

# Novel applications of Lattice QCD

## Parton Distributions, proton charge radius and neutron electric dipole moment



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XII<sup>TH</sup> QUARK & THE HADRON SPECTRUM CONFINEMENT

XII<sup>TH</sup> Quark Confinement and the Hadron Spectrum

from 28 August 2016 to 4 September 2016  
Europe/Athens timezone

# Outline

1

## Introduction

- Current status of simulations
- Evaluation of form factors in lattice QCD

2

## Nucleon Electromagnetic form factors and radii

- Electromagnetic form factors
- Strange EM form factors
- EM radii  $\langle r_E^2 \rangle, \langle r_M^2 \rangle$

3

## Parton Distributions

- First moments:  $\langle x \rangle_q, \langle x \rangle_{\Delta q}, \langle x \rangle_{\delta q}$
- Nucleon spin
- Direct evaluation

4

## Electric Dipole Moment

5

## Conclusions

# Quantum ChromoDynamics (QCD)

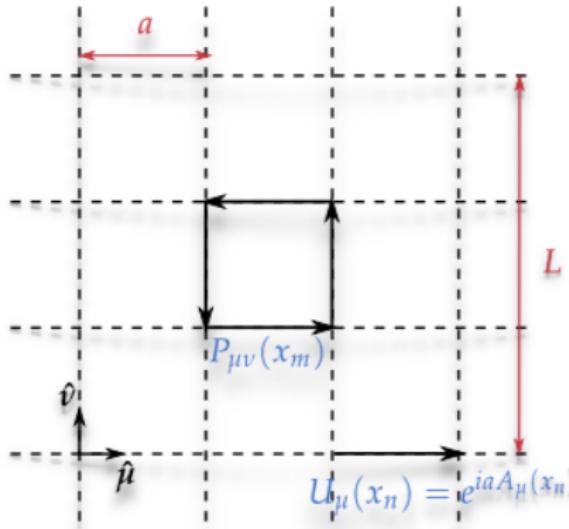
QCD-Gauge theory of the strong interaction

Lagrangian: formulated in terms of quarks and gluons

$$\mathcal{L}_{QCD} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \sum_{f=u,d,s,c,b,t} \bar{\psi}_f (i\gamma^\mu D_\mu - m_f) \psi_f$$

$$D_\mu = \partial_\mu - ig \frac{\lambda^a}{2} A_\mu^a$$

Choice of fermion discretisation scheme e.g. Clover, Twisted Mass, Staggered, Overlap, Domain Wall  
Each has its advantages and disadvantages



Eventually,

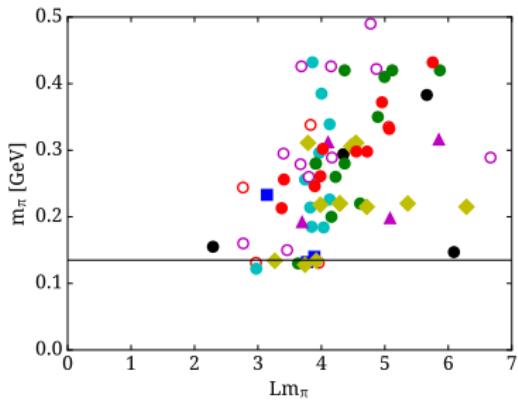
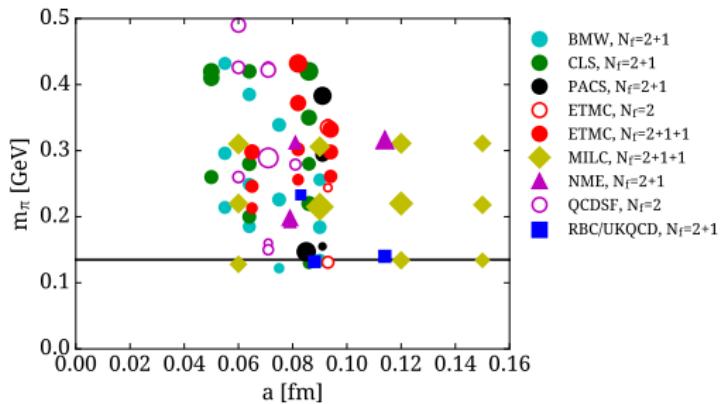
- all discretization schemes must agree in the continuum limit  $a \rightarrow 0$
- observables extrapolated to the infinite volume limit  $L \rightarrow \infty$

## Why nucleon structure?

With simulations at the physical value of the pion mass there is a number of interesting questions we want to address:

- Can we reproduce known quantities?
- Can we reproduce the excited spectrum of the nucleon and its associated resonances?
- Can we resolve the long-standing issue of the spin content of the nucleon?
- Can we determine accurately enough the charge radius of the proton?
- Can we provide input for experimental searches for new physics?

