Integrating out resonances
in strongly-coupled electroweak scenarios

Ignasi Rosell
Universidad CEU Cardenal Herrera
València (Spain)

In collaboration with:
A. Pich (IFIC, UV-CSIC, València, Spain)
J. Santos (IFIC, UV-CSIC, València, Spain)
J.J. Sanz-Cillero (IFT, UAM-CSIC, Madrid, Spain)

Work in progress and about to be sent to arXiv.
PRD 93 (2016) no.5, 055041 [arXiv: 1510.03114]
OUTLINE

1) Motivation

2) Building the Lagrangian
   1) Low energies (no resonances)
   2) High energies: Proca vs. antisymmetric formalism

3) Estimation of the LECs

4) Short-distance constraints and the purely bosonic sector

5) Conclusions
1. Motivation

i) The **Standard Model** (SM) provides an extremely successful description of the **electroweak** and **strong** interactions.

ii) A **key feature** is the particular mechanism adopted to break the electroweak gauge symmetry to the electroweak subgroup, \( SU(2)_L \times U(1)_Y \rightarrow U(1)_{\text{QED}} \), so that the W and Z bosons become massive. The LHC discovered a new particle around 125 GeV∗.

iii) What if this new particle is **not a standard Higgs boson**? Or a **scalar resonance**? We should look for alternative mechanisms of mass generation.

iv) **Strongly-coupled models**: usually they do contain resonances.

* CMS and ATLAS Collaborations.  

---

Integrating out resonances in strongly-coupled electroweak scenarios, I. Rosell
Resonance Lagrangians can be used to estimate the Low Energy Couplings (LECs) of the Electroweak Effective Theory (EWET). Short-distance constraints are fundamental in order to reduce the number of resonance parameters. What values for resonance masses are required from phenomenology?
**Similarities to Chiral Symmetry Breaking in QCD**

i) **Custodial symmetry**: The Lagrangian is approximately invariant under global SU(2)_L x SU(2)_R transformations. *Electroweak Symmetry Breaking* (EWSB) turns to be SU(2)_L x SU(2)_R \(\rightarrow\) SU(2)_{L+R}.

ii) Similar to the **Chiral Symmetry Breaking** (ChSB) occurring in QCD. So the same pion Lagrangian describes the Goldstone boson dynamics associated with the EWSB, being replaced \(f_\pi\) by \(v=1/\sqrt{(2G_F)}=246\) GeV. Similar to **Chiral Perturbation Theory** (ChPT)*^.

\[
\Delta \mathcal{L}^{(2)}_{\text{ChPT}} = \frac{f_\pi^2}{4} \langle u_\mu u^{\mu} \rangle \quad \rightarrow \quad \Delta \mathcal{L}^{(2)}_{\text{EWET}} = \frac{v^2}{4} \langle u_\mu u^{\mu} \rangle
\]

iii) We can introduce the **resonance fields** needed in strongly-coupled models in a similar way as in ChPT: **Resonance Chiral Theory** (RChT)**.

✓ Note the implications of a naïve rescaling from QCD to EW:

\[
\begin{aligned}
&f_\pi = 0.090\text{ GeV} \quad \rightarrow \quad v = 0.246\text{ TeV} \\
&M_\rho = 0.770\text{ GeV} \quad \rightarrow \quad M_V = 2.1\text{ TeV} \\
&M_{a1} = 1.260\text{ GeV} \quad \rightarrow \quad M_A = 3.4\text{ TeV}
\end{aligned}
\]

The determination of the Electroweak LECs is similar to the ChPT case**.

As in QCD, the assumed high-energy constraints are fundamental.

---

* Weinberg ’79  
* Gasser and Leutwyler ’84 ’85  
* Bijnens et al. ’99 ’00  
^Dobado, Espriu and Herrero ’91  
^Espriu and Herrero ’92  
^Herrero and Ruiz-Morales ’94  
**Ecker et al. ’89  
** Cirigliano et al. ’06  

Integrating out resonances in strongly-coupled electroweak scenarios, I. Rosell
Looking at the phenomenology*

- Oblique electroweak observables** (S and T)
- Dispersive relations for both S** and T*
- Short-distance constraints: two-Goldstone VFF, Higgs-Goldstone VFF, Weinberg Sum Rules

\[ S_{LO} \]

\[ M_V (\text{TeV}) \]

\[ M_V^\text{LO} < 6.0 \text{ TeV} \]

\[ 0 < \kappa_W < 1 \]

\[ 0.2 < M_V/M_A < 1 \]

\[ 0.02 < M_V/M_A < 0.2 \]

\[ \kappa_W \approx 1 \]

\[ M_R \approx \text{TeV} \]

---

* Pich, IR and Sanz-Cillero ’12 ’13 ’14
** Peskin and Takeuchi ’92

Integrating out resonances in strongly-coupled electroweak scenarios, I. Rosell

6/17
2. Building the Lagrangian

✓ Two strongly coupled Lagrangians for two energy regions:
  ✓ Electroweak Effective Theory (EWET) at low energies (without resonances).
  ✓ Resonance Theory at high energies* (with resonances).

✓ The aim of this work:

   Estimation of the Low-Energy Couplings (LECs) in terms of resonance parameters

✓ Steps:

1. Building the EWET and resonance Lagrangian
2. Matching the two effective theories

✓ High-energy constraints

1. From QCD we know the importance of sum-rules and form factors at large energies.
2. Operators with a large number of derivatives tend to violate the asymptotic behaviour.
3. The constraints are required to reduce the number of unknown resonance parameters.
4. The underlying theory is less known than in the case of QCD.

✓ This program works pretty well in QCD: estimation of the LECs (Chiral Perturbation Theory)** and importance of short-distance constraints***.

