Scheme variations of the QCD coupling

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Strong coupling

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\[
\alpha_s(Q^2) = \frac{1}{\pi} \frac{d}{dq} \ln \frac{1}{\bar{c}(q)}
\]

\[QCD \quad \alpha_s(M_Z) = 0.1181 \pm 0.0013\]

- \( \tau \) decays (N^3LO)
- DIS jets (NLO)
- Heavy Quarkonia (NLO)
- e^+e^- jets & shapes (res. NNLO)
- e.w. precision fits (NNLO)
- \( p\bar{p} \rightarrow \text{jets} \) (NLO)
- \( pp \rightarrow \text{tt} \) (NNLO)

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Scale evolution

Scale evolution of $\alpha_s$ is given by the $\beta$-function:

$$- Q \frac{d a_Q}{d Q} \equiv \beta(a_Q) = \beta_1 a_Q^2 + \beta_2 a_Q^3 + \beta_3 a_Q^4 + \ldots .$$

with $a_Q = \alpha_s / \pi$.

The scale invariant parameter $\Lambda$ can be defined by:

$$\Lambda \equiv Q e^{-\frac{1}{\beta_1 a_Q}} [a_Q]^{-\frac{\beta_2}{\beta_1^2}} \exp \left\{ \int_0^{a_Q} \frac{d a}{\tilde{\beta}(a)} \right\},$$

where

$$\frac{1}{\tilde{\beta}(a)} \equiv \frac{1}{\beta(a)} - \frac{1}{\beta_1 a^2} + \frac{\beta_2}{\beta_1^2 a}$$

is free of singularities at $a \to 0$. 
The coupling $\hat{a}$

However, $\Lambda$ depends on the renormalisation scheme.

$$a' \equiv a + c_1 \ a^2 + c_2 \ a^3 + c_3 \ a^4 + \ldots$$

Then, $\Lambda$ transforms as: (Celmaster, Gonsalves 1979)

$$\Lambda' = \Lambda e^{c_1/\beta_1}.$$  

This suggests to define a new coupling $\hat{a}_Q$:

$$\frac{1}{\hat{a}_Q} + \frac{\beta_2}{\beta_1} \ln \hat{a}_Q \equiv \beta_1 \left( \ln \frac{Q}{\Lambda} + \frac{C}{2} \right)$$

$$= \frac{1}{a_Q} + \frac{\beta_1}{2} C + \frac{\beta_2}{\beta_1} \ln a_Q - \beta_1 \int_0^{a_Q} \frac{da}{\tilde{\beta}(a)}$$

(Boito, MJ, Miravitllas 2016)
The $\beta$-function of $\hat{a}_Q$ takes the simple form:

$$-Q\frac{d\hat{a}_Q}{dQ} \equiv \hat{\beta}(\hat{a}_Q) = \frac{\beta_1 \hat{a}_Q^2}{\left(1 - \frac{\beta_2}{\beta_1} \hat{a}_Q\right)}.$$  

Perturbatively, $\hat{a}$ and $a$ are related by:

$$\hat{a}(a) = a - \frac{9}{4} C a^2 - \left(\frac{3397}{2592} + 4C - \frac{81}{16} C^2\right) a^3$$

$$- \left(\frac{741103}{186624} + \frac{233}{192} C - \frac{45}{2} C^2 + \frac{729}{64} C^3 + \frac{445}{144} \zeta_3\right) a^4$$

$$- \left(\frac{727240925}{80621568} - \frac{869039}{41472} C - \frac{26673}{512} C^2 + \frac{351}{4} C^3 - \frac{6561}{256} C^4\right) a^5$$

$$- \frac{445}{32} \zeta_3 C + \frac{10375693}{373248} \zeta_3 - \frac{1335}{256} \zeta_4 - \frac{534385}{20736} \zeta_5\right) a^5 + \mathcal{O}(a^6)$$
The coupling $\hat{a}$ as a function of $C$ for $\alpha_s(M_\tau) = 0.316(10)$. 
Adler function

\[ 4\pi^2 D(a_Q) - 1 \equiv \hat{D}(a_Q) = \sum_{n=1}^{\infty} c_{n,1} a_Q^n \]

\[ = a_Q + 1.640 a_Q^2 + 6.371 a_Q^3 + 49.08 a_Q^4 + \ldots \]

