times color \times flavor$$

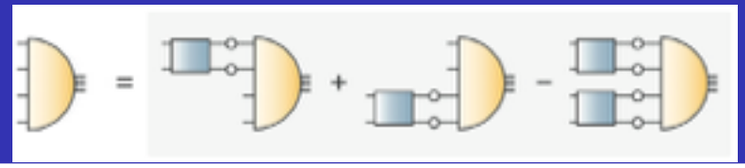
9 Lorentz scalars (built from P,p,q,k)

256 tensor structures (scalar tetra)

$3 \otimes \bar{3}, 6 \otimes \bar{6}$ or $1 \otimes 1, 8 \otimes 8$

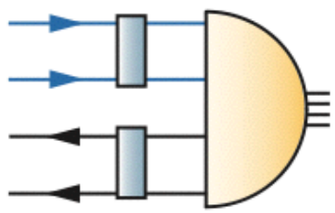
- good approximation: keep s-waves only; 16 tensor structures

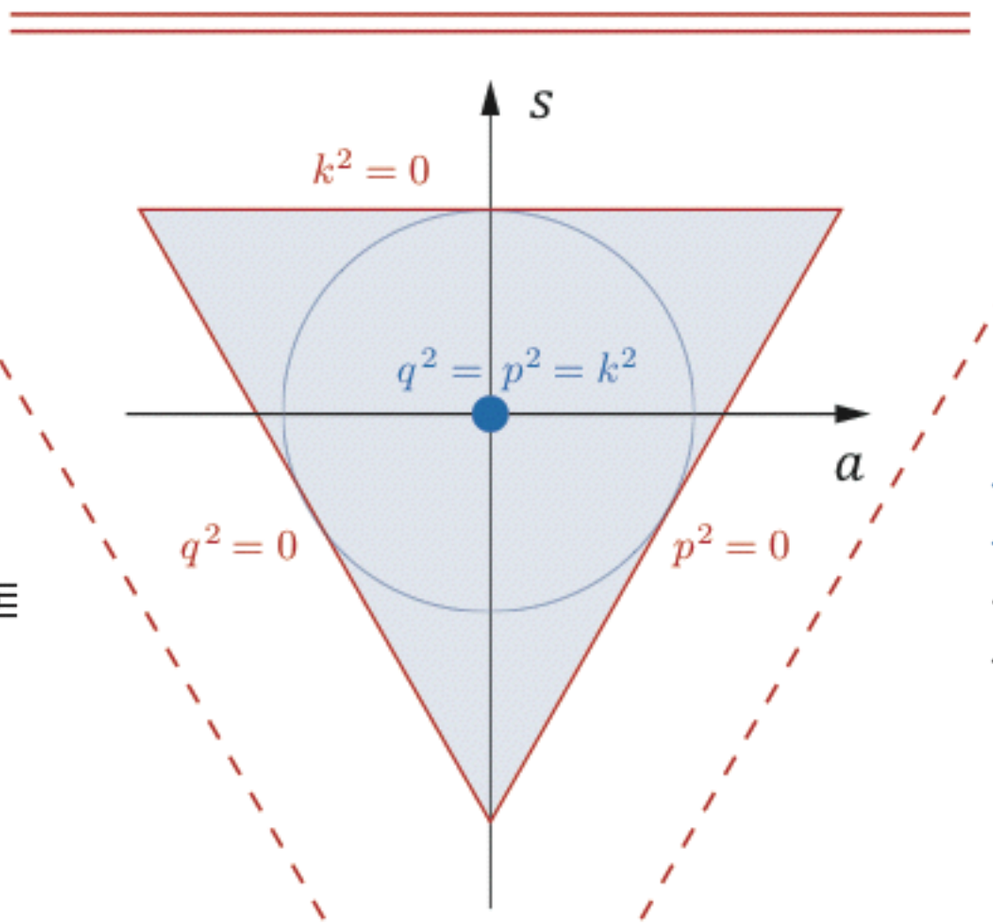
Four-body equation:



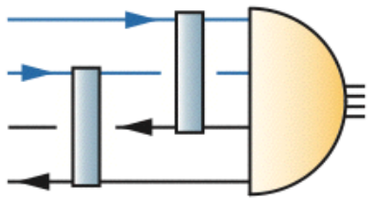
Organise Dirac-Lorentz-tensors into multiplets of **S4**

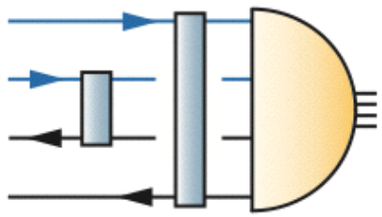
- Singlet: $S_0 = (p^2 + q^2 + k^2)/4$, **carries overall scale**
- Doublet: $a = \sqrt{3}(q^2 - p^2)/(4S_0)$; $s = (p^2 + q^2 - 2k^2)/(4S_0)$





diquark pole





meson poles

- Two triplets

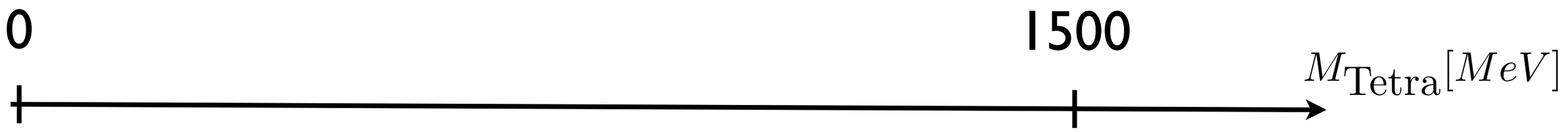
Eichman, CF, Heupel, PLB 753 (2016) 282-287

