OPE of Green functions in the odd sector of QCD

Tomáš Kadavý
with Karol Kampf and Jiří Novotný

Institute of Particle and Nuclear Physics
Faculty of Mathematics and Physics, Charles University in Prague

12^{th} Quark Confinement and the Hadron Spectrum
Thessaloniki, Greece, August 29 – September 4, 2016

Supported by the Charles University in Prague, project GA UK no. 700214

August 30, 2016

Groundwork of this talk

- **Based on:**
  - **[K. Kampf and J. Novotný '11 (arXiv: 1104.3137)]**
    Resonance saturation in the odd-intrinsic parity sector of low-energy QCD
  - **[T. Kadavý, K. Kampf and J. Novotný (in preparation)]**
    Operator product expansion of VVA and AAA Green functions of chiral currents
  - **[T. Kadavý, K. Kampf and J. Novotný (in preparation)]**
    Three-point Green functions of chiral currents in the odd-intrinsic parity sector of QCD up to $\mathcal{O}(p^6)$

OPE and Green functions

Operator Product Expansion (OPE)

- OPE is a framework to study short-distance behaviour of Green functions.
  - The OPE is equivalent to an assumption that at large external momentum p, the two-point Green function of the operators above can be rewritten in the form
$$i\int \mathrm{d}^4x\, e^{ipx}\langle 0|\mathrm{T}A(x)B(0)|0\rangle = \sum_n C_n^{AB}(p^2)\langle 0|\mathcal{O}_n|0\rangle\,,$$
- QCD condensates with dimension ≤ 6:

$$\begin{aligned}\mathcal{O}_1 &= 1\,, & \mathcal{O}_5 &= \langle 0|\overline{q}\sigma_{\mu\nu}G^{\mu\nu}q|0\rangle\,,\\ \mathcal{O}_3 &= \langle 0|\overline{q}q|0\rangle\,, & \mathcal{O}_6^q &= \langle 0|(\overline{q}\Gamma Tq)(\overline{q}\Gamma Tq)|0\rangle\,,\\ \mathcal{O}_4 &= \langle 0|G_{\mu\nu}G^{\mu\nu}|0\rangle\,, & \mathcal{O}_6^G &= \langle 0|G_{\mu\nu}G_\sigma^\nu G^{\sigma\mu}|0\rangle\,.\end{aligned}$$

- $\Gamma = \{\mathbb{1}_4, \gamma_5, \gamma^\mu, \gamma_5\gamma^\mu, \sigma^{\mu\nu}\}, T = \{\mathbb{1}_3, T^a\}$.

Green functions of (chiral) currents

- QCD introduces an octet of
  - vector and axial-vector currents:

$$V^a_\mu = \overline{q}(x)\gamma_\mu T^a q(x)\,, \qquad A^a_\mu = \overline{q}(x)\gamma_\mu\gamma_5 T^a q(x)\,,$$

  and
  - scalar and pseudoscalar densities:

$$S^a = \overline{q}(x) T^a q(x)\,, \qquad P^a = i\overline{q}(x)\gamma_5 T^a q(x)\,.$$

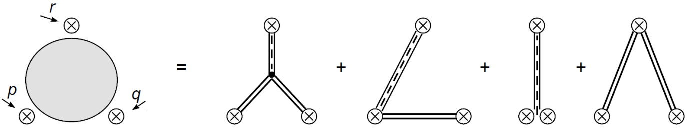
- The amplitudes of physical processes can be computed using LSZ reduction formula from the Green functions, the time ordered products of quantum fields (the group and Lorentz indices are suppresed):

$$\int \mathrm{d}^4 x_1 \int \mathrm{d}^4 x_2\, e^{i(p_1 x_1 + p_2 x_2)} \left\langle 0 \middle| \mathrm{T}\left[\mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(0)\right] \middle| 0 \right\rangle .$$

- Only five nontrivial Green functions in the odd-intrinsic parity sector of QCD.
  - VVP, VAS, AAP, VVA and AAA.

