Recent results on the meson and baryons spectrum from lattice QCD

Daniel Mohler

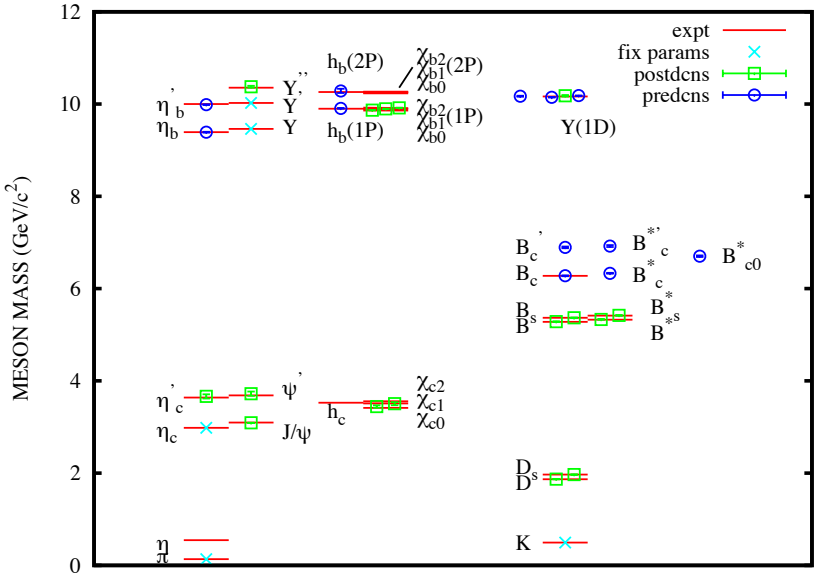
Thessaloniki,
August 29, 2016

Outline

1. Spectroscopy and properties of states stable under QCD
2. Spectroscopy of resonances/ close-to-threshold states
   - Lüscher's finite volume method
   - Lattice systematics: A practitioner's point-of-view
   - Meson-meson scattering and resonances/ bound states
     - Example 1: The ρ meson
     - Example 2: $D_{s0}^*(2317)$ and $D_{s1}(2460)$ and their b-quark cousins
     - Example 3: Coupled channel scattering and light scalar mesons
     - A lesson from meson-meson scattering
   - Baryon resonances in Meson-Baryon scattering
3. Summary and outlook

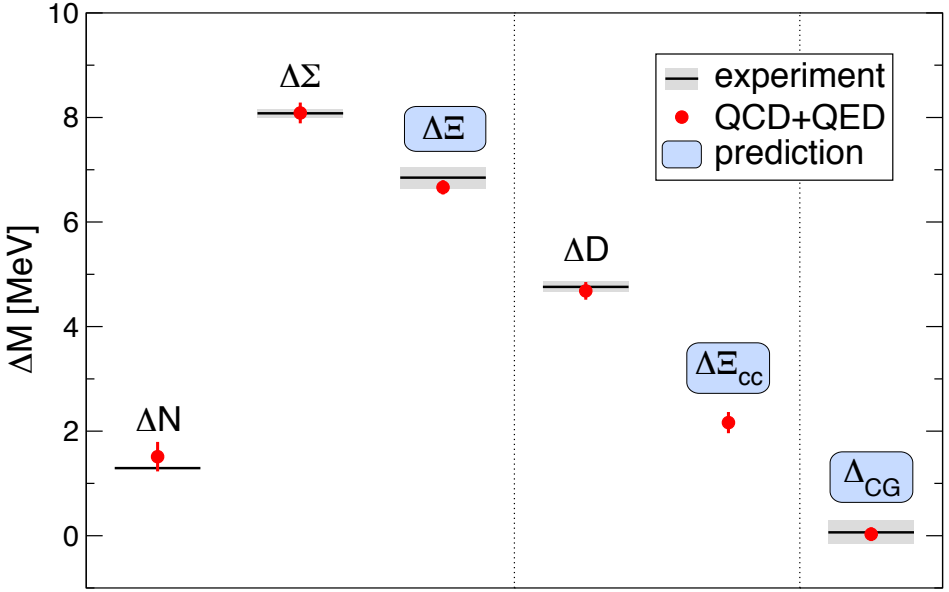
Stable hadron states: A lattice success story

Heavy mesons



Example from HPQCD
Dowdall *et al.* PRD 86 094510 (2012)

QCD+QED Isospin splittings

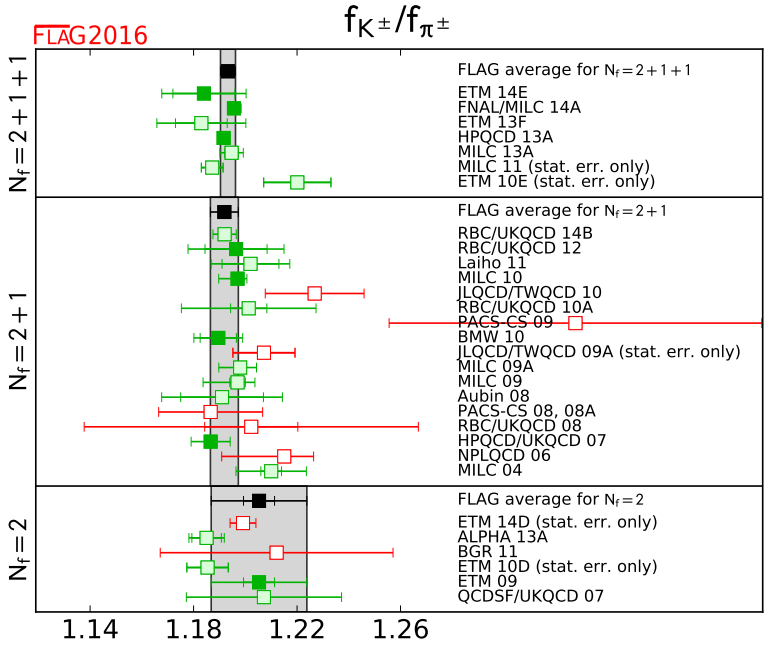


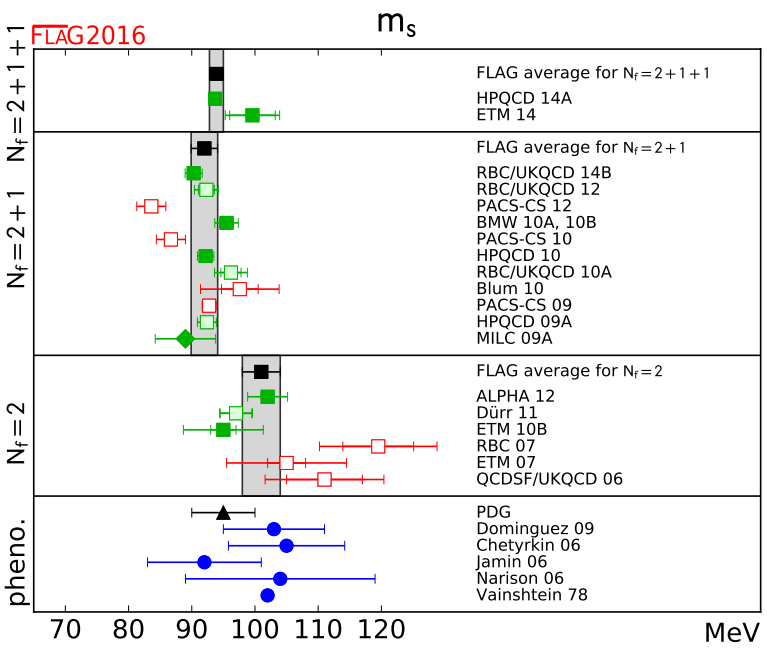
Example from BMW
Borsanyi *et al.* Science 347 1452 (2015)

- Hadrons stable under QCD: full control of systematic uncertainties
- Goal: Extend this success to hadron resonances

Example: Precision flavor physics

- FLAG review aims to answer: "What is currently the best lattice value for a particular quantity?"
- Uses symbols derived from rigorous quality criteria and covers precision results
- Can be found at http://itpwiki.unibe.ch/flag/
- Heavy quark observables: talk by Alexei Bazavov Mo. 16:20



