Recent results on the meson and baryons spectrum from lattice QCD

Daniel Mohler

Thessaloniki,
August 29, 2016
Outline

1 Spectroscopy and properties of states stable under QCD

2 Spectroscopy of resonances/ close-to-threshold states
   - Lüscher’s finite volume method
   - Lattice systematics: A practitioner’s point-of-view
   - Meson-meson scattering and resonances/ bound states
     - Example 1: The $\rho$ meson
     - Example 2: $D_{s0}^*(2317)$ and $D_{s1}(2460)$ and their $b$-quark cousins
     - Example 3: Coupled channel scattering and light scalar mesons
     - A lesson from meson-meson scattering
   - Baryon resonances in Meson-Baryon scattering

3 Summary and outlook
Stable hadron states: A lattice success story

Heavy mesons

Example from HPQCD
Dowdall et al. PRD 86 094510 (2012)

 Hadrons stable under QCD: full control of systematic uncertainties

Goal: Extend this success to hadron resonances

Example from BMW
Example: Precision flavor physics

- FLAG review aims to answer: “What is currently the best lattice value for a particular quantity?”
- Uses symbols derived from rigorous quality criteria and covers precision results
- Can be found at [http://itpwiki.unibe.ch/flag/](http://itpwiki.unibe.ch/flag/)
- Heavy quark observables: talk by Alexei Bazavov Mo. 16:20

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Mesons and baryons from lattice QCD
Thessaloniki, August 29, 2016 4 / 27
Ground states vs. excited states

Observables from Euclidean correlators:

\[
\left\langle \hat{O}_2(t)\hat{O}_1(0) \right\rangle_T \propto \sum_n e^{-tE_n} < 0|\hat{O}_2|n > < n|\hat{O}_1|0 >
\]

- Tower of states with chosen quantum numbers (i.e. \(IJ^{PC}\))
- Ground state is dominant at large \(t\)
- Exitated states appear as sub-leading exponentials
- Noisy background from limited statistics

Extracting excited states needs advanced methods!

Tower of states does include multi-hadron states
Basic observation: In finite volume, multi-particle energies are shifted with regard to the free energy levels due to the interaction

\[ E = E(p_1) + E(p_2) + \Delta_E \]

Computational strategy:

1. Extract energy levels \( E_n(L) \) in a finite box
2. Lüscher formula → phase shift of the continuum scattering amplitude
   Elastic s-wave scattering:
   \[ p_{cm} \cot \delta(p_{cm}) = \frac{2}{\sqrt{\pi}L} Z_{00} \left( 1; \left( \frac{L}{2\pi p_{cm}} \right)^2 \right) \]
3. Extract resonance parameters (similar to experiment)
Calculational strategies

- Original idea: Rest-frame calculation in multiple spatial volumes $L^3$
- Moving frames → additional data points (however: reduced symmetry; mixing)
- Calculations in multiple asymmetric boxes i.e. $L^2 \times L_z$

Beyond elastic scattering

- Any number of 2-hadron channels well understood
- $2 \leftrightarrow 1$ and $2 \leftrightarrow 2$ transitions well understood (example $\pi\pi \rightarrow \pi\gamma^*$)
- Progress on 3 hadron scattering
  → Talk by M. Hansen on Tuesday at 18:00

For hadron resonances and excitations, results currently of a qualitative nature; Usually no chiral/continuum extrapolation

Method relies on determining small volume-dependent energy shifts → high statistics needed
→ multiple lattice volumes crucial

Need to use all-to-all propagator methods

→ need at least $O(10)$ more Matrix inversions

Computational cost of traditional lattice calculations

Gauge field generation $\gg$ Propagators $\gg$ Observables

Computational cost of more complicated observables

Gauge field generation $\approx$ Propagators $\approx$ Observables
The $\rho$ resonance - a benchmark calculation

From Lang, DM, Prelovsek, Vidmar, PRD 84 054503 (2011); erratum ibid;