* Pich, IR, Santos and Sanz-Cillero '16 [and in progress]
** Cirigliano et al. '06
*** Ecker et al. '89

Integrating out resonances in strongly-coupled electroweak scenarios, I. Rosell
How do we build the Lagrangian?

✓ Custodial symmetry

✓ Degrees of freedom:
  ✓ At low energies: bosons $\chi$ (EW goldstones, gauge bosons, $h$), fermions $\psi$
  ✓ At high energies: previous dof + resonances (V,A,S,P triplets and singlets)

✓ *Chiral* power counting*

\[
\frac{\chi}{v} \sim \mathcal{O}(p^0) \quad \frac{\psi}{v} \sim \mathcal{O}(p) \quad \partial_\mu, m \sim \mathcal{O}(p) \quad T \sim \mathcal{O}(p)
\]

✓ So

✓ At low energies: $\mathcal{L}_{\text{EWET}} = \mathcal{L}_2 + \mathcal{L}_4 + \ldots$

✓ At high energies: $\mathcal{L}_R = c_R R \mathcal{O}_{p^2}[\chi, \psi] + \ldots$

---

* Weinberg ’79
* Appelquist and Bernand ‘80
* Longhitano ‘80, ‘81
* Manohar, and Georgi ‘84
* Gasser and Leutwyler ‘84 ’85
* Hirn and Stern ’05
* Alonso et al. ’12
* Buchalla, Catá and Krause ’13
* Brivio et al. ’13
* Delgado et al. ’14
* Pich, IR, Santos and Sanz-Cillero ‘16 [and in progress]

Integrating out resonances in strongly-coupled electroweak scenarios, I. Rosell
2.1. Low energies (no resonances)*

\[
\mathcal{L}_{\text{EWET}}^{(4)} = \sum_{i=1}^{11} F_i \mathcal{O}_i + \sum_{i=1}^{3} \tilde{F}_i \tilde{\mathcal{O}}_i + \sum_{i=1}^{7} \mathcal{F}_i^{\psi^2} \mathcal{O}_i^{\psi^2} + \sum_{i=1}^{3} \tilde{F}_i^{\psi^2} \tilde{\mathcal{O}}_i^{\psi^2} + \sum_{i=1}^{10} \mathcal{F}_i^{\psi^4} \mathcal{O}_i^{\psi^4} + \sum_{i=1}^{2} \tilde{F}_i^{\psi^4} \tilde{\mathcal{O}}_i^{\psi^4}
\]

<table>
<thead>
<tr>
<th>$i$</th>
<th>$\mathcal{O}_i$</th>
<th>$\tilde{\mathcal{O}}_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\frac{1}{4} \langle f_+^{\mu \nu} f_+^{\mu \nu} - f_+^{\mu \nu} f_-^{\mu \nu} \rangle$</td>
<td>$\frac{1}{2} \langle f_+^{\mu \nu} [u_\mu, u_\nu] \rangle$</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{1}{2} \langle f_+^{\mu \nu} [u_\mu, u_\nu] \rangle$</td>
<td>$\langle f_+^{\mu \nu} \rangle$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{i}{2} \langle f_+^{\mu \nu} \rangle$</td>
<td>$\frac{\partial \mu}{v} \langle f_+^{\mu \nu} \rangle$</td>
</tr>
<tr>
<td>4</td>
<td>$\langle u_\mu u_\nu \rangle$</td>
<td>$\langle u_\mu u_\nu \rangle$</td>
</tr>
<tr>
<td>5</td>
<td>$(\partial_\mu h)(\partial_\nu h) \langle u_\nu \rangle$</td>
<td>$(\partial_\mu h)(\partial_\nu h) \langle u_\nu \rangle$</td>
</tr>
<tr>
<td>6</td>
<td>$(\partial_\mu h)(\partial_\nu h) \langle u_\nu \rangle$</td>
<td>$(\partial_\nu h)(\partial_\mu h) \langle u_\nu \rangle$</td>
</tr>
<tr>
<td>7</td>
<td>$(\partial_\mu h)(\partial_\nu h) \langle u_\nu \rangle$</td>
<td>$(\partial_\nu h)(\partial_\mu h) \langle u_\nu \rangle$</td>
</tr>
<tr>
<td>8</td>
<td>$(\partial_\mu h)(\partial_\nu h) \langle u_\nu \rangle$</td>
<td>$(\partial_\nu h)(\partial_\mu h) \langle u_\nu \rangle$</td>
</tr>
<tr>
<td>9</td>
<td>$(\partial_\mu h)(\partial_\nu h) \langle u_\nu \rangle$</td>
<td>$(\partial_\nu h)(\partial_\mu h) \langle u_\nu \rangle$</td>
</tr>
<tr>
<td>10</td>
<td>$(\partial_\mu h)(\partial_\nu h) \langle u_\nu \rangle$</td>
<td>$(\partial_\nu h)(\partial_\mu h) \langle u_\nu \rangle$</td>
</tr>
<tr>
<td>11</td>
<td>$(\partial_\mu h)(\partial_\nu h) \langle u_\nu \rangle$</td>
<td>$(\partial_\nu h)(\partial_\mu h) \langle u_\nu \rangle$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$i$</th>
<th>$\mathcal{O}_i^{\psi^2}$</th>
<th>$\tilde{\mathcal{O}}_i^{\psi^2}$</th>
<th>$\mathcal{O}_i^{\psi^4}$</th>
<th>$\tilde{\mathcal{O}}_i^{\psi^4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\langle J_S \rangle$</td>
<td>$\langle J_S \rangle$</td>
<td>$\langle J_S \rangle$</td>
<td>$\langle J_S \rangle$</td>
</tr>
<tr>
<td>2</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
</tr>
<tr>
<td>3</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
</tr>
<tr>
<td>4</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
</tr>
<tr>
<td>5</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
</tr>
<tr>
<td>6</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
</tr>
<tr>
<td>7</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
</tr>
<tr>
<td>8</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
</tr>
<tr>
<td>9</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
</tr>
<tr>
<td>10</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
<td>$\langle J_T^{\mu \nu} \rangle$</td>
</tr>
</tbody>
</table>