Bound state masses

- Different levels of approximations:

Bound state masses

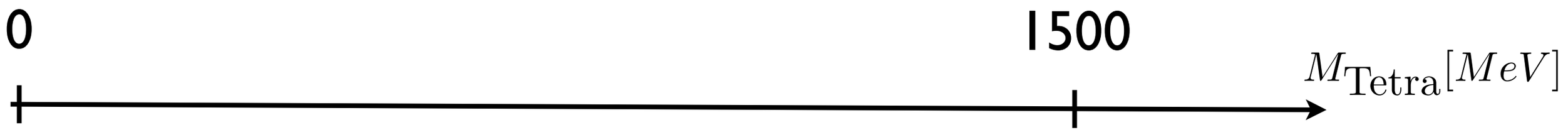
- Different levels of approximations:



Singlet only

Bound state masses

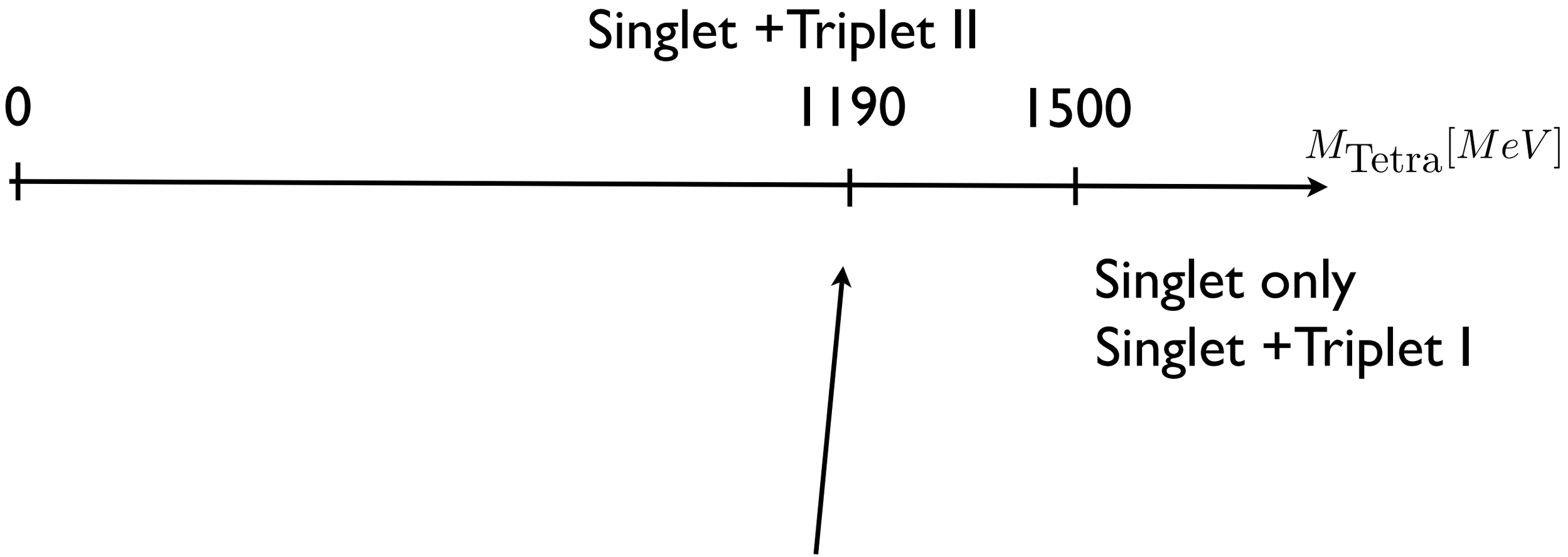
- Different levels of approximations:



Singlet only
Singlet +Triplet I

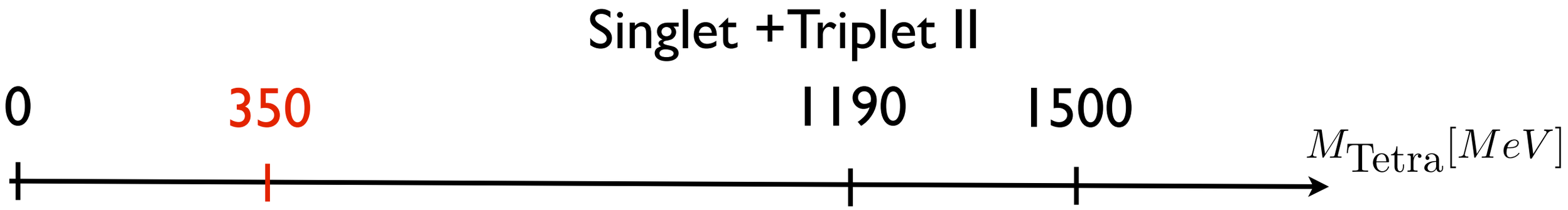
Bound state masses

- Different levels of approximations:

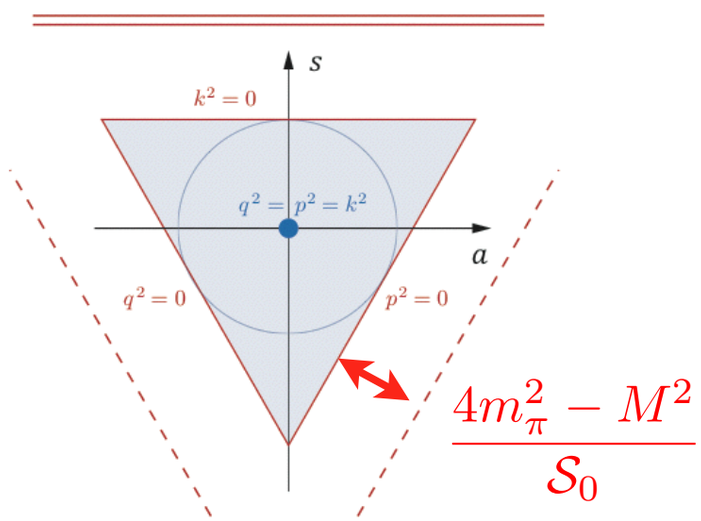


Bound state masses

- Different levels of approximations:



Singlet + Doublet



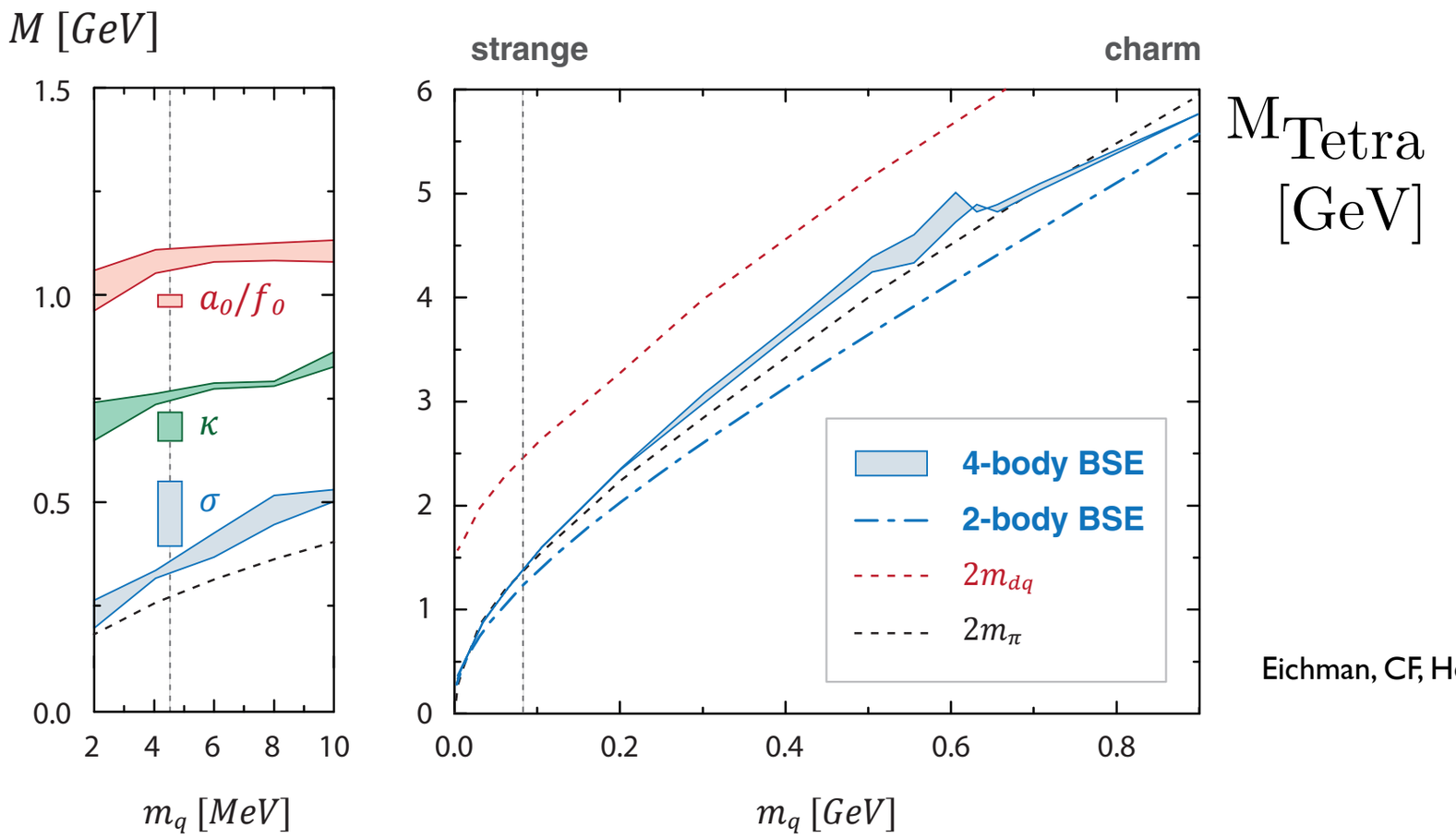
Two-pion resonance

Singlet only

Singlet +Triplet I

Bound state of
four massive quarks

Mass evolution of tetraquark



Eichman, CF, Heupel, PLB 753 (2016) 282-287

- Resonance becomes bound state for large m_q
- Dynamical decision: meson clusters, not diquarks
- Results:

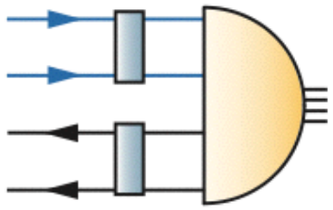
$$m_\sigma \sim 350\,\mathrm{MeV}$$
$$m_\kappa \sim 750\,\mathrm{MeV} \qquad m_{ss\bar{s}\bar{s}} \sim 1.5\,\mathrm{GeV}$$
$$m_{a_0,f_0} \sim 1080\,\mathrm{MeV} \qquad m_{cc\bar{c}\bar{c}} \sim 5.7\,\mathrm{GeV}$$

qualitatively similar to two-body framework

Heupel, Eichman, CF, PLB 718 (2012) 545-549

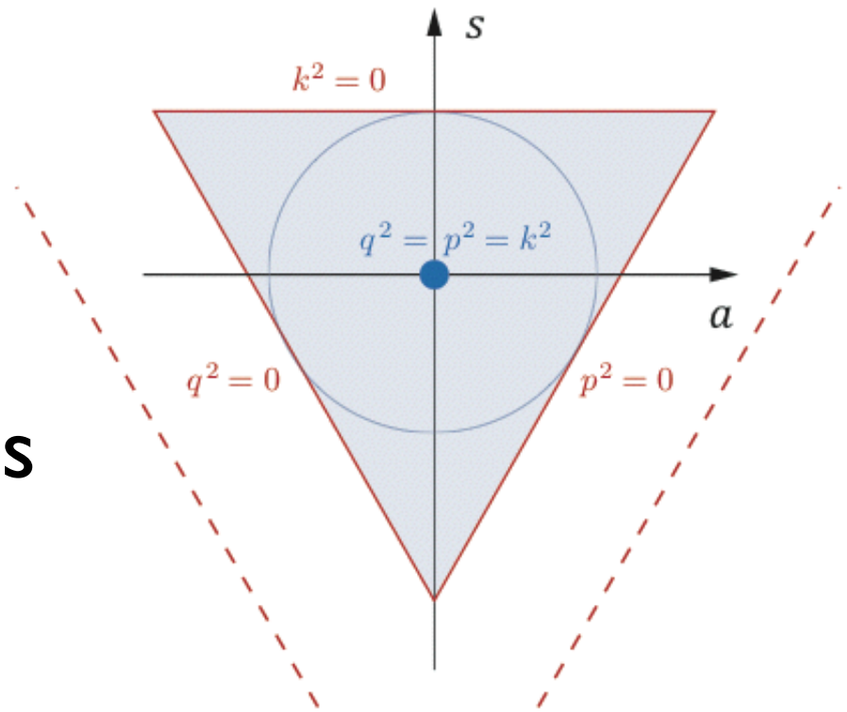
Outlook: heavy-light systems

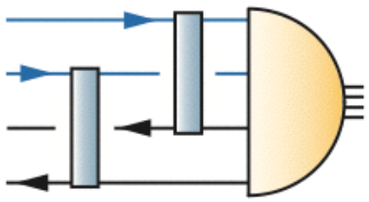
Dynamical situation in **S4**-doublet:



heavy-light diquarks

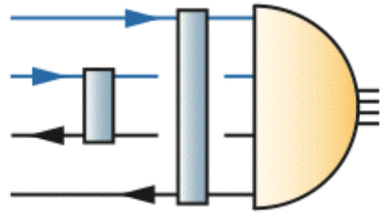
$$cq - \bar{c}\bar{q}$$





meson-molecules

$$c\bar{q} - \bar{c}q$$



hadro-charmonium

$$c\bar{c} - q\bar{q}$$

Dynamical decision of most important clustering!

Summary and outlook

Summary

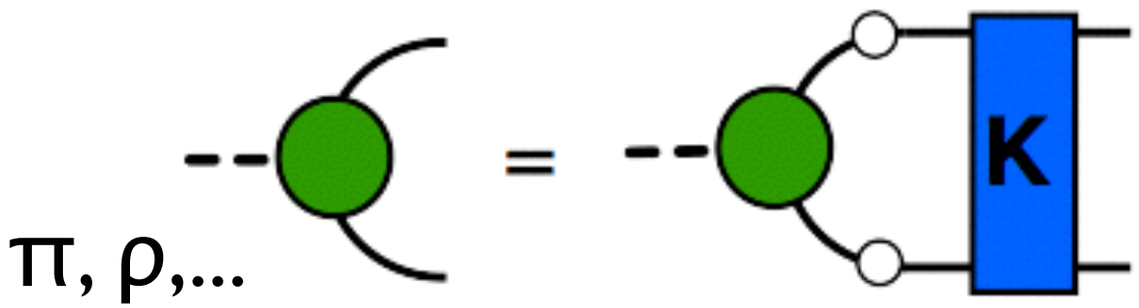
- Baryon spectrum: good agreement with experiment!
- Three-body vs diquark-quark: fair agreement
- Tetraquarks dominated by internal meson-meson configurations
- Dynamical description of σ as π-π resonance

Outlook

- Baryons: strange sector
- Baryons: transition form factors
- Tetraquarks: explore heavy-light systems

Backup Slides

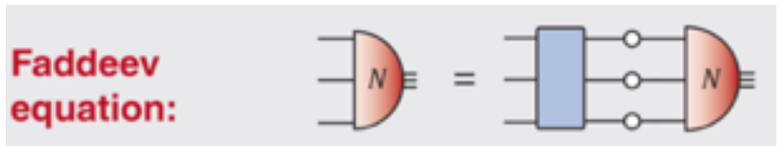
Strategy II: Beyond rainbow ladder

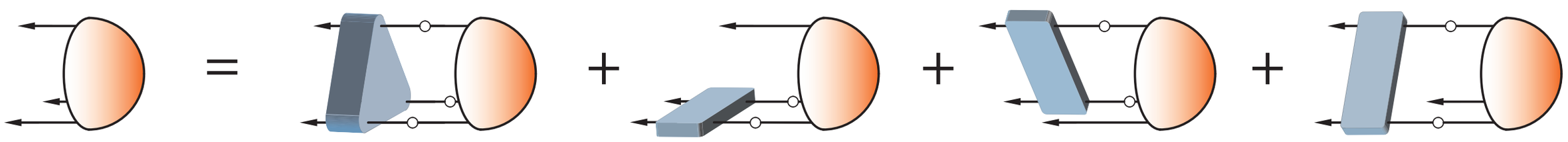


Recent improvements beyond rainbow-ladder:

- treat Yang-Mills sector explicitly — CF, Watson, Cassing, PRD 72 (2005)
- include gauge effects in vertex — Chang, Roberts, PRL 103 (2009); Heupel, Goecke, CF, EPJA 50 (2014)
- anomalous quark magnetic moment — Chang, Liu, Roberts, PRL 106 (2011)
- include gluon self-interaction effects — CF, Williams, PRL 103 (2009)
- include pion cloud effects — CF, Nickel, Wambach PRD 76 (2007)

Faddeev - equation





- irreducible three-body forces

Sanchis-Alepuz, Williams, work in progress…

- two-body interactions:
  - non-perturbative gluon exchange

Eichmann, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)

  - meson exchange

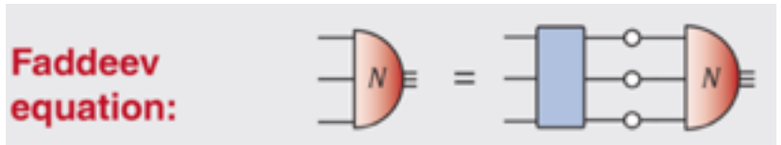
Sanchis-Alepuz, CF, Kubrak, PLB 733 (2014)

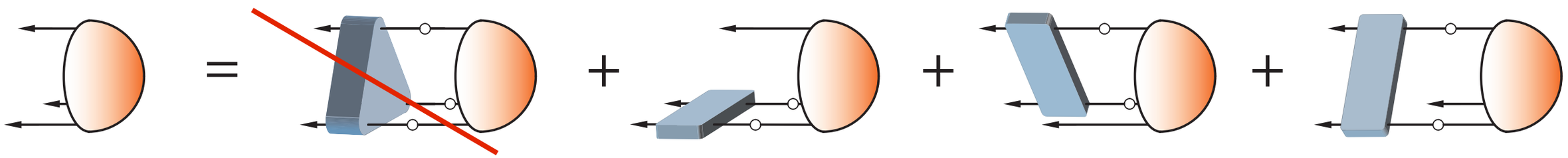
  - two-body forces beyond one-particle exchange

Sanchis-Alepuz, Williams, PLB 749 (2015) 592

- numerically expensive but manageable !

Faddeev - equation





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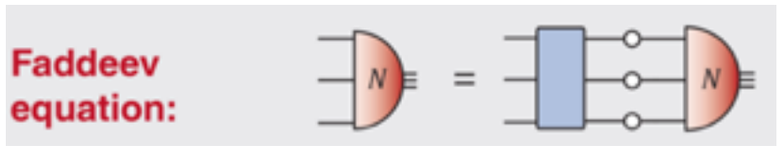

