Decays of neutral pions

Electromagnetic form factors and radiative corrections

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Decay modes of neutral pion: $\pi^0 \rightarrow \gamma \gamma$, $\pi^0 \rightarrow e^+e^- \gamma$, $\pi^0 \rightarrow e^+e^-e^-e^-$, ...

**Rare** decay $\pi^0 \rightarrow e^+e^-$

- precise measurements of branching ratio
  - $K\text{TeV}$ experiment at Fermilab ([Abouzaid et al., PRD 75 (2007)])
    \[ B^{K\text{TeV}}(\pi^0 \rightarrow e^+e^-(\gamma), \ x_D > 0.95) = (6.44 \pm 0.25 \pm 0.22) \times 10^{-8} \]
- Standard Model theoretical prediction
  - $3.3 \sigma$ disagreement ([Dorokhov and Ivanov, PRD 75 (2007)])
- discrepancy not satisfactorily explained yet
- very fashionable to ascribe eventual discrepancies to effects of new physics

**BUT**

- first, look for more conventional solution (i.e. within SM)

  → radiative corrections (usually very important)
  → form factor modeling
- pions are complicated composite objects
  \[ \rightarrow \text{elementary interactions are not point-like} \]

- electromagnetic pion transition form factor \( F_{\pi^0 \gamma^* \gamma^*} \) describes this complexity

\[
\begin{align*}
\text{LO contribution} & \quad \Rightarrow \quad \text{its representation} \\
\text{in QED expansion} & \quad \Rightarrow \quad \text{as the LO of } \chi \text{PT}
\end{align*}
\]

- free parameter \( \chi^{(r)}(\mu) \) appears in the finite part of the counter term

\[
\chi = [\text{UV-divergent part}] + \chi^{(r)}(\mu)
\]

\[ \rightarrow \text{unique for every form factor, e.g. } \chi^{(r)}_{KTeV}(M_\rho) = 6.0 \pm 1.0 \]
- calculated by *Vaško and Novotný, JHEP 1110 (2011)*
Bremsstrahlung

- compensation of infrared divergences in 2-loop contributions

→ TH, Kampf and Novotný, EPJC 74 (2014)
Size of the radiative corrections (**newly calculated**)

\[ \delta^{\text{NLO}}(0.95) \equiv \delta^{\text{virt.}} + \delta^{\text{BS}}(0.95) = (-5.5 \pm 0.2)\% \]

- can be thought as model-independent
- differs **significantly** from previous **approximate** calculations

- original KTeV vs. SM discrepancy reduced to the 2\(\sigma\) level or less
  \[ \chi^{(r)}_{\text{KTeV}}(M_\rho) = 4.5 \pm 1.0 \]

- LMD model (**Knecht et al., PRL 83 (1999)**)
  \[ \chi^{(r)}_{\text{LMD}}(M_\rho) = 2.2 \pm 0.9 \]

NLO radiative corrections in the QED sector did not solve the discrepancy
→ back to LO, but use different model
Chiral Perturbation Theory ($\chi$PT)

Resonance Chiral Theory ($R\chi$T)
1) Ansatz for Pseudoscalar-Vector-Vector (PVV) correlator
   - Two-Hadron-Saturation (THS) - 2 meson multiplets per channel

\[ \Pi^{\text{THS}}(r^2, p^2, q^2) \sim \frac{1}{r^2(r^2 - M_P^2)} \frac{P(r^2; p^2, q^2)}{(p^2 - M_{V_1}^2)(p^2 - M_{V_2}^2)(q^2 - M_{V_1}^2)(q^2 - M_{V_2}^2)} \]

- in numerator stands general polynomial symmetrical in \( p^2 \) and \( q^2 \)
  → correlator must drop at large momenta
  → 22 free parameters

\[ P(r^2; p^2, q^2) = c_0 p^2 q^2 + c_1 [(p^2)^3 q^2 + (q^2)^3 p^2] + c_2 (r^2)^2 p^2 q^2 + \ldots \]

2) Use high- and low-energy limits to constrain the parameters
   - Operator product expansion (OPE)
   - Brodsky–Lepage (BL) quark counting rules
   - chiral anomaly
Form factor is in general related to PVV correlator as

$$F_{\pi^0 \gamma^* \gamma^*}(p^2, q^2) \sim \lim_{r^2 \to 0} r^2 \Pi(r^2; p^2, q^2)$$

→ in our case complicated, but with only one free parameter

$$F_{\pi^0 \gamma^* \gamma^*}^{\text{THS}}(p^2, q^2) = -\frac{N_c}{12\pi^2 F} \left[ \frac{M_{V_1}^4 M_{V_2}^4}{(p^2 - M_{V_1}^2)(q^2 - M_{V_2}^2)} \right] \left[ \frac{M_{V_1}^4 M_{V_2}^4}{(p^2 - M_{V_1}^2)(q^2 - M_{V_2}^2)} \right]$$

$$\times \left\{ 1 + \frac{\kappa}{2N_c (4\pi F)^4} - \frac{4\pi^2 F^2(p^2 + q^2)}{N_c M_{V_1}^2 M_{V_2}^2} \left[ 6 + \frac{p^2 q^2}{M_{V_1}^2 M_{V_2}^2} \right] \right\}$$