* Longhitano '80 '81
* Buchalla and Catà '12 '14
* Pich, IR, Santos and Sanz-Cillero '16 [and in progress]
* Alonso et al. '13

Integrating out resonances in strongly-coupled electroweak scenarios, I. Rosell
2.2. High energies (with resonances)*

\[ \mathcal{L}_{RT} = \mathcal{L}_R[R, \chi, \psi] + \mathcal{L}_{\text{non-R}}[\chi, \psi] \]

\[ \mathcal{L}_{\text{non-R}}^{(P)} = \sum_{i=1}^{11} F_i^{(P)} O_i + \sum_{i=1}^{3} \tilde{F}_i^{(P)} \tilde{O}_i + \sum_{i=1}^{7} F_i^{\psi^2,(P)} O_i^{\psi^2} + \sum_{i=1}^{3} \tilde{F}_i^{\psi^2,(P)} \tilde{O}_i^{\psi^2} + \sum_{i=1}^{10} F_i^{\psi^4,(P)} O_i^{\psi^4} + \sum_{i=1}^{2} \tilde{F}_i^{\psi^4,(P)} \tilde{O}_i^{\psi^4} \]

\[ \mathcal{L}_{\text{non-R}}^{(A)} = \sum_{i=1}^{11} F_i^{(A)} O_i + \sum_{i=1}^{3} \tilde{F}_i^{(A)} \tilde{O}_i + \sum_{i=1}^{7} F_i^{\psi^2,(A)} O_i^{\psi^2} + \sum_{i=1}^{3} \tilde{F}_i^{\psi^2,(A)} \tilde{O}_i^{\psi^2} + \sum_{i=1}^{10} F_i^{\psi^4,(A)} O_i^{\psi^4} + \sum_{i=1}^{2} \tilde{F}_i^{\psi^4,(A)} \tilde{O}_i^{\psi^4} \]

i) Spin-0 (S,S_1,P,P_1)

\[ \mathcal{L}_R = \frac{1}{2} <\nabla^\mu R \nabla_\mu R - M_R^2 R^2 > + <R \chi_R> \quad (R = S, P), \]

\[ \mathcal{L}_{R_1} = \frac{1}{2} (\partial^\mu R_1 \partial_\mu R_1 - M_{R_1}^2 R_1^2) + R_1 \chi_{R_1} \quad (R_1 = S_1, P_1). \]

\[ \chi_{S_1} = \lambda h_{S_1} v h^2 + \frac{c_d}{\sqrt{2}} <u_\mu u^\mu> + \frac{c_{S_1}}{\sqrt{2}} <J_S> \]

\[ \chi_{P_1} = \frac{c_{P_1}}{\sqrt{2}} <J_P> \]

\[ \chi_S = c_1^S J_S \]

\[ \chi_P = c_1^P J_P + d_P \frac{(\partial_\mu h)}{v} u^\mu \]

* Pich, IR, Santos and Sanz-Cillero ‘16 [and in progress]
ii) Spin-1 ($V,V_1,A,A_1$) with Proca formalism

\[
\mathcal{L}^{(P)}_{\hat{R}} = -\frac{1}{4} \left< \hat{R}_{\mu\nu} \hat{R}^{\mu\nu} - 2 M_R^2 \hat{R}_\mu \hat{R}^\mu \right> + \left< \hat{R}_\mu \hat{\chi}_R^{\mu} + \hat{R}_{\mu\nu} \hat{\chi}_R^{\mu\nu} \right> \quad (\hat{R} = \hat{V}, \hat{A}),
\]

\[
\mathcal{L}^{(P)}_{\hat{R}_1} = -\frac{1}{4} \left( \hat{R}_{1\mu\nu} \hat{R}_1^{\mu\nu} - 2 M_{R_1}^2 \hat{R}_{1\mu} \hat{R}_1^{\mu} \right) + \hat{R}_{1\mu} \hat{\chi}_{R_1}^{\mu} + \hat{R}_{1\mu\nu} \hat{\chi}_{R_1}^{\mu\nu} \quad (\hat{R}_1 = \hat{V}_1, \hat{A}_1),
\]

\[
\hat{R}_{\mu\nu} = \nabla_\mu \hat{R}_\nu - \nabla_\nu \hat{R}_\mu
\]

\[
\hat{\chi}_V^{\mu\nu} = \frac{f_{\hat{V}}}{2\sqrt{2}} f_+^{\mu\nu} + \frac{i g_{V}}{2\sqrt{2}} [u^\mu, u^\nu] + \frac{\tilde{f}_{\hat{V}}}{2\sqrt{2}} f_-^{\mu\nu} + \frac{\tilde{\lambda}_{h}\hat{V}}{\sqrt{2}} \left[ (\partial^\mu h) u^\nu - (\partial^\nu h) u^\mu \right] + c_{\hat{V}} \hat{J}_T^{\mu\nu}
\]

\[
\hat{\chi}_A^{\mu\nu} = \frac{f_{\hat{A}}}{2\sqrt{2}} f_+^{\mu\nu} + \frac{\lambda_{h}\hat{A}}{\sqrt{2}} \left[ (\partial^\mu h) u^\nu - (\partial^\nu h) u^\mu \right] + \frac{\tilde{f}_{\hat{A}}}{2\sqrt{2}} f_-^{\mu\nu} + \frac{\tilde{\lambda}_{h}\hat{A}}{\sqrt{2}} \left[ u^\mu, u^\nu \right] + c_{\hat{A}} \hat{J}_T^{\mu\nu}
\]