## Status of simulations



Size of the symbols according to the value of  $m_\pi L$ : smallest value  $m_\pi L \sim 3$  and largest  $m_\pi L \sim 6.7$ .

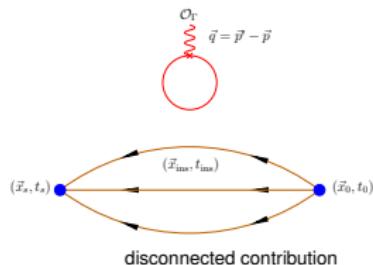
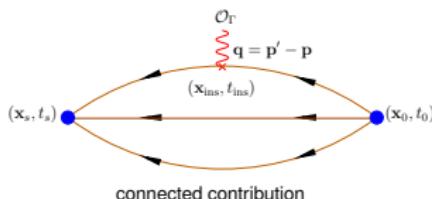
In this talk I will discuss three topics:

- Nucleon form factors and radii - using simulations with pion mass close to its physical value
- Parton distributions - moments using simulations with pion mass close to its physical value, direct evaluation using heavier pions
- Neutron electric dipole moment - using simulations with heavier than physical pion mass

# Evaluation of matrix elements

Three-point functions:

$$G^{\mu\nu}(\Gamma, \vec{q}, t_s, t_{\text{ins}}) = \sum_{\vec{x}_s, \vec{x}_{\text{ins}}} e^{i\vec{x}_{\text{ins}} \cdot \vec{q}} \Gamma_{\beta\alpha} \langle J_\alpha(\vec{x}_s, t_s) \mathcal{O}_\Gamma^{\mu\nu}(\vec{x}_{\text{ins}}, t_{\text{ins}}) \bar{J}_\beta(\vec{x}_0, t_0) \rangle$$



- Plateau method:

$$R(t_s, t_{\text{ins}}, t_0) \xrightarrow{(t_{\text{ins}} - t_0)\Delta \gg 1} \mathcal{M}[1 + \dots e^{-\Delta(\mathbf{p})(t_{\text{ins}} - t_0)} + \dots e^{-\Delta(\mathbf{p}')}(t_s - t_{\text{ins}})]$$

- Summation method: Summing over  $t_{\text{ins}}$ :

$$\sum_{t_{\text{ins}}=t_0}^{t_s} R(t_s, t_{\text{ins}}, t_0) = \text{Const.} + \mathcal{M}[(t_s - t_0) + \mathcal{O}(e^{-\Delta(\mathbf{p})(t_s - t_0)}) + \mathcal{O}(e^{-\Delta(\mathbf{p}')}(t_s - t_0))].$$

Excited state contributions are suppressed by exponentials decaying with  $t_s - t_0$ , rather than  $t_s - t_{\text{ins}}$  and/or  $t_{\text{ins}} - t_0$

However, one needs to fit the slope rather than to a constant or take differences and then fit to a constant

L. Maiani, G. Martinelli, M. L. Paciello, and B. Taglienti, Nucl. Phys. B293, 420 (1987); S. Capitani *et al.*, arXiv:1205.0180

- Fit keeping the first excited state, T. Bhattacharya *et al.*, arXiv:1306.5435

All should yield the same answer in the end of the day!

## Nucleon isovector charges: $g_A$ , $g_S$ , $g_T$

- axial-vector operator:  $\mathcal{O}_A^a = \bar{\psi}(x)\gamma^\mu\gamma_5\frac{\tau^a}{2}\psi(x)$
- scalar operator:  $\mathcal{O}_S^a = \bar{\psi}(x)\frac{\tau^a}{2}\psi(x)$
- pseudoscalar:  $\mathcal{O}_P^a = \bar{\psi}(x)\gamma_5\frac{\tau^a}{2}\psi(x)$
- tensor operator:  $\mathcal{O}_T^a = \bar{\psi}(x)\sigma^{\mu\nu}\frac{\tau^a}{2}\psi(x)$

⇒ extract from matrix element:  $\langle N(\vec{p}') \mathcal{O}_X N(\vec{p}) \rangle|_{q^2=0}$