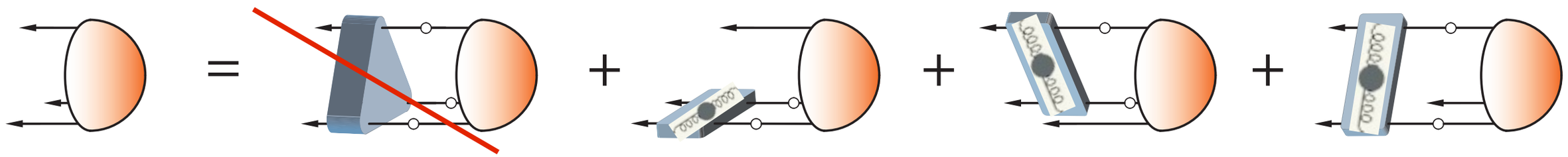

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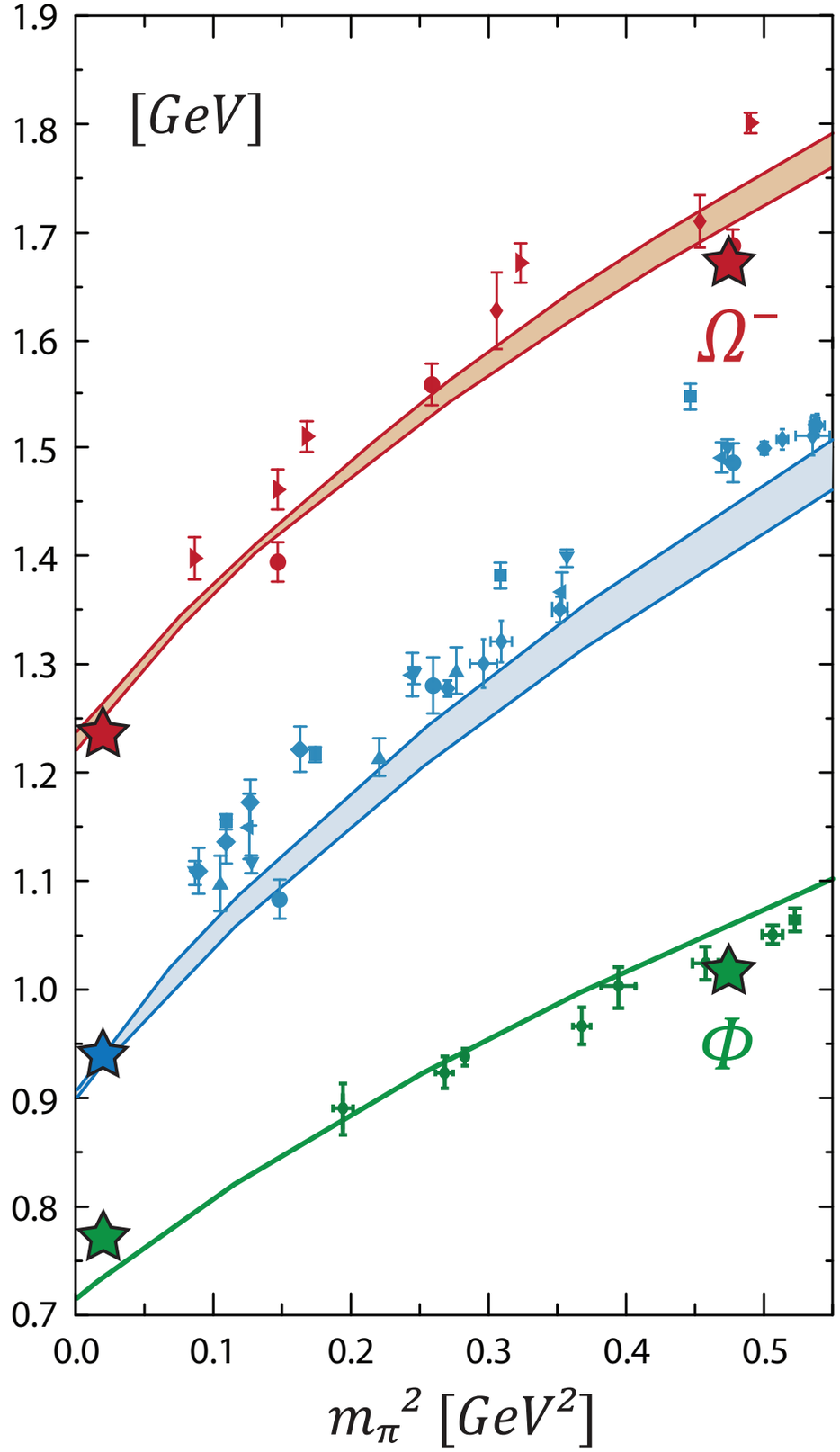
Sanchis-Alepuz, Williams, PLB 749 (2015) 592

- numerically expensive but manageable !

Baryon masses - gluon exchange only

- first covariant three-body calculations !
- grosso modo: consistent description of mesons and baryons
- wave functions contain sizable p-wave contributions



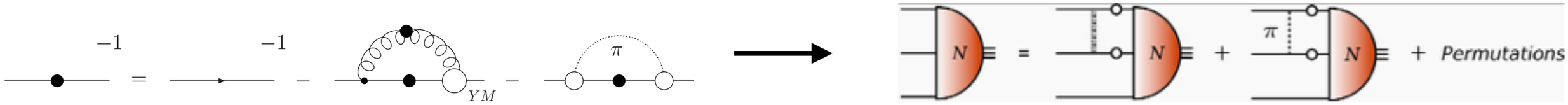
Delta:
Sanchis-Alepuz et al.,
PRD 84 (2011)

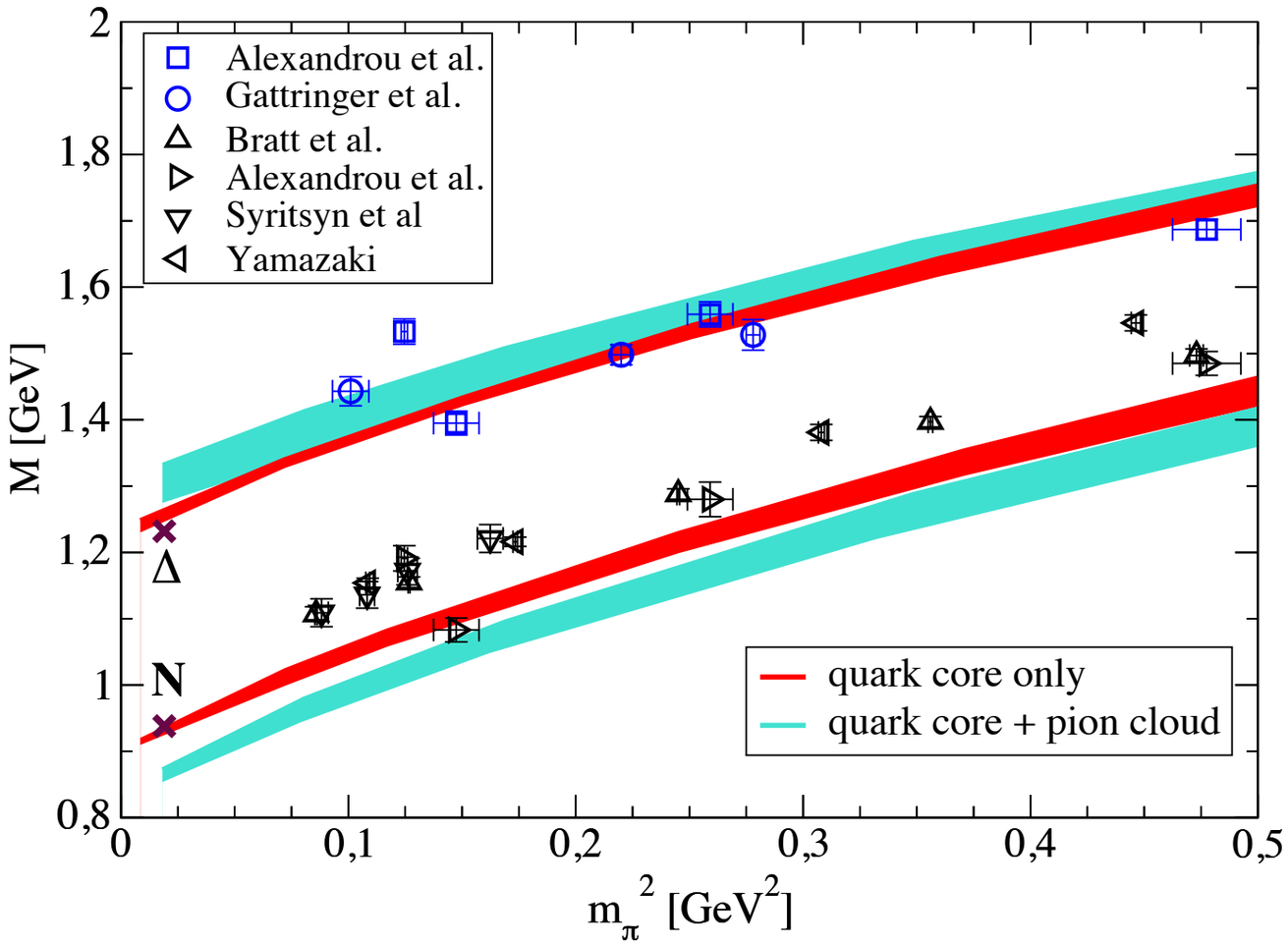
Nucleon:
Eichmann et al.,
PRL 104 (2010),
PRD 84 (2011)