\[
\hat{\chi}_{V_1}^{\mu\nu} = f_{V_1} X^{\mu\nu} + \frac{c_0}{\sqrt{2}} \hat{J}_T^{\mu\nu} \quad \hat{\chi}_{A_1}^{\mu\nu} = \tilde{f}_{A_1} X^{\mu\nu} + \frac{\tilde{c}_0}{\sqrt{2}} \hat{J}_T^{\mu\nu}
\]

\[
\hat{\chi}_V^{\mu} = c_{\hat{V}}^{\mu} J_1^{\mu} + \tilde{c}_{\hat{V}}^{\mu} J_1^{\mu}
\]

\[
\hat{\chi}_A^{\mu} = c_{\hat{A}}^{\mu} J_1^{\mu} + \tilde{c}_{\hat{A}}^{\mu} J_1^{\mu}
\]

\[
\hat{\chi}_{V_1}^{\mu} = \tilde{c}_{\hat{V}}^{\mu} < u^\mu T > + \frac{c_1}{\sqrt{2}} \hat{J}_V^{\mu} \quad \hat{\chi}_{A_1}^{\mu} = c_{\hat{A}}^{\mu} < u^\mu T > + \frac{\tilde{c}_1}{\sqrt{2}} \hat{J}_V^{\mu}
\]

* Pich, IR, Santos and Sanz-Cillero '16 [and in progress]
iii) Spin-1 ($V,V_1,A,A_1$) with antisymmetric formalism

\[
\mathcal{L}^{(A)}_R = -\frac{1}{2} < \nabla^\lambda R_{\lambda\mu} \nabla_\sigma R^{\sigma\mu} > - \frac{1}{2} M^2_R R_{\mu\nu} R^{\mu\nu} > + < R_{\mu\nu} \chi_{R}^{\mu\nu} > \quad (R = V, A), \\
\mathcal{L}^{(A)}_{R_1} = -\frac{1}{2} \left( \partial^\lambda R_{1\lambda\mu} \partial_\sigma R_{1}^{\sigma\mu} - \frac{1}{2} M^2_{R_1} R_{1\mu\nu} R_{1}^{\mu\nu} \right) + R_{1\mu\nu} \chi_{R_1}^{\mu\nu} \quad (R_1 = V_1, A_1),
\]

\[
\begin{align*}
\chi_{V}^{\mu\nu} & \bigg|_{\text{Bos}} = \frac{F_V}{2\sqrt{2}} f_{+}^{\mu\nu} + \frac{i G_V}{2\sqrt{2}} [u^\mu, u^\nu] + \tilde{F}_V \frac{1}{\sqrt{2}} f_{+}^{\mu\nu} + \frac{\lambda_1^{V}}{\sqrt{2}} \left[ (\partial^\mu h) u^\nu - (\partial^\nu h) u^\mu \right] \\
\chi_{A}^{\mu\nu} & \bigg|_{\text{Bos}} = \frac{F_A}{2\sqrt{2}} f_{-}^{\mu\nu} + \lambda_1^{A} \frac{1}{\sqrt{2}} \left[ (\partial^\mu h) u^\nu - (\partial^\nu h) u^\mu \right] + \tilde{F}_A \frac{1}{\sqrt{2}} f_{+}^{\mu\nu} + \frac{i G_A}{2\sqrt{2}} [u^\mu, u^\nu] \\
\chi_{V_1}^{\mu\nu} & \bigg|_{\text{Bos}} = F_{V_1} X^{\mu\nu} + \frac{G_T}{2} \left( \partial^\mu < u^\nu T > - \partial^\nu < u^\mu T > \right) \\
\chi_{A_1}^{\mu\nu} & \bigg|_{\text{Bos}} = \tilde{F}_{A_1} X^{\mu\nu} + \frac{G_{T_1}}{2} \left( \partial^\mu < u^\nu T > - \partial^\nu < u^\mu T > \right)
\end{align*}
\]

\[
\begin{align*}
\chi_{V}^{\mu\nu} & \bigg|_{\text{Fer}} = C_0^V J^{\mu\nu}_{T} + \frac{C_1^V}{2} \left( \nabla^\mu J^\nu_{V} - \nabla^\nu J^\mu_{V} \right) + \frac{\tilde{C}_1^V}{2} \left( \nabla^\mu J^\nu_{A} - \nabla^\nu J^\mu_{A} \right) \\
\chi_{A}^{\mu\nu} & \bigg|_{\text{Fer}} = \tilde{C}_0^A J^{\mu\nu}_{T} + \frac{C_1^A}{2} \left( \nabla^\mu J^\nu_{A} - \nabla^\nu J^\mu_{A} \right) + \frac{\tilde{C}_1^A}{2} \left( \nabla^\mu J^\nu_{V} - \nabla^\nu J^\mu_{V} \right) \\
\chi_{V_1}^{\mu\nu} & \bigg|_{\text{Fer}} = \frac{C_0^{V_1}}{\sqrt{2}} < J^{\mu\nu}_{T} > + \frac{C_1^{V_1}}{2\sqrt{2}} < \partial^\mu J^\nu_{V} - \partial^\nu J^\mu_{V} > + \frac{\tilde{C}_1^{V_1}}{2\sqrt{2}} < \partial^\mu J^\nu_{A} - \partial^\nu J^\mu_{A} > \\
\chi_{A_1}^{\mu\nu} & \bigg|_{\text{Fer}} = \frac{\tilde{C}_0^{A_1}}{\sqrt{2}} < J^{\mu\nu}_{T} > + \frac{C_1^{A_1}}{2\sqrt{2}} < \partial^\mu J^\nu_{A} - \partial^\nu J^\mu_{A} > + \frac{\tilde{C}_1^{A_1}}{2\sqrt{2}} < \partial^\mu J^\nu_{V} - \partial^\nu J^\mu_{V} >
\end{align*}
\]

