# Scheme variations of the QCD coupling

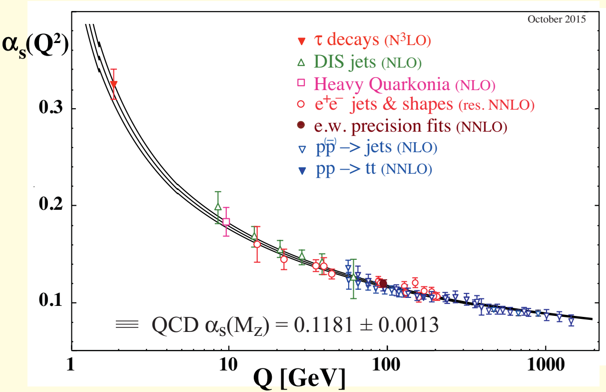
Matthias Jamin
ICREA @ IFAE
Universitat Autònoma de Barcelona



Strong coupling
Scale evolution
The coupling $\hat{a}$
Adler function
Tau decay
Higgs decay
Summary

# Strong coupling

PDG 2015




Strong coupling
Scale evolution
The coupling â
Adler function
Tau decay
Higgs decay
Summary

# Scale evolution

Scale evolution of $\alpha_s$ is given by the $\beta$-function:

$$-Q\,\frac{\mathrm{d}a_Q}{\mathrm{d}Q} \equiv \beta(a_Q) = \beta_1\,a_Q^2 + \beta_2\,a_Q^3 + \beta_3\,a_Q^4 + \ldots .$$

with $a_Q = \alpha_s/\pi$.

The scale invariant parameter $\Lambda$ can be defined by:

$$\Lambda \equiv Q\mathrm{e}^{-\frac{1}{\beta_1 a_Q}}\,[a_Q]^{-\frac{\beta_2}{\beta_1^2}}\exp\left\{\int_0^{a_Q}\frac{\mathrm{d}a}{\tilde{\beta}(a)}\right\},$$

where

$$\frac{1}{\tilde{\beta}(a)} \equiv \frac{1}{\beta(a)} - \frac{1}{\beta_1 a^2} + \frac{\beta_2}{\beta_1^2 a}$$

is free of singularites at $a \to 0$.



Strong coupling
Scale evolution
The coupling $\hat{a}$
Adler function
Tau decay
Higgs decay
Summary

# The coupling $\hat{a}$

However, $\Lambda$ depends on the renormalisation scheme.

$$a' \equiv a + c_1\, a^2 + c_2\, a^3 + c_3\, a^4 + \ldots .$$

Then, $\Lambda$ transforms as: (Celmaster, Gonsalves 1979)

$$\Lambda' = \Lambda\, \mathrm{e}^{c_1/\beta_1} .$$

This suggests to define a new coupling $\hat{a}_Q$:

$$\frac{1}{\hat{a}_Q} + \frac{\beta_2}{\beta_1} \ln \hat{a}_Q \equiv \beta_1 \left( \ln \frac{Q}{\Lambda} + \frac{C}{2} \right)$$

$$= \frac{1}{a_Q} + \frac{\beta_1}{2} C + \frac{\beta_2}{\beta_1} \ln a_Q - \beta_1 \int\limits_0^{a_Q} \frac{\mathrm{d}a}{\tilde{\beta}(a)}$$

(Boito, MJ, Miravitllas 2016)



Strong coupling
Scale evolution
The coupling $\hat{a}$
Adler function
Tau decay
Higgs decay
Summary

The $\beta$-function of $\hat{a}_Q$ takes the simple form:

$$-Q\,\frac{d\hat{a}_Q}{dQ} \equiv \hat{\beta}(\hat{a}_Q) = \frac{\beta_1\,\hat{a}_Q^2}{\left(1-\frac{\beta_2}{\beta_1}\,\hat{a}_Q\right)}\,.$$

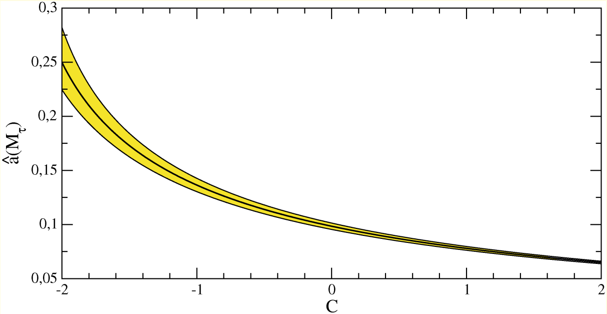
Perturbatively, $\hat{a}$ and $a$ are related by:

$$\begin{aligned}\hat{a}(a) &= a-\tfrac{9}{4}\,C\,a^2-\left(\tfrac{3397}{2592}+4C-\tfrac{81}{16}\,C^2\right)a^3\\ &- \left(\tfrac{741103}{186624}+\tfrac{233}{192}\,C-\tfrac{45}{2}\,C^2+\tfrac{729}{64}\,C^3+\tfrac{445}{144}\zeta_3\right)a^4\\ &- \left(\tfrac{727240925}{80621568}-\tfrac{869039}{41472}\,C-\tfrac{26673}{512}\,C^2+\tfrac{351}{4}\,C^3-\tfrac{6561}{256}\,C^4\right.\\ &- \left.\tfrac{445}{32}\zeta_3 C+\tfrac{10375693}{373248}\zeta_3-\tfrac{1335}{256}\zeta_4-\tfrac{534385}{20736}\zeta_5\right)a^5+\mathcal{O}(a^6)\end{aligned}$$



Strong coupling
Scale evolution
The coupling $\hat{a}$
Adler function
Tau decay
Higgs decay
Summary

# The coupling $\hat{a}$




The coupling $\hat{a}$ as a function of $C$ for $\alpha_s(M_\tau) = 0.316(10)$.



Strong coupling
Scale evolution
The coupling $\hat{a}$
Adler function
Tau decay
Higgs decay
Summary

# Adler function

$$4\pi^2 D(a_Q) - 1 \;\equiv\; \widehat{D}(a_Q) = \sum_{n=1}^{\infty} c_{n,1}\, a_Q^n$$
$$= a_Q + 1.640\, a_Q^2 + 6.371\, a_Q^3 + 49.08\, a_Q^4 + \ldots$$

Expressed in terms of the coupling $\hat{a}$:

$$\begin{aligned}\widehat{D}(\hat{a}_Q) &= \hat{a}_Q + (1.640 + 2.25C)\,\hat{a}_Q^2 \\ &+ (7.682 + 11.38C + 5.063C^2)\,\hat{a}_Q^3 \\ &+ (61.06 + 72.08C + 47.40C^2 + 11.39C^3)\,\hat{a}_Q^4 + \ldots\end{aligned}$$



Strong coupling

Scale evolution

The coupling $\hat{a}$

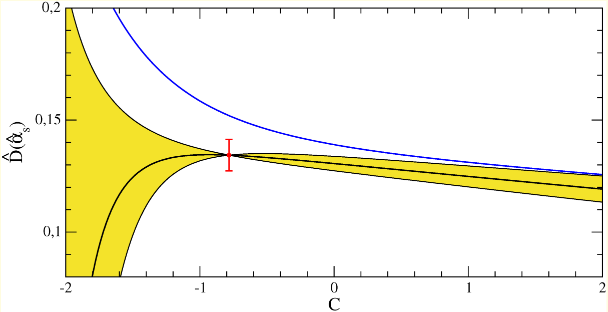
Adler function

Tau decay

Higgs decay

Summary

# Adler function




$$\widehat{D}(\hat{a}_{M_\tau}, C = -0.78) = 0.1343 \pm 0.0070 \pm 0.0067$$



Strong coupling
Scale evolution
The coupling $\hat{a}$
Adler function
Tau decay
Higgs decay
Summary

# Tau decay

The hadronic $\tau$ decay rate takes the form:

$$R_\tau = 3\, S_{\rm EW}(|V_{ud}|^2 + |V_{us}|^2)\,(1 + \delta^{(0)} + \cdots)$$

The perturbative part in fixed-order PT reads:

$$\delta_{\rm FO}^{(0)}(a_Q) = a_Q + 5.202\, a_Q^2 + 26.37\, a_Q^3 + 127.1\, a_Q^4 + \ldots$$

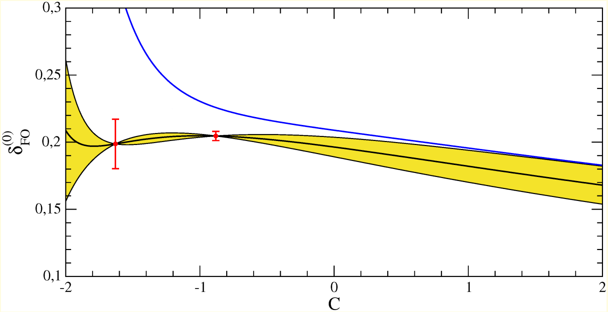
Expressed in terms of the coupling $\hat{a}$:

$$\begin{aligned} \delta_{\rm FO}^{(0)}(\hat{a}_Q) &= \hat{a}_Q + (5.202 + 2.25C)\,\hat{a}_Q^2 \\ &+ (27.68 + 27.41\,C + 5.063\,C^2)\,\hat{a}_Q^3 \\ &+ (148.4 + 235.5\,C + 101.5\,C^2 + 11.39\,C^3)\,\hat{a}_Q^4 + \ldots \end{aligned}$$



Strong coupling
Scale evolution
The coupling $\hat{a}$
Adler function
Tau decay
Higgs decay
Summary

# Tau decay




$$\delta^{(0)}_{\rm FO}(\hat{a}_{M_\tau}, C = -0.88) = 0.2047 \pm 0.0034 \pm 0.0133$$

Strong coupling
Scale evolution
The coupling $\hat{a}$
Adler function
Tau decay
Higgs decay
Summary

# Higgs decay

The decay rate of $H$ into $\bar{q}q$ is given by:

$$\Gamma(H \to q\bar{q}) = \frac{\sqrt{2}\,G_F}{M_H}\,\mathrm{Im}\Psi\left(M_H^2 + i0\right) \equiv \frac{N_c\,G_F M_H}{4\sqrt{2}\pi}\,\widehat{m}_q^2\,\widehat{R}(\alpha_s(M_H))$$

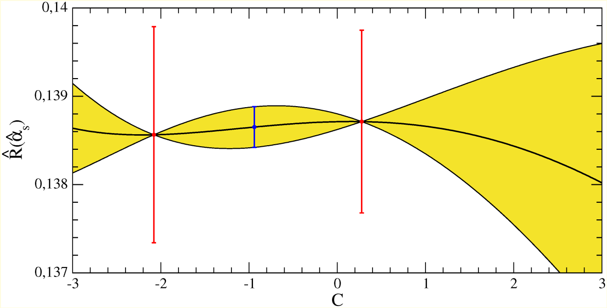
$\widehat{R}$ is only a function of the coupling: (MJ, Miravitllas 2016)

$$\begin{aligned}
\widehat{R}(\widehat{\alpha}_s) &= [\widehat{\alpha}_s(Q)]^{24/23}\{\,1 + (8.0176 + 2\,C)\,\hat{a}_Q \\
&+ (46.732 + 33.924\,C + 3.9167\,C^2)\,\hat{a}_Q^2 \\
&+ (141.19 + 315.38\,C + 103.88\,C^2 + 7.6157\,C^3)\,\hat{a}_Q^3 \\
&- (524.03 - 1491.9\,C - 1353.1\,C^2 - 277.97\,C^3 \\
&\qquad - 14.756\,C^4)\,\hat{a}_Q^4 + \ldots\,\}
\end{aligned}$$



Strong coupling
Scale evolution
The coupling $\hat{a}$
Adler function
Tau decay
Higgs decay
Summary

# Higgs decay




$$\widehat{R}(C = -0.94) = 0.1387 \pm 0.0002 \pm 0.0020 = 0.1387 \pm 0.0020$$

Strong coupling
Scale evolution
The coupling $\hat{a}$
Adler function
Tau decay
Higgs decay
Summary

# Summary

- The scheme dependence of the novel coupling $\hat{a}$ can be parameterised by a single parameter $C$.
- Its corresponding $\beta$-function turns out to be manifestly scheme invariant.
- The coupling $\hat{a}$ allows to easily study scheme dependencies, and to optimise the perturbative expansion.
- This appears to be particularly useful in observables that contain global multiplicative factors of the coupling.



Strong coupling
Scale evolution
The coupling $\hat{a}$
Adler function
Tau decay
Higgs decay
Summary

# Summary

- The scheme dependence of the novel coupling $\hat{a}$ can be parameterised by a single parameter $C$.
- Its corresponding $\beta$-function turns out to be manifestly scheme invariant.
- The coupling $\hat{a}$ allows to easily study scheme dependencies, and to optimise the perturbative expansion.
- This appears to be particularly useful in observables that contain global multiplicative factors of the coupling.

**Thank You!**




Strong coupling

Scale evolution

The coupling $\hat{a}$

Adler function

Tau decay

Higgs decay

Summary