ρ–meson:
Maris & Tandy,
PRC 60 (1999)

Eichmann, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)
Eichmann, PRD 84 (2011)
Sanchis-Alepuz ,Eichmann, Villalba-Chavez, Alkofer, PRD (2012)

Baryon masses- including pion cloud





Sanchis-Alepuz, CF, Kubrak, PLB 733 (2014) [1401.3183]

- fix Λ by f_π, vary η s.t. f_π still ok
- effects of the order of 50-100 MeV
- missing: gluon self-interaction effects

$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right) e^{-\eta^2\left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)$$

Pion cloud effects in baryons: structure

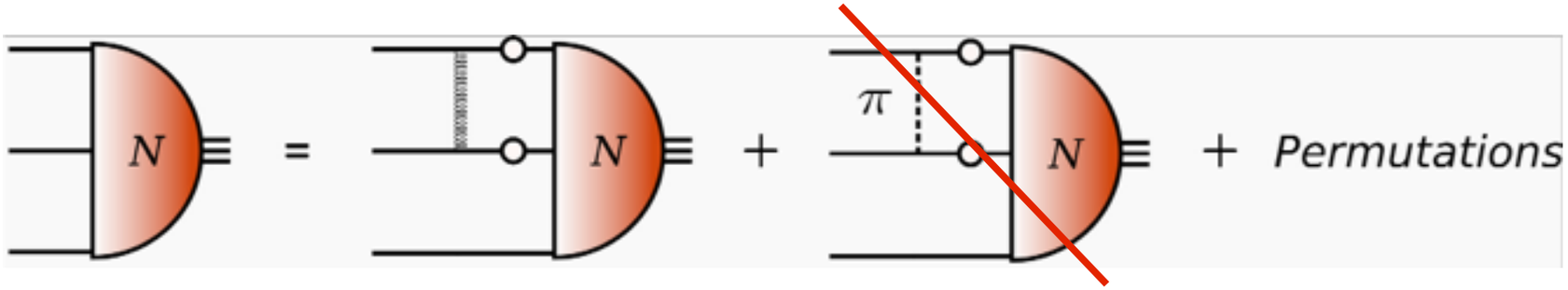
	Nucleon			Delta			
	s-wave	p-wave	d-wave	s-wave	p-wave	d-wave	f-wave
quark core	75	24	1	61	31	7	0,2
quark core plus pion cloud	75	24	1	60	31	8	0,2

$$\sigma_{\pi N} = 30(3)\,\mathrm{MeV} \quad (\text{quark core only})$$
$$\sigma_{\pi N} = 31(3)\,\mathrm{MeV} \quad (\text{quark core + pion cloud})$$

Sanchis-Alepuz, CF, Kubrak, PLB 733 (2014) [1401.3183]

- pion cloud does not change shape of nucleon: uniform skin
- sigma-term probably too small...

Octet-Decuplet-states



Rainbow-ladder result (flavour independent):

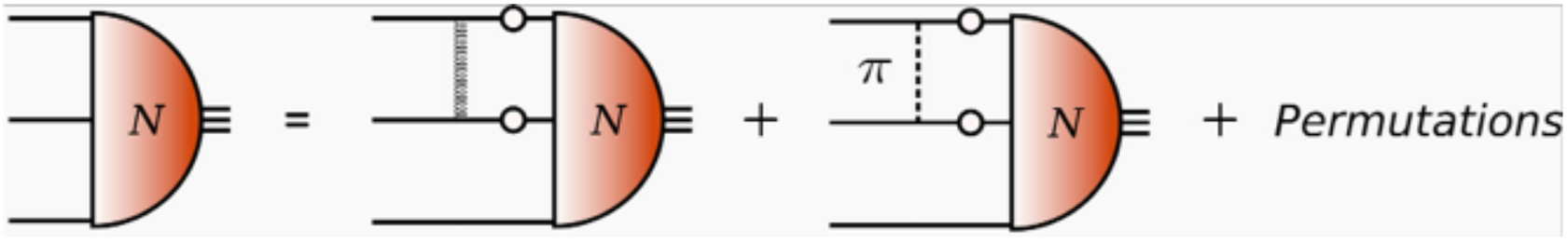
$^1/_2^+$	N	Σ	Λ	Ξ
Faddeev	0.930 (3)	1.073 (1)	1.073 (1)	1.235 (5)
Experiment	0.938	1.189	1.116	1.315
Relative difference	< 1 %	10 %	4 %	6 %