* Pich, IR, Santos and Sanz-Cillero ‘16 [and in progress]
Proca vs. antisymmetric formalism*

✓ By using path integral and changes of variables both formalisms are proven to be equivalent:

✓ The following set of relations between resonance parameters emerges:

\[
F_R = f_R M_R, \quad G_R = g_R M_R, \quad \lambda_1^{hR} = \lambda_1^{hR} M_R, \quad C_0^R = c_0^R M_R,
\]

\[
\tilde{F}_R = \tilde{f}_R M_R, \quad \tilde{G}_R = \tilde{g}_R M_R, \quad \tilde{\lambda}_1^{hR} = \tilde{\lambda}_1^{hR} M_R, \quad \tilde{C}_0^R = \tilde{c}_0^R M_R,
\]

\[
C_T^R = c_T^R / M_R, \quad \tilde{C}_T^R = \tilde{c}_T^R / M_R, \quad C_1^R = c_1^R / M_R, \quad \tilde{C}_1^R = \tilde{c}_1^R / M_R.
\]

✓ The couplings of the non-resonant operators are different:

\[\mathcal{L}_{\text{non-R}}^{(P)} \neq \mathcal{L}_{\text{non-R}}^{(A)}\]

✓ By using high-energy behaviour:

\[
\Re^V_{\varphi \varphi}(s) = \begin{cases} 
1 + \frac{F_V G_V}{v^2} \frac{s}{M_V^2 - s} + \frac{\tilde{F}_A \tilde{G}_A}{v^2} \frac{s}{M_A^2 - s} - 2 \mathcal{F}_3^{\text{SDA}} \frac{s}{v^2} & (A) \\
1 + \frac{f_V g_V}{v^2} \frac{s}{M_V^2 - s} + \frac{\tilde{f}_A \tilde{g}_A}{v^2} \frac{s}{M_A^2 - s} - 2 \mathcal{F}_3^{\text{SDP}} \frac{s}{v^2} & (P)
\end{cases}
\]

\[\mathcal{F}_3^{\text{SDA}} = 0 \quad \mathcal{F}_3^{\text{SDP}} = -\frac{f_V g_V}{2} - \frac{\tilde{f}_A \tilde{g}_A}{2}\]

* Ecker et al. '89
* Bijnens and Pallante '96
* Kampf, Novotny and Trnka '07
* Pich, IR, Santos and Sanz-Cillero ‘16 [and in progress]
**Proca vs. antisymmetric formalism**

- By using **path integral** and **changes of variables** both formalisms are proven to be equivalent:
  
  - The following **set of relations between resonance parameters** emerges:

    
    \[
    \begin{align*}
    F_R &= f_R M_R, & G_R &= g_R M_R, & \chi^h_R &= \chi^h R M_R, & C^R_0 &= c^R_0 M_R, \\
    \tilde{F}_R &= \tilde{f}_R M_R, & \tilde{G}_R &= \tilde{g}_R M_R, & \tilde{\chi}^h_R &= \tilde{\chi}^h R M_R, & \tilde{C}^R_0 &= \tilde{c}^R_0 M_R, \\
    C^R_T &= c^R_T / M_R, & \tilde{C}^R_T &= \tilde{c}^R_T / M_R, & C_1^R &= c_1^R / M_R, & \tilde{C}_1^R &= \tilde{c}_1^R / M_R.
    \end{align*}
    \]

  - The couplings of the **non-resonant operators** are different:

    \[
    \mathcal{L}^{(P)}_{\text{non-R}} \neq \mathcal{L}^{(A)}_{\text{non-R}}
    \]

- By using **high-energy** behaviour:

  - **LECs with resonance contributions** coming from $\chi^{\mu \nu}_R$ do not contain local contributions, $\mathcal{F}_i^{\text{SDA}} = \tilde{\mathcal{F}}_i^{\text{SDA}} = 0$, so then the **antisymmetric formalism** is the best choice.

  - **LECs with resonance contributions** coming from $\hat{\chi}^\mu_R$ do not contain local contributions, $\mathcal{F}_i^{\text{SDP}} = \tilde{\mathcal{F}}_i^{\text{SDP}} = 0$, so then **Proca** is the best choice.

* Ecker et al. '89
* Bijnens and Pallante '96
* Kampf, Novotny and Trnka '07
* Pich, IR, Santos and Sanz-Cillero '16 [and in progress]
3. Estimation of the LECs

- Integration of the heavy modes
- Similar to the ChPT case*
- Results**

\[ e^{iS_{\text{eff}}[\chi,\psi]} = \int [d\mathbf{R}] e^{iS[\chi,\psi,\mathbf{R}]} \]

* Ecker et al. '89
** Pich, IR, Santos and Sanz-Cillero '16 [and in progress]
3. Estimation of the LECs

- Integration of the heavy modes
- Similar to the ChPT case*
- Results**
  - Purely bosonic operators

\[
e^{iS_{\text{eff}}[\chi, \psi]} = \int [d\mathbf{R}] e^{iS[\chi, \psi, \mathbf{R}]}
\]