- Axial charge  $g_A$
- Scalar charge  $g_S$
- Pseudoscalar charge  $g_P$ , • Tensor charge  $g_T$

(i) isovector combination has no disconnect contributions; (ii)  $g_A$  well known experimentally, Goldberger-Treiman relation yields  $g_P$ ,  $g_T$  to be measured at JLab, Predict  $g_S$

See talks:

- Monday Section B: S. Collins and C.A.
- Friday Section E: M. Constantinou

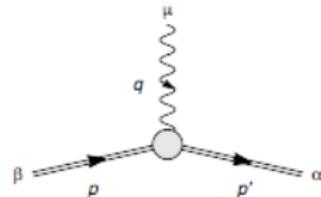
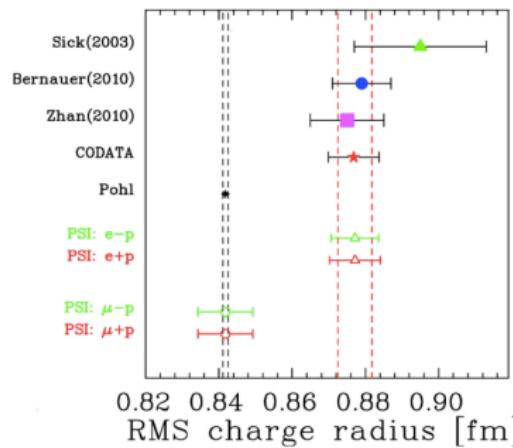
## Electromagnetic form factors

# Electromagnetic form factors

$$\langle N(p', s') | j^\mu(0) | N(p, s) \rangle = \bar{u}_N(p', s') \left[ \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} F_2(q^2) \right] u_N(p, s)$$

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_N^2} F_2(q^2)$$

$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$

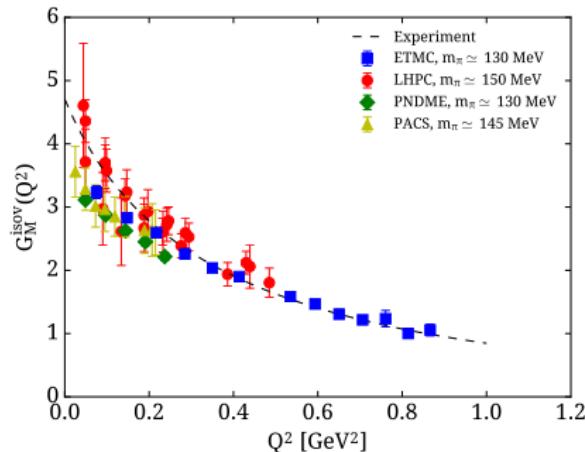
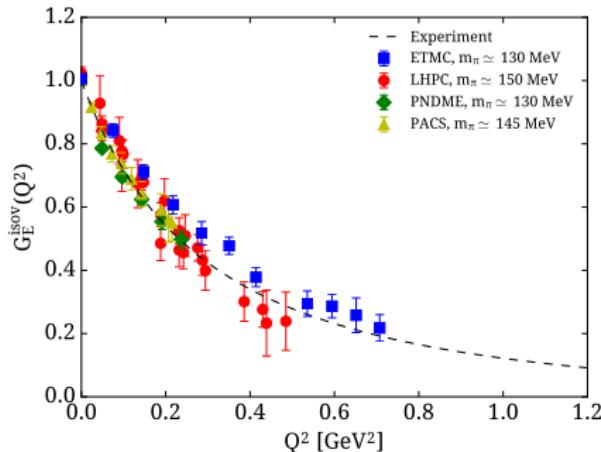


E. J. Downie, EPJ Conf. 113 (2016) 05021

- Proton radius extracted from muonic hydrogen is  $7.9\sigma$  different from the one extracted from electron scattering, R. Pohl *et al.*, Nature 466 (2010) 213
- Muonic measurement is ten times more accurate and a reanalysis of electron scattering data may give agreement with muonic measurement
- The Mainz A1 collaboration at MAMI has measured at low  $Q^2$  and find  $r_p = 0.879(5)_{\text{stat}}(4)_{\text{syst}}(2)_{\text{model}}(4)_{\text{group}}$  fm in agreement with the CODATA06 value of 0.8768(69) fm J. C. Bernauer *et al.*, Phys. Rev. C 90, 015206 (2014)
- Other analyses of electron scattering data that include the Mainz data yield consistency with muonic results e.g. K. Griffioen, C. Carlson, S. Maddox, PRC 93 (2016) 015204; I. T. Lorenz, H.-W. Hammer, and Ulf-G. Meissner, EPJ A (2012), arXiv:1205.6628; D. W. Higinbotham *et al.*, 1510.01293 - See review talk by R. Hill