$^3/_2^+$	Δ	Σ^*	Ξ^*	Ω
Faddeev	1.21 (2)	1.33 (2)	1.47 (3)	1.65 (4)
Experiment	1.232 (1)	1.385 (2)	1.533 (2)	1.672
Relative difference	2 %	4 %	4 %	1 %

- reasonable, but no Σ-Λ-splitting

Sanchis-Alepuz, CF, PRD, 90 (2014) 9

Octett-Decuplett-states



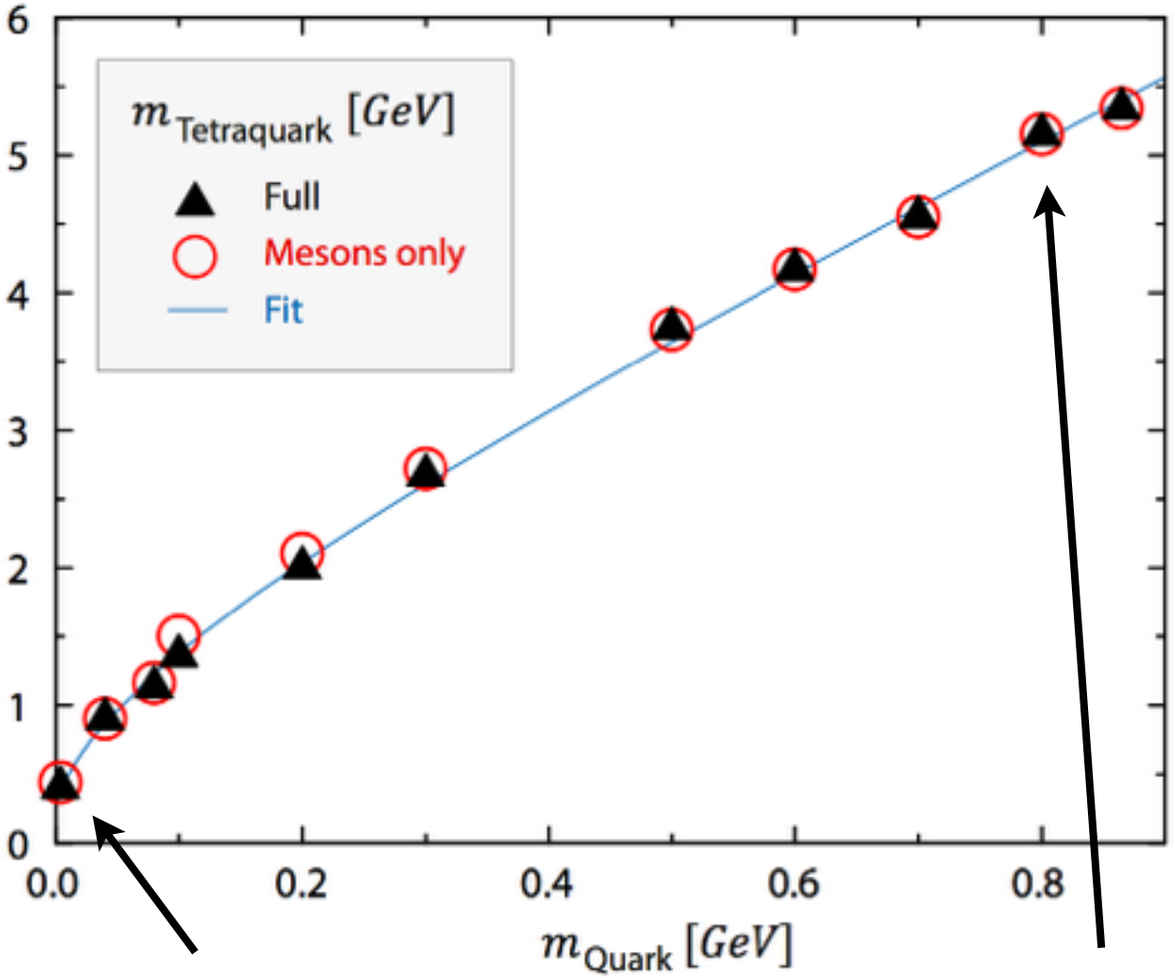
Include pion cloud effects:

	Λ	Σ	Ξ
RL (flavour indep.):	1.073 (1)	1.073 (1)	1.235 (5)
RL (flavour dep.):	1.070 (10)	1.070 (10)	1.220 (10)
RL + pion cloud:	1.161 (7)	1.164 (9)	
Exp.	1.116	1.189	1.315

- small Σ-Λ-splitting
- correct sign
- need kaon cloud as well...

Sanchis-Alepuz, CF, PRD, 90 (2014) 9

Results: scalar tetraquarks



up/down

charm

Heupel, Eichman, CF, PLB 718 (2012) 545-549

- Pion-Pion-contribution dominates ! } $f_0(500)$
- $m(0^{++}) = 403$ MeV

see also Caprini, Colangelo and Leutwyler, PRL. 96 (2006) 132001
Parganlija, Kovacs, Wolf, Giacosa and Rischke, PRD 87 (2013) 014011

- Narrow scalar $cc\overline{cc}$: $m(0^{++}) = 5.3 \pm (0.5)$ GeV