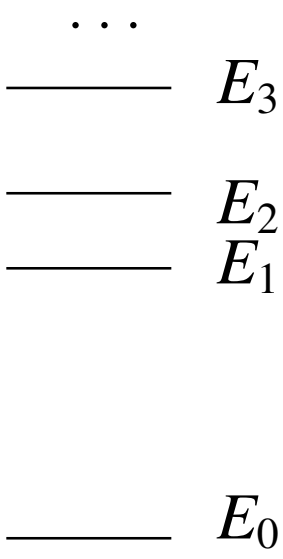


Ground states vs. excited states

Observables from Euclidean correlators:

$$\left\langle \hat{O}_2(t)\hat{O}_1(0) \right\rangle_T \propto \sum_n e^{-tE_n} < 0|\hat{O}_2|n >< n|\hat{O}_1|0 >$$

- Tower of states with chosen quantum numbers (i.e. IJ^{PC})
- Ground state is dominant at large t
- Exited states appear as sub-leading exponentials
- Noisy background from limited statistics



- Extracting excited states needs advanced methods!
- Tower of states does include multi-hadron states

Scattering in finite volume: The Lüscher method I

M. Lüscher Commun. Math. Phys. 105 (1986) 153;
Nucl. Phys. B 354 (1991) 531; Nucl. Phys. B 364 (1991) 237.

Basic observation: In finite volume, multi-particle energies are shifted with regard to the free energy levels due to the interaction

$$E = E(p_1) + E(p_2) + \Delta_E$$

Computational strategy:

(1) Extract energy levels $E_n(L)$ in a finite box

(2) Lüscher formula $\rightarrow$ phase shift of the continuum scattering amplitude
Elastic s-wave scattering:

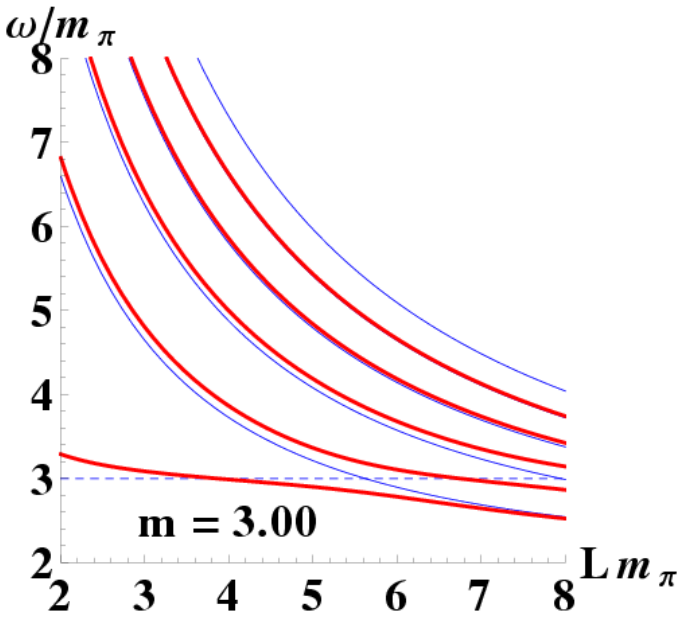
$$p_{cm} \cot \delta(p_{cm}) = \frac{2}{\sqrt{\pi} L} Z_{00} \left(1; \left(\frac{L}{2\pi} p_{cm} \right)^2 \right)$$

(3) Extract resonance parameters (similar to experiment)

Scattering in finite volume: The Lüscher method II

Calculational strategies

- Original idea: Rest-frame calculation in multiple spatial volumes L^3
- Moving frames $\rightarrow$ additional data points (however: reduced symmetry; mixing)
- Calculations in multiple asymmetric boxes i.e. $L^2 \times L_z$



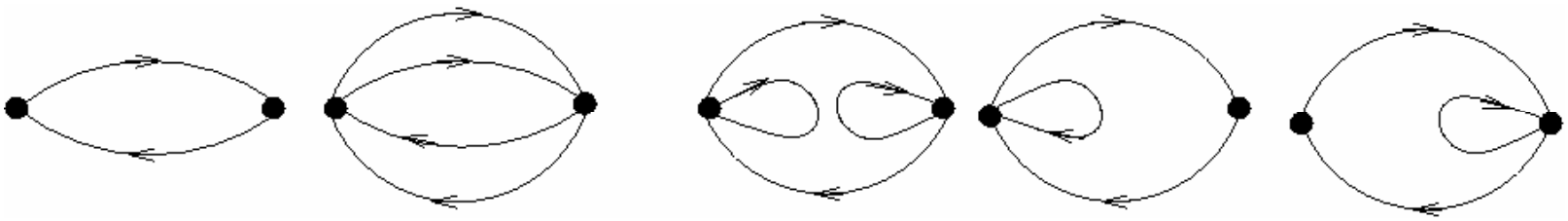
Beyond elastic scattering

- Any number of 2-hadron channels well understood
- $2 \leftrightarrow 1$ and $2 \leftrightarrow 2$ transitions well understood (example $\pi\pi \rightarrow \pi\gamma^*$)
- Progress on 3 hadron scattering
  $\rightarrow$ Talk by M. Hansen on Tuesday at 18:00

Reviews: LATTICE plenaries by Raúl A. Briceño arXiv:1411.6944
and Max Hansen arXiv:1511.04737

Lattice systematics: A practitioner's point-of-view

- For hadron resonances and excitations, results currently of a qualitative nature; Usually no chiral/continuum extrapolation
- Method relies on determining small volume-dependent energy shifts
  $\rightarrow$ high statistics needed
  $\rightarrow$ multiple lattice volumes crucial
- Need to use all-to-all propagator methods



  $\rightarrow$ need at least $O(10)$ more Matrix inversions
- Computational cost of traditional lattice calculations

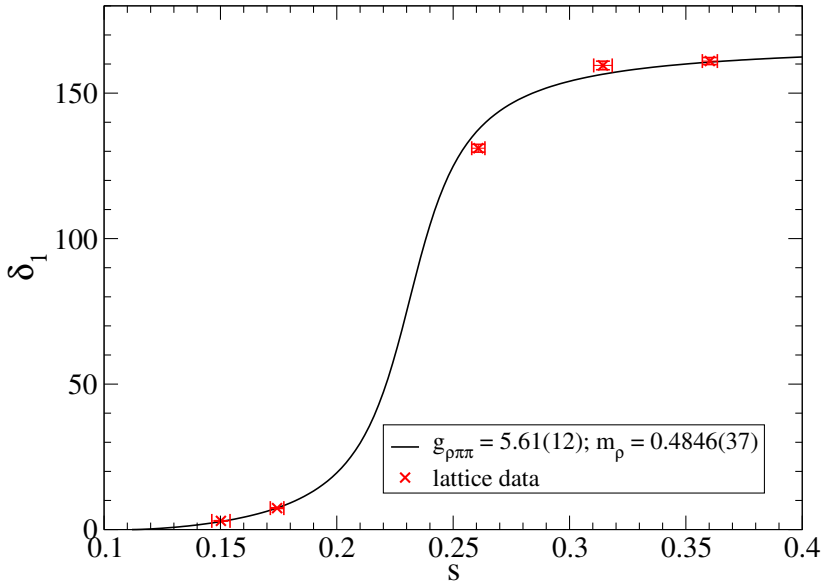
$$\text{Gauge field generation} \gg \text{Propagators} \gg \text{Observables}$$

- Computational cost of more complicated observables

$$\text{Gauge field generation} \approx \text{Propagators} \approx \text{Observables}$$

The ρ resonance - a benchmark calculation

From Lang, DM, Prelovsek, Vidmar, PRD 84 054503 (2011); erratum ibid;



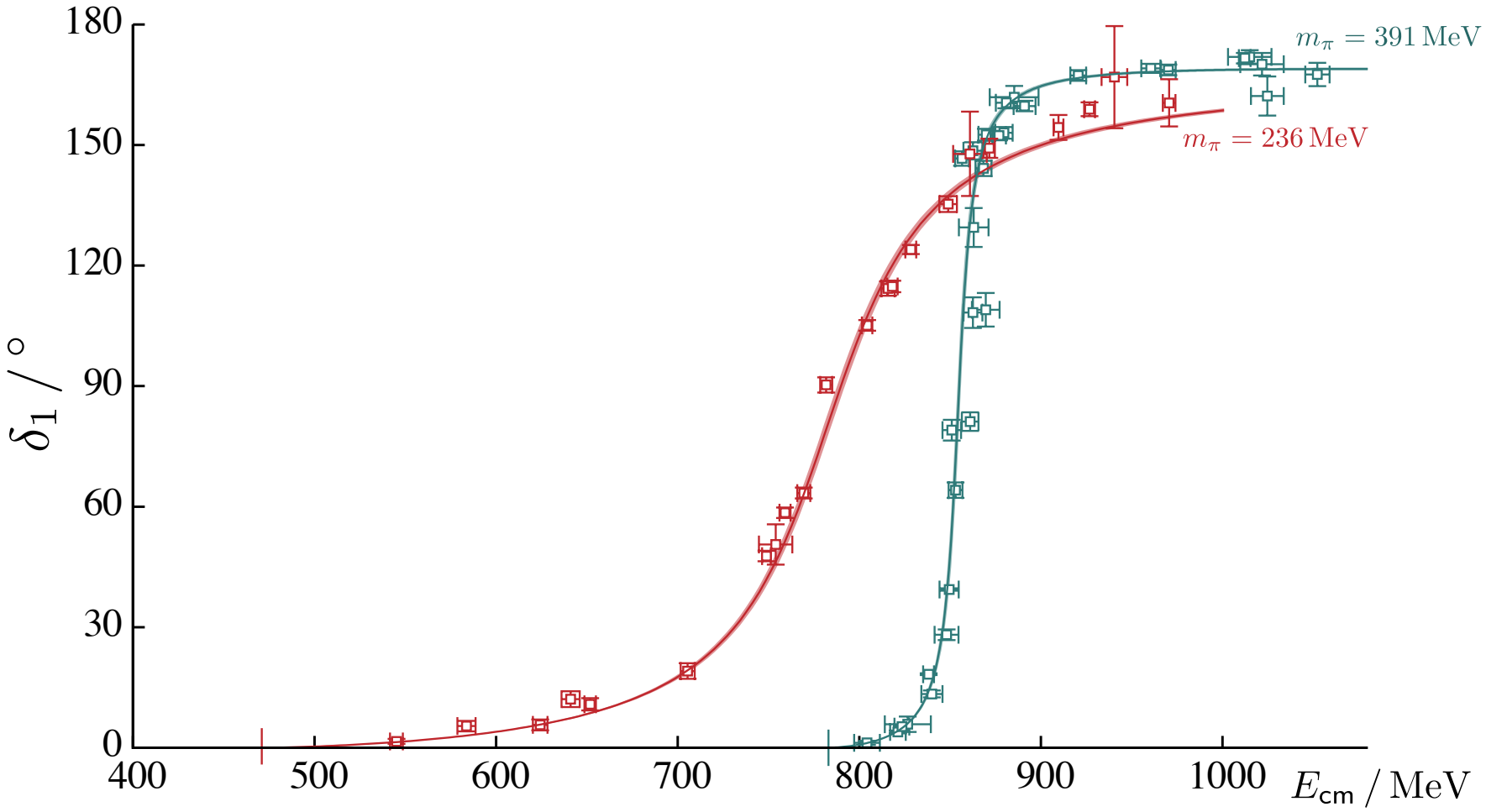
- We extract $g_{\rho\pi\pi}$ rather than Γ

$$\Gamma(s) = \frac{p^{\star 3}}{s} \frac{g^2_{\rho\pi\pi}}{6\pi}$$

- Results for $m_\pi = 266(3)(3)$MeV

$$g_{\rho\pi\pi} = 5.61(12) \qquad m_\rho = 772(6)(8)\ \text{MeV}$$

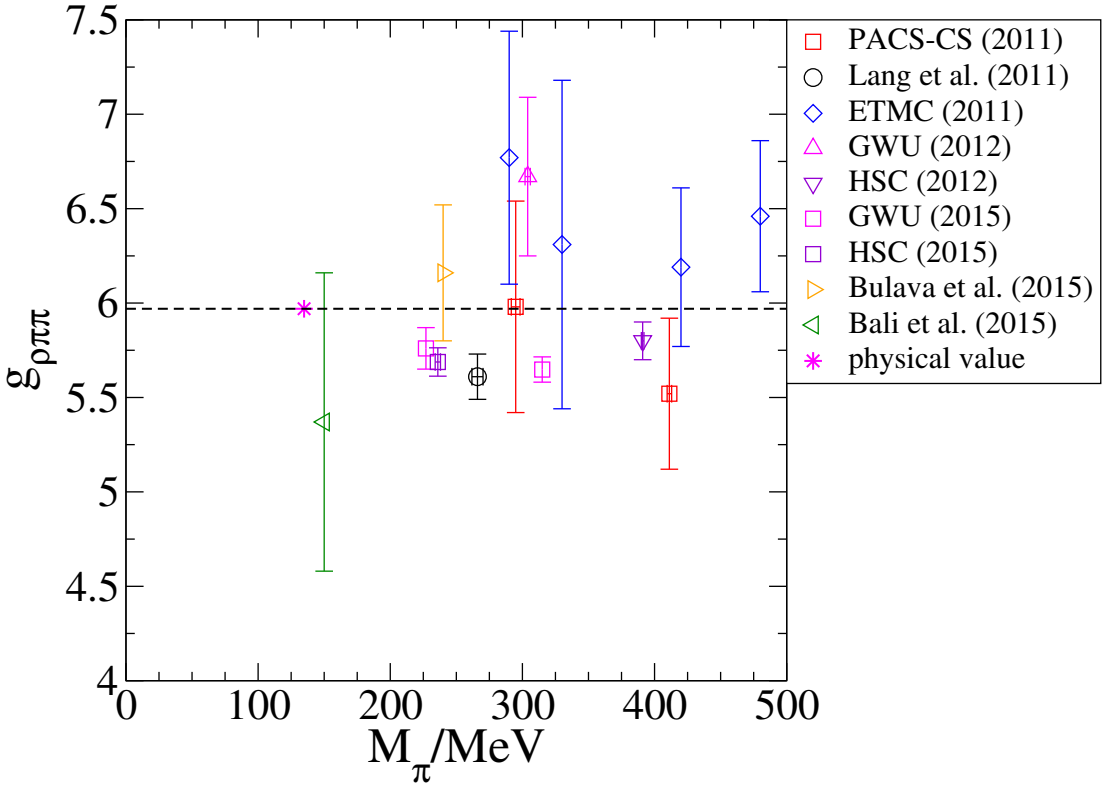
The ρ: More recent results:



Wilson *et al.* PRD 92 094502 (2015)

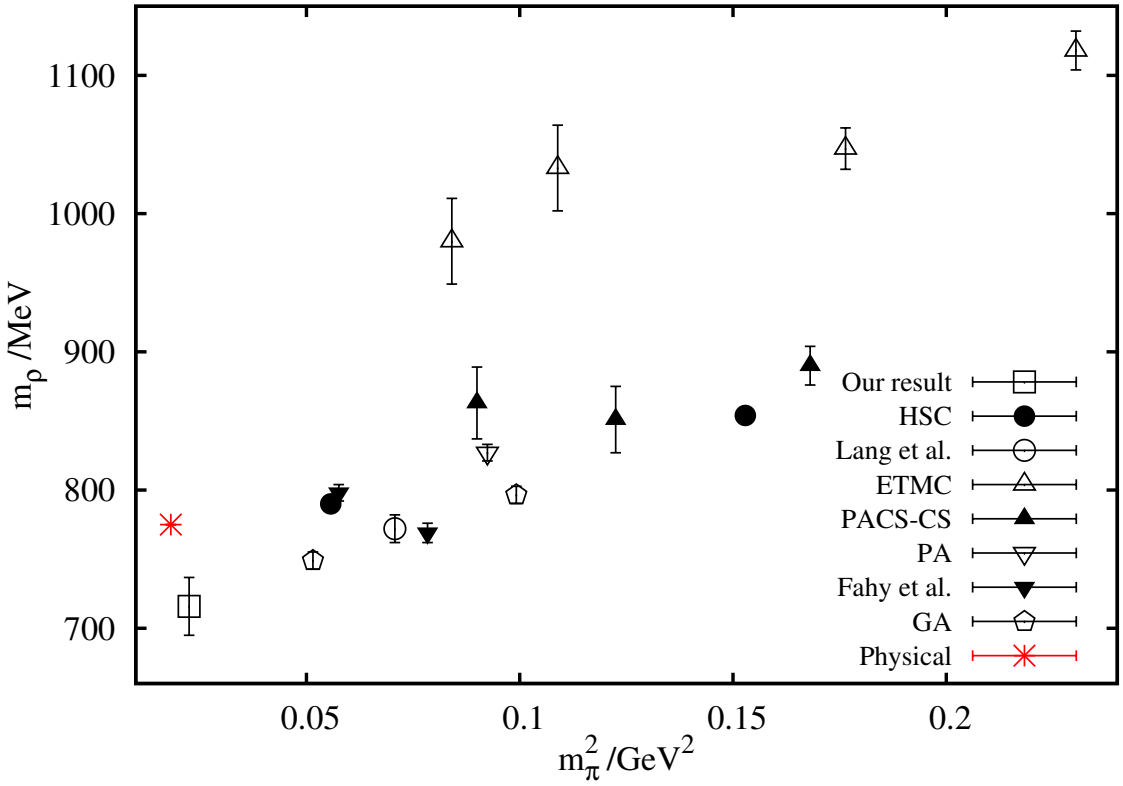
The ρ resonance - comparing results for the coupling

$$g_{\rho\pi\pi}^{(phys)} \approx 5.97 \qquad m_\rho = 775.11(34)\ \text{MeV}$$



- Caution: To date no simulation with full control of systematics
- Many more preliminary results at Lattice2016
- In general good agreement at larger than physical pion mass

The ρ resonance - comparing results for the masses



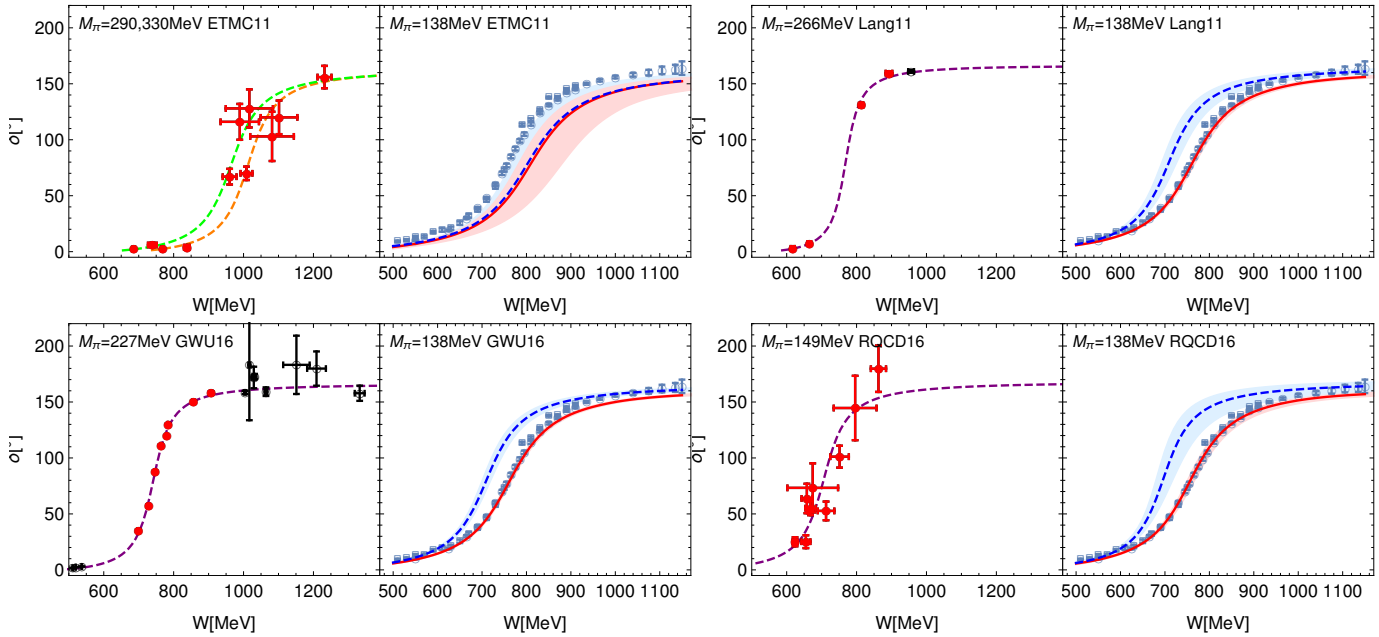
Plot from RQCD, Bali *et al.* PRD 93 054509 (2016)

- Quite a large spread in the lattice data
- Can one learn something from looking at the current collection of data?

The ρ resonance: 2 vs. 2+1 flavor

Hu, Molina, Döring, Alexandru arXiv:1605.04823, accepted by PRL

- Analysis of $N_f = 2$ lattice scattering data using Unitarized χPT
- Use fit to experiment data to go from $N_f = 2 \rightarrow N_f = 2 + 1$.
- Main claim: The missing $K\bar{K}$ channel explains low 2 flavor lattice data



→ Talk by Raquel Molina on Thursday at 15:50

The ρ-resonance and $F_\pi(E)$

- In the elastic region the form factor $F_\pi(E)$ is

$$R(s) = \frac{1}{4}\left(1 - \frac{4m_\pi^2}{s}\right)^{\frac{3}{2}} |F_\pi(\sqrt{s})|^2 = \frac{\sigma(e^+e^- \to hadrons)}{4\pi\alpha(s)^2/(3s)}$$

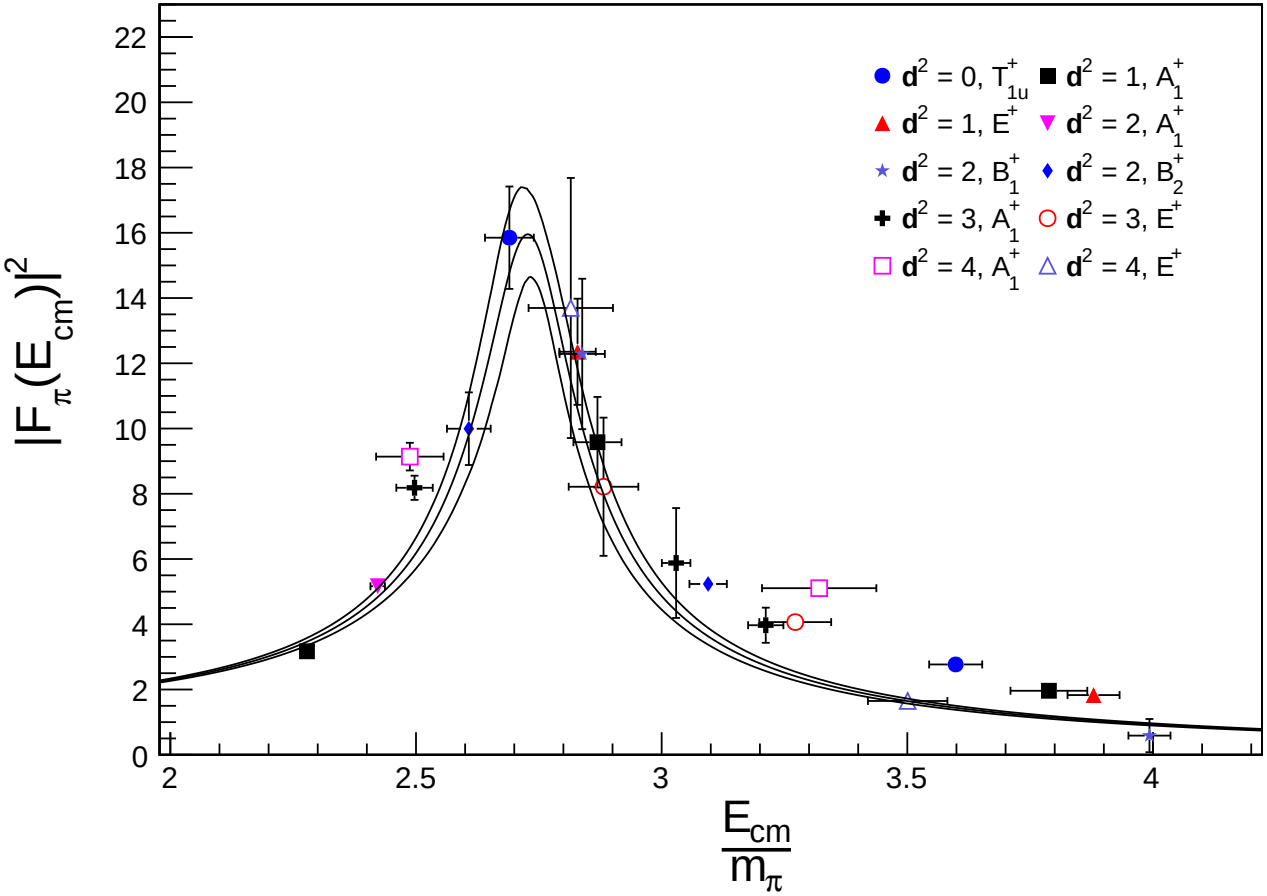
- Is of phenomenological importance for lattice determinations of the hadronic vacuum polarization (HVP) contribution to $(g-2)_\mu$.
- It can be calculated from a finite-volume matrix element

H. B. Meyer PRL 107 072002 (2011)

- The derivative of the phase shift $\partial\delta_1(k)/\partial k$ enters the determination

The ρ-resonance and $F_\pi(E)$

Bulava *et al.* arXiv:1511.02351



- Results at a single lattice spacing and $m_\pi = 280$MeV
- Curve is the Gounaris-Sakurai parametrization (not a fit)

Exotic D_s and B_s candidates

Established s and p-wave states:

D_s ($J^P = 0^-$) and D_s^* (1^-)	B_s ($J^P = 0^-$) and B_s^* (1^-)
$D_{s0}^*(2317)$ (0^+), $D_{s1}(2460)$ (1^+), $D_{s1}(2536)$ (1^+), $D_{s2}^*(2573)$ (2^+)	$B_{s1}(5830)$ (1^+), $B_{s2}^*(5840)$ (2^+)

- Peculiarity: $M_{c\bar{s}} \approx M_{c\bar{d}}$ $\rightarrow$ exotic structure? (tetraquark, molecule)
- Traditional lattice studies (using single hadron operators) tend get too large or badly determined masses

- B_s cousins of the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ not (yet) seen in experiment
- LHCb should be able to see these

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$D_{s1}(2536)$ (1^+), $D_{s2}^*(2573)$ (2^+)	$B_{s1}(5830)$ (1^+), $B_{s2}^*(5840)$ (2^+)

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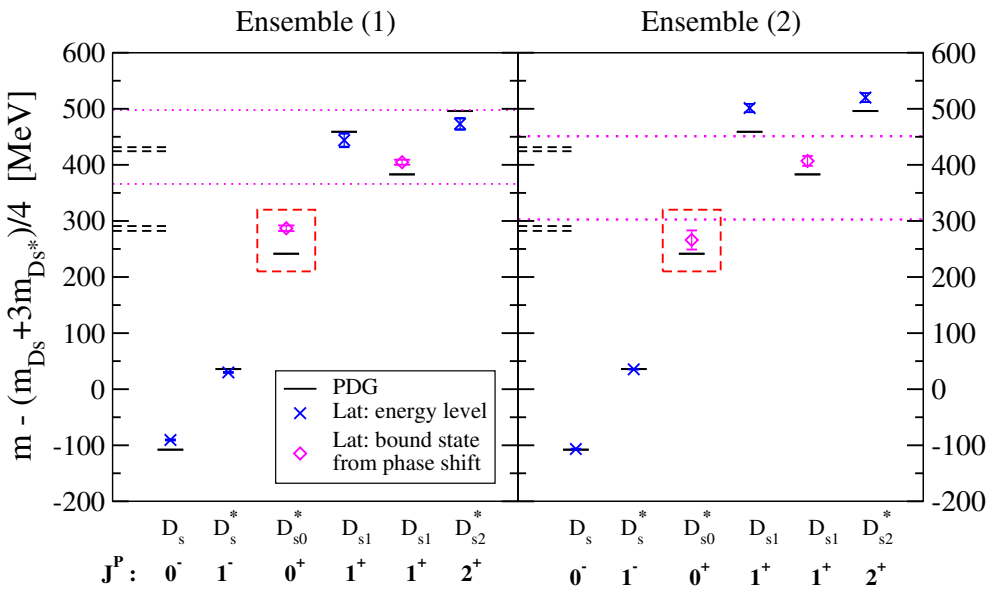
Spectrum results for D_s

Mohler *et al.* PRL 111 222001 (2013)

Lang, DM *et al.* PRD 90 034510 (2014)



- Discretization uncertainties sizeable for charm
- Many improvements possible for the D_s states

For more D and D_s results

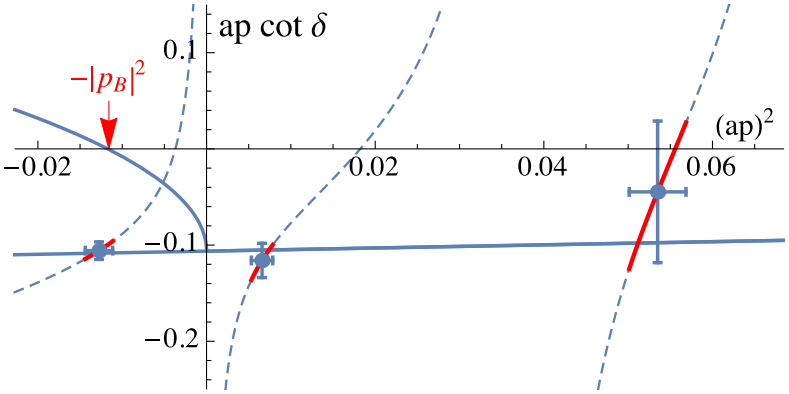
→ Talk by Gunnar Bali Monday 15:20

→ Talk by Graham Moir Monday 16:40

B^*_{s0} and B_{s1}: Results

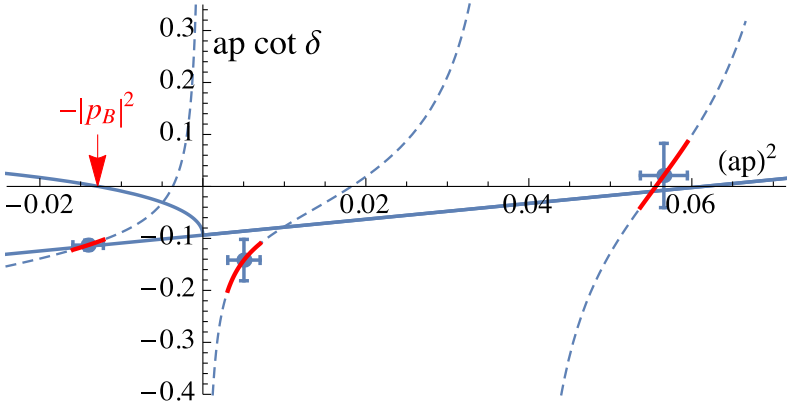
Lang, DM, Prelovsek, Woloshyn PLB 750 17 (2015)

B^*_{s0}



$$a_0^{BK} = -0.85(10)\,\text{fm}$$
$$r_0^{BK} = 0.03(15)\,\text{fm}$$
$$M_{B^*_{s0}} = 5.711(13)\,\text{GeV}$$

B_{s1}

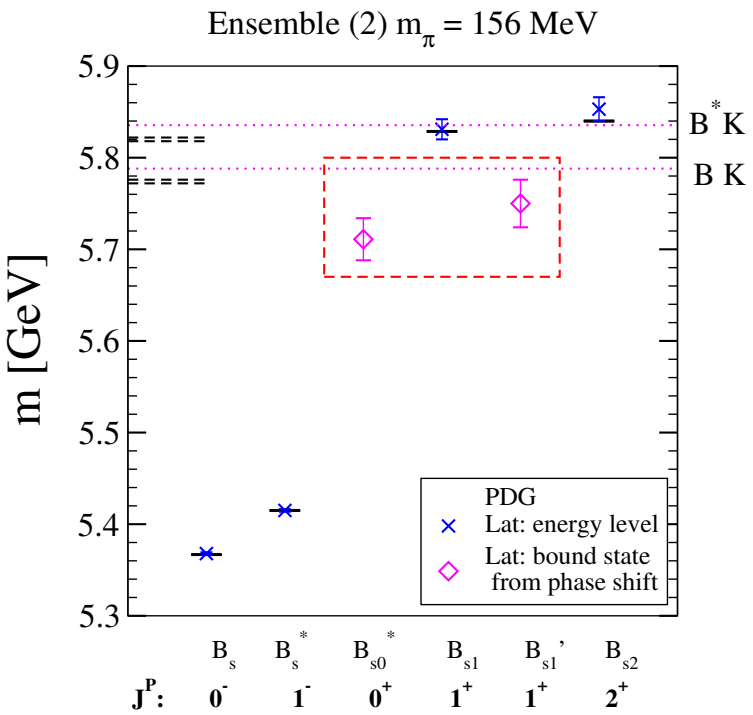


$$a_0^{B^*K} = -0.97(16)\,\text{fm}$$
$$r_0^{B^*K} = 0.28(15)\,\text{fm}$$
$$M_{B_{s1}} = 5.750(17)\,\text{GeV}$$

- Energy from the difference to the $B^{(*)}K$ threshold

Spectrum results

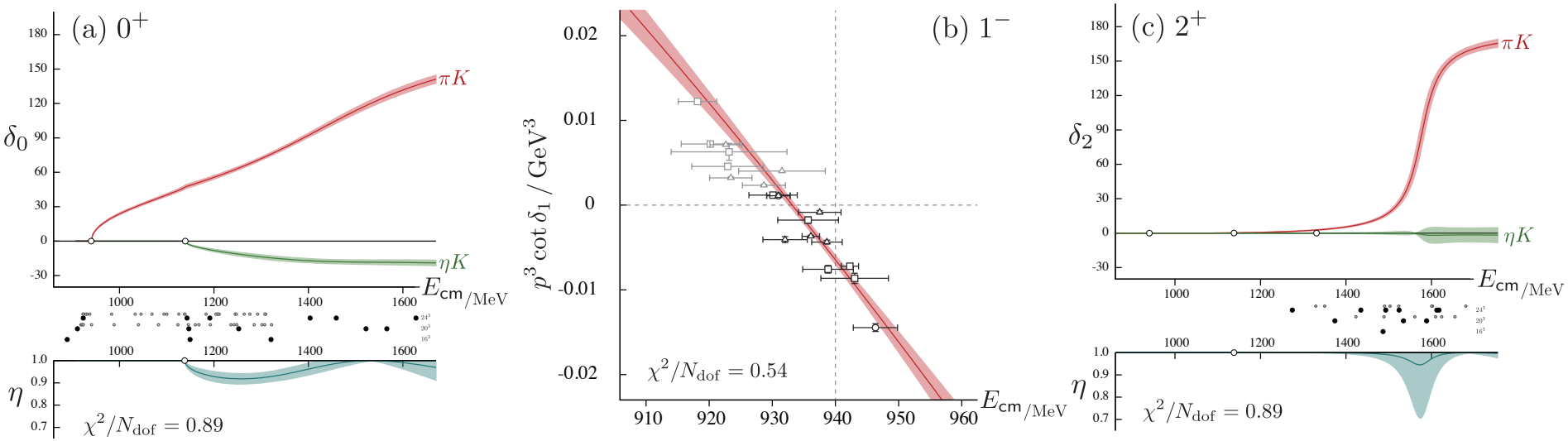
Lang, DM, Prelovsek, Woloshyn PLB 750 17 (2015)



- Full uncertainty estimate only for magenta B_s states
- Prediction of exotic states from Lattice QCD!

Coupled channel $\pi K - \eta K$ scattering

PRL 113 182001 (2014)



- First coupled channel study in Lattice QCD
- Channels are mostly decoupled
- For recent coupled-channel results:

→ Plenary talk by Christopher Thomas

→ Talk by Raúl Briceño Tuesday at 17:30

Coupled channel $\pi K - \eta K$ scattering

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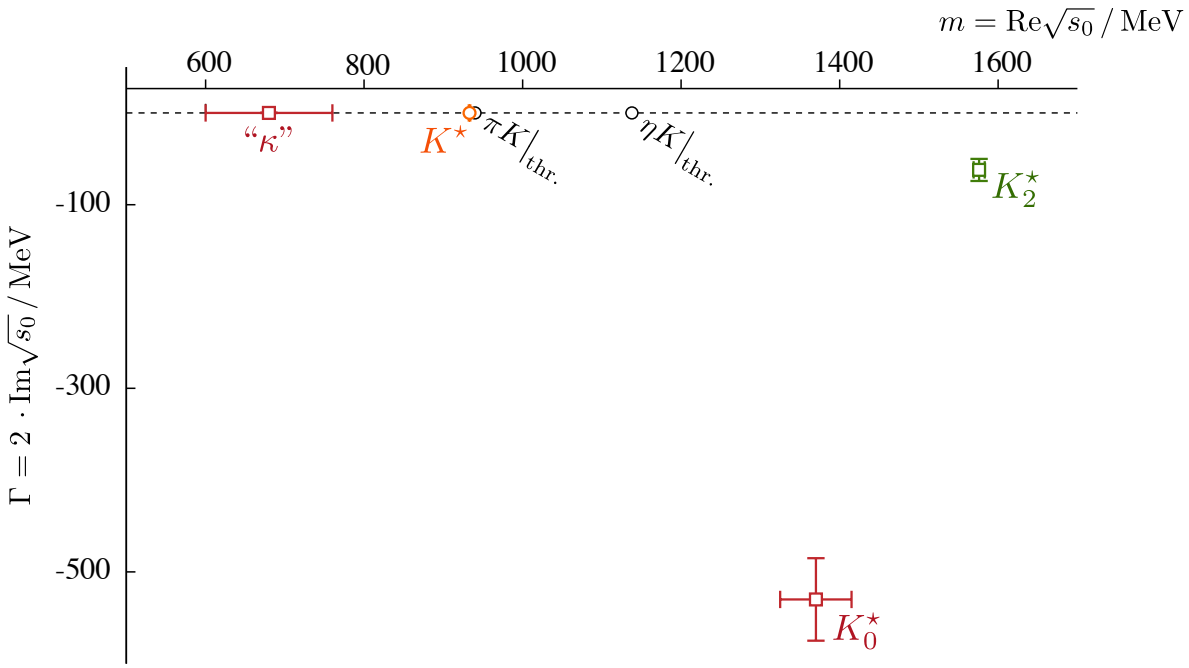

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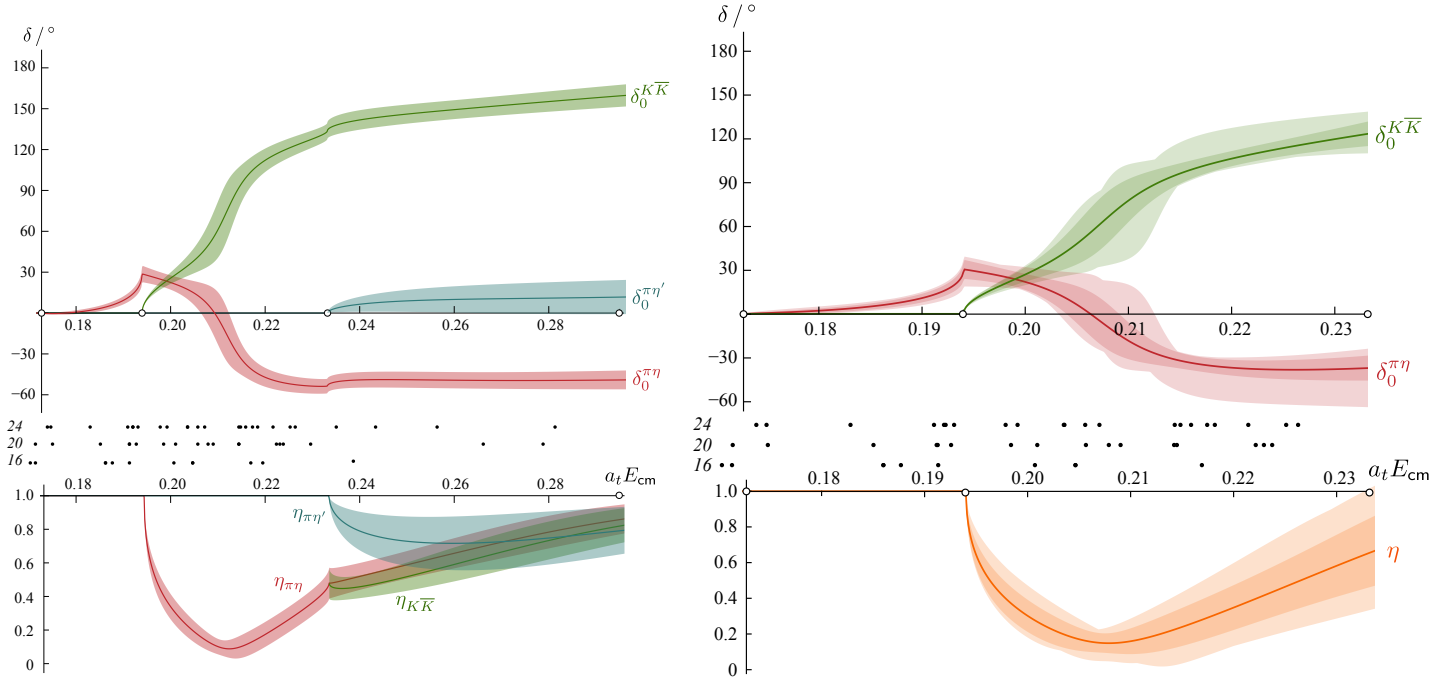
Coupled channel $\pi\eta - \bar{K}K$ scattering

J. Dudek et al. PRD 93 094506 (2016)



- Channels are tightly coupled
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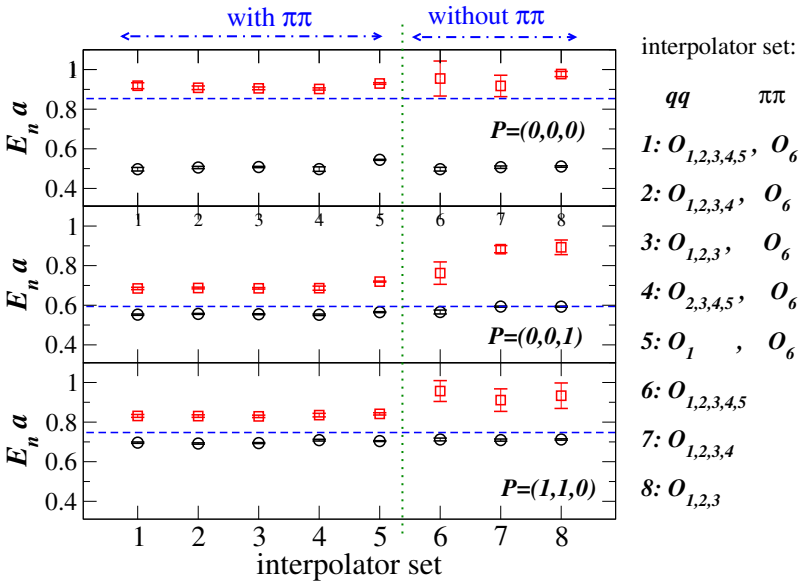
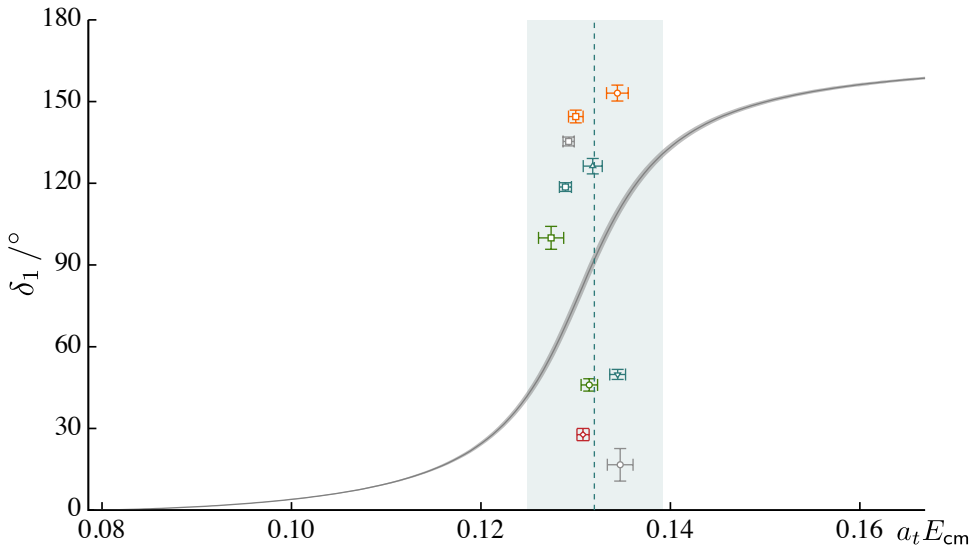
A lesson from meson-meson scattering

Plots from Lang, DM, Prelovsek, Vidmar, PRD 84 054503 (2011); and Wilson *et al.* PRD 92 094502 (2015)

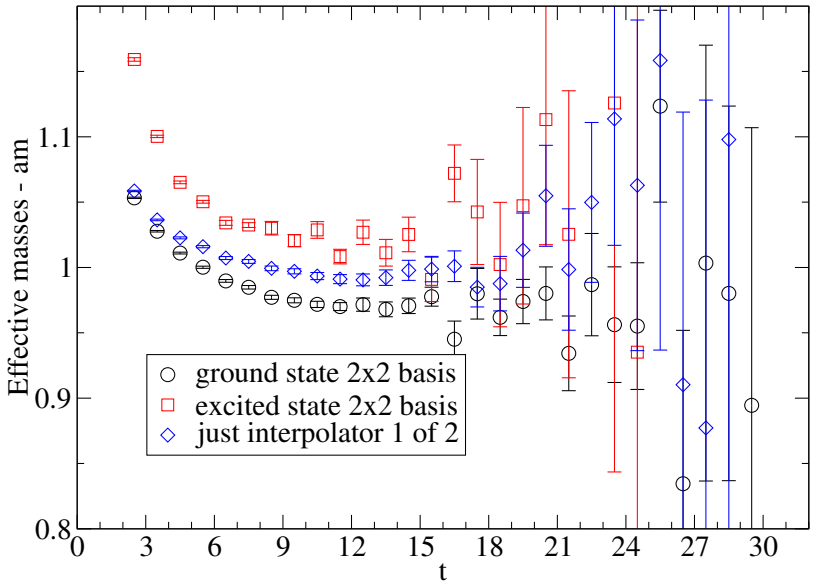


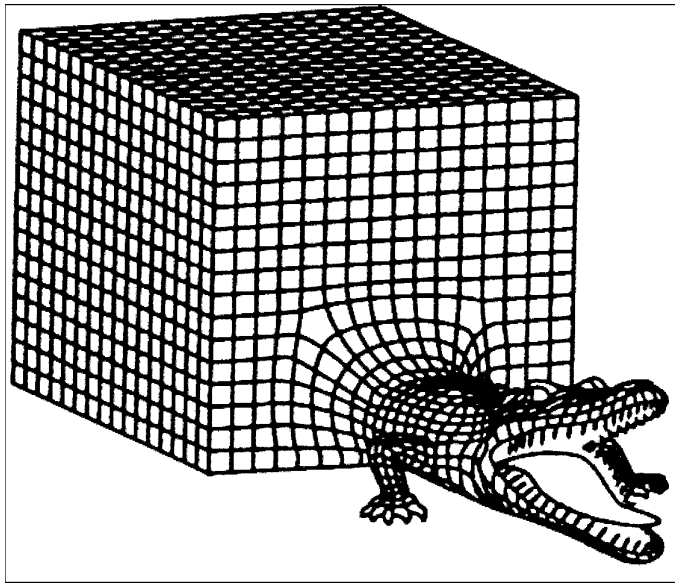


- A diverse interpolator basis is vital to determine the true spectrum!
- Beware: Effective energies may seem to reach a plateau (with good fit χ^2)

A lesson from meson-meson scattering

Data from Mohler *et al.* PRL 111 222001 (2013)





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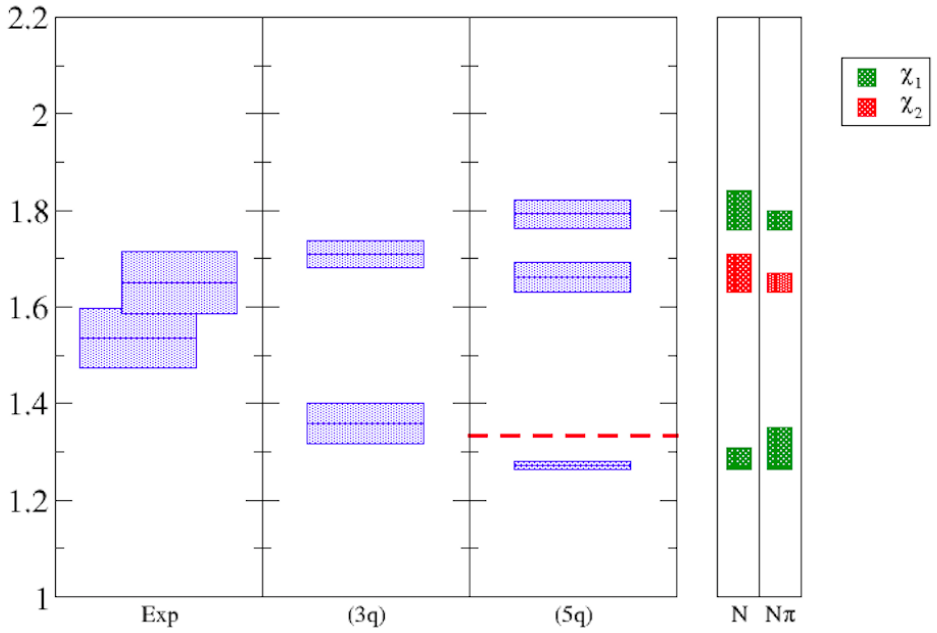
Baryon resonances in Meson-Baryon scattering

- Studies of the baryon spectrum more difficult
  - Signal to noise much worse
  - Observables are more expensive
  - Parity mixing in moving frames
- A large number of energy levels has been extracted using 3-quark interpolators
- However: Only very few studies using baryon – meson or 5-quark interpolators exist
- Multi-hadron levels are missing from the spectrum
- Based on meson experience: How trustworthy are the existing results?

Nucleon–pion scattering for the negative parity Nucleons

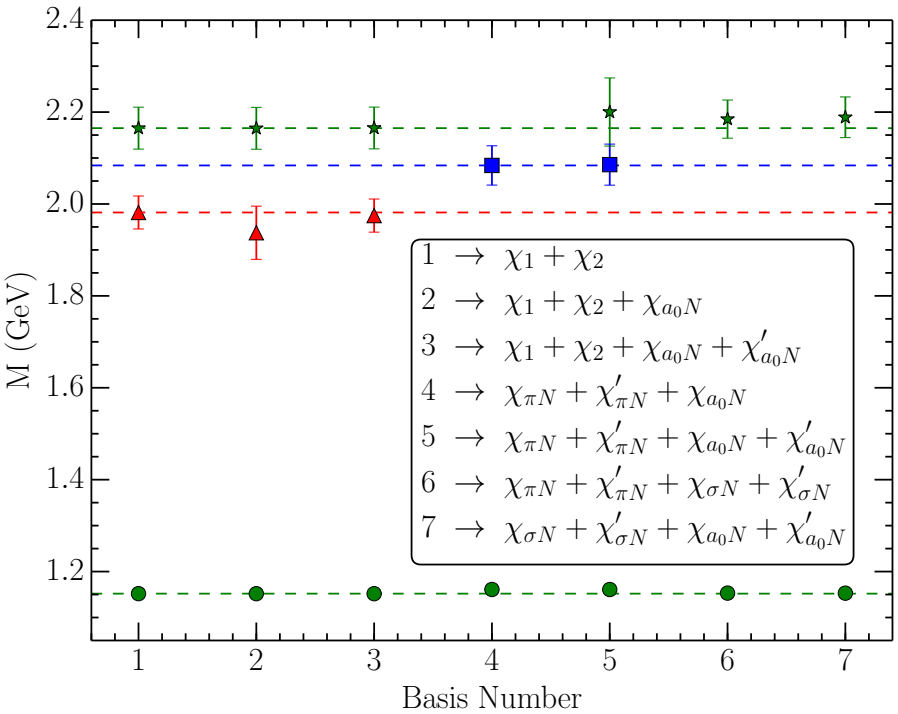
Lang and Verduci, PRD 87 054502 (2013);
plot from arXiv:1412.0701.



- Clear issue with extracting the correct spectrum using only 3-quark operators
- Only energy levels – not yet enough data for a (coupled channel) phase-shift analysis

The Roper channel using local 5-quark interpolators

Kiratidis *et al.* arXiv:1608.03051



- 3-quark interpolators plus local 5-quark interpolators with $N\pi$, $N\sigma$ and Na_0 structures
- How does this relate to previous claims about the Roper?
- Authors say it flavors dynamically generated coupled-channel resonance
- Caution: Extracted spectrum is clearly incomplete

Summary and outlook

- A lot of progress and activity in QCD resonance and bound-state studies
- A number of studies of resonances in meson-meson scattering
  Could only cover selected examples. Interesting results also for
  - $D^{(*)}D^{(*)}$-scattering and charmonium results
  - Heavy-quark exotics

  See for example review by S. Prelovsek arXiv:1508.07322
- First coupled channel simulations

  → Plenary talk by Christopher Thomas
  → Talk by Raúl Briceño Tuesday at 17:30
- Studies of systematic uncertainties will start to appear for the simplest of resonances
- Meson-baryon studies are more difficult (and still scarce)

Thank you!

Baryon – Baryon bound states and resonances

- $\rightarrow$Talk by P. Junnarkar Tuesday at 19:00

Backup Slides

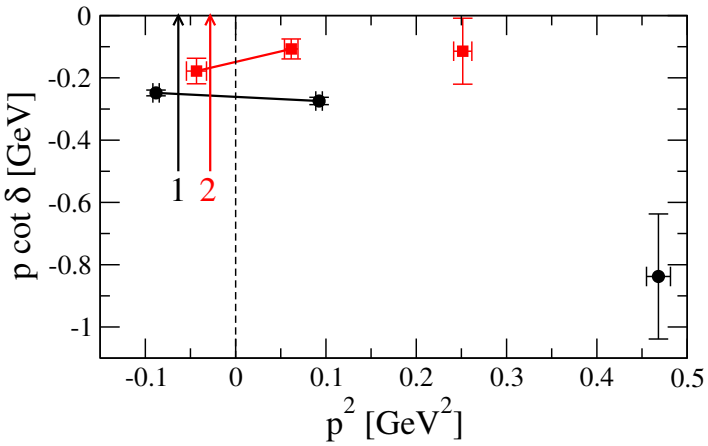
$D_{s0}^*(2317)$: D-meson – Kaon s-wave scattering

M. Lüscher Commun. Math. Phys. 105 (1986) 153;
Nucl. Phys. B 354 (1991) 531; Nucl. Phys. B 364 (1991) 237.

$$p \cot \delta(p) = \frac{2}{\sqrt{\pi} L} Z_{00}(1; q^2)$$
$$\approx \frac{1}{a_0} + \frac{1}{2} r_0 p^2$$

Mohler *et al.* PRL 111 222001 (2013)
Lang, DM *et al.* PRD 90 034510 (2014)

Results for ensembles (1) and (2)



$$a_0 = -0.756 \pm 0.025\text{fm} \quad (1)$$
$$r_0 = -0.056 \pm 0.031\text{fm}$$
$$a_0 = -1.33 \pm 0.20\text{fm} \quad (2)$$
$$r_0 = 0.27 \pm 0.17\text{fm}$$

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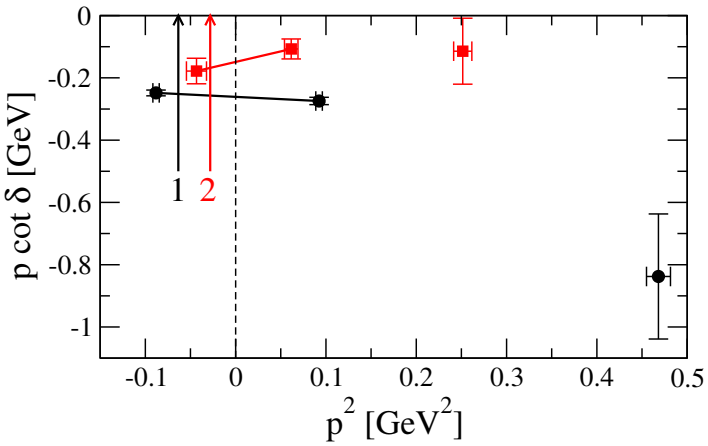

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