# Recent results on the electric and magnetic form factors

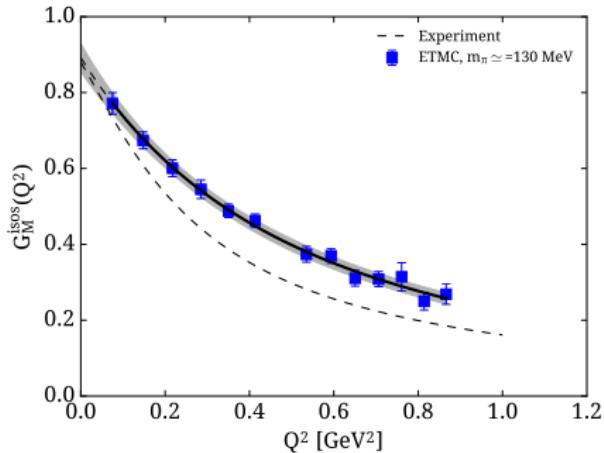
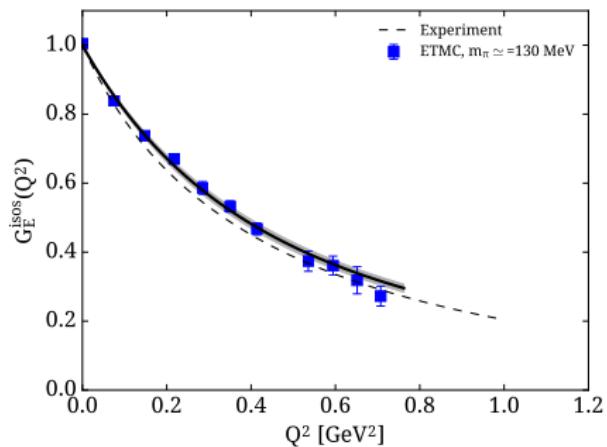
## Isovector form factors



- ETMC using  $N_f = 2$  twisted mass fermions (TMF),  $a = 0.093$  fm,  $48^3 \times 96$   $G_E$  with  $t_s = 1.7$  fm and 66,000 statistics,  $G_M$  with  $t_s = 1.3$  fm and 9,300 statistics
- LHPC using  $N_f = 2 + 1$  clover fermions,  $a = 0.116$  fm,  $48^4$ , summation method with 3 values of  $t_s$  from 0.9 fm to 1.4 fm and  $\sim 7,800$  statistics, 1404.4029
- PNDME mixed action HISQ  $N_f = 2 + 1 + 1$  and clover valence,  $a = 0.087$  fm,  $64^3 \times 96$ , summation method with 3 values of  $t_s$  from 0.9 fm to 1.4 fm and  $\sim 7,000$  HP and  $\sim 85,000$  NP, Yong-Chull Jang, Lattice 2016
- PACS using  $N_f = 2 + 1$  clover fermions,  $a = 0.085$  fm,  $96^3 \times 192$ ,  $t_s = 1.3$  fm, 9,300 statistics, Y. Kuramashi, Lattice 2016