OPE and Green functions of (chiral) currents

- Topology of the Feynman diagrams (the crossing is implicitly assumed):



- OPE's for VVP, VAS and AAP Green functions are easily obtained with two contractions as the lowest order contributions:

$\langle 0|$  $|0\rangle \neq 0\,.$

- The results:

$$VVP: \quad \Pi^{\mathrm{OPE}}_{VVP}\big((\lambda p)^2, (\lambda q)^2; (\lambda r)^2\big) = \frac{B_0 F^2}{2\lambda^4}\,\frac{p^2+q^2+r^2}{p^2q^2r^2} + \mathcal{O}\left(\frac{1}{\lambda^6}\right),$$

$$VAS: \quad \Pi^{\mathrm{OPE}}_{VAS}\big((\lambda p)^2, (\lambda q)^2; (\lambda r)^2\big) = \frac{B_0 F^2}{2\lambda^4}\,\frac{p^2-q^2-r^2}{p^2q^2r^2} + \mathcal{O}\left(\frac{1}{\lambda^6}\right),$$

$$AAP: \quad \Pi^{\mathrm{OPE}}_{AAP}\big((\lambda p)^2, (\lambda q)^2; (\lambda r)^2\big) = \frac{B_0 F^2}{2\lambda^4}\,\frac{p^2+q^2-r^2}{p^2q^2r^2} + \mathcal{O}\left(\frac{1}{\lambda^6}\right).$$

Odd-intrinsic parity sector of QCD

Operator basis of $\mathcal{O}(p^6)$

- The Lagrangian of $\mathcal{O}(p^6)$, relevant in the odd-intrinsic parity sector, was formulated for the first time in [K. Kampf and J. Novotný '11]

$$\mathcal{L}_{\mathrm{R}\chi\mathrm{T}}^{(6,\mathrm{odd})} = \sum_X \sum_i \kappa_i^X \mathcal{O}_i^X \,, \qquad \mathcal{O}_i^X = \varepsilon^{\mu\nu\alpha\beta} \widehat{\mathcal{O}}^X_{i\,\mu\nu\alpha\beta} \,.$$

- X represents the single fields V, A, S, P, the combinations of two fields $VV, AA, SA, SV, VA, PA, PV$ and three fields VVP, VAS, AAP.
- Example: a set of operators with one vector resonance field

i	$\widehat{\mathcal{O}}^V_{i\,\mu\nu\alpha\beta}$	i	$\widehat{\mathcal{O}}^V_{i\,\mu\nu\alpha\beta}$
1	$\mathrm{i}\langle V^{\mu\nu}(h^{\alpha\sigma}u_\sigma u^\beta - u^\beta u_\sigma h^{\alpha\sigma})\rangle$	10	$\langle V^{\mu\nu}u^\alpha \chi_- u^\beta\rangle$
2	$\mathrm{i}\langle V^{\mu\nu}(u_\sigma h^{\alpha\sigma}u^\beta - u^\beta h^{\alpha\sigma}u_\sigma)$	11	$\langle V^{\mu\nu}\{f_+^{\alpha\rho}, f_-^{\beta\sigma}\}\rangle g_{\rho\sigma}$
3	$\mathrm{i}\langle V^{\mu\nu}(u_\sigma u^\beta h^{\alpha\sigma} - h^{\alpha\sigma}u^\beta u_\sigma)\rangle$	12	$\langle V^{\mu\nu}\{f_+^{\alpha\rho}, h^{\beta\sigma}\}\rangle g_{\rho\sigma}$
4	$\mathrm{i}\langle [V^{\mu\nu}, \nabla^\alpha \chi_+]u^\beta\rangle$	13	$\mathrm{i}\langle V^{\mu\nu} f_+^{\alpha\beta}\rangle\langle\chi_-\rangle$
5	$\mathrm{i}\langle V^{\mu\nu}[f_-^{\alpha\beta}, u_\sigma u^\sigma]\rangle$	14	$\mathrm{i}\langle V^{\mu\nu}\{f_+^{\alpha\beta}, \chi_-\}\rangle$
6	$\mathrm{i}\langle V^{\mu\nu}(f_-^{\alpha\sigma}u^\beta u_\sigma - u_\sigma u^\beta f_-^{\alpha\sigma})\rangle$	15	$\mathrm{i}\langle V^{\mu\nu}[f_-^{\alpha\beta}, \chi_+]\rangle$
7	$\mathrm{i}\langle V^{\mu\nu}(u_\sigma f_-^{\alpha\sigma}u^\beta - u^\beta f_-^{\alpha\sigma}u_\sigma)\rangle$	16	$\langle V^{\mu\nu}\{\nabla^\alpha f_+^{\beta\sigma}, u_\sigma\}\rangle$
8	$\mathrm{i}\langle V^{\mu\nu}(f_-^{\alpha\sigma}u_\sigma u^\beta - u^\beta u_\sigma f_-^{\alpha\sigma})\rangle$	17	$\langle V^{\mu\nu}\{\nabla_\sigma f_+^{\alpha\sigma}, u^\beta\}\rangle$
9	$\langle V^{\mu\nu}\{\chi_-, u^\alpha u^\beta\}\rangle$	18	$\langle V^{\mu\nu}u^\alpha u^\beta\rangle\langle\chi_-\rangle$