κ determined from fit to ω-π transition form factor measurements

$$\kappa = 21 \pm 3$$

$$M_{V_1} \sim \rho, \omega$$ vector-meson mass

$$M_{V_2} \sim$$ between physical masses of first and second vector-meson excitations

$$M_{V_2} \in [1400, 1740] \text{ MeV}$$
VMD and LMD models

Examples of other approaches

- **Vector-Meson Dominance (VMD)**

\[
F_{\pi^0 \gamma^* \gamma^*}^{\text{VMD}}(p^2, q^2) = -\frac{N_c}{12\pi^2 F} \left[ \frac{M_{V_1}^4}{(p^2 - M_{V_1}^2)(q^2 - M_{V_1}^2)} \right]
\]

→ violates OPE: \( F_{\pi^0 \gamma^* \gamma^*}(q^2, q^2) \propto \frac{1}{q^2}, \; q^2 \to -\infty \)

- **Lowest-Meson Dominance (LMD)**

\[
F_{\pi^0 \gamma^* \gamma^*}^{\text{LMD}}(p^2, q^2) = F_{\pi^0 \gamma^* \gamma^*}^{\text{VMD}}(p^2, q^2) \left\{ 1 - \frac{4\pi^2 F^2 (p^2 + q^2)}{N_c M_{V_1}^4} \right\}
\]

→ violates BL: \( F_{\pi^0 \gamma^* \gamma^*}(0, q^2) \propto \frac{1}{q^2}, \; q^2 \to -\infty \)

- none of the models used two meson multiplets in both channels

→ vector and pseudoscalar
Theoretical prediction within THS model

\[ B^{\text{THS}}(\pi^0 \rightarrow e^+ e^- (\gamma), \ x_D > 0.95) = (5.8 \pm 0.2) \times 10^{-8} \]

- recall experimental value: \( B^{K\text{TeV}} = (6.44 \pm 0.33) \times 10^{-8} \)
  \( \rightarrow \) disagreement at the level of only 1.8 \( \sigma \)
- matching on LO \( \chi \)PT gives \( \chi^{(r)}_{\text{THS}}(M_\rho) = 2.2 \pm 0.7 \)
- if KTeV result confirmed \( \rightarrow \) two scenarios are conceivable:
  a) some aspects of the THS approach not well-suited for \( \pi^0 \rightarrow e^+ e^- \)
  b) beyond-Standard Model physics influences the rare pion decay significantly
- under the present circumstances the current discrepancy is inconclusive

Quantity really measured by KTeV

\[ \frac{\Gamma(\pi^0 \rightarrow e^+ e^- (\gamma), \ x > 0.95)}{\Gamma(\pi^0 \rightarrow e^+ e^- \gamma(\gamma), \ x > 0.2319)} \bigg|_{K\text{TeV}} = (1.685 \pm 0.064 \pm 0.027) \times 10^{-4} \]

\( \rightarrow \) Dalitz decay comes into play
Dalitz decay radiative corrections

- corrections to the Dalitz plot in the form of a table of values
  → *Mikaelian and Smith*, PRD 5 (1972)

- new calculations motivated by needs of NA48/NA62 experiments at CERN
  → measure the slope $a$ of $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0, q^2)$

- unlike before no approximation was used
  → can be used also for related decays $\eta \rightarrow \ell^+\ell^-\gamma$ etc.

- C++ code returns the correction for any given $x$ and $y$
  → propagated into simulation software of NA62 experiment

Pseudoscalar decays

- $\chi^{(r)}$ universal for $P \rightarrow l^+ l^-$ processes up to $\mathcal{O}(m_l^2/\Lambda^{2}_{\chi_{PT}})$

Muon $g - 2$: hadronic light-by-light scattering

- pseudoscalar meson exchange contribution requires hadron-physics input
All NLO QED radiative corrections for discussed processes are now available → can be taken into account in future experimental analyses

- $\pi^0 \rightarrow e^+ e^-$
  
  Vaško and Novotný, JHEP 1110 (2011)
  
  TH, Kampf and Novotný, EPJC 74 (2014)

- $\pi^0 \rightarrow e^+ e^− γ$
  
  TH, Kampf and Novotný, PRD 92 (2015)

THS model for $\mathcal{F}_{\pi^0 γ^* γ^*}(p^2, q^2)$

- phenomenologically successful
- satisfies all main theoretical constraints
- TH and S. Leupold, EPJC 75 (2015)

Altogether, we get reasonable SM prediction → differs from KTeV by 1.8 $σ$
Goodbye

Thank you for your attention!