## Recent results on the electric and magnetic form factors

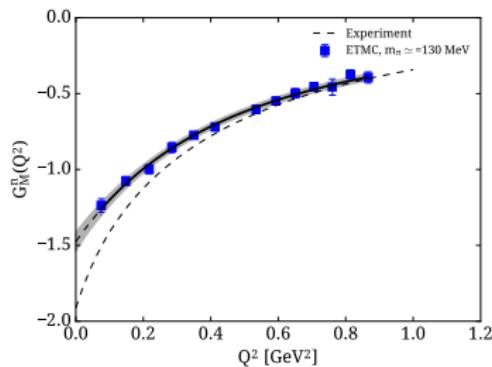
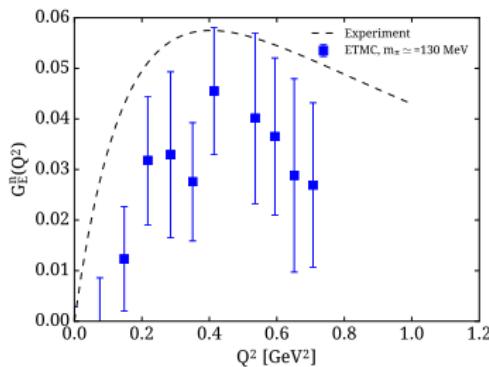
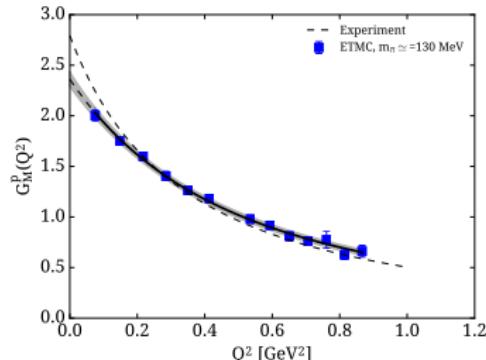
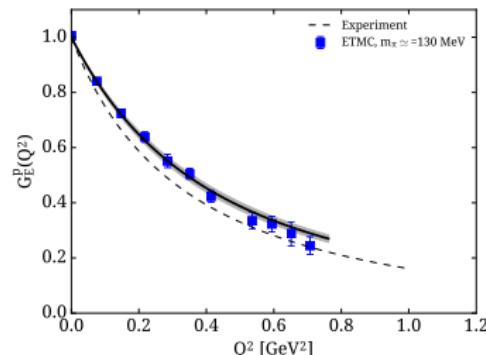
Isoscalar form factors - connected contributions



- ETMC using  $N_f = 2$  twisted mass fermions (TMF),  $a = 0.093 \text{ fm}$ ,  $48^3 \times 96$   $G_E$  with  $t_s = 1.7 \text{ fm}$  and 66,000 statistics,  $G_M$  with  $t_s = 1.3 \text{ fm}$  and 9,300 statistics

# Recent results on the electric and magnetic form factors

## Isoscalar form factors - connected contributions



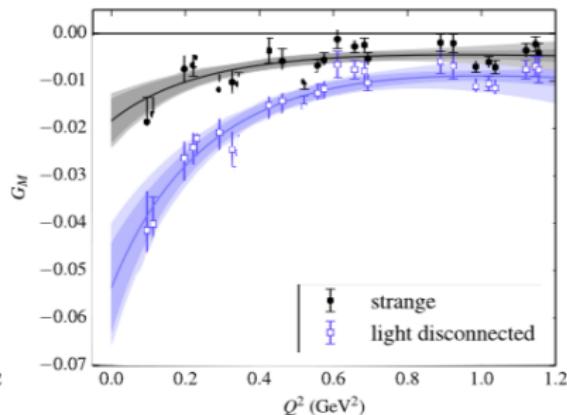
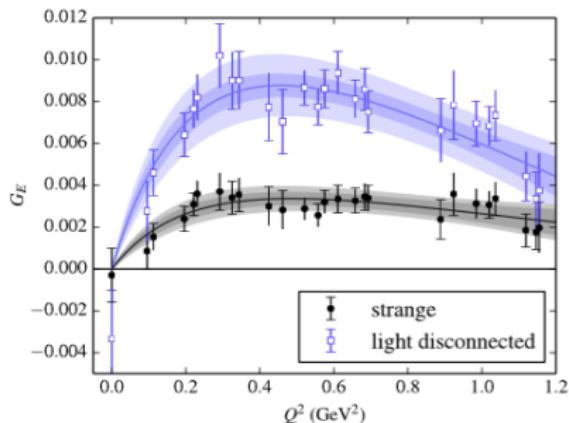
- ETMC using  $N_f = 2$  twisted mass fermions (TMF),  $a = 0.093 \text{ fm}$ ,  $48^3 \times 96$   $G_E$  with  $t_s = 1.7 \text{ fm}$  and 66,000 statistics,  $G_M$  with  $t_s = 1.3 \text{ fm}$  and 9,300 statistics

# Strange Electromagnetic form factors

Experimental determination: Parity violating  $e - N$  scattering

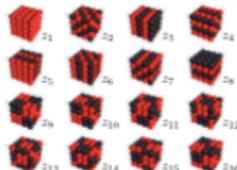
HAPPEX experiment finds  $G_M^S(0.62) = -0.070(67)$

New methods for disconnected fermion loops: hierarchical probing, A. Stathopoulos, J. Laeuchli, K. Orginos, arXiv:1302.4018