VVP Green function

- A tensor structure:

$$\left(\Pi_{VVP}(p,q;r)\right)^{abc}_{\mu\nu} = \Pi_{VVP}(p^2,q^2,r^2)d^{abc}\varepsilon_{\mu\nu(p)(q)}\,.$$

- Comparison with the calculation in χPT leads to an isolation of two low-energy constants C_7^W and C_{22}^W in terms of κ-couplings.
- OPE dictates the coupling constants constraints: $\kappa_5^P = 0$ and

$$\kappa_{14}^V = \frac{N_C}{256\sqrt{2}\pi^2 F_V}\,, \quad \kappa_{16}^V + 2\kappa_{12}^V = -\frac{N_C}{32\sqrt{2}\pi^2 F_V}\,, \quad \kappa_{17}^V = -\frac{N_C}{64\sqrt{2}\pi^2 F_V}\,,$$
$$\kappa_2^{VV} = \frac{F^2 + 16\sqrt{2}d_m F_V \kappa_3^{PV}}{32F_V^2} - \frac{N_C M_V^2}{512\pi^2 F_V^2}\,, \quad 8\kappa_2^{VV} - \kappa_3^{VV} = \frac{F^2}{8F_V^2}\,.$$

- $\Pi_{VVP}^{R\chi T}(p^2,q^2,r^2)$: substituing the constraints into $\Pi_{VVP}(p^2,q^2,r^2)$.

VVP Green function: $\mathcal{F}^{\mathrm{R}\chi\mathrm{T}}_{\pi^0\to\gamma\gamma}$ formfactor

- $\mathcal{F}^{\mathrm{R}\chi\mathrm{T}}_{\pi^0\to\gamma\gamma}$ formfactor:

$$\mathcal{F}^{R\chi T}_{\pi^0\to\gamma\gamma}(p^2,q^2,r^2) = \frac{2r^2}{3B_0F}\Pi^{R\chi T}_{VVP}(p^2,q^2,r^2)\,.$$

- The Brodsky-Lepage behaviour for large momentum [G. P. Lepage and S. J. Brodsky '80, '81]:

$$\mathcal{F}^{R\chi T}_{\pi^0\to\gamma\gamma}(0,-Q^2,m_\pi^2) \sim -\frac{1}{Q^2} \text{ for } Q^2\to\infty\,.$$

- B-L behaviour leads to the constraint

$$\kappa_3^{PV} = -\frac{F^2}{32\sqrt{2}d_mF_V}\,.$$

- BABAR measurement shows phenomenological disagreement with this condition that leads to the deviation with $\delta_{\mathrm{BL}} = -0,055\pm 0.025\,,$

$$\kappa_3^{PV} = -\frac{F^2}{32\sqrt{2}d_mF_V}(1+\delta_{\mathrm{BL}})\,.$$

VVP Green function: $\mathcal{F}^{\mathrm{R}\chi\mathrm{T}}_{\pi^0\to\gamma\gamma}$ formfactor