- We extract $g_{\rho\pi\pi}$ rather than $\Gamma$

$$\Gamma(s) = \frac{p^*^3}{s} \frac{g_{\rho\pi\pi}^2}{6\pi}$$

- Results for $m_\pi = 266(3)(3)\text{MeV}$

$$g_{\rho\pi\pi} = 5.61(12) \quad m_\rho = 772(6)(8)\text{ MeV}$$
The $\rho$: More recent results:

Wilson et al. PRD 92 094502 (2015)

$m_\pi = 391$ MeV

$m_\pi = 236$ MeV
The $\rho$ resonance - comparing results for the coupling

$g^{(\text{phys})}_{\rho\pi\pi} \approx 5.97 \quad m_\rho = 775.11(34) \text{ MeV}$

- Caution: To date no simulation with full control of systematics
- Many more preliminary results at Lattice2016
- In general good agreement at larger than physical pion mass
The $\rho$ resonance - comparing results for the masses

Plot from RQCD, Bali et al. PRD 93 054509 (2016)

- Quite a large spread in the lattice data
- Can one learn something from looking at the current collection of data?
The $\rho$ resonance: 2 vs. 2+1 flavor

Hu, Molina, Döring, Alexandru arXiv:1605.04823, accepted by PRL

- Analysis of $N_f = 2$ lattice scattering data using Unitarized $\chi$PT
- Use fit to experiment data to go from $N_f = 2 \rightarrow N_f = 2 + 1$.
- Main claim: The missing $K\bar{K}$ channel explains low 2 flavor lattice data

→ Talk by Raquel Molina on Thursday at 15:50
The $\rho$-resonance and $F_\pi(E)$

- In the elastic region the form factor $F_\pi(E)$ is
  \[
  R(s) = \frac{1}{4} \left( 1 - \frac{4m^2_\pi}{s} \right)^{\frac{3}{2}} |F_\pi(\sqrt{s})|^2 = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{4\pi\alpha(s)^2/(3s)}
  \]

- Is of phenomenological importance for lattice determinations of the hadronic vacuum polarization (HVP) contribution to $(g - 2)_\mu$.

- It can be calculated from a finite-volume matrix element

  H. B. Meyer PRL 107 072002 (2011)

- The derivative of the phase shift $\partial\delta_1(k)/\partial k$ enters the determination
The $\rho$-resonance and $F_\pi(E)$

Bulava et al.  arXiv:1511.02351

- Results at a single lattice spacing and $m_\pi = 280$MeV
- Curve is the Gounaris-Sakurai parametrization (not a fit)
Exotic $D_s$ and $B_s$ candidates

Established $s$ and $p$-wave states:

$D_s (J^P = 0^-)$ and $D_s^* (1^-)$

$D_{s0}^*(2317) (0^+)$, $D_{s1} (2460) (1^+)$,
$D_{s1} (2536) (1^+)$, $D_{s2}^* (2573) (2^+)$

$B_s (J^P = 0^-)$ and $B_s^* (1^-)$

$B_{s1} (5830) (1^+)$, $B_{s2}^* (5840) (2^+)$

- Peculiarity: $M_{c\bar{s}} \approx M_{c\bar{d}} \rightarrow$ exotic structure? (tetraquark, molecule)
- Traditional lattice studies (using single hadron operators) tend get too large or badly determined masses
- $B_s$ cousins of the $D_{s0}^* (2317)$ and $D_{s1} (2460)$ not (yet) seen in experiment
- LHCb should be able to see these
Exotic $D_s$ and $B_s$ candidates

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Spectrum results for $D_s$

Mohler et al. PRL 111 222001 (2013)

Lang, DM et al. PRD 90 034510 (2014)

- Discretization uncertainties sizeable for charm
- Many improvements possible for the $D_s$ states

For more $D$ and $D_s$ results

→ Talk by Gunnar Bali Monday 15:20
→ Talk by Graham Moir Monday 16:40
\( B_{s0}^* \) and \( B_{s1} \): Results

Lang, DM, Prelovsek, Woloshyn PLB 750 17 (2015)

\[ a_0^{BK} = -0.85(10) \text{ fm} \]
\[ r_0^{BK} = 0.03(15) \text{ fm} \]
\[ M_{B_{s0}^*} = 5.711(13) \text{ GeV} \]

\[ a_0^{B^*K} = -0.97(16) \text{ fm} \]
\[ r_0^{B^*K} = 0.28(15) \text{ fm} \]
\[ M_{B_{s1}} = 5.750(17) \text{ GeV} \]