$N_f = 2 + 1$  clover fermions,  $m_\pi \sim 320$  MeV, J. Green et al., Phys.Rev. D92 (2015) 3,

031501, arXiv: 1505.01803

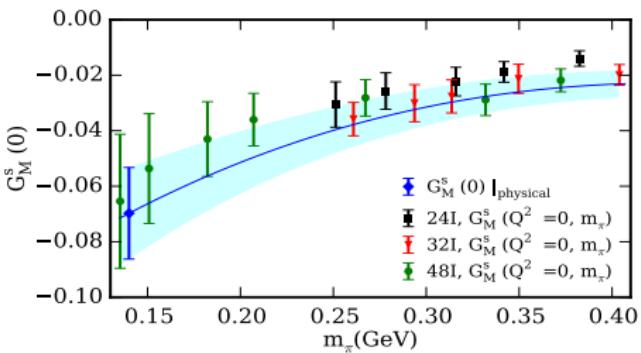
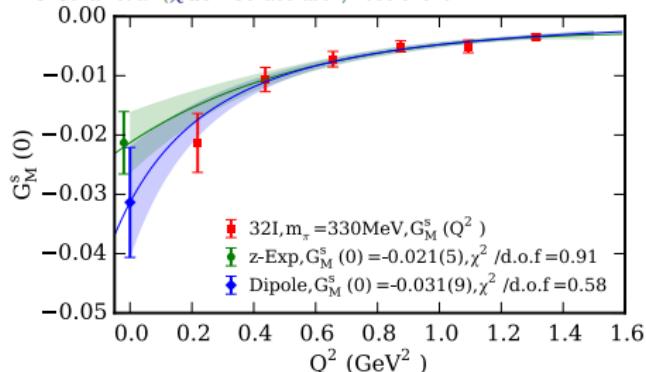


Sampling of the fermion propagator using site colouring schemes

# Strange Electromagnetic form factors

Experimental determination: Parity violating  $e - N$  scattering  
HAPPEX experiment finds  $G_M^s(0.62) = -0.070(67)$

R. S. Sufian et al. (QCDSF Collaboration) 1606.07075



Overlap valence on  $N_f = 2 + 1$  domain wall fermions,  $24^3 \times 64$ ,  $a = 0.11 \text{ fm}$ ,  $m_\pi = 330 \text{ MeV}$ ;  $32^3 \times 64$ ,  $a = 0.083 \text{ fm}$ ,  $m_\pi = 300 \text{ MeV}$  and  $48^3$ ,  $a=0.11 \text{ fm}$ ,  $m_\pi = 139 \text{ MeV}$

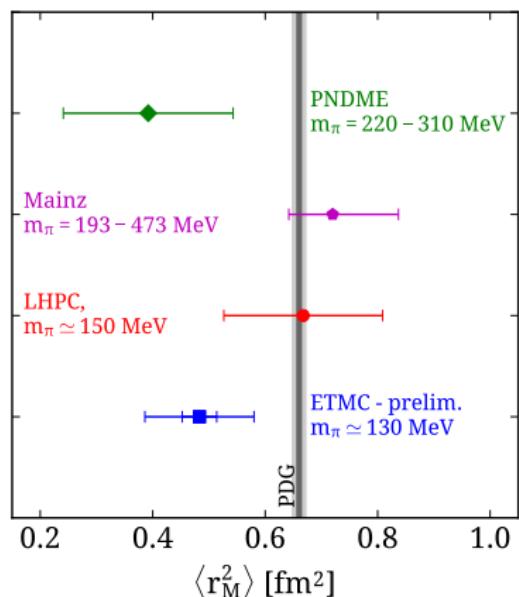
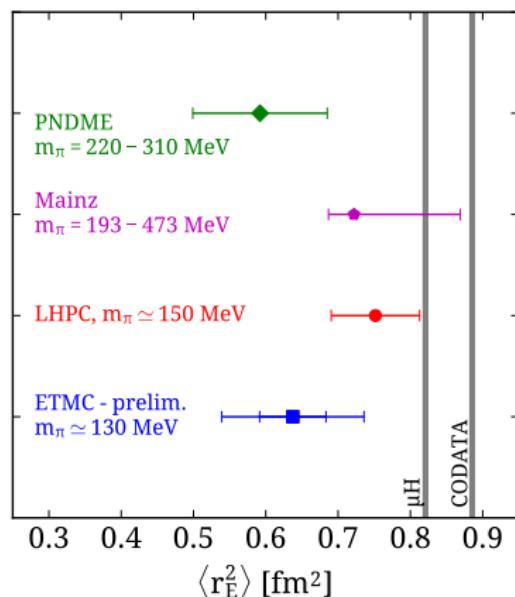
## Electromagnetic radii