Expressed in terms of the coupling \( \hat{a} \):

\[ \hat{D}(\hat{a}_Q) = \hat{a}_Q + (1.640 + 2.25C) \hat{a}_Q^2 \]

\[ + \ (7.682 + 11.38C + 5.063C^2) \hat{a}_Q^3 \]

\[ + \ (61.06 + 72.08C + 47.40C^2 + 11.39C^3) \hat{a}_Q^4 + \ldots \]
Adler function

\[ \hat{D}(\hat{a}_{\tau}, C = -0.78) = 0.1343 \pm 0.0070 \pm 0.0067 \]
The hadronic $\tau$ decay rate takes the form:

$$R_\tau = 3 S_{EW}(|V_{ud}|^2 + |V_{us}|^2) (1 + \delta^{(0)} + \ldots)$$

The perturbative part in fixed-order PT reads:

$$\delta^{(0)}_{FO}(a_Q) = a_Q + 5.202 \hat{a}_Q^2 + 26.37 \hat{a}_Q^3 + 127.1 \hat{a}_Q^4 + \ldots$$

Expressed in terms of the coupling $\hat{a}$:

$$\delta^{(0)}_{FO}(\hat{a}_Q) = \hat{a}_Q + (5.202 + 2.25C) \hat{a}_Q^2 + (27.68 + 27.41C + 5.063C^2) \hat{a}_Q^3 + (148.4 + 235.5C + 101.5C^2 + 11.39C^3) \hat{a}_Q^4 + \ldots$$
\[ \delta_{FO}^{(0)}(\hat{a}_{M\tau}, C = -0.88) = 0.2047 \pm 0.0034 \pm 0.0133 \]
Higgs decay

The decay rate of $H$ into $\bar{q}q$ is given by:

$$\Gamma(H \rightarrow q\bar{q}) = \frac{\sqrt{2} G_F}{M_H} \text{Im} \psi(M_H^2 + i0) \equiv \frac{N_c G_F M_H}{4\sqrt{2}\pi} \hat{m}_q^2 \hat{R}(\hat{\alpha}_s(M_H))$$

$\hat{R}$ is only a function of the coupling: (MJ, Miravitllas 2016)

$$\hat{R}(\hat{\alpha}_s) = [\hat{\alpha}_s(Q)]^{24/23} \{ 1 + (8.0176 + 2 C) \hat{a}_Q$$

$$+ (46.732 + 33.924 C + 3.9167 C^2) \hat{a}_Q^2$$

$$+ (141.19 + 315.38 C + 103.88 C^2 + 7.6157 C^3) \hat{a}_Q^3$$

$$- (524.03 - 1491.9 C - 1353.1 C^2 - 277.97 C^3$$

$$- 14.756 C^4) \hat{a}_Q^4 + \ldots \}$$
\[ \hat{R}(C = -0.94) = 0.1387 \pm 0.0002 \pm 0.0020 = 0.1387 \pm 0.0020 \]
Summary

- The scheme dependence of the novel coupling $\hat{\alpha}$ can be parameterised by a single parameter $C$.

- Its corresponding $\beta$-function turns out to be manifestly scheme invariant.

- The coupling $\hat{\alpha}$ allows to easily study scheme dependencies, and to optimise the perturbative expansion.

- This appears to be particularly useful in observables that contain global multiplicative factors of the coupling.
Summary

- The scheme dependence of the novel coupling \( \hat{a} \) can be parameterised by a single parameter \( C \).
- Its corresponding \( \beta \)-function turns out to be manifestly scheme invariant.
- The coupling \( \hat{a} \) allows to easily study scheme dependencies, and to optimise the perturbative expansion.
- This appears to be particularly useful in observables that contain global multiplicative factors of the coupling.

Thank You!