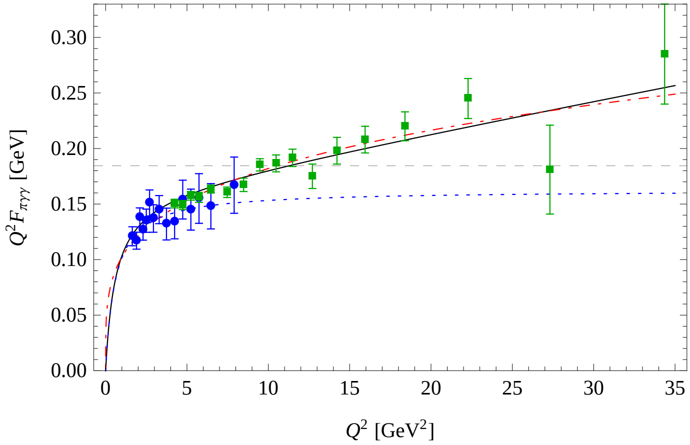


Figure: A plot of BABAR (green) and CLEO (blue) data fitted with the formfactor $\mathcal{F}^{\mathrm{R}\chi\mathrm{T}}_{\pi^0\gamma\gamma}(0,-Q^2;0)$ using the modified Brodsky-Lepage condition. The full black line represents fit with $\delta_{\mathrm{BL}} = -0.055$, and blue dotted line is a fit with standard $\delta_{\mathrm{BL}} = 0$.

VVP Green function: Decay of $\rho \to \pi\gamma$

- An amplitude: $\mathcal{A}_{\pi^0\to\gamma\gamma} = e^2 \mathcal{F}_{\pi^0\to\gamma\gamma}(0,0,0)\,.$
- A decay width ($\pi^0(p) \to \gamma(k) + \gamma(l)$):

$$\Gamma_{\pi^0\to\gamma\gamma} = \frac{1}{32\pi m_{\pi^0}} \sum_{\text{pol}} |\mathcal{A}_{\pi^0\to\gamma\gamma} \varepsilon^{\mu\nu\alpha\beta} k_\alpha l_\beta \epsilon^*_\mu(k) \epsilon^*_\nu(l)|^2\,.$$

- The connection of $\rho^+(q) \to \pi^+(p) + \gamma(k)$ with the previous process:

$$\mathcal{A}_{\rho^+\to\pi^+\gamma} = \frac{e}{2F_V M_V} \lim_{q^2\to M_V^2} (q^2 - M_V^2) \mathcal{F}_{\pi^0\to\gamma\gamma}(0,q^2,0)\,.$$

- A decay width:

$$\Gamma_{\rho^+\to\pi^+\gamma} = \frac{m_\rho^2 - m_\pi^2}{48\pi m_\rho^3} \sum_{\text{pol}} |\mathcal{A}_{\rho^+\to\pi^+\gamma} \varepsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta \epsilon_\mu(p) \epsilon^*_\nu(k)|^2\,.$$

with [B. Moussallam '95]

$$\frac{2eF_V}{M_V} \left| \frac{\mathcal{A}_{\rho^+\to\pi^+\gamma}}{\mathcal{A}_{\rho^0\to\gamma\gamma}} \right| \equiv 1 + x\,.$$

- RχT: $x = -0.010 \pm 0.005$ and $\Gamma_{\rho^+\to\pi^+\gamma} = 67.0 \pm 2.3\,\text{keV}$.