Slope at  $Q^2 \rightarrow 0$  yields the radius :  $\langle r_{EM}^2 \rangle = -\frac{6}{G_{EM}(0)} \frac{dG_{EM}(Q^2)}{dQ^2} \Big|_{Q^2=0}$

We need:

- An Ansatz for the  $Q^2$ -dependence of the form factors → dipole fit:  $\frac{G_0}{(1+Q^2/M^2)^2}$ ,  $z$ -expansion
- Low values of  $Q^2$ , lowest momentum  $2\pi/L \rightarrow$  large spatial length  $L$

Only connected



Need further study and better accuracy

## Position methods

- Avoid model dependence-fits
- Application to Sachs form factors → nucleon isovector magnetic moment  $G_M^{\text{isov}}(0)$

$$\lim_{t \rightarrow \infty} \lim_{t_s - t \rightarrow \infty} \frac{C^{3pt\mu}(t_s, t, \vec{q}, \Gamma_\nu)}{C^{2pt}s} = \Pi^\mu(\vec{q}, \Gamma_\nu),$$

$G_M$  is extracted from:

$$\Pi_i(\vec{q}, \Gamma_k) = -C \frac{1}{4m_N} \epsilon_{ijk} q_j G_M(Q^2)$$

⇒ Due to the factor  $q_j$  the magnetic moment  $G_M(0)$  cannot be extracted directly

Use instead

$$\lim_{q^2 \rightarrow 0} \frac{\partial}{\partial q_j} \Pi_i(t, \vec{q}, \Gamma_k) = \frac{1}{2m_N} \epsilon_{ijk} G_M(0).$$

- Check with  $G_E$ :

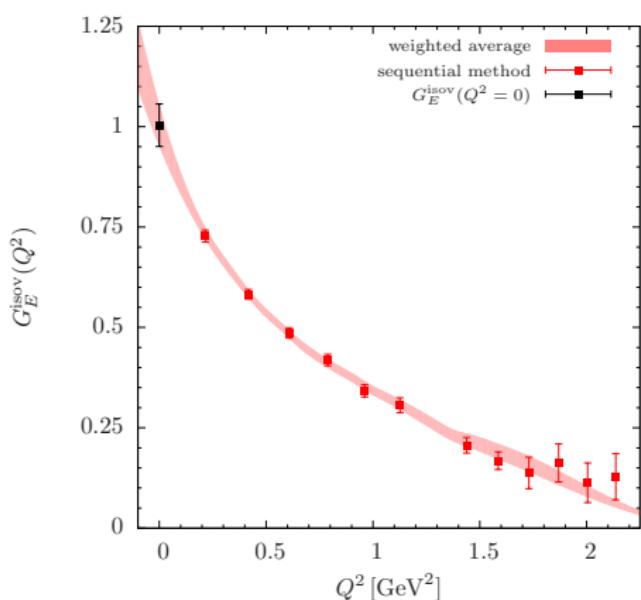
$$\Pi_i(\vec{q}, \Gamma_0) = -C \frac{i}{2m_N} q_i G_E(Q^2)$$

- Then apply to Isovector rms charge radius of the nucleon and the Neutron electric dipole moment

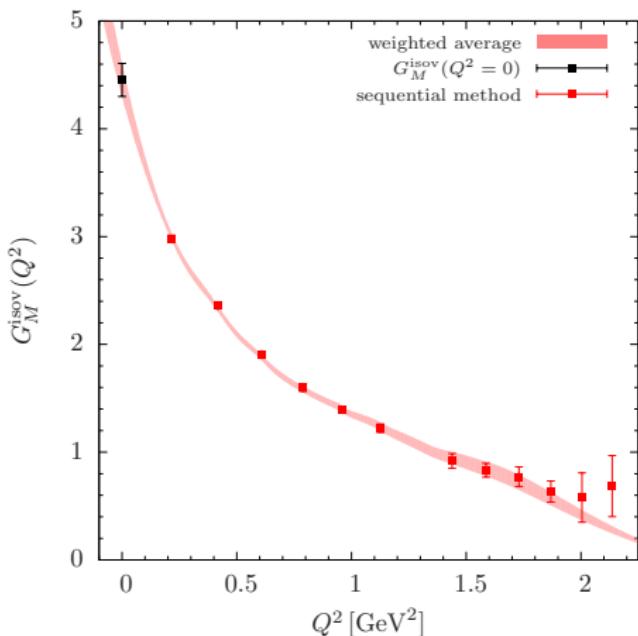
As a first step we calculated  $G_M(0)$  (equivalently  $F_2(0)$ ) at  $m_\pi = 373$  MeV.