- Energy from the difference to the \( B^{(*)} \text{K} \) threshold
Spectrum results

Lang, DM, Prelovsek, Woloshyn PLB 750 17 (2015)

Full uncertainty estimate only for magenta $B_s$ states
Prediction of exotic states from Lattice QCD!
Coupled channel $\pi K - \eta K$ scattering

- First coupled channel study in Lattice QCD
- Channels are mostly decoupled
- For recent coupled-channel results:
  - → Plenary talk by Christopher Thomas
  - → Talk by Raúl Briceño Tuesday at 17:30
First coupled channel study in Lattice QCD
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Channels are tightly coupled

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A lesson from meson-meson scattering

Plots from Lang, DM, Prelovsek, Vidmar, PRD 84 054503 (2011); and Wilson et al. PRD 92 094502 (2015)

- A diverse interpolator basis is vital to determine the true spectrum!
- Beware: Effective energies may seem to reach a plateau (with good fit $\chi^2$)
A lesson from meson-meson scattering

Data from Mohler et al. PRL 111 222001 (2013)

- A diverse interpolator basis is vital to determine the true spectrum!
- Beware: Effective energies may seem to reach a plateau (with good fit $\chi^2$)
Studies of the baryon spectrum more difficult
- Signal to noise much worse
- Observables are more expensive
- Parity mixing in moving frames

A large number of energy levels has been extracted using 3-quark interpolators

However: Only very few studies using baryon – meson or 5-quark interpolators exist

Multi-hadron levels are missing from the spectrum

Based on meson experience: How trustworthy are the existing results?
Nucleon–pion scattering for the negative parity Nucleons

Lang and Verduci, PRD 87 054502 (2013); plot from arXiv:1412.0701.

- Clear issue with extracting the correct spectrum using only 3-quark operators
- Only energy levels – not yet enough data for a (coupled channel) phase-shift analysis
3-quark interpolators plus local 5-quark interpolators with $N\pi$, $N\sigma$ and $Na_0$ structures

How does this relate to previous claims about the Roper?

Authors say it flavors dynamically generated coupled-channel resonance

Caution: Extracted spectrum is clearly incomplete
A lot of progress and activity in QCD resonance and bound-state studies

A number of studies of resonances in meson-meson scattering
Could only cover selected examples. Interesting results also for
  - $D^{(*)}D^{(*)}$-scattering and charmonium results
  - Heavy-quark exotics

See for example review by S. Prelovsek arXiv:1508.07322

First coupled channel simulations
  → Plenary talk by Christopher Thomas
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Studies of systematic uncertainties will start to appear for the simplest of resonances

Meson-baryon studies are more difficult (and still scarce)
Thank you!
Baryon – Baryon bound states and resonances

→ Talk by P. Junnarkar Tuesday at 19:00
Backup Slides
$D_{s0}^*(2317)$: D-meson – Kaon s-wave scattering


\[ p \cot \delta(p) = \frac{2}{\sqrt{\pi L}} Z_{00}(1; q^2) \]
\[ \approx \frac{1}{a_0} + \frac{1}{2} r_0 p^2 \]

Mohler et al. PRL 111 222001 (2013)
Lang, DM et al. PRD 90 034510 (2014)

Results for ensembles (1) and (2)

\[ a_0 = -0.756 \pm 0.025 \text{fm} \quad (1) \]
\[ r_0 = -0.056 \pm 0.031 \text{fm} \]
\[ a_0 = -1.33 \pm 0.20 \text{fm} \quad (2) \]
\[ r_0 = 0.27 \pm 0.17 \text{fm} \]
$D_s^*(2317)$: D-meson – Kaon s-wave scattering


\[ p \cot \delta(p) = \frac{2}{\sqrt{\pi} L} Z_{00}(1; q^2) \approx \frac{1}{a_0} + \frac{1}{2} r_0 p^2 \]

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Results for ensembles (1) and (2)

\[
\begin{align*}
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\]