VVP Green function: Decays of $\pi(1300)$

- Two channels studied: $\pi(1300) \to \gamma\gamma$ and $\pi(1300) \to \rho\gamma$.
- The amplitudes:

$$\mathcal{A}^{R\chi T}_{\pi' \to \gamma\gamma} = e^2 \frac{8\sqrt{2}}{3} F_V \frac{2\sqrt{2}\kappa_3^{PV} M_V^2 - F_V \kappa^{VVP}}{M_V^4} ,$$

$$\mathcal{A}^{R\chi T}_{\pi' \to \rho\gamma} = -e \frac{4\sqrt{2}}{3M_V} \frac{\sqrt{2}\kappa_3^{PV} M_V^2 - F_V \kappa^{VVP}}{M_V^2} .$$

- Belle collaboration [K. Abe et al. '06]: $\Gamma_{\pi' \to \gamma\gamma} < 72\,\mathrm{eV}$.
- An experimental bound on $\Gamma_{\pi' \to \gamma\gamma}$ can be used to get the estimate:

$$\kappa^{VVP} \approx (-0.57 \pm 0.13)\,\mathrm{GeV} .$$

VVP Green function: Decays of $\pi(1300)$

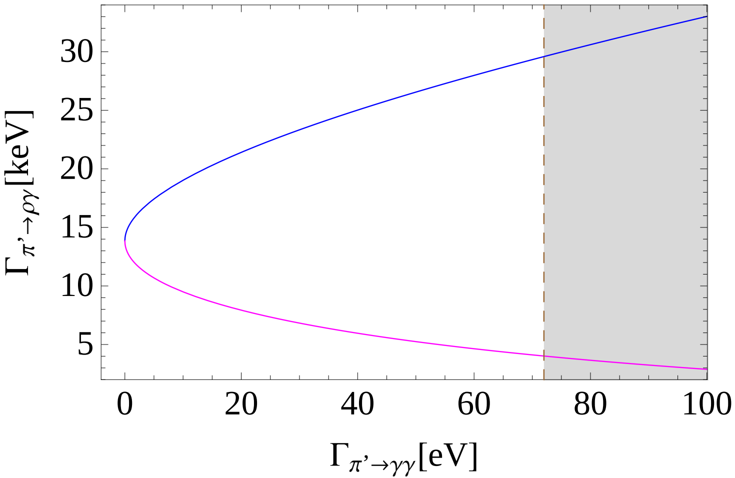


Figure: The connection of decay widths for $\pi(1300) \to \gamma\gamma$ and $\pi(1300) \to \rho\gamma$. The dashed line denotes the Belle collaboration limit. [K. Abe et al. '06].

VVP Green function: The muon $g-2$ factor

- Hadronic contributions: hadronic light-by-light scattering.
  - The main source of theoretical error in the SM prediction.
- The four point Green function $\langle VVVV \rangle$ can be simplified into:
  - $\pi^{\pm}$ and $K^{\pm}$ loops,
  - π^0, η, η' exchanges: the $\langle VVP \rangle$ case etc.
- Using the fully off-shell $\mathcal{F}^{R\chi T}_{\pi^0 \to \gamma\gamma}(p^2, q^2, r^2)$ formfactor we get:

$$a_\mu^{\mathrm{LbyL},\pi^0} = (65.8 \pm 1.2) \cdot 10^{-11} \,.$$

- The updated result [P. Roig, A. Guevara and G. L. Castro '14]:

$$a_\mu^{\pi^0} = (66.6 \pm 2.1) \cdot 10^{-11} \,.$$

VAS Green function

- A tensor structure:

$$\left(\Pi_{VAS}(p,q;r)\right)_{\mu\nu}^{abc} = \Pi_{VAS}(p^2,q^2,r^2) f^{abc} \varepsilon_{\mu\nu(p)(q)} \,.$$

- At low energies, up to $\mathcal{O}(p^6)$: $\Pi(p^2,q^2,r^2) = -32 B_0 C_{11}^W$.
- An experiment: decay $K^+ \to l^+ \nu \gamma$ suggests [A. A. Poblaguev et al. '02], [R. Unterdorfer and H. Pichl '08]:

$$C_{11}^W = (0.68 \pm 0.21) \cdot 10^{-3}\,\mathrm{GeV}^{-2}$$

and

$$\kappa^{VAS} = (0.61 \pm 0.40)\,\mathrm{GeV}\,.$$

VVA and AAA Green functions

- VVA: 6 topologies of diagrams (9 variants due to $v_\mu^a \leftrightarrow v_\nu^b$).
- AAA: 5 topologies of diagrams (18 variants due to $a_\mu^a \leftrightarrow a_\nu^b \leftrightarrow a_\rho^c$).
- The Ward identities restrict the general decomposition of the tensor part of VVA into four terms

$$\left(\Pi_{VVA}(p,q;r)\right)_{\mu\nu\rho}^{abc} = d^{abc}\Pi_{\mu\nu\rho}(p,q;r)\,,$$
$$\Pi_{\mu\nu\rho}(p,q;r) = w_L \varepsilon_{\mu\nu(p)(q)} r_\rho + w_T^{(1)}\Pi^{(1)}_{\mu\nu\rho} + w_T^{(2)}\Pi^{(2)}_{\mu\nu\rho} + w_T^{(3)}\Pi^{(3)}_{\mu\nu\rho}\,.$$

- The tensor part is nontrivial [M. Knecht, S. Peris, M. Perrottet and E. de Rafael '04]

$$\Pi^{(1)}_{\mu\nu\rho} = p_\nu \varepsilon_{\mu\rho(p)(q)} - q_\mu \varepsilon_{\nu\rho(p)(q)} - \frac{p^2+q^2-r^2}{r^2}\varepsilon_{\mu\nu(p)(q)} r_\rho + \frac{p^2+q^2-r^2}{2}\varepsilon_{\mu\nu\rho(p-q)}\,,$$
$$\Pi^{(2)}_{\mu\nu\rho} = \varepsilon_{\mu\nu(p)(q)}(p-q)_\rho + \frac{p^2-q^2}{r^2}\varepsilon_{\mu\nu(p)(q)} r_\rho\,,$$
$$\Pi^{(3)}_{\mu\nu\rho} = p_\nu \varepsilon_{\mu\rho(p)(q)} + q_\mu \varepsilon_{\nu\rho(p)(q)} - \frac{p^2+q^2-r^2}{2}\varepsilon_{\mu\nu\rho(r)}\,.$$

VVA Green function

- Extracted formfactors [T. Kadavý, K. Kampf and J. Novotný '16]:

$$
\begin{aligned}
w_L &= \frac{N_c}{8\pi^2 r^2}\,,\\
w_T^{(1)} &= -\frac{2\sqrt{2}F_V\left[\kappa_{17}^V(p^2+q^2-2M_V^2)-\sqrt{2}F_V\kappa_3^{VV}\right]}{(p^2-M_V^2)(q^2-M_V^2)}\,,\\
w_T^{(2)} &= -\frac{2\sqrt{2}F_V(p^2-q^2)(2\kappa_{12}^V+\kappa_{16}^V-\kappa_{17}^V)}{(p^2-M_V^2)(q^2-M_V^2)}\,,\\
w_T^{(3)} &= \frac{2\sqrt{2}F_V(p^2-q^2)}{(p^2-M_V^2)(q^2-M_V^2)}\left(2\kappa_{11}^V+2\kappa_{12}^V-\kappa_{17}^V-\frac{\sqrt{2}F_A\kappa_5^{VA}}{r^2-M_A^2}\right).
\end{aligned}
$$

- Phenomenologically important formfactor $w_T(Q^2)$:

$$
w_T(Q^2) = -16\pi^2\left[w_T^{(1)}(-Q^2,0,-Q^2)+w_T^{(3)}(-Q^2,0,-Q^2)\right].
$$

VVA Green function: Coupling constants constraints

- Soft-wall AdS/QCD and OPE [J. J. Sanz-Cillero '12] and [P. Colangelo, F. De Fazio, J. J. Sanz-Cillero, F. Giannuzzi and S. Nicotri '12]:

$$w_T(Q^2) = \frac{N_c}{Q^2} + \frac{128\pi^3\alpha_s\chi\langle\overline{q}q\rangle^2}{9Q^6} + \mathcal{O}\left(\frac{1}{Q^8}\right).$$

- By a comparison it is possible to extract the coupling constants constraints:

$$\kappa_{11}^V + \kappa_{12}^V = -\frac{N_c}{64\sqrt{2}\pi^2 F_V}\,, \quad \kappa_3^{VV} = -\frac{N_c M_V^4}{64\pi^2 M_A^2 F_V^2}\,, \quad \kappa_5^{VA} = \kappa_3^{VV}\frac{F_V}{F_A}\,.$$

- Numerically: $\kappa_5^{VA} = -0.086$; decay $f_1(1285) \to \rho\gamma$: $\kappa_5^{VA} = -0.062 \pm 0.030$.
- Using the constraints for VVP we can also determine:

$$\kappa_2^{VV} = \frac{1}{64\pi^2}\left(F^2 - \frac{N_c M_V^4}{8\pi^2 M_A^2}\right), \quad \kappa_3^{PV} = -\frac{F^2}{32\sqrt{2}d_m F_V}\left[1 + \frac{N_c M_V^2}{8\pi^2 F^2}\left(\frac{M_V^2}{M_A^2} - 1\right)\right].$$

- Therefore $\delta_{\rm BL} = -1.342$.

VVA Green function: $\mathcal{F}^{\mathrm{R}\chi\mathrm{T}}_{\pi^0\to\gamma\gamma}$ formfactor revisited

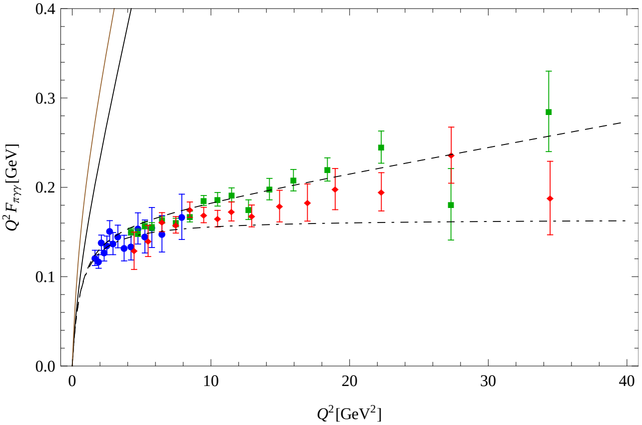


Figure: A plot of BABAR (green), BELLE (red) and CLEO (blue) data fitted with the formfactor $\mathcal{F}^{\mathrm{R}\chi\mathrm{T}}_{\pi^0\gamma\gamma}(0,-Q^2;0)$ using the modified Brodsky-Lepage condition. The full black line represents our fit with $\delta_{\mathrm{BL}} = -1.342$, and the full brown line is a fit using the LMD formfactor. The dashed line stands for $\delta_{\mathrm{BL}} = -0.055$ and the dot-dashed line for $\delta_{\mathrm{BL}} = 0$.

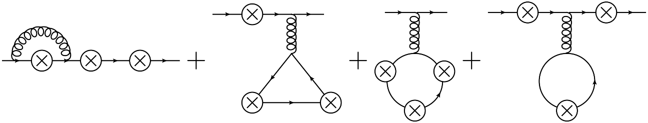
VVA and AAA Green functions: OPE

- Neither VVA nor AAA have the leading order contribution to the OPE, i.e.

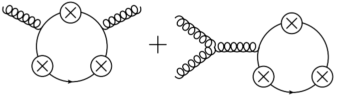
$\langle 0|$  $|0\rangle = 0\,.$

- Therefore, one needs to include other contributions from QCD condensates [T. Kadavý, K. Kampf and J. Novotný '16]:

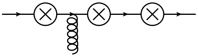
$D = 3:$



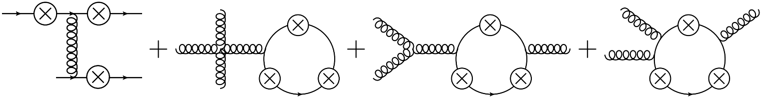
$D = 4:$



$D = 5:$



$D = 6:$



Conclusion

Thank you!