# Magnetic moment $G_M^{\text{isov}}(0)$

- In principle, values at larger  $Q^2$  have very little influence
- Value for  $G_M^{\text{isov}} = 4.45(15)_{\text{stat}}$  larger than result from dipole fit  $3.99(9)_{\text{stat}}$
- Closer to exp. value (4.71)



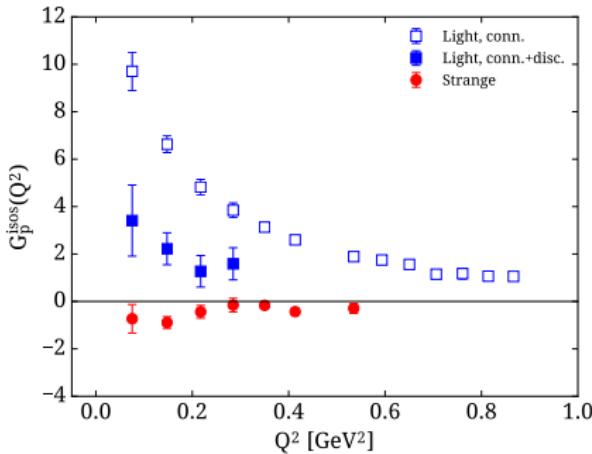
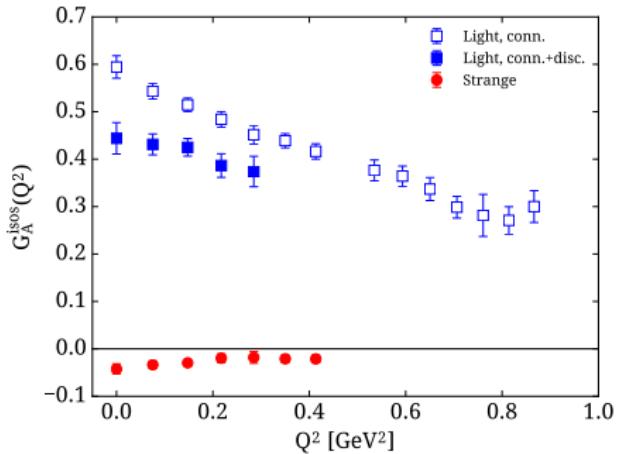
$G_E(0) = 1$  confirmed



$G_M^{\text{isov}}(0)$  from  $\mathcal{O}(4700)$  gauge confs of B55;  $t_s/a = 14$

## Recent results on nucleon axial form factors

e.g. isoscalar form factors from ETMC



- ETMC using  $N_f = 2$  twisted mass fermions (TMF),  $a = 0.093$  fm,  $48^3 \times 96$   $G_E$  with  $t_s = 1.7$  fm and 66,000 statistics,  $G_M$  with  $t_s = 1.3$  fm and 9,300 statistics
- See talk by Sara Collins, Section B, Monday 29th Aug.

# Parton Distributions

# Moments of Generalized Parton Distributions

Factorization leads to matrix elements of local operators:

- vector operator

$$\mathcal{O}_{V^a}^{\mu_1 \dots \mu_n} = \bar{\psi}(x) \gamma^{\{\mu_1} i \overset{\leftrightarrow}{D}^{\mu_2} \dots i \overset{\leftrightarrow}{D}^{\mu_n\}} \frac{\tau^a}{2} \psi(x)$$

- axial-vector operator

$$\mathcal{O}_{A^a}^{\mu_1 \dots \mu_n} = \bar{\psi}(x) \gamma^{\{\mu_1} i \overset{\leftrightarrow}{D}^{\mu_2} \dots i \overset{\leftrightarrow}{D}^{\mu_n\}} \gamma_5 \frac{\tau^a}{2} \psi(x)$$

- tensor operator

$$\mathcal{O}_{T^a}^{\mu_1 \dots \mu_n} = \bar{\psi}(x) \sigma^{\{\mu_1, \mu_2} i \overset{\leftrightarrow}{D}^{\mu_3} \dots i \overset{\