<table>
<thead>
<tr>
<th>(i)</th>
<th>(\Delta \mathcal{F}_i)</th>
<th>(\Delta \tilde{\mathcal{F}}_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(-\frac{F_V^2 - \tilde{F}_V^2}{4M_V^2} + \frac{F_A^2 - \tilde{F}_A^2}{4M_A^2})</td>
<td>(-\frac{\tilde{F}_V G_V}{2M_V^2} - \frac{F_A \tilde{G}_A}{2M_A^2})</td>
</tr>
<tr>
<td>2</td>
<td>(-\frac{F_V^2 + \tilde{F}_V^2}{8M_V^2} - \frac{F_A^2 + \tilde{F}_A^2}{8M_A^2})</td>
<td>(-\frac{F_V \tilde{F}_V}{4M_V^2} - \frac{F_A \tilde{F}_A}{4M_A^2})</td>
</tr>
<tr>
<td>3</td>
<td>(-\frac{F_V G_V}{2M_V^2} - \frac{\tilde{F}_A \tilde{G}_A}{2M_A^2})</td>
<td>(-\frac{F_V \tilde{\lambda}_V^{HA} v}{M_V^2} - \frac{F_A \tilde{\lambda}_A^{HA} v}{M_A^2})</td>
</tr>
<tr>
<td>4</td>
<td>(\frac{G_V^2}{4M_V^2} + \frac{\tilde{G}_A^2}{4M_A^2})</td>
<td>--</td>
</tr>
<tr>
<td>5</td>
<td>(-\frac{c_d^2}{4M_S^2} - \frac{G_V^2}{4M_V^2} - \frac{\tilde{G}_A^2}{4M_A^2})</td>
<td>--</td>
</tr>
<tr>
<td>6</td>
<td>(-\frac{\tilde{\lambda}_V^{HA} v^2}{M_V^2} - \frac{\tilde{\lambda}_A^{HA} v^2}{M_A^2})</td>
<td>--</td>
</tr>
<tr>
<td>7</td>
<td>(\frac{d_p^2}{2M_p^2} + \frac{\tilde{\lambda}_V^{HA} v^2}{M_A^2} + \frac{\tilde{\lambda}_V^{HA} v^2}{M_V^2})</td>
<td>--</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>--</td>
</tr>
<tr>
<td>9</td>
<td>(-\frac{F_A \tilde{\lambda}_V^{HA} v}{M_A^2} - \frac{\tilde{F}_V \tilde{\lambda}_V^{HA} v}{M_V^2})</td>
<td>--</td>
</tr>
<tr>
<td>10</td>
<td>(-\frac{(c_T^1)^2}{2M_V^2} - \frac{(c_T^1)^2}{2M_A^2})</td>
<td>--</td>
</tr>
<tr>
<td>11</td>
<td>(-\frac{F_V^2}{M_V^2} - \frac{\tilde{F}_A^2}{M_A^2})</td>
<td>--</td>
</tr>
</tbody>
</table>

* Ecker et al. ’89
** Pich, IR, Santos and Sanz-Cillero ’16 [and in progress]
3. Estimation of the LECs

- Integration of the heavy modes
- Similar to the ChPT case*
- Results**

** Purely bosonic operators

** Two-fermion operators

\[
e^{iS_{\text{eff}}[\chi, \psi]} = \int [dR] e^{iS[\chi, \psi, R]}\]

<table>
<thead>
<tr>
<th>(i)</th>
<th>(\Delta F_{i}^{\psi^2})</th>
<th>(\Delta \bar{F}_{i}^{\psi^2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(\frac{c_d c_{S_i}}{2 M_{S_i}})</td>
<td>(-\frac{F_v C_0^V}{\sqrt{2} M_V^2} - \frac{F_A C_0^A}{\sqrt{2} M_A^2})</td>
</tr>
<tr>
<td>2</td>
<td>(-\frac{G_v}{\sqrt{2} M_V^2} - \frac{G_A}{\sqrt{2} M_A^2})</td>
<td>(-2 \frac{v \lambda_{hV}^b}{M_V^2} C_0^V - 2 \frac{\sqrt{2} v \lambda_{hA}^a C_0^A}{M_A^2})</td>
</tr>
<tr>
<td>3</td>
<td>(-\frac{F_v C_0^V}{\sqrt{2} M_V^2} - \frac{F_A C_0^A}{\sqrt{2} M_A^2})</td>
<td>(-\frac{\xi^V F_1}{\sqrt{2} M_V^2} c_1^V - \frac{c_1^A}{\sqrt{2} M_A^2})</td>
</tr>
<tr>
<td>4</td>
<td>(-\frac{\sqrt{2} F_v C_1^{V_1}}{M_{V_1}^2} - \frac{\sqrt{2} F_A c_1^{A_1}}{M_{A_1}^2})</td>
<td>(---)</td>
</tr>
<tr>
<td>5</td>
<td>(\frac{d_p c_1^P}{M_p^2})</td>
<td>(---)</td>
</tr>
<tr>
<td>6</td>
<td>(-\frac{c_7^P c_1^V}{\sqrt{2} M_{V_1}^2} - \frac{c_1^A c_1^{A_1}}{\sqrt{2} M_{A_1}^2})</td>
<td>(---)</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>(---)</td>
</tr>
</tbody>
</table>

* Ecker et al. '89
** Pich, IR, Santos and Sanz-Cillero ’16 [and in progress]
3. Estimation of the LECs

- Integration of the heavy modes
- Similar to the ChPT case*

Results**

<table>
<thead>
<tr>
<th>$i$</th>
<th>$\Delta F_i^{\nu^4}$</th>
<th>$\Delta \bar{F}_i^{\nu^4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\frac{(c_1^S)^2}{2M_3^2}$</td>
<td>$-\frac{c_1^V c_1^A}{M_3^2}$</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{(c_1^P)^2}{2M_3^2}$</td>
<td>$\frac{c_1^V c_1^A}{2M_3^2}$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{(c_1^S)^2}{4M_3^2}$</td>
<td>$\frac{c_1^V c_1^A}{4M_3^2}$</td>
</tr>
<tr>
<td>4</td>
<td>$-\frac{(c_1^P)^2}{4M_3^2}$</td>
<td>$\frac{c_1^V c_1^A}{4M_3^2}$</td>
</tr>
<tr>
<td>5</td>
<td>$\frac{(c_1^V)^2}{2M_3^2}$</td>
<td>$-\frac{(c_1^A)^2}{2M_3^2}$</td>
</tr>
<tr>
<td>6</td>
<td>$-\frac{(c_1^P)^2}{2M_3^2}$</td>
<td>$-\frac{(c_1^A)^2}{2M_3^2}$</td>
</tr>
<tr>
<td>7</td>
<td>$\frac{(c_1^V)^2}{4M_3^2}$</td>
<td>$-\frac{(c_1^A)^2}{4M_3^2}$</td>
</tr>
<tr>
<td>8</td>
<td>$\frac{(c_1^V)^2}{4M_3^2}$</td>
<td>$-\frac{(c_1^A)^2}{4M_3^2}$</td>
</tr>
<tr>
<td>9</td>
<td>$\frac{(c_1^V)^2}{2M_3^2}$</td>
<td>$-\frac{(c_1^A)^2}{2M_3^2}$</td>
</tr>
<tr>
<td>10</td>
<td>$\frac{(c_1^V)^2}{2M_3^2}$</td>
<td>$-\frac{(c_1^A)^2}{2M_3^2}$</td>
</tr>
</tbody>
</table>

* Ecker et al. '89
** Pich, IR, Santos and Sanz-Cillero '16 [and in progress]

Integrating out resonances in strongly-coupled electroweak scenarios, I. Rosell
4. Short-distance constraints and the purely bosonic sector*

- Only P-even bosonic operators
- Short-distance constraints coming from two-Goldstone and Higgs-Goldstone vector form factors and Weinberg Sum Rules.
- Results in terms of a few resonance parameters:

\[
\begin{align*}
\mathcal{F}_1 &= \frac{F_A^2}{4M_A^2} - \frac{F_V^2}{4M_V^2} = -\frac{v^2}{4} \left( \frac{1}{M_V^2} + \frac{1}{M_A^2} \right) \\
\mathcal{F}_2 &= -\frac{F_A^2}{8M_A^2} - \frac{F_V^2}{8M_V^2} = -\frac{v^2(M_A^4 + M_V^4)}{8M_V^2 M_A^2 (M_A^2 - M_V^2)} \\
\mathcal{F}_3 &= -\frac{F_V G_V}{2M_V^2} = -\frac{v^2}{2M_V^2} \\
\mathcal{F}_4 &= \frac{G_V^2}{4M_V^2} = \frac{(M_A^2 - M_V^2)v^2}{4M_V^2 M_A^2} \\
\mathcal{F}_5 &= \frac{c_1^2}{4M_A^2 S_1} - \frac{G_V^2}{4M_V^2} = \frac{c_2^2}{4M_A^2 S_1} - \frac{(M_A^2 - M_V^2)v^2}{4M_V^2 M_A^2} \\
\mathcal{F}_6 &= -\frac{(\lambda_1^A)^2 v^2}{M_A^2} = -\frac{M_V^2 (M_A^2 - M_V^2)v^2}{M_A^6} \\
\mathcal{F}_7 &= \frac{d_P^2}{2M_P^2} + \frac{(\lambda_1^A)^2 v^2}{M_A^2} = \frac{d_P^2}{2M_P^2} + \frac{M_V^2 (M_A^2 - M_V^2)v^2}{M_A^6} \\
\mathcal{F}_8 &= 0 \\
\mathcal{F}_9 &= -\frac{F_A \lambda_1^A v}{M_A^2} = -\frac{M_A^2 v^2}{M_A^4} 
\end{align*}
\]

* Pich, IR, Santos and Sanz-Cillero '16

Integrating out resonances in strongly-coupled electroweak scenarios, I. Rosell
Integrating out resonances in strongly-coupled electroweak scenarios, I. Rosell
5. Conclusions

1. What? Electroweak Strongly Coupled Models

2. Why? What if this new particle around 125 GeV is not a SM Higgs boson?
   - We should look for alternative ways of mass generation: strongly-coupled models.
   - They can be used to determine the LECs

3. Where? Effective approach
   - a) EWSB: $SU(2)_L \times SU(2)_R \rightarrow SU(2)_{L+R}$: similar to ChSB in QCD: ChPT.
   - b) Strongly-coupled models: similar to resonances in QCD: RChT.
   - c) Chiral power counting and short-distance constraints.
5. Conclusions

1. What?  
**Electroweak Strongly Coupled Models**

2. Why?  
What if this new particle around 125 GeV is not a SM Higgs boson?

- We should look for alternative ways of mass generation: strongly-coupled models.
- They can be used to determine the LECs


3. Where?  
**Effective approach**

- a) EWSB: $SU(2)_L \times SU(2)_R \rightarrow SU(2)_{L+R}$: similar to ChSB in QCD: ChPT.
- b) Strongly-coupled models: similar to resonances in QCD: RChT.
- c) *Chiral power counting* and short-distance constraints.

---

**Estimation of the LECs**

1. Building the low-energy and high-energy Lagrangian  
2. Equivalence between Proca and antisymmetric formalism  
3. Integrating out the resonances  
4. Estimation of the LECs  
5. Short-distance constraints

---

Integrating out resonances in strongly-coupled electroweak scenarios, I. Rosell
Backup slides: calculation of S and T

i) The Lagrangian

Let us consider a low-energy effective theory containing the SM gauge bosons coupled to the electroweak Goldstones, one light-scalar state \( h \) (the Higgs) and the lightest vector and axial-vector resonances:

\[
\mathcal{L} = \frac{v^2}{4} \langle u_\mu u^{\mu} \rangle \left( 1 + \frac{2\kappa_W}{v} h \right) + \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f^{\mu\nu}_+ \rangle + \frac{i G_V}{2\sqrt{2}} \langle V_{\mu\nu} [u^\mu, u^\nu] \rangle + \frac{F_A}{2\sqrt{2}} \langle A_{\mu\nu} f^{\mu\nu}_- \rangle + \sqrt{2} \lambda_1^{hA} \partial_\mu h \langle A^{\mu\nu} u^\nu \rangle
\]

Seven resonance parameters: \( \kappa_W, F_V, G_V, F_A, \lambda_1^{hA}, M_V \) and \( M_A \).

\( \kappa_W = \kappa_Z = a = \omega = 1 \) recovers the SM vertex

\( \pi \) and \( h \) sector

\( \pi \) and \( V \) sector

\( \pi, h \) and \( A \) sector

The high-energy constraints are fundamental.

ii) At leading-order (LO)*

\[
S_{LO} = 4\pi \left( \frac{F_V^2}{M_V^2} - \frac{F_A^2}{M_A^2} \right)
\]

\( T_{LO} = 0 \)

* Peskin and Takeuchi '92.

Integrating out resonances in strongly-coupled electroweak scenarios, I. Rosell
iii) At next-to-leading order (NLO)*

- Dispersive relations
- Only lightest two-particles cuts have been considered, since higher cuts are supposed to be suppressed**.

iv) High-energy constraints

- We have seven resonance parameters: importance of short-distance information.
- In contrast to QCD, the underlying theory is not known.
- Weinberg Sum-Rules (WSR)***:

\[
\Pi_{30}(s) = \frac{g^2 \tan \theta_W}{4} s \left[ \Pi_{VV}(s) - \Pi_{AA}(s) \right]
\]

\[
\begin{align*}
\frac{1}{\pi} \int_0^\infty dt \left[ \text{Im}\Pi_{VV}(t) - \text{Im}\Pi_{AA}(t) \right] &= v^2 \\
\frac{1}{\pi} \int_0^\infty dt t \left[ \text{Im}\Pi_{VV}(t) - \text{Im}\Pi_{AA}(t) \right] &= 0
\end{align*}
\]

- We have 7 resonance parameters and up to 5 constraints:
  - With both, the 1st and the 2nd WSR: \( \kappa_W \) and \( M_V \) as free parameters
  - With only the 1st WSR: \( \kappa_W, M_V \) and \( M_A \) as free parameters

* Barbieri et al.'08
* Cata and Kamenik '08
* Orgogozo and Rynchov '11 '12
* Pich, IR and Sanz-Cillero '12
*** Weinberg '67
*** Bernard et al. '75.

Integrating out resonances in strongly-coupled electroweak scenarios, I. Rosell
iii) At next-to-leading order (NLO)*

- Dispersive relations
- Only lightest two-particles cuts have been considered, since higher cuts are supposed to be suppressed**.

iv) High-energy constraints

- We have seven resonance parameters: importance of short-distance information.
- In contrast to QCD, the underlying theory is not known.
- Weinberg Sum-Rules (WSR)***:

\[ F_V^2 M_V^2 - F_A^2 M_A^2 = 0 \]

1st WSR at LO

\[ F_V G_V = v^2 \]

1st WSR at NLO (= VFF^ and AFF^^):

\[ F_A \lambda_1^{hA} = \kappa_W v \]

2nd WSR at LO

\[ F_V^2 - F_A^2 = v^2 \]

2nd WSR at NLO:

\[ \kappa_W = \frac{M_V^2}{M_A^2} \]

- We have 7 resonance parameters and up to 5 constraints:
  - With both, the 1st and the 2nd WSR: \( \kappa_W \) and \( M_V \) as free parameters
  - With only the 1st WSR: \( \kappa_W, M_V \) and \( M_A \) as free parameters

---

* Barbieri et al.'08
* Cata and Kamenik '08
* Orgogozo and Rynchov '11 '12

** Pich, IR and Sanz-Cillero '12

*** Weinberg '67

^ Ecker et al. '89

^^Pich, IR and Sanz-Cillero '08

Integrating out resonances in strongly-coupled electroweak scenarios, I. Rosell
Backup slides: S and T at LO and at NLO

\[ S = 0.03 \pm 0.10 \text{ (} M_H = 0.126 \text{ TeV)} \]
\[ T = 0.05 \pm 0.12 \text{ (} M_H = 0.126 \text{ TeV)} \]

i) LO results

i.i) 1st and 2nd WSRs**

\[ S_{LO} = \frac{4\pi v^2}{M_V^2} \left( 1 + \frac{M_V^2}{M_A^2} \right) \]

\[ \frac{4\pi v^2}{M_V^2} < S_{LO} < \frac{8\pi v^2}{M_V^2} \]

i.ii) Only 1st WSR***

\[ S_{LO} = 4\pi \left\{ \frac{v^2}{M_V^2} + F_A^2 \left( \frac{1}{M_V^2} - \frac{1}{M_A^2} \right) \right\} \]

\[ S_{LO} > \frac{4\pi v^2}{M_V^2} \]

At LO \( M_A > M_V > 1.5 \text{ TeV} \) at 95% CL

* Gfitter
* LEP EWWG
* Zfitter

** Peskin and Takeuchi ’92
*** Pich, IR and Sanz-Cillero ’12

Integrating out resonances in strongly-coupled electroweak scenarios, I. Rosell
ii) NLO results: 1st and 2nd WSRs*

\[1 > \kappa_W > 0.94\]
\[M_A \approx M_V > 5\, \text{TeV} \quad (68\%\, \text{CL})\]

iii) NLO results: 1st WSR and \(M_V < M_A^*\)


iv) Preliminary results: inclusion of fermion cut doesn’t change appreciably the results**.

✓ A moderate resonance-mass splitting implies \(\kappa_W \approx 1\).
✓ \(M_V < 1\, \text{TeV}\) implies large resonance-mass splitting.
✓ In any scenario \(M_A > 1.5\, \text{TeV}\) at 68% CL.

---

* Pich, IR and Sanz-Cillero ’13 ’14
** Pich, IR, Santos and Sanz-Cillero [in progress]