Infrared behaviors of SU(2) gauge theory

HELSINKI INSTITUTE OF PHYSICS

Kimmo Tuominen
Helsinki University

XIIth Confinement
Thessaloniki 28.8.-3.9.2016

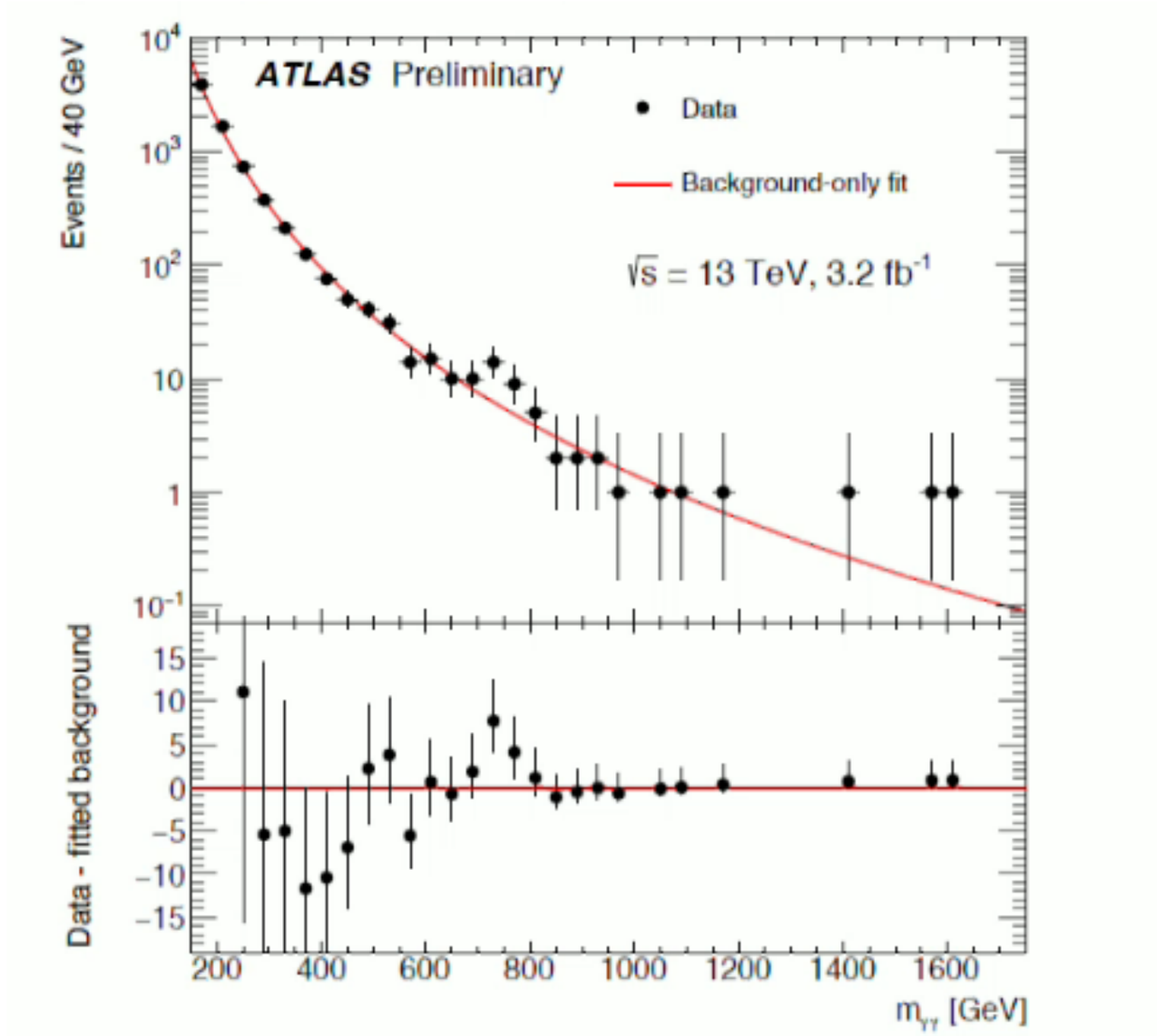
Talk based on work done with:

Alessanro Amato, Viljami Leino, Jarno Rantaharju, Teemu Rantalaiho, Kari Rummukainen, Joni Suorsa, Sara Tähtinen.

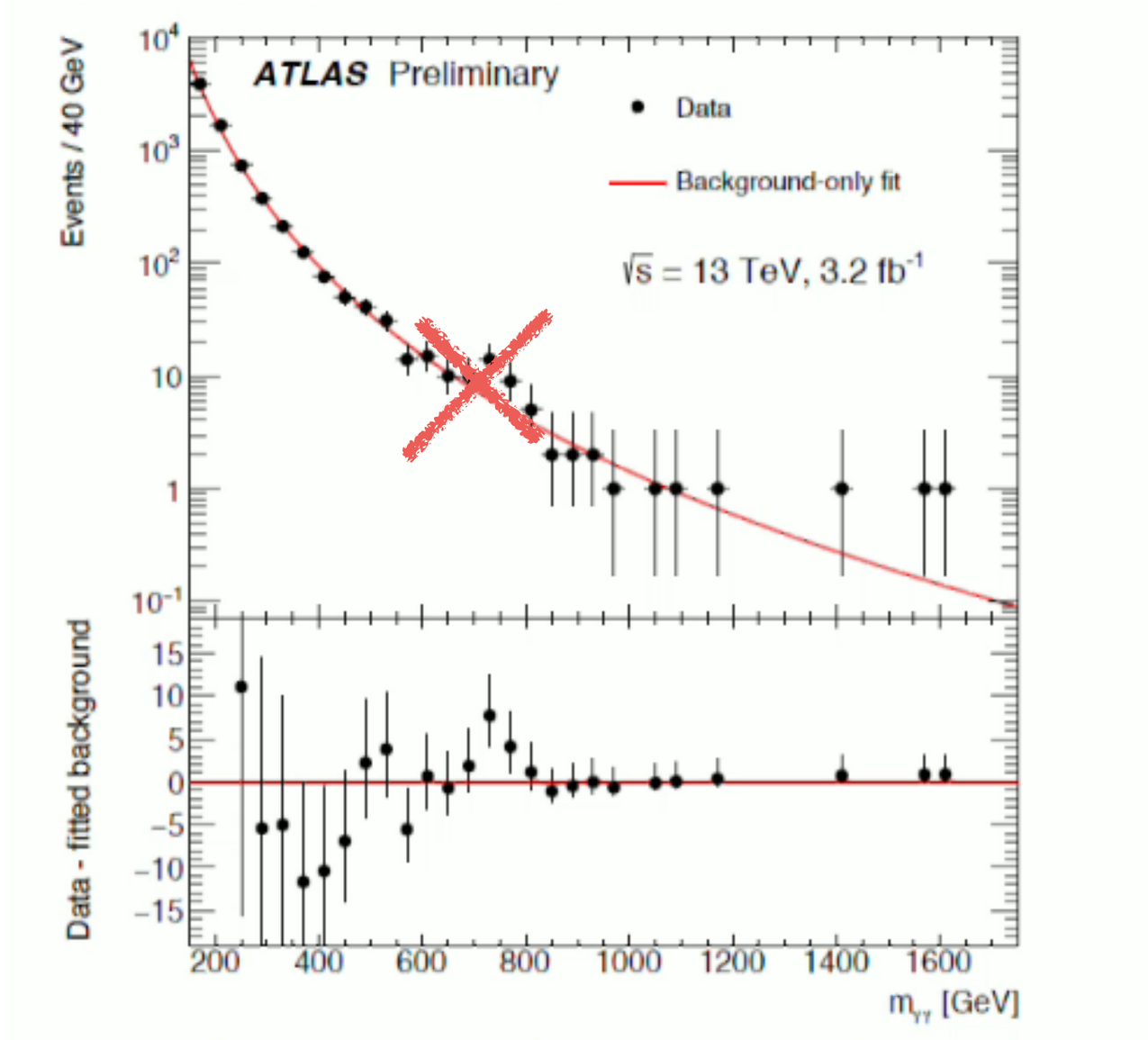
Strong dynamics beyond QCD

Motivated by
- Understanding strong dynamics
- Phases of gauge theory
- Possible applications to particle pheno

Composite dynamics marches on…



Composite dynamics marches on…



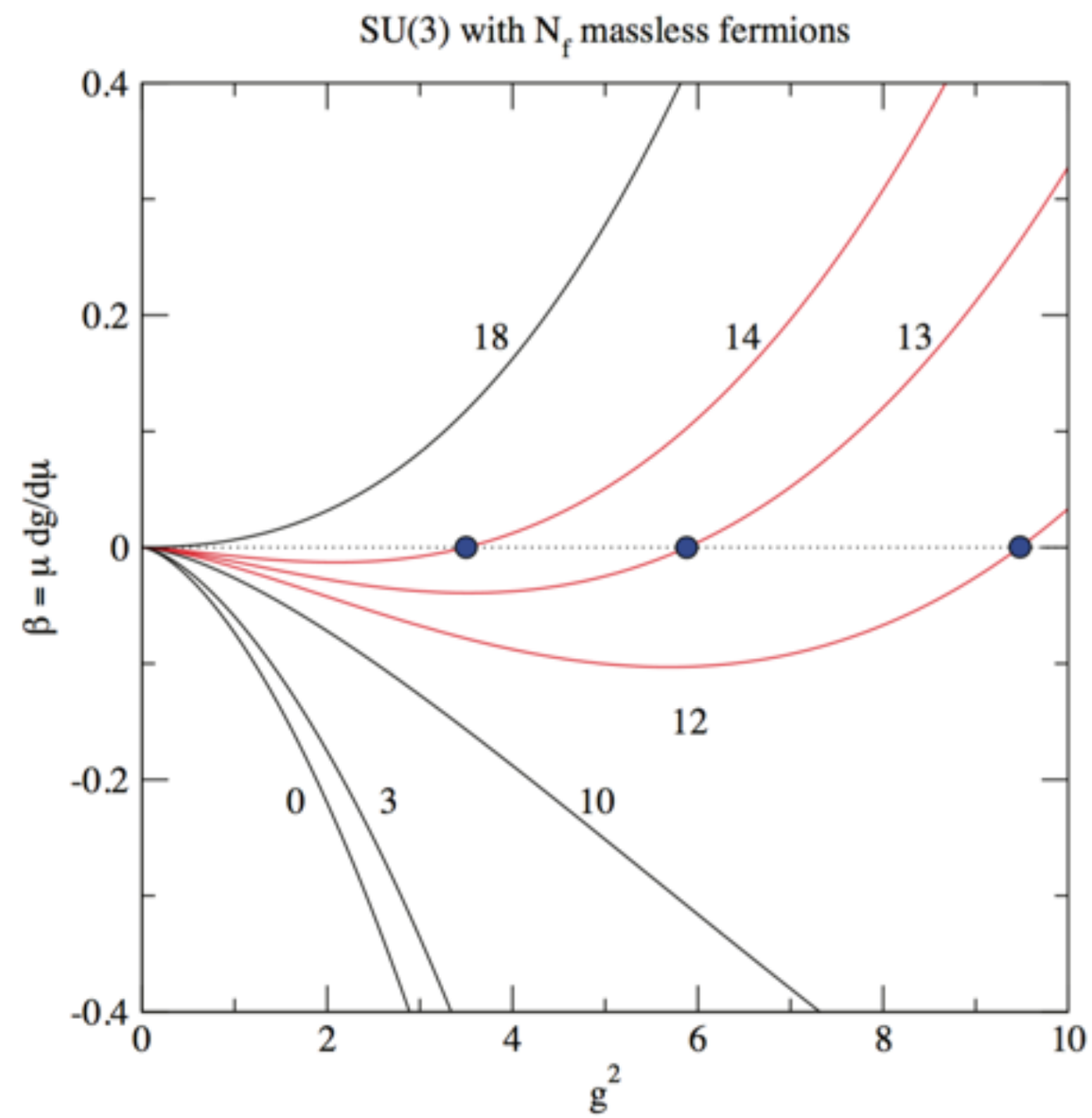
"This excess is no more.
It has ceased to be.
It's expired and gone to meet its maker.
It's a stiff.
Bereft of life, it rests in peace.
It's pushing up the daisies.
It's rung down the curtain
and joined the choir invisible.

This is an ex-excess."

Introduction: Conformal window

RG evolution @ 2-loops:

$$\beta(g) \equiv \mu \frac{dg}{d\mu} = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)}$$



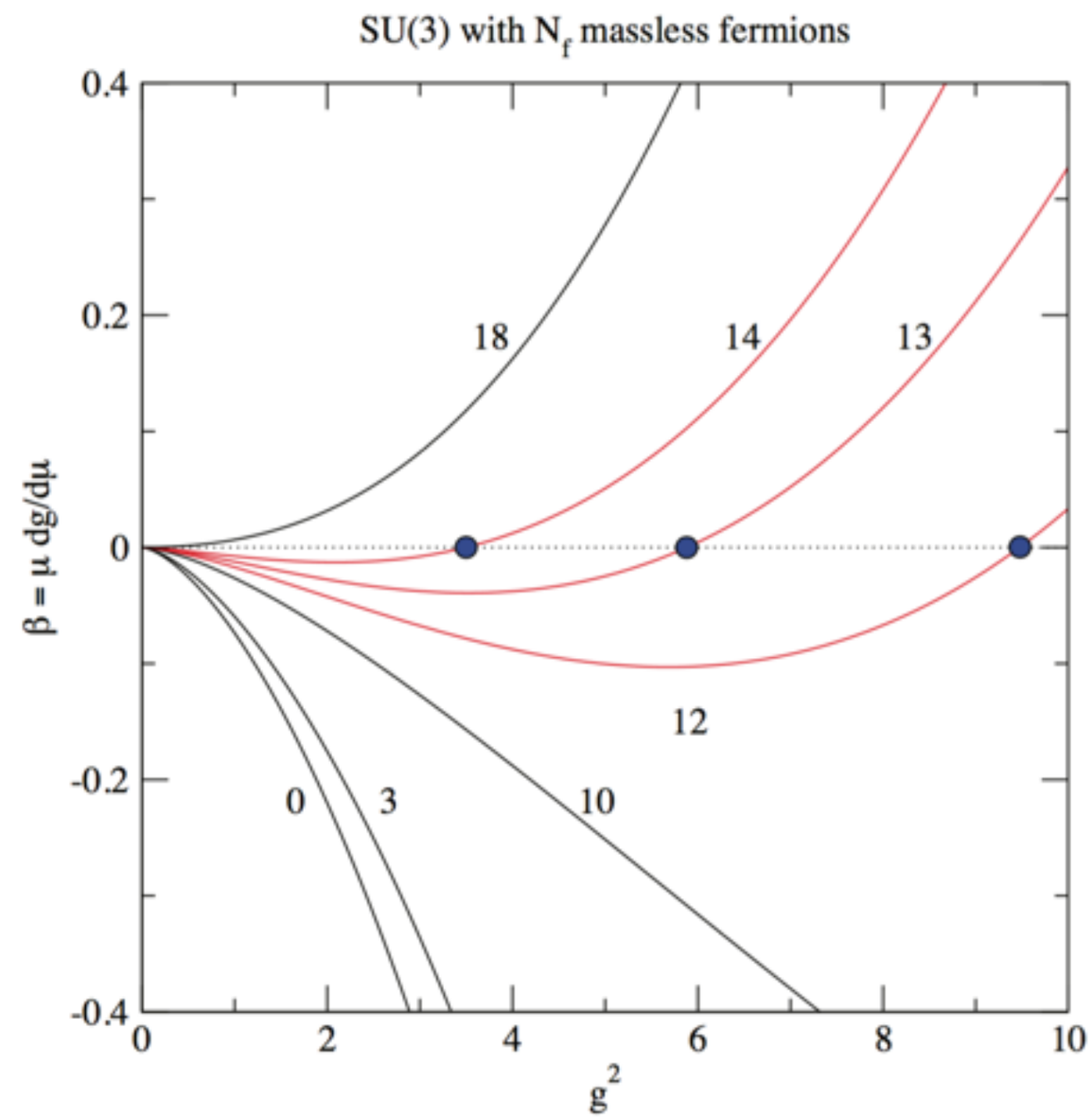
Introduction: Conformal window

RG evolution @ 2-loops:

$$\beta(g) \equiv \mu \frac{dg}{d\mu} = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)}$$

Small N_f : $\beta_0 > 0,\ \beta_1 > 0$

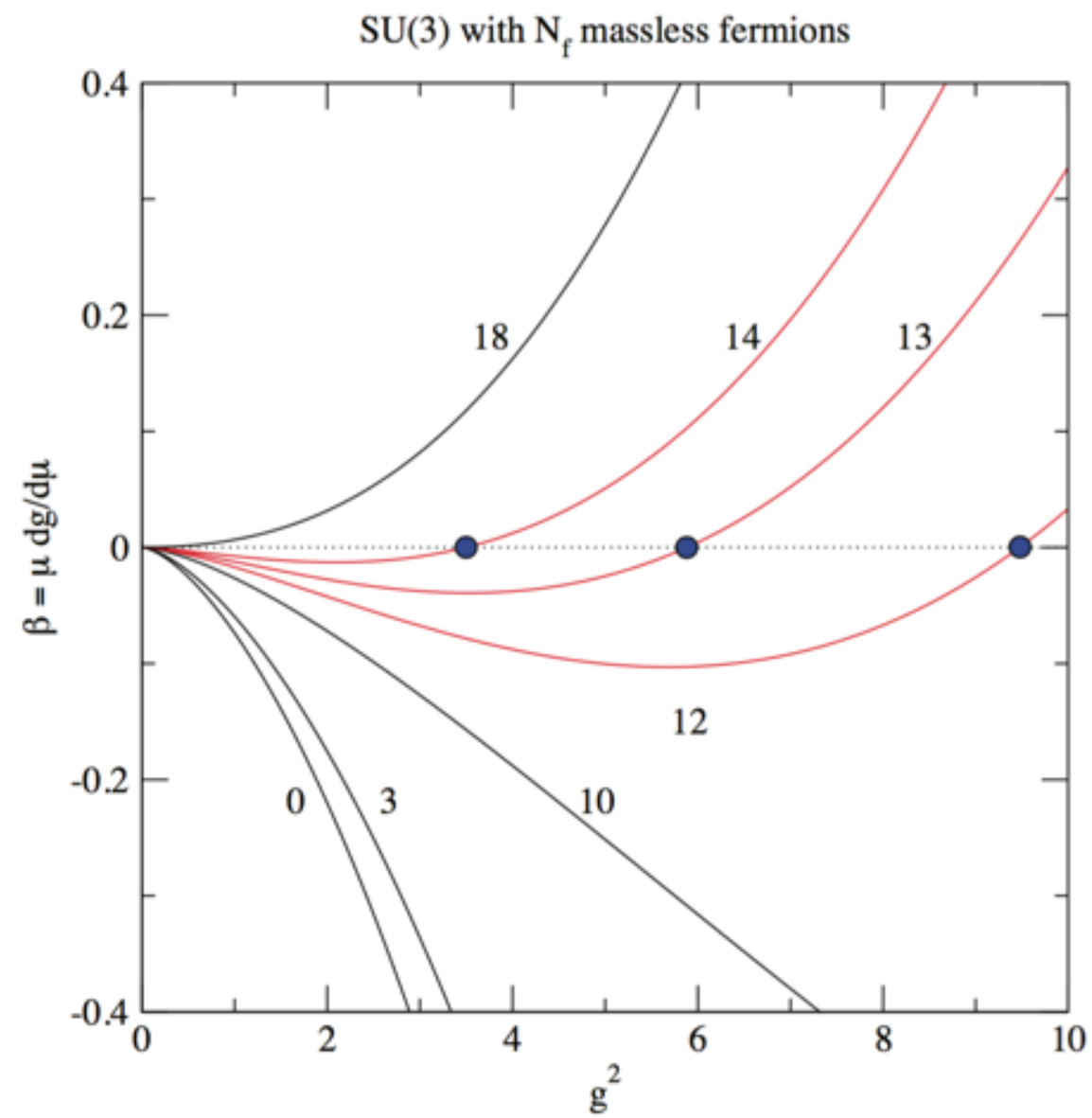
Chiral broken,
running coupling,
QCD-like



Introduction: Conformal window

RG evolution @ 2-loops:

$$\beta(g) \equiv \mu \frac{dg}{d\mu} = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)}$$



Small N_f : $\beta_0 > 0,\ \beta_1 > 0$

Chiral broken,
running coupling,
QCD-like

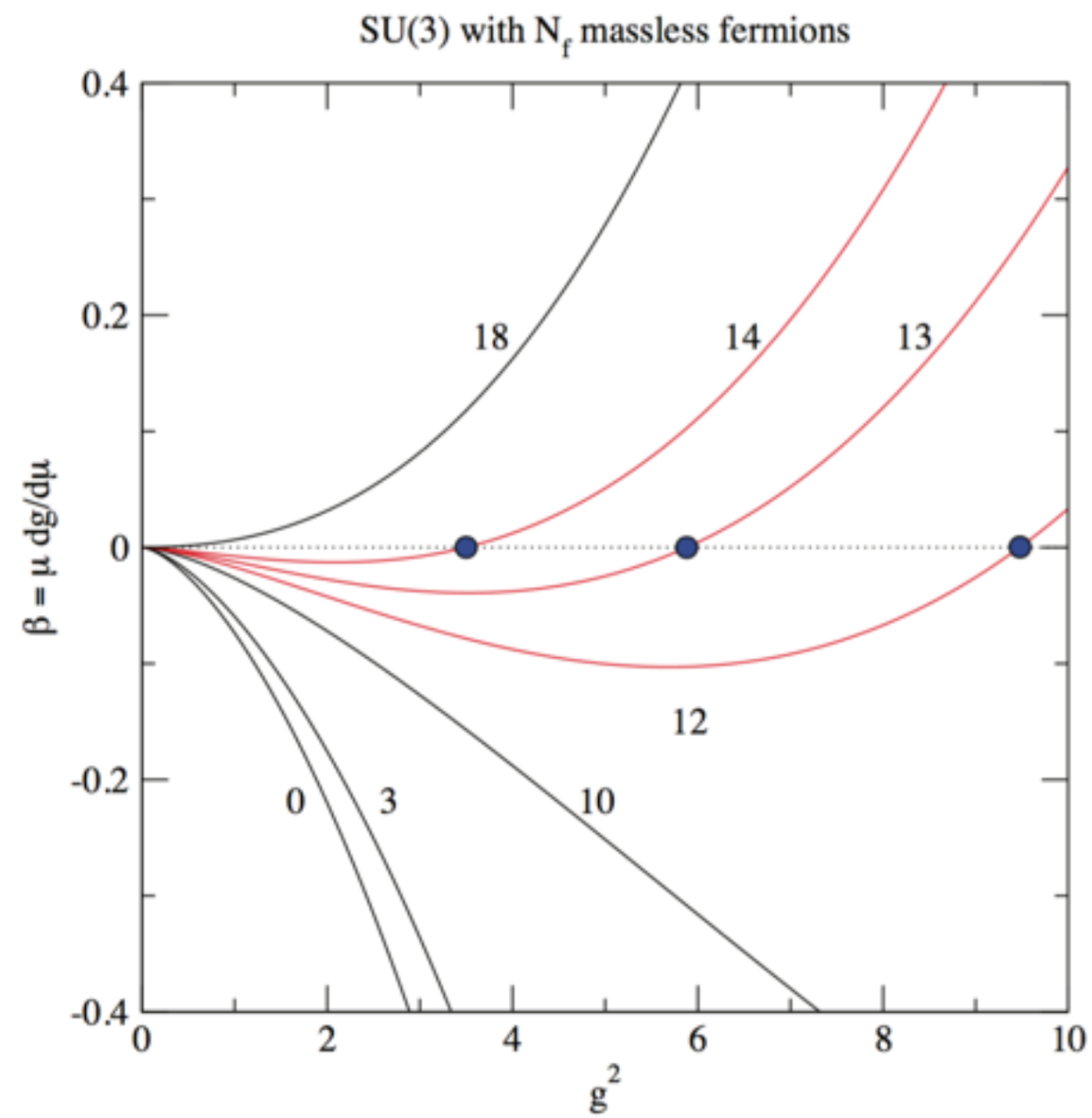
Large N_f : $\beta_0 < 0,\ \beta_1 < 0$

As. freedom lost

Introduction: Conformal window

RG evolution @ 2-loops:

$$\beta(g) \equiv \mu \frac{dg}{d\mu} = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)}$$



Small N_f : $\beta_0 > 0,\ \beta_1 > 0$

Chiral broken,
running coupling,
QCD-like

Medium N_f : $\beta_0 > 0,\ \beta_1 < 0$

IR fixed point,
no chiral symm. breaking

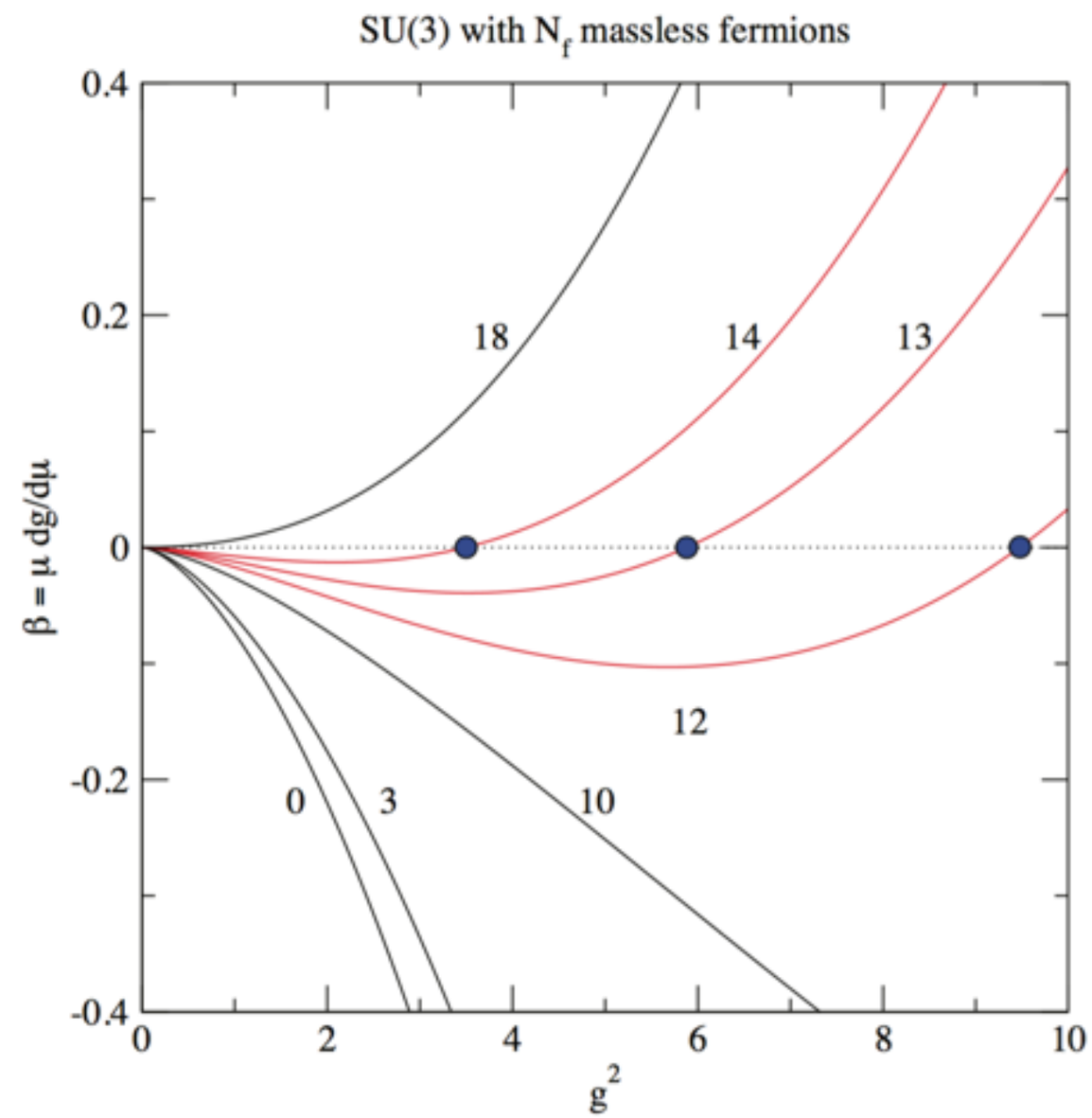
Large N_f : $\beta_0 < 0,\ \beta_1 < 0$

As. freedom lost

Introduction: Conformal window

RG evolution @ 2-loops:

$$\beta(g) \equiv \mu \frac{dg}{d\mu} = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)}$$



Small N_f : $\beta_0 > 0,\ \beta_1 > 0$

Chiral broken,
running coupling,
QCD-like

Medium N_f : $\beta_0 > 0,\ \beta_1 < 0$

IR fixed point,
no chiral symm. breaking

Large N_f : $\beta_0 < 0,\ \beta_1 < 0$

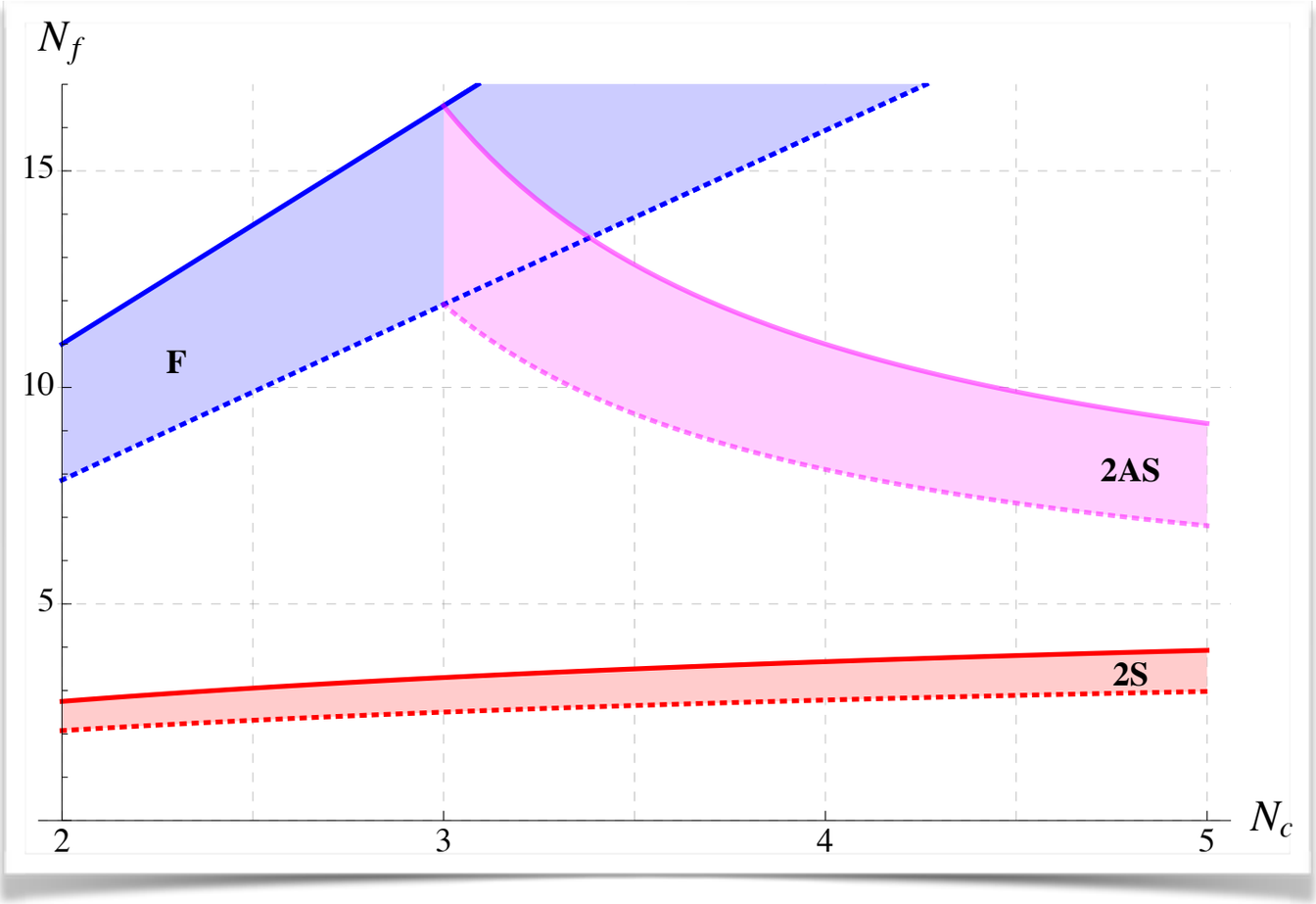
As. freedom lost

Conformal window:
N_f s.t. IRFP exists

Introduction: Conformal window in SU(N)

[Miransky & Yamawaki, Appelquist et al.]

[Sannino & Tuominen]



Conformal window:
N_f s.t. IRFP exists

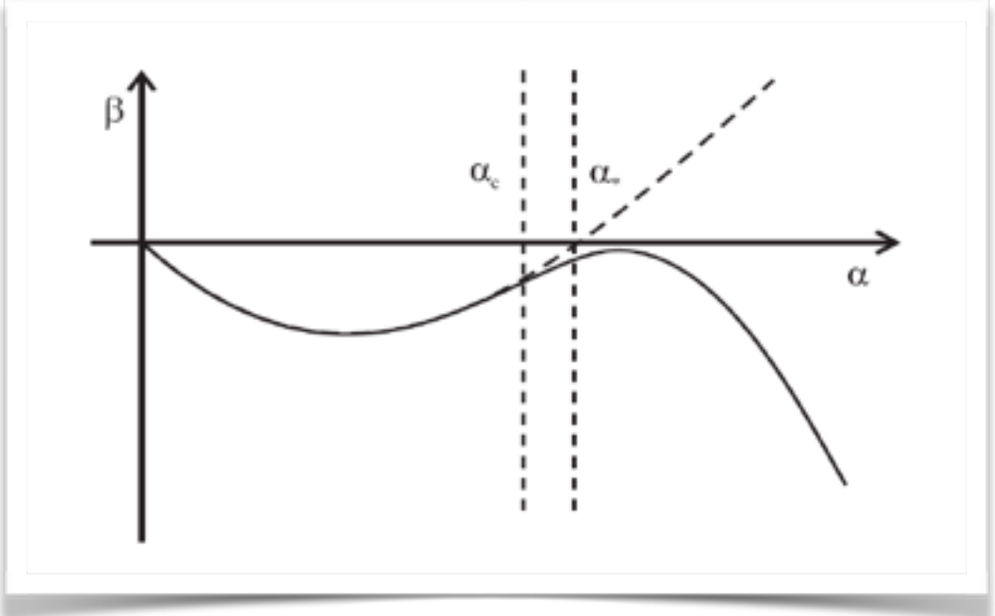
F: fundamental

2AS: 2-ind. antisymmetric

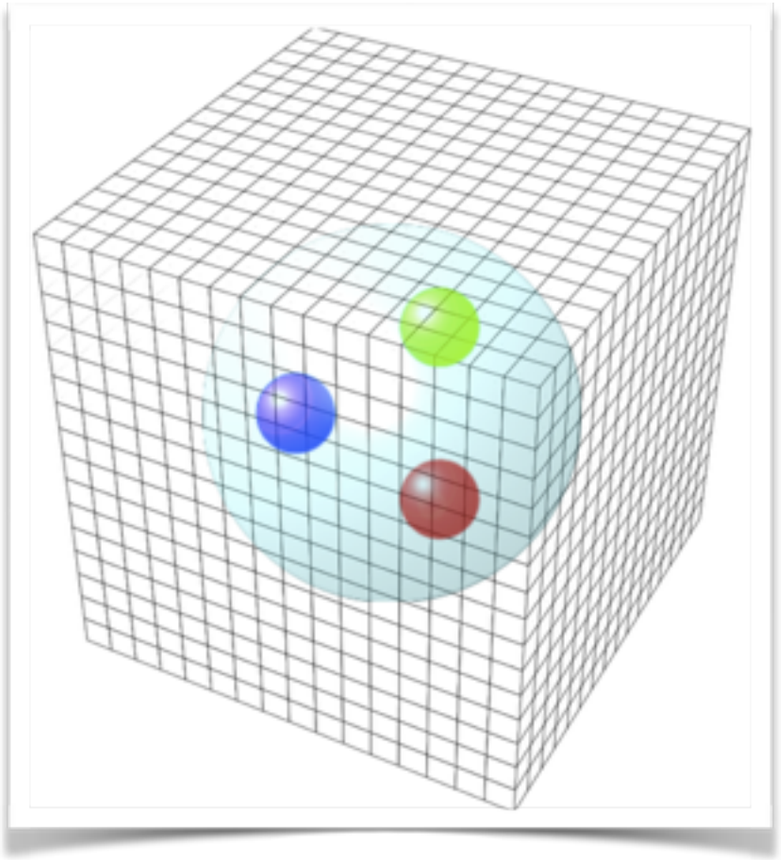
2S: 2-ind. symmetric

Upper lines: loss of as. freedom

Lower lines: chiral symm. breaks
(ladder appro)



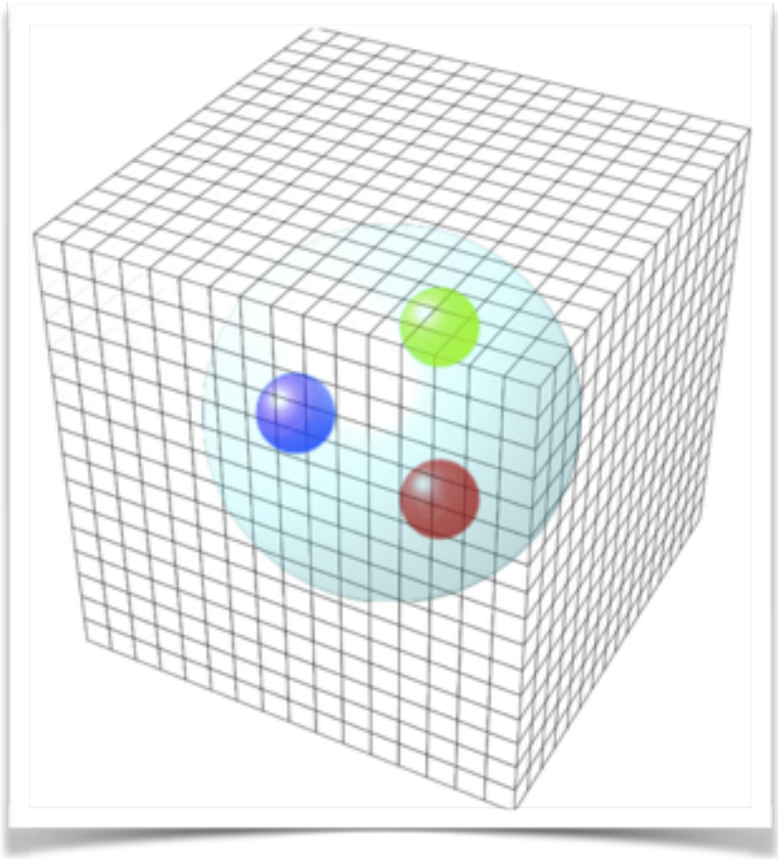
Introduction: QCD-like vs. conformal



Precocious asymptotic freedom:

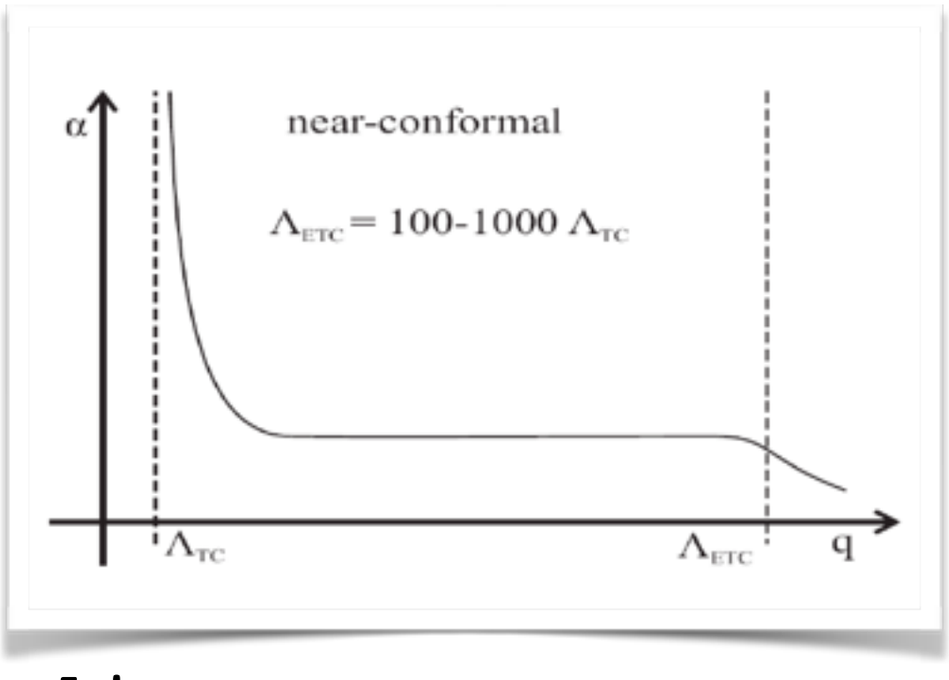
Single scale theory,
Clean cut between strong/ weak coupling

Introduction: QCD-like vs. conformal



Precocious asymptotic freedom:

Single scale theory,
Clean cut between strong/ weak coupling



(almost) conformal theories:

coupling equally strong
almost everywhere

* must live with strong lattice coupling,
* need appropriate tools

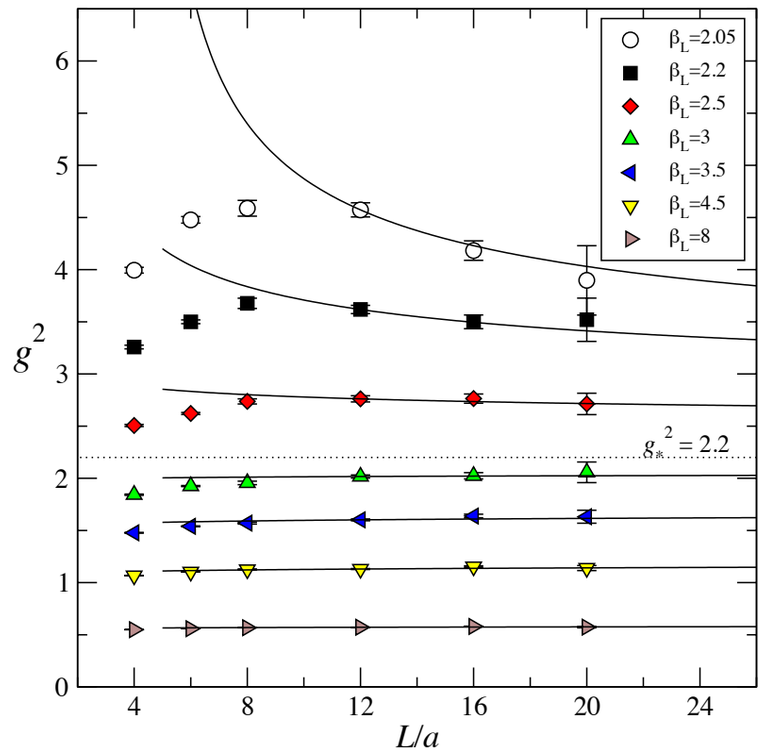
Example: SU(2) + 2 adjoint fermions

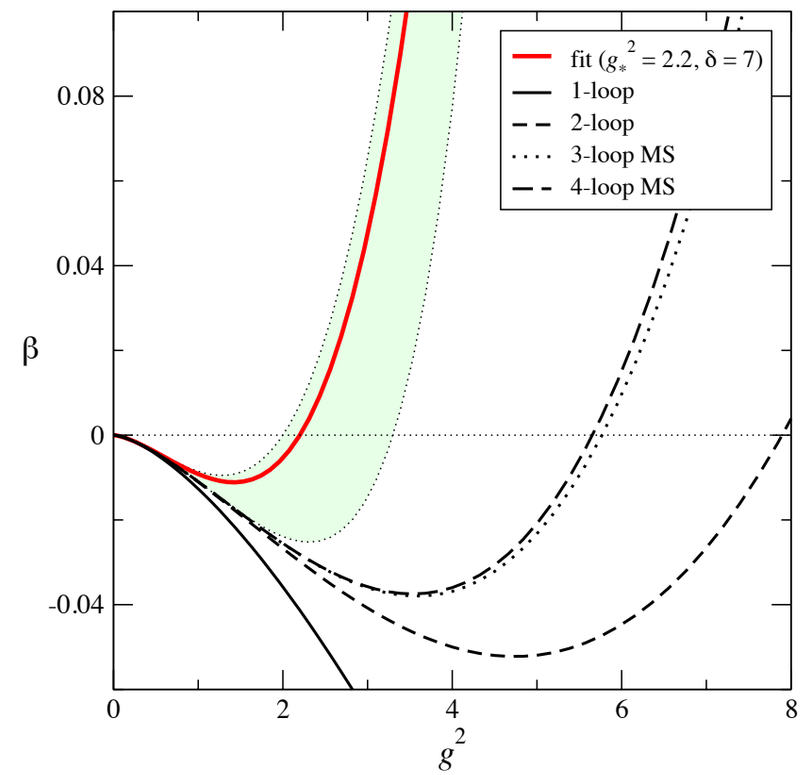
[Hietanen et al. (2009)]

Non-improved Wilson fermions: IRFP exists

Confirmed by several groups.

[Catterall, Sannino]
[Del Debbio et al.]
[De Grand et al.]





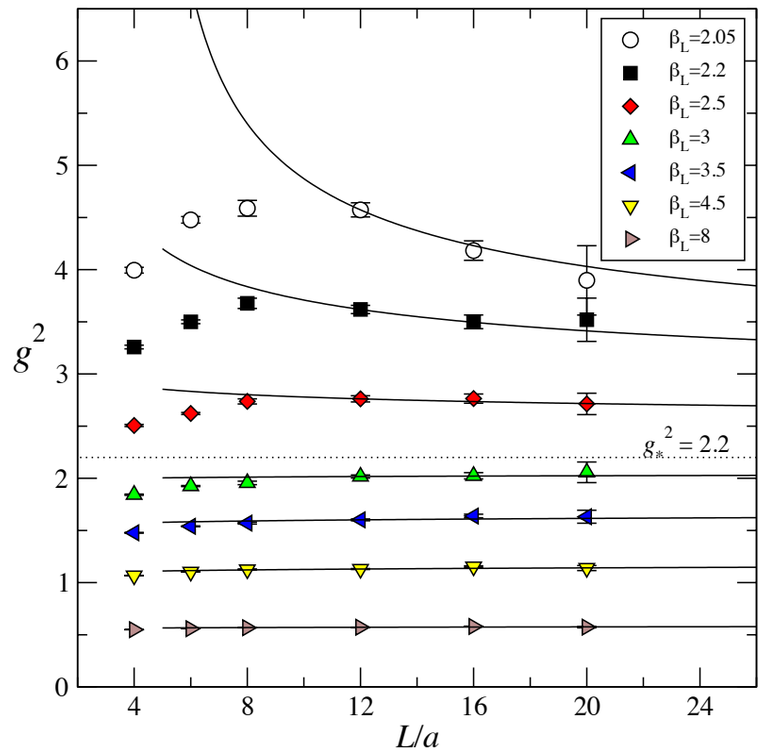
Example: SU(2) + 2 adjoint fermions

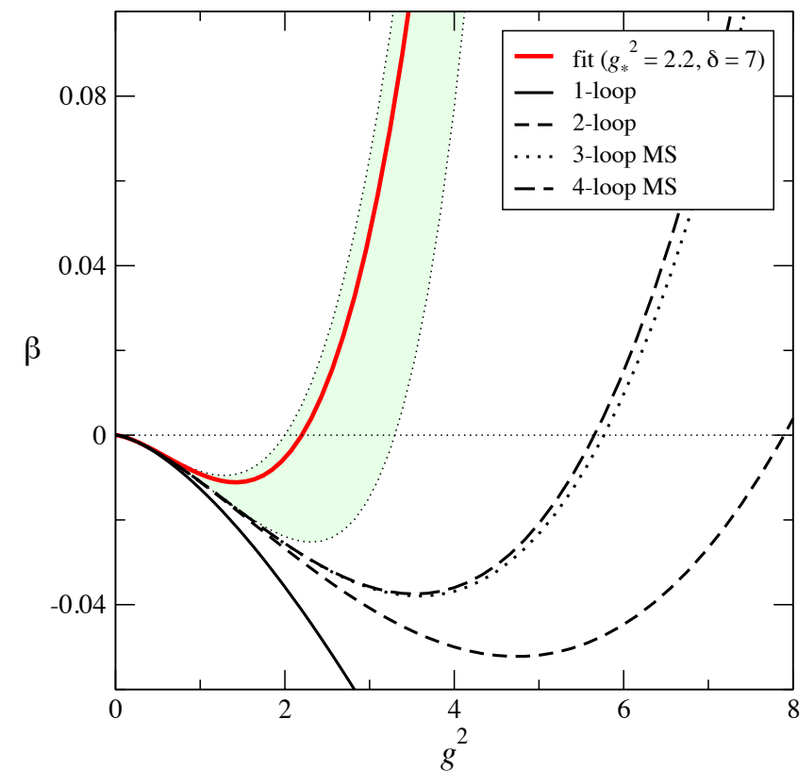
[Hietanen et al. (2009)]

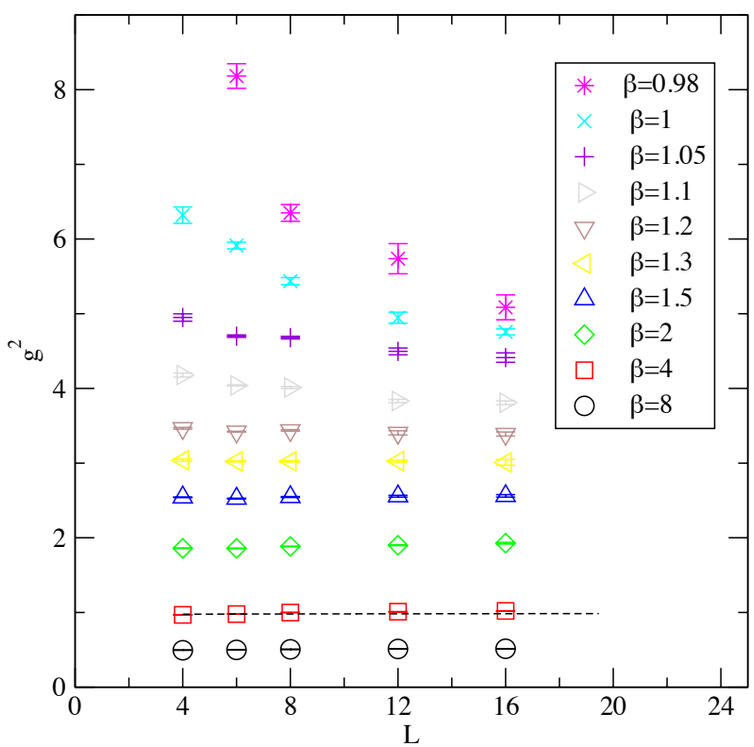
Non-improved Wilson fermions: IRFP exists

Confirmed by several groups.

[Catterall, Sannino]
[Del Debbio et al.]
[De Grand et al.]







[Rantaharju et al. (2016)]

HEX smeared gauge & Wilson-clover action

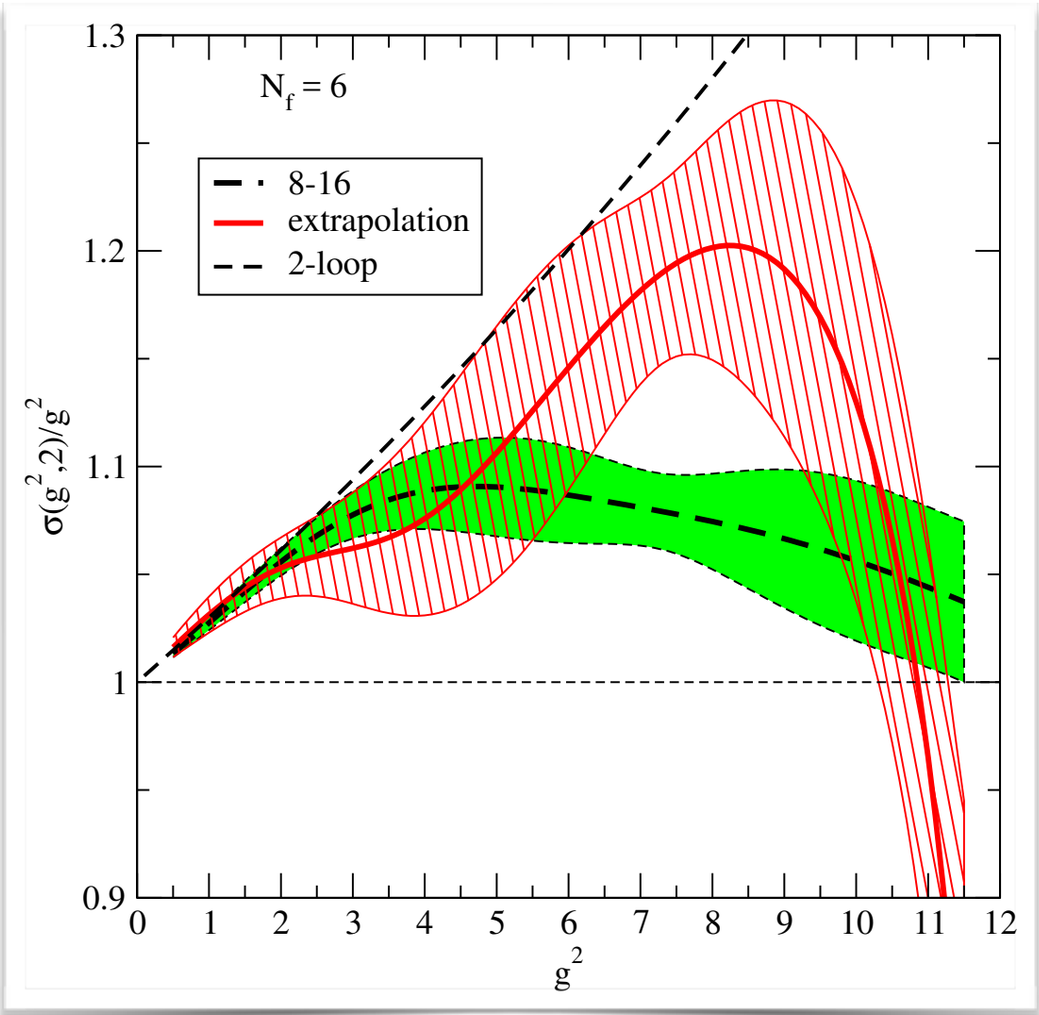
Also measured anomalous dimension of the quark mass operator: $\gamma^* \simeq 0.2$

SU(2) + N_f fundamental flavors

Asymptotic freedom lost: $N_f = 11$

Expect:

- $N_f = 8$ inside CW.
- $N_f = 2$ QCD-like, $N_f = 4$ a bit less so.
- $N_f = 6$ borderline (?), possibly inside CW.



Earlier study inconclusive.
[Karavirta et al. (2011)]
[similar results Appelquist et al. (2014)]

*Schrödinger functional step scaling
- noisy, prevents large lattices

*Improved Wilson-clover action

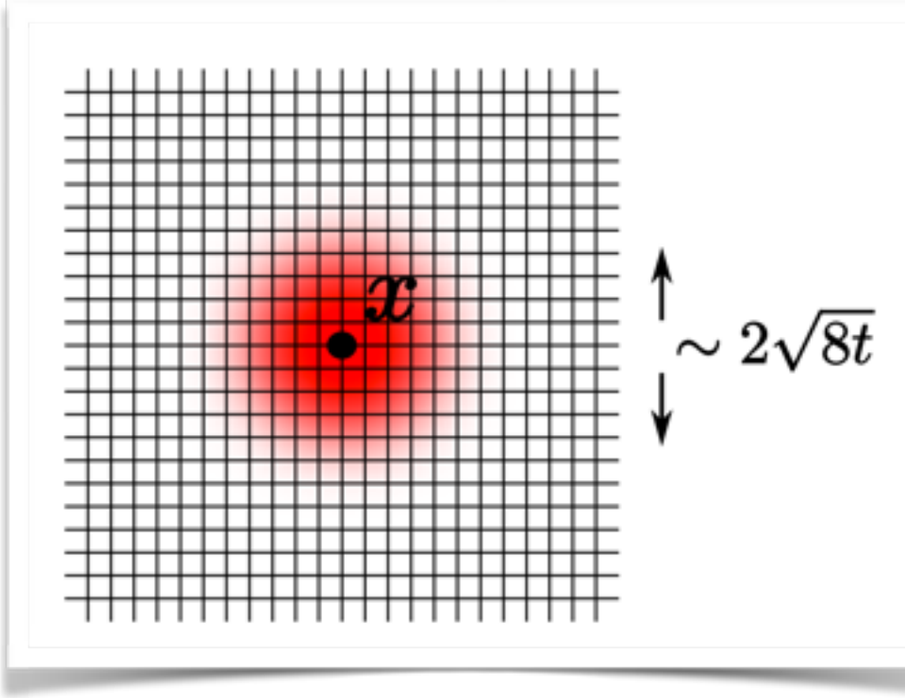
Now use instead:
- Gradient flow step scaling
- HEX-smeared Wilson-clover with mixed gauge action.

Results:

$$N_f = 8$$

Coupling: definitions

- Fixed trivial boundaries, no background field
- Tune fermion mass to zero using axial Ward identity



- Run Wilson flow to scale

$$\mu^{-1} = \sqrt{8t} = cL, \qquad c = 0.4$$

- Define the coupling

$$g^2_{\mathrm{GF}} = \frac{t^2}{\mathcal{N}} \langle E(t + \tau_0 a^2) \rangle \qquad (\tau_0 \text{ tunable parameter})$$

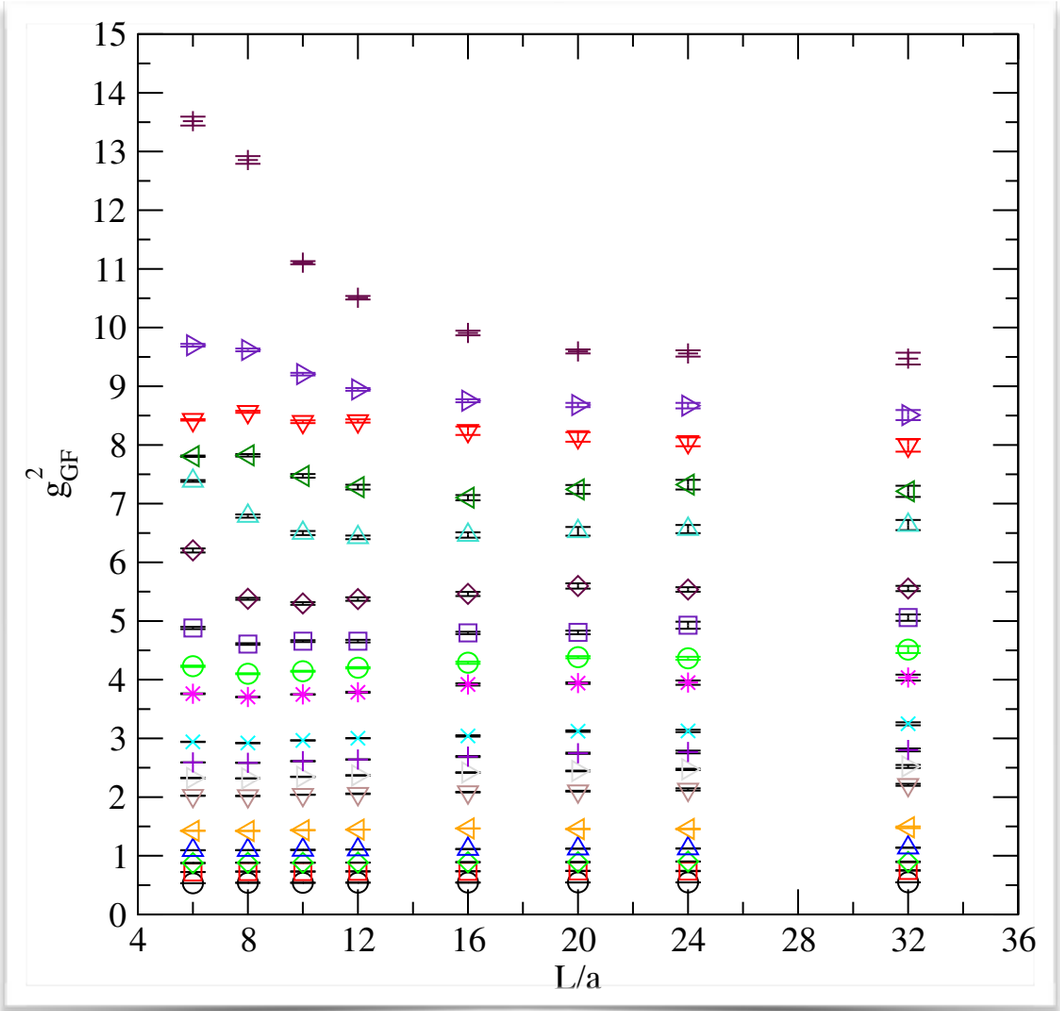
- Step scaling

$$\Sigma(u, s, L/a) = g^2_{\mathrm{GF}}(g_0^2, sL/a)|_{g^2_{\mathrm{GF}}(g_0^2, L/a) = u}$$

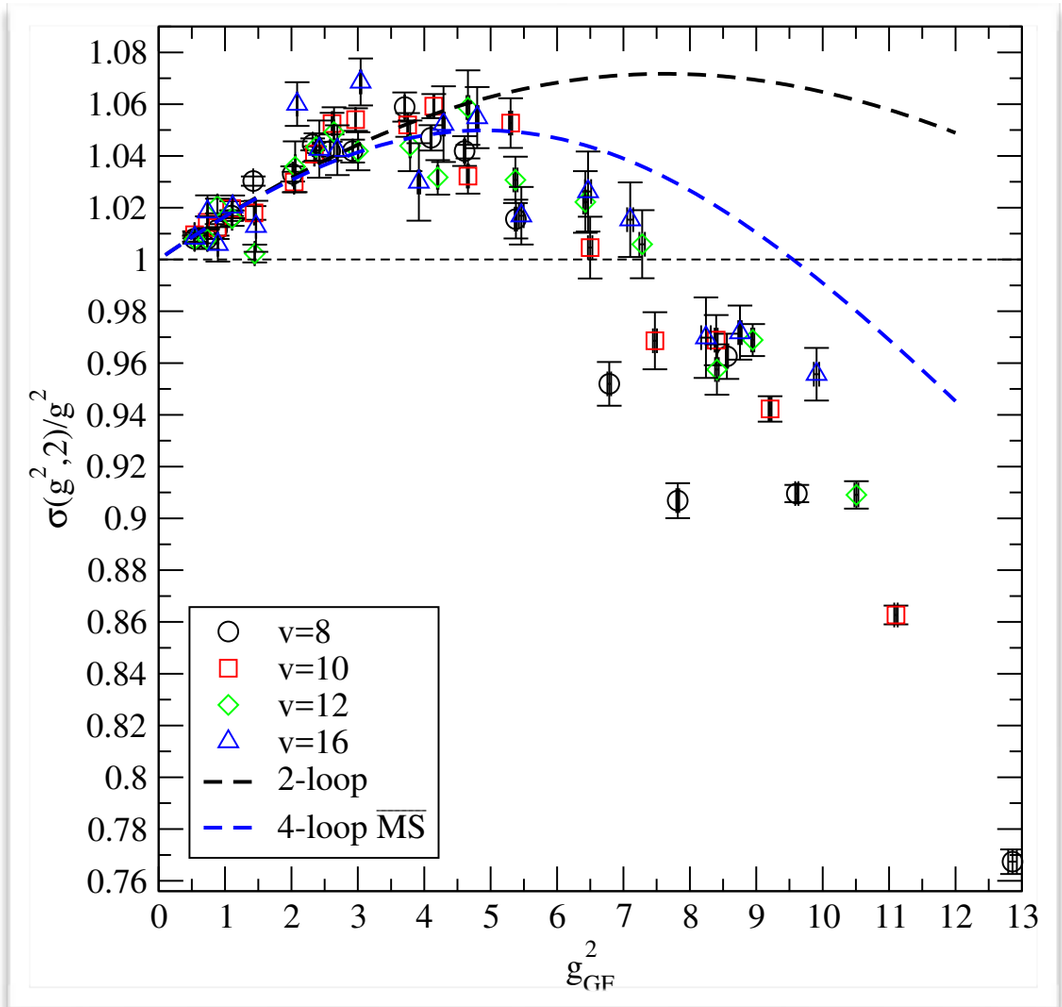
$$\sigma(u, s) = \lim_{a/L \to 0} \Sigma(u, s, L/a)$$

- Use rational interpolation for $g_{\mathrm{GF}}(g_0^2, L/a)$

Coupling: $N_f = 8$

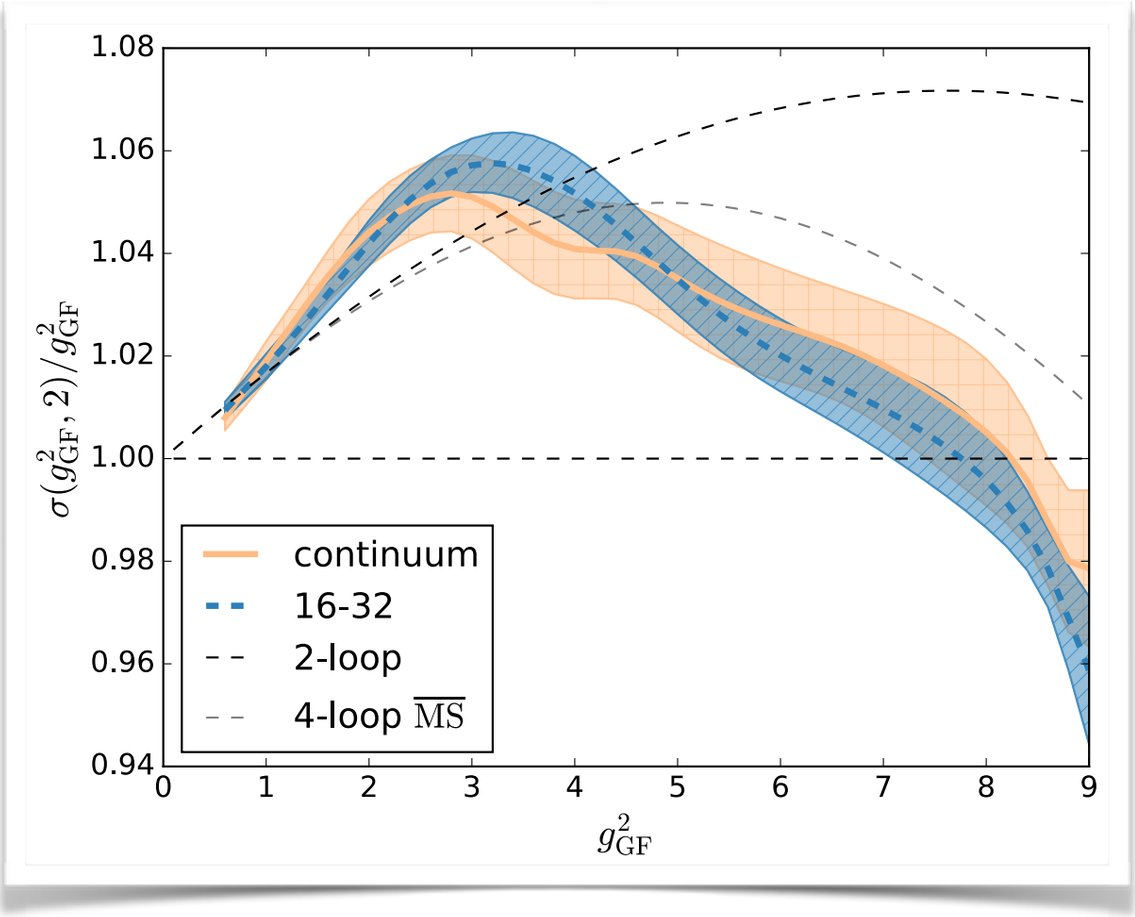


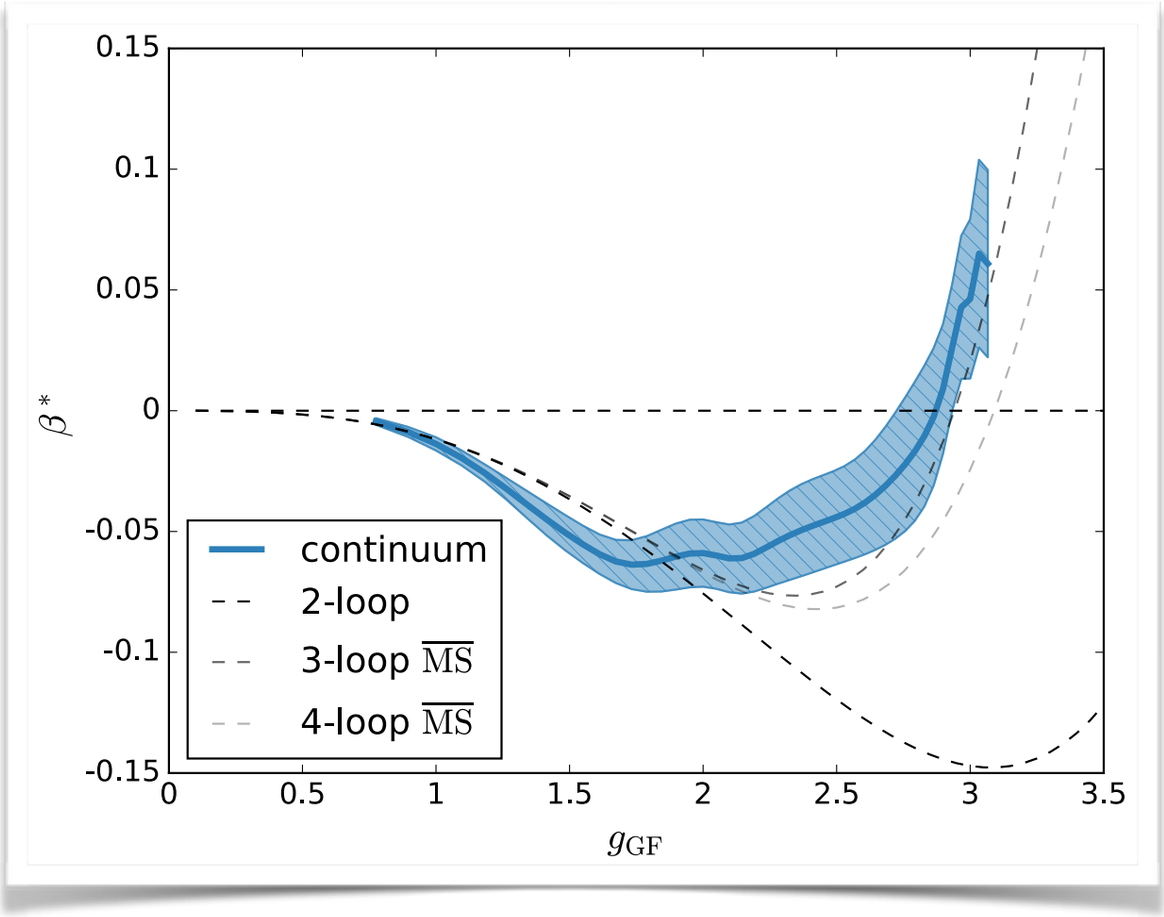
Raw data



Step scaling

Coupling: $N_f = 8$, continuum extrapolation

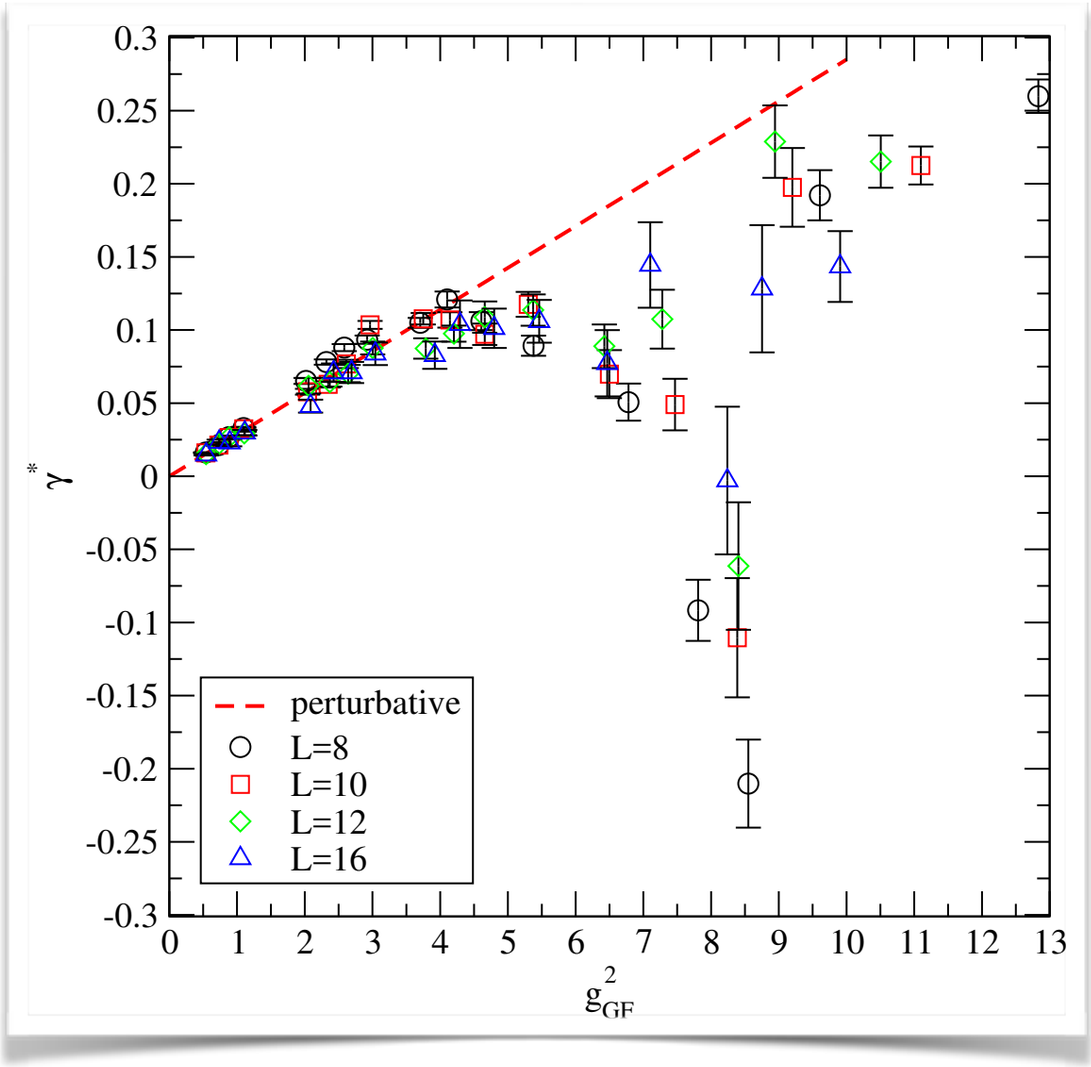




IRFP at $g^2_{\rm GF} \approx 8$

Anomalous dimension: $N_f = 8$

Using same configs as for the coupling measurement:



At IRFP

$$g^2_{\mathrm{GF}} \approx 8 \Rightarrow \gamma^* \approx 0.15$$

Step scaling method (Ward identities). [Luscher, Weisz]

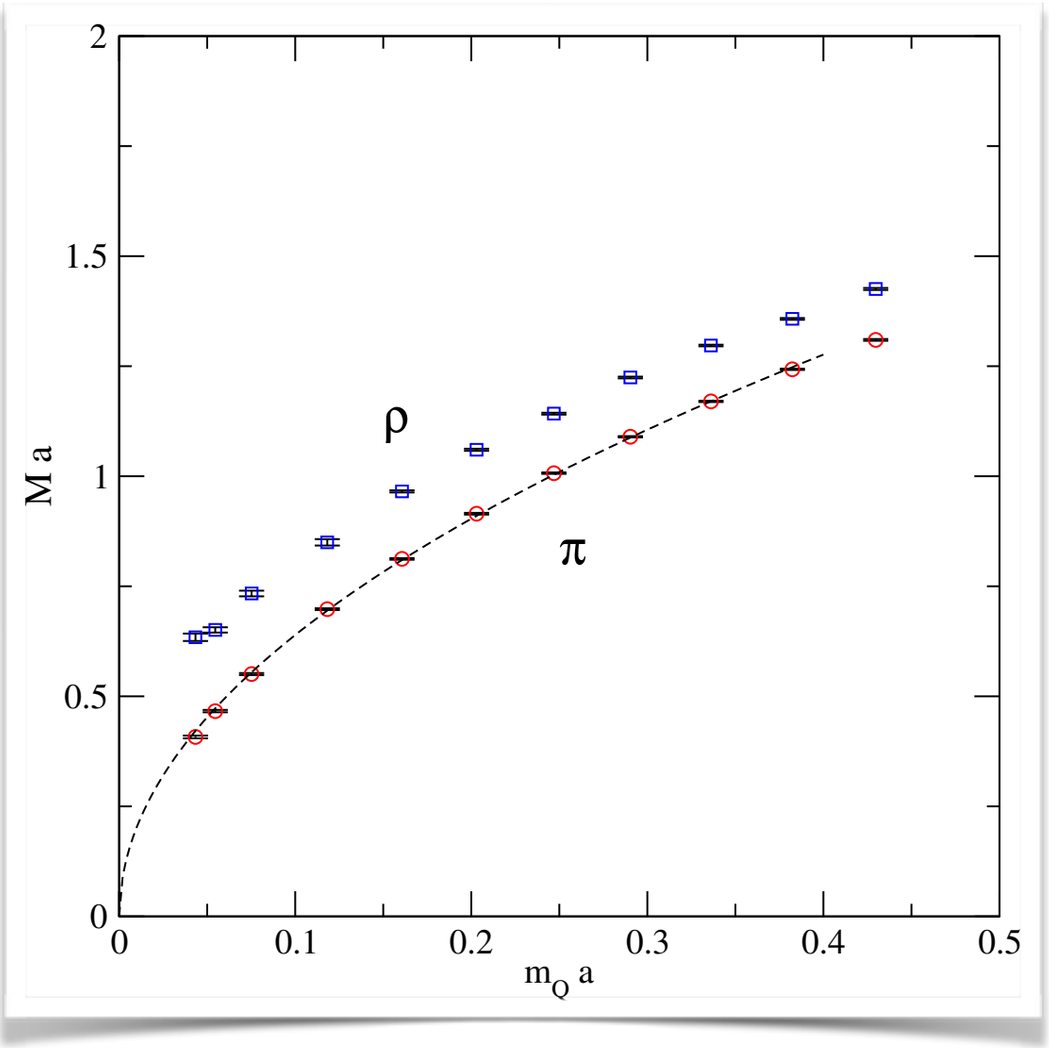
Similar result also with Dirac mode number density.

[Patella]

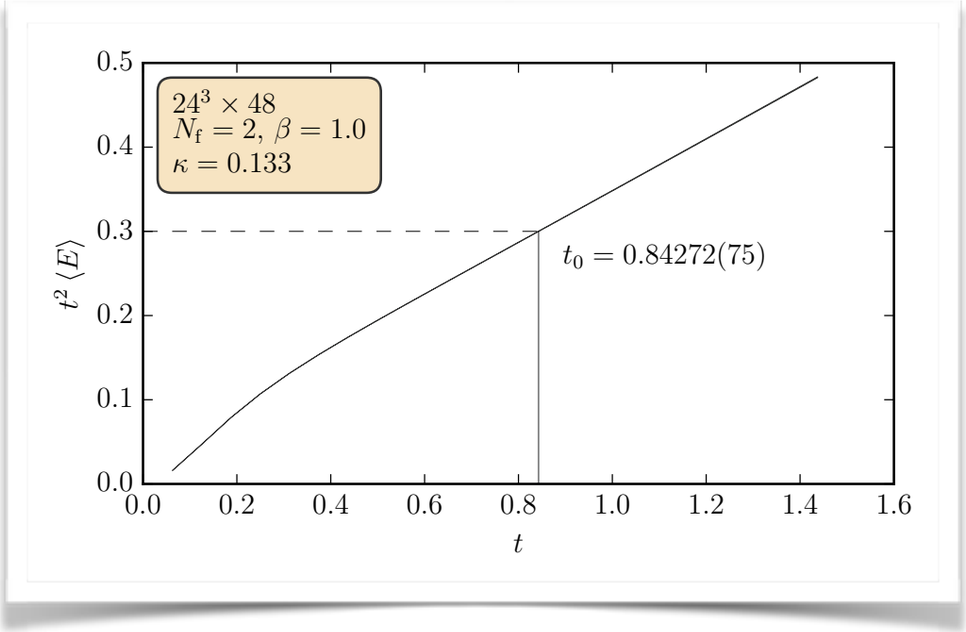
Results:

$$N_f = 2, 4, 6$$

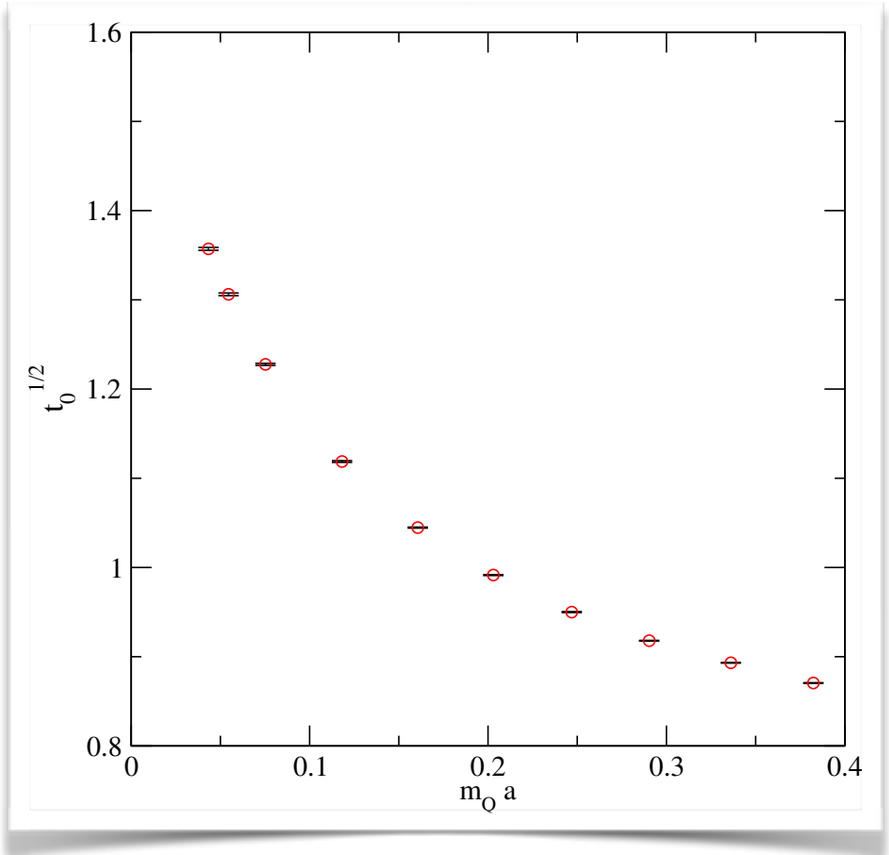
$N_f = 2$ $\quad 24^3 \times 48$ $\quad \beta_L = 1.0$



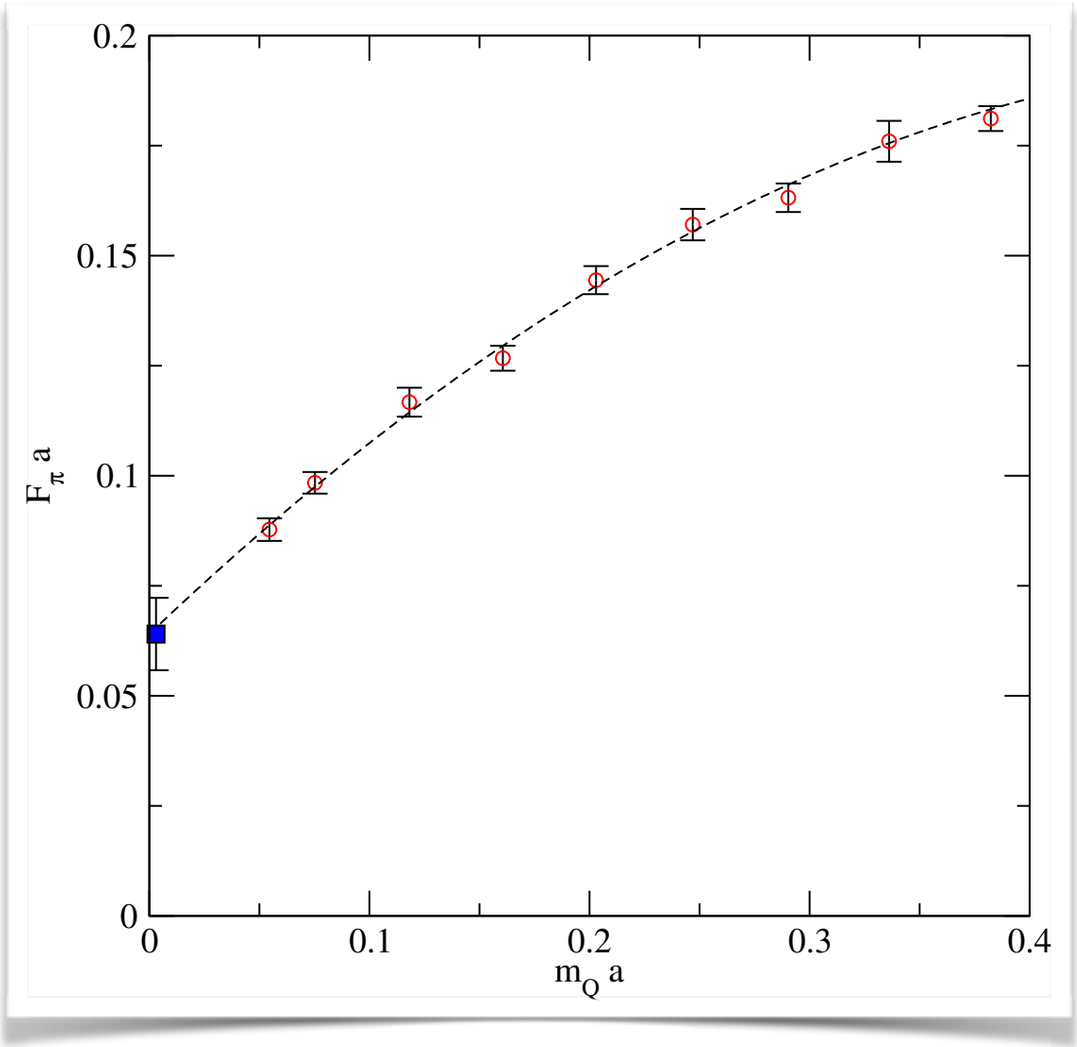
Scale from W-flow:

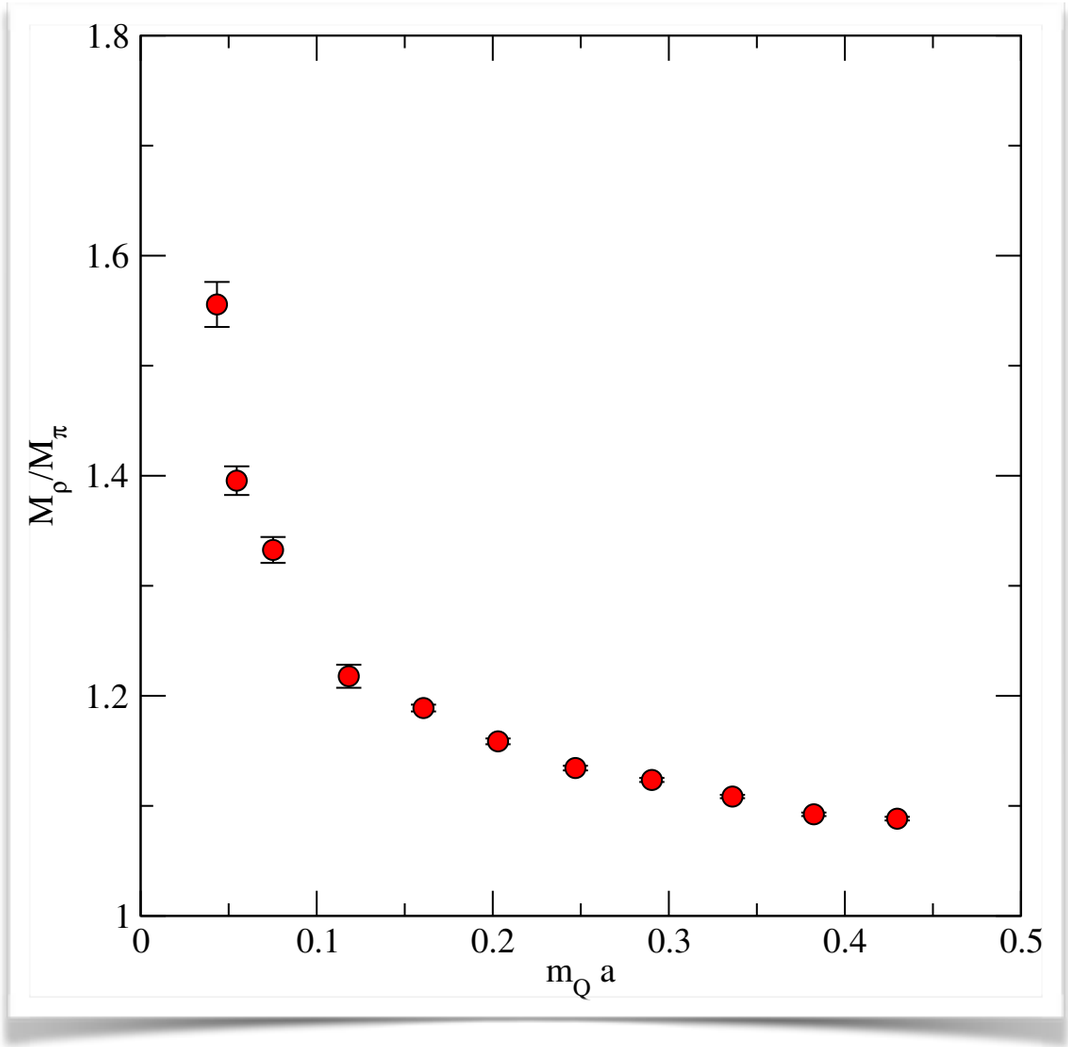


$$\sqrt{t_0} \sim 1/M_\rho$$

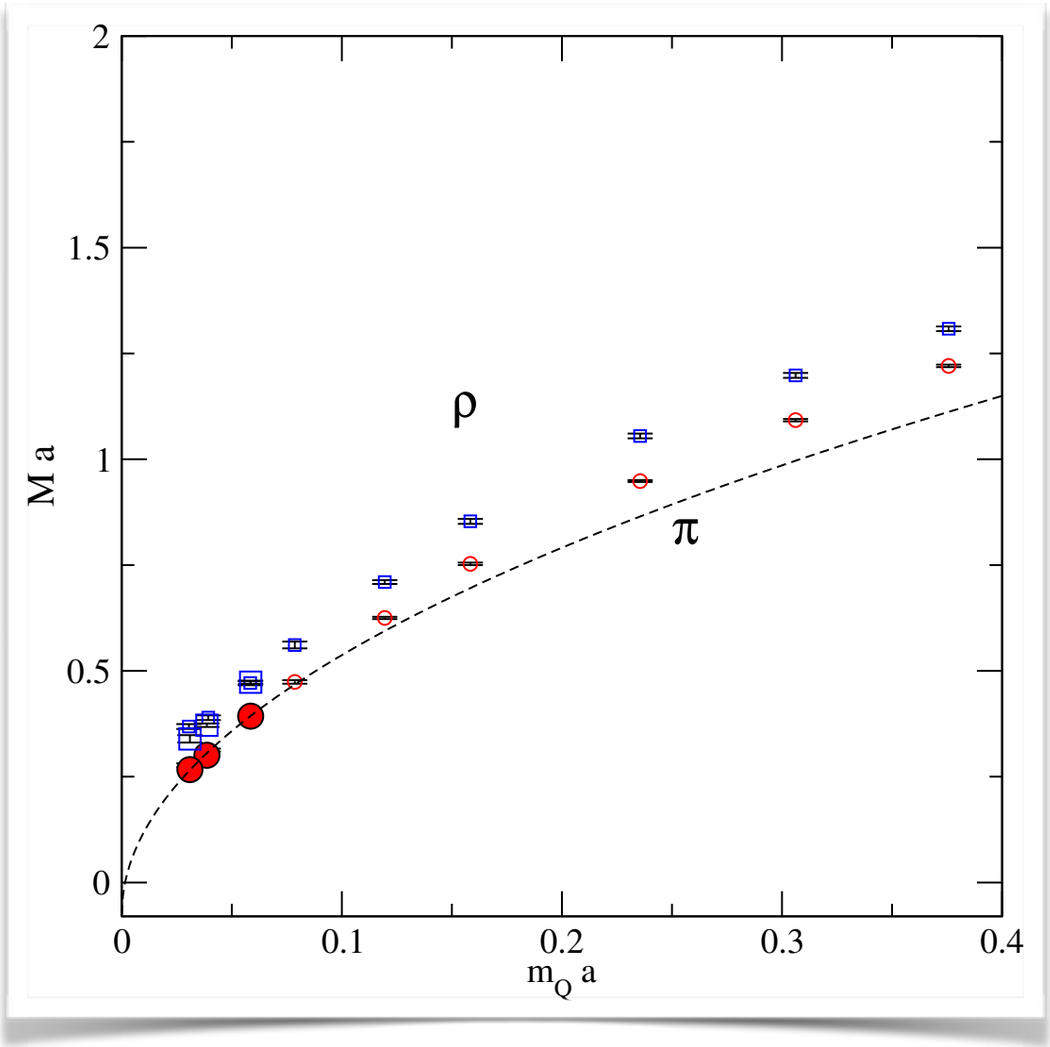


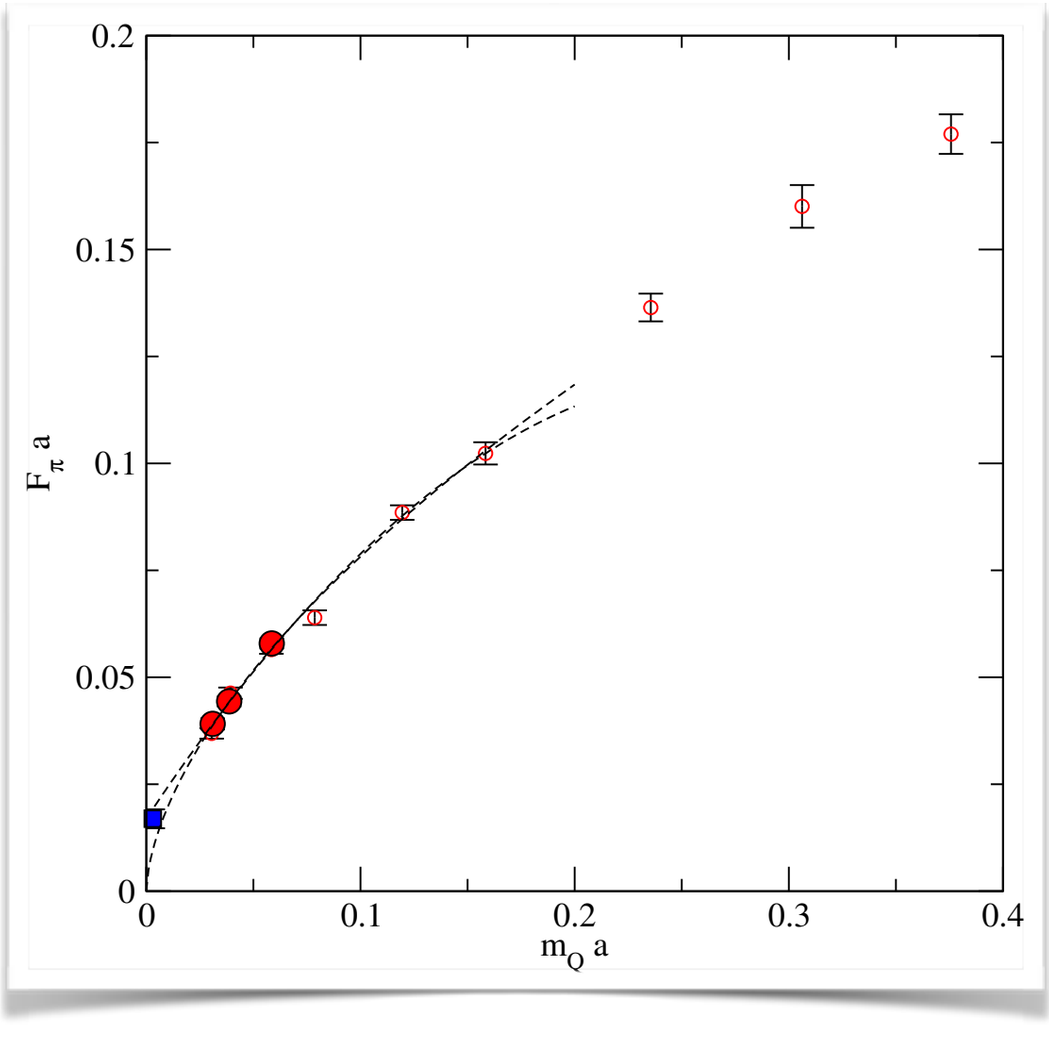
$N_f = 2$ $24^3 \times 48$ $\beta_L = 1.0$



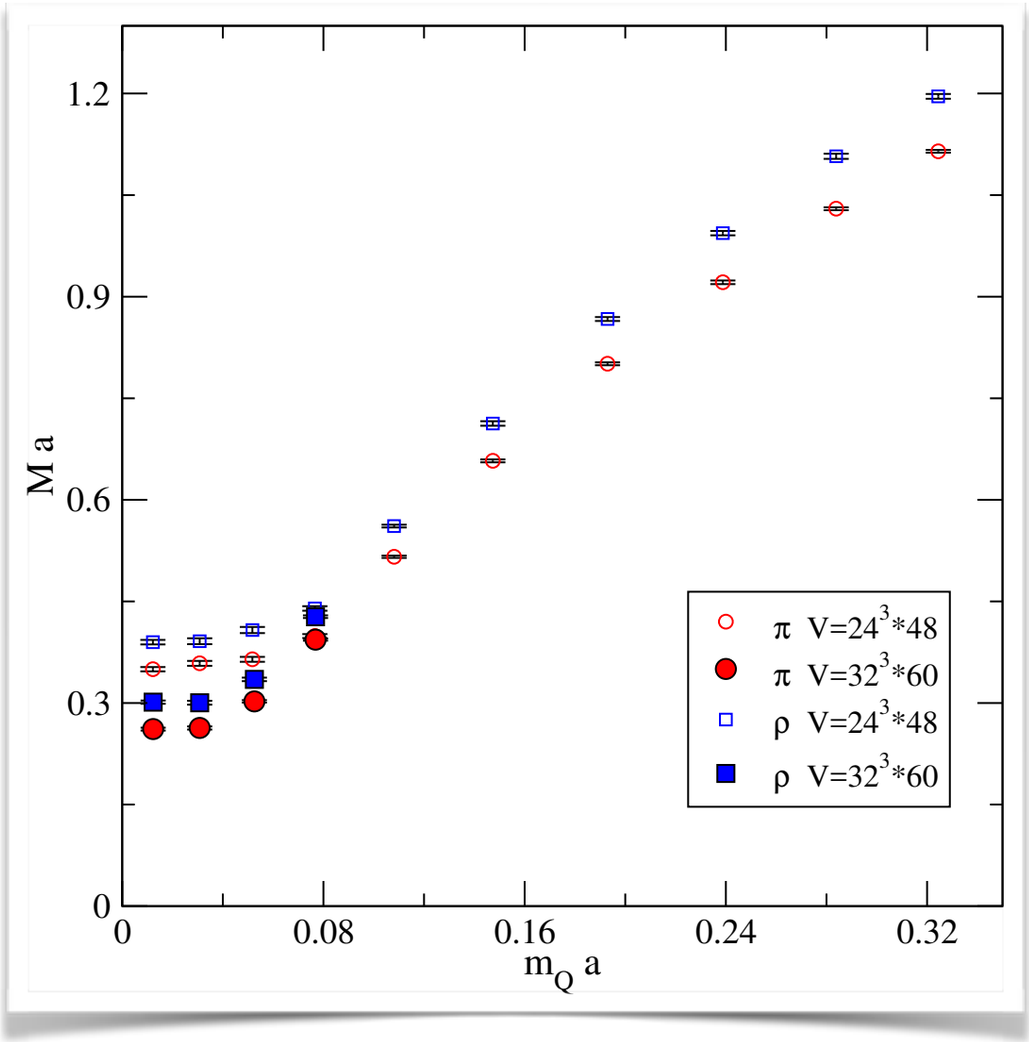


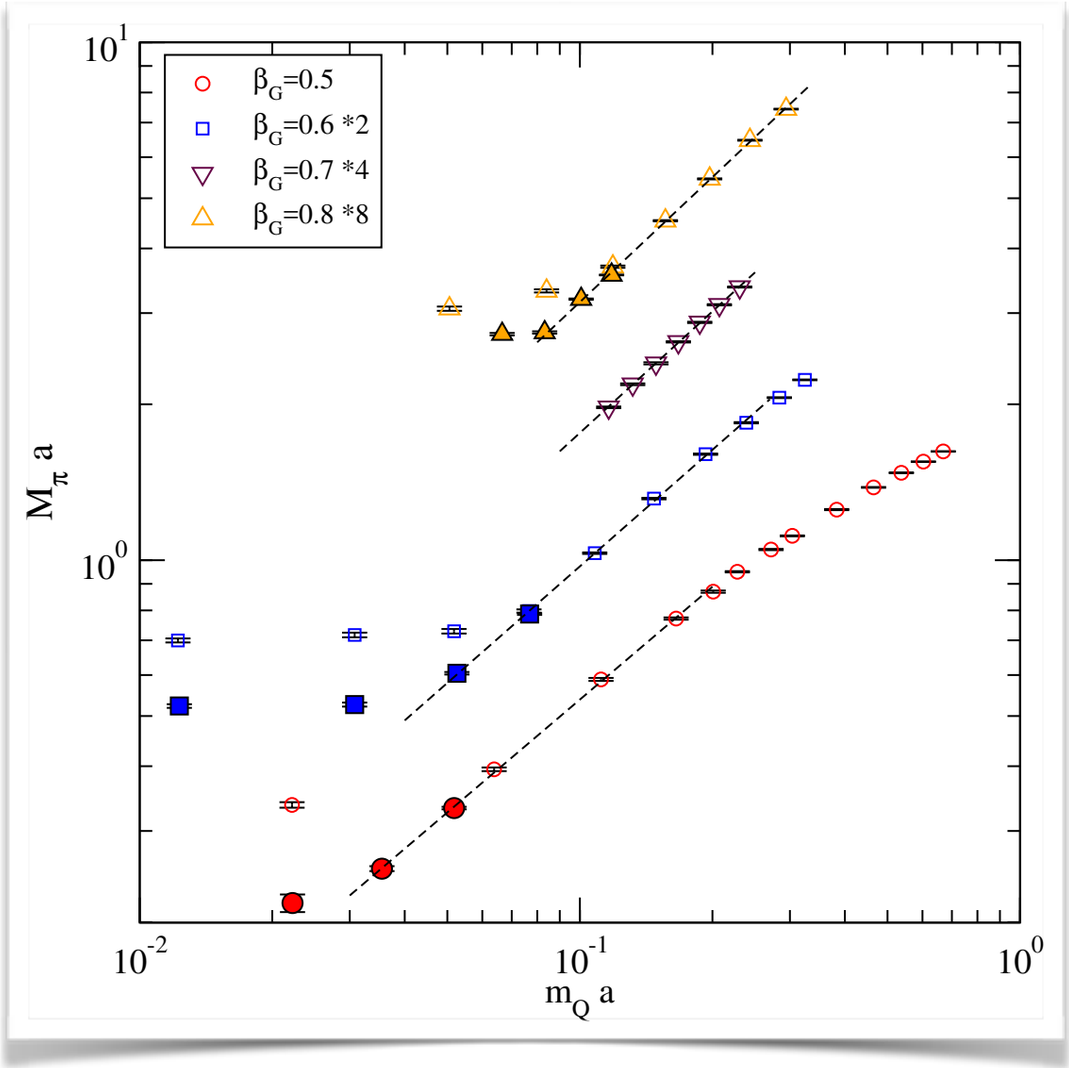
$N_f = 4$ $24^3 \times 48$ $32^3 \times 60$ $\beta_L = 0.8$





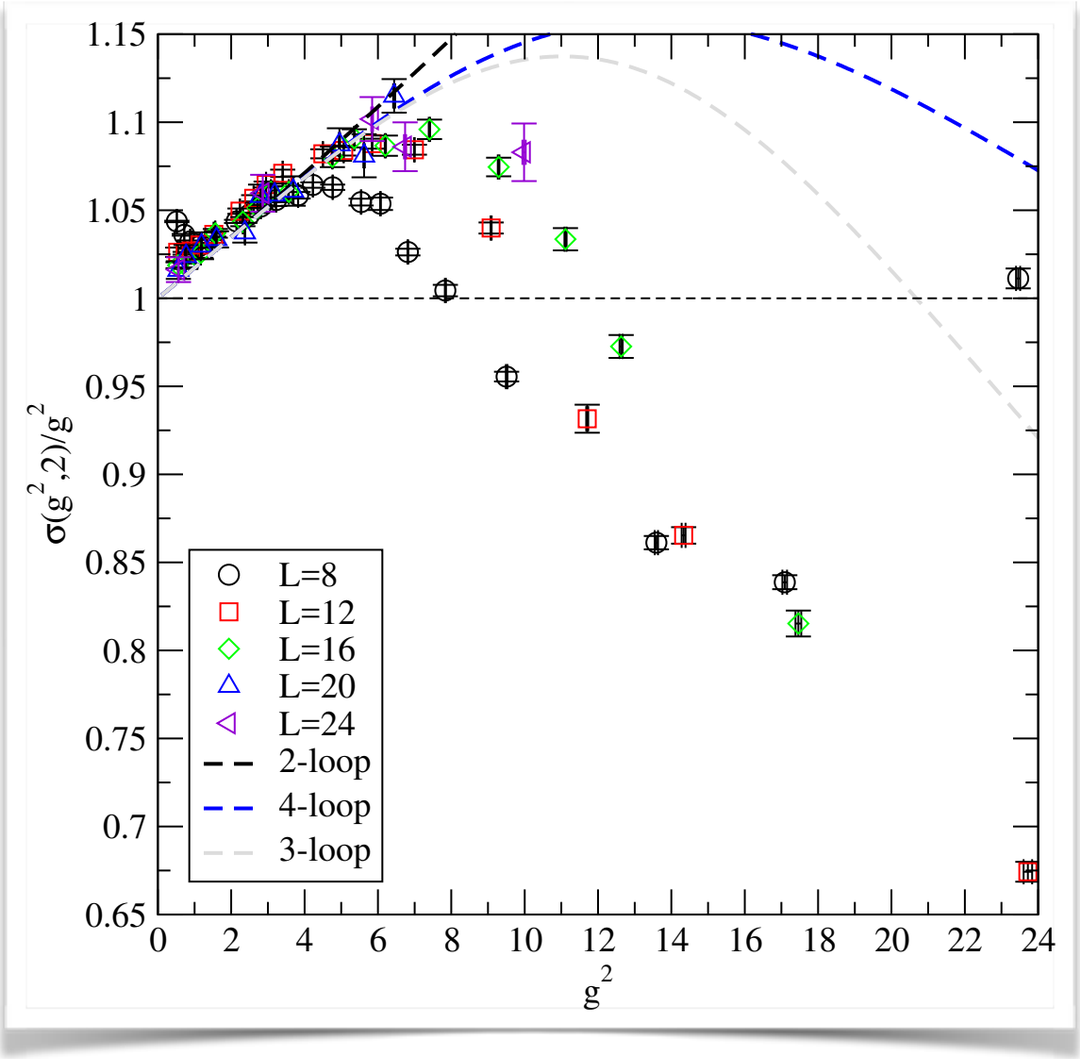
$N_f = 6$

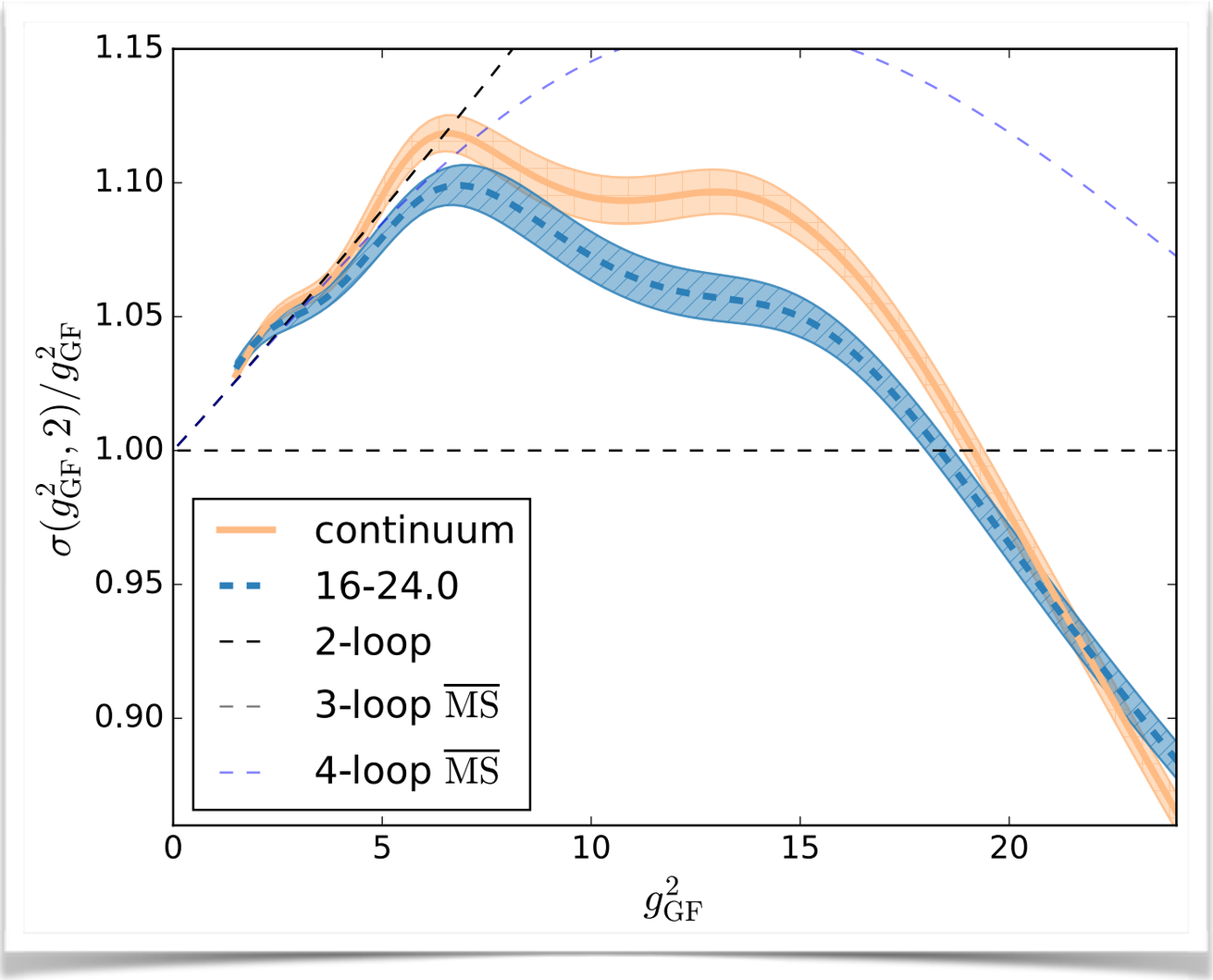




Strong finite volume effects.

Step scaling for $N_f = 6$ (Preliminary)

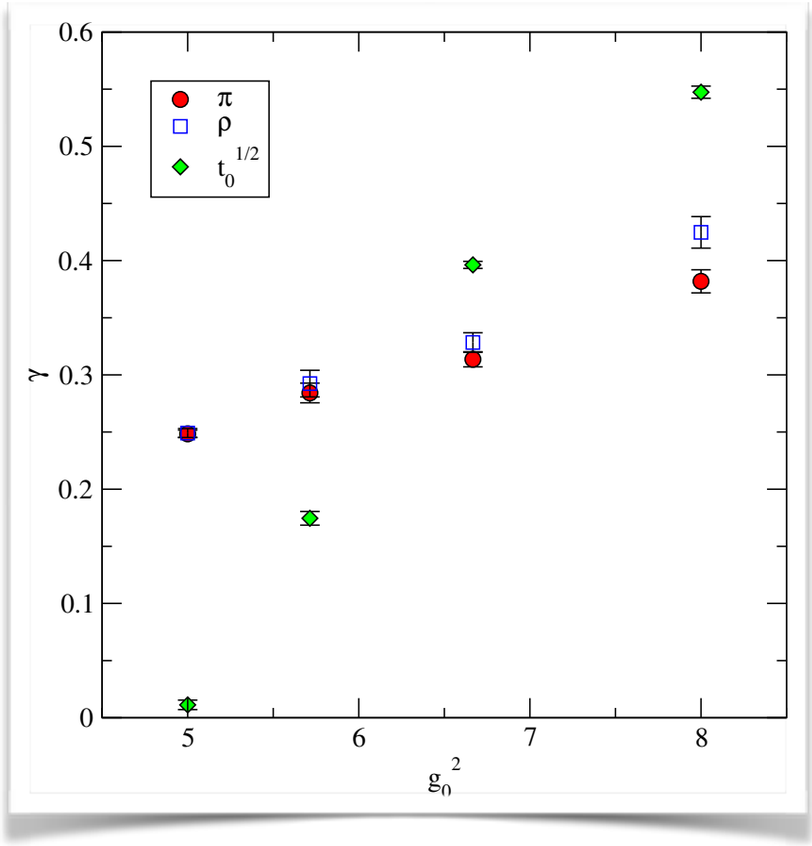




IRFP at strong coupling

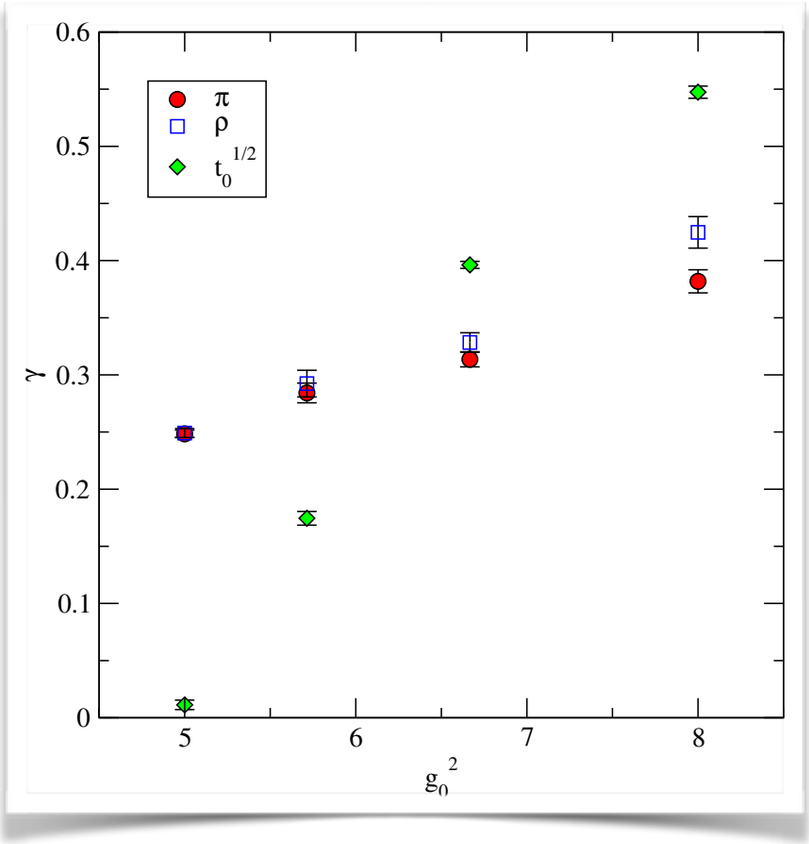
Anomalous dimension: from spectrum? $N_f = 6$

$$M \propto m_Q^{1/(1+\gamma)}$$

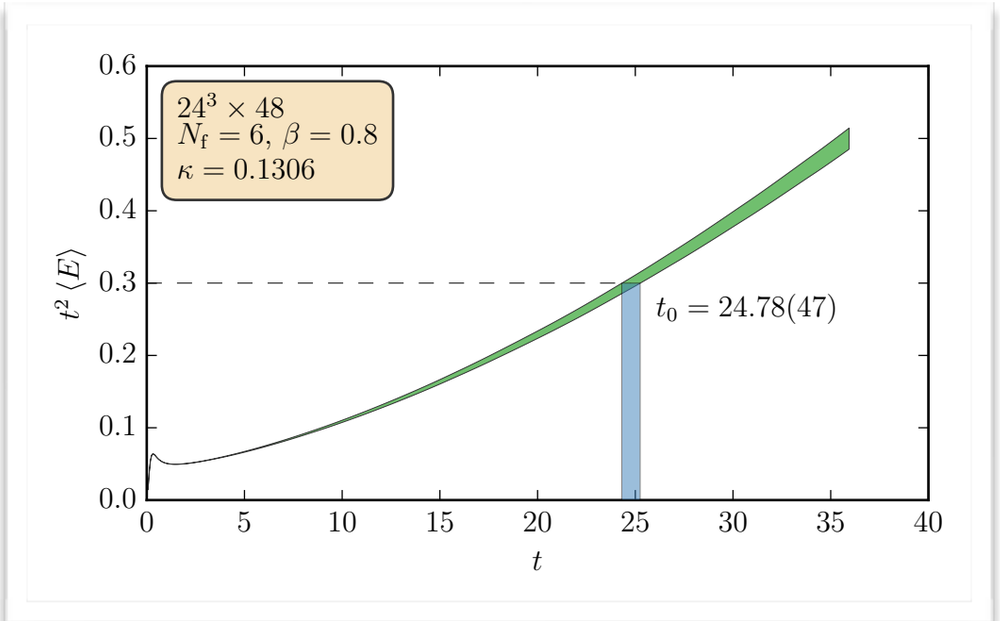


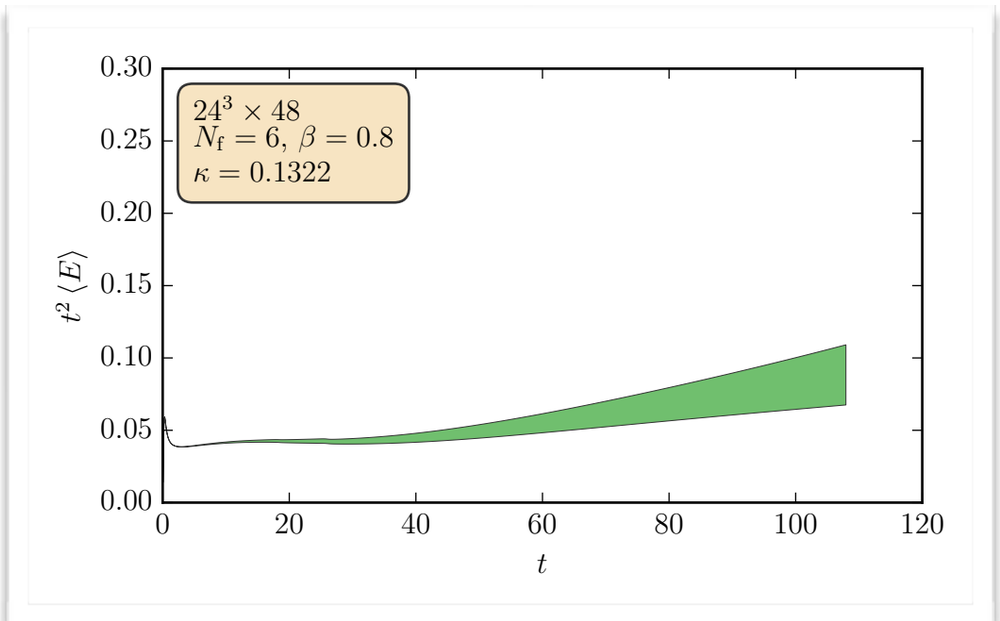
Anomalous dimension: from spectrum? $N_f = 6$

$$M \propto m_Q^{1/(1+\gamma)}$$



The flow runs out of lattice at small m_Q





Conclusions

Consistent results for SU(2), fundamental fermions:

- chiral symmetry broken for $N_f \leq 4$
- IRFP for $N_f \geq 6$

Gradient flow step scaling:

- works fine at strong coupling

Universal anomalous dimension:

- stretching the limits of the lattice
- dependence on the simulation parameters?