Infrared behaviors of SU(2) gauge theory

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Talk based on work done with:

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Strong dynamics beyond QCD

Motivated by
- Understanding strong dynamics
- Phases of gauge theory
- Possible applications to particle pheno
Composite dynamics marches on...
Composite dynamics marches on...

“This excess is no more. It has ceased to be. It’s expired and gone to meet its maker. It’s a stiff. Berest of life, it rests in peace. It’s pushing up the daisies. It’s rung down the curtain and joined the choir invisible.

This is an ex-excess.”
Introduction: Conformal window

RG evolution @ 2-loops:

\[ \beta(g) \equiv \mu \frac{dg}{d\mu} = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)} \]
Introduction: Conformal window

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Small $N_f$: $\beta_0 > 0, \beta_1 > 0$

Chiral broken, running coupling, QCD-like
Introduction: Conformal window

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Large \( N_f \): \( \beta_0 < 0, \beta_1 < 0 \)

As. freedom lost
**Introduction: Conformal window**

**RG evolution @ 2-loops:**

\[
\beta(g) \equiv \mu \frac{dg}{d\mu} = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)}
\]

**Small \(N_f\):** \(\beta_0 > 0, \beta_1 > 0\)
- Chiral broken, running coupling, QCD-like

**Medium \(N_f\):** \(\beta_0 > 0, \beta_1 < 0\)
- IR fixed point, no chiral symmetry breaking

**Large \(N_f\):** \(\beta_0 < 0, \beta_1 < 0\)
- Asymmetry freedom lost
Introduction: Conformal window

RG evolution @ 2-loops:
\[ \beta(g) \equiv \mu \frac{dg}{d\mu} = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{16\pi^2} \]

Small \( N_f \): \( \beta_0 > 0, \beta_1 > 0 \)
Chiral broken, running coupling, QCD-like

Medium \( N_f \): \( \beta_0 > 0, \beta_1 < 0 \)
IR fixed point, no chiral symm. breaking

Large \( N_f \): \( \beta_0 < 0, \beta_1 < 0 \)
As. freedom lost

Conformal window: \( N_f \) s.t. IRFP exists
**Introduction:** Conformal window in SU(N)

[Miransky & Yamawaki, Appelquist et al.]
[Sannino & Tuominen]

Conformal window: $N_f$ s.t. IRFP exists

**F:** fundamental

**2AS:** 2-ind. antisymmetric

**2S:** 2-ind. symmetric

Upper lines: loss of as. freedom
Lower lines: chiral symm. breaks
(ladder appro)
Introduction: QCD-like vs. conformal

Precocious asymptotic freedom:
Single scale theory,
Clean cut between strong/ weak coupling
Introduction: QCD-like vs. conformal

Precocious asymptotic freedom:
Single scale theory,
Clean cut between strong/weak coupling

(almost) conformal theories:
coupling equally strong
almost everywhere

* must live with strong lattice coupling,
* need appropriate tools
Example: \(SU(2) + 2\) adjoint fermions

[Hiertanen et al. (2009)]
Non-improved
Wilson fermions: IRFP exists

Confirmed by several groups.
[Catterall, Sannino]
[Del Debbio et al.]
[De Grand et al.]

\[g^2 = 2.2\]
\[\beta_{L} = 2.05\]
\[\beta_{L} = 2.2\]
\[\beta_{L} = 2.5\]
\[\beta_{L} = 3\]
\[\beta_{L} = 3.5\]
\[\beta_{L} = 4.5\]
\[\beta_{L} = 8\]
Example: SU(2) + 2 adjoint fermions

[Hietanen et al. (2009)]
Non-improved Wilson fermions: IRFP exists
Confirmed by several groups.
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[Hex et al. (2016)]
HEX smeared gauge & Wilson-clover action
Also measured anomalous dimension of the quark mass operator: $\gamma^* \simeq 0.2$
SU(2) + $N_f$ fundamental flavors

Asymptotic freedom lost: $N_f = 11$

Expect:

- $N_f = 8$ inside CW.
- $N_f = 2$ QCD-like, $N_f = 4$ a bit less so.
- $N_f = 6$ borderline (?), possibly inside CW.

Earlier study inconclusive.
[Karavirta et al. (2011)]
[similar results Appelquist et al. (2014)]

*Schrödinger functional step scaling
- noisy, prevents large lattices

*Improved Wilson-clover action

Now use instead:
- Gradient flow step scaling
- HEX-smeared Wilson-clover with mixed gauge action.
Results:

\[ N_f = 8 \]
Coupling: definitions

- Fixed trivial boundaries, no background field
- Tune fermion mass to zero using axial Ward identity

- Run Wilson flow to scale

\[ \mu^{-1} = \sqrt{8t} = cL, \quad c = 0.4 \]

- Define the coupling

\[ g_{GF}^2 = \frac{t^2}{N} \langle E(t + \tau_0 a^2) \rangle \quad (\tau_0 \text{ tunable parameter}) \]

- Step scaling

\[ \Sigma(u, s, L/a) = g_{GF}^2(g_0^2, sL/a)|_{g_{GF}^2(g_0^2, L/a) = u} \]

\[ \sigma(u, s) = \lim_{a/L \to 0} \Sigma(u, s, L/a) \]

- Use rational interpolation for \( g_{GF}(g_0^2, L/a) \)
Coupling: $N_f = 8$

Raw data  

Step scaling
**Coupling:** \( N_f = 8, \) continuum extrapolation

**IRFP at** \( g_{GF}^2 \approx 8 \)
**Anomalous dimension:** \( N_f = 8 \)

Using same configs as for the coupling measurement:

At IRFP

\[ g_{GF}^2 \approx 8 \implies \gamma^* \approx 0.15 \]

Step scaling method (Ward identities). [Luscher, Weisz]

Similar result also with Dirac mode number density. [Patella]
Results:

\[ N_f = 2, 4, 6 \]
\[ N_f = 2 \quad 24^3 \times 48 \quad \beta_L = 1.0 \]

\[ \sqrt{t_0} \sim \frac{1}{M_\rho} \]
\[ N_f = 2 \quad 24^3 \times 48 \quad \beta_L = 1.0 \]
\( N_f = 4 \quad 24^3 \times 48 \)

\( 32^3 \times 60 \quad \beta_L = 0.8 \)
$N_f = 6$

Strong finite volume effects.
Step scaling for $N_f = 6$ (Preliminary)

IRFP at strong coupling
Anomalous dimension: from spectrum? \( N_f = 6 \)

\[ M \propto m_Q^{1/(1+\gamma)} \]
Anomalous dimension: from spectrum? $N_f = 6$

$M \propto m_Q^{1/(1+\gamma)}$

The flow runs out of lattice at small $m_Q$.
Conclusions

Consistent results for SU(2), fundamental fermions:
• chiral symmetry broken for $N_f \leq 4$
• IRFP for $N_f \geq 6$

Gradient flow step scaling:
• works fine at strong coupling

Universal anomalous dimension:
• stretching the limits of the lattice
• dependence on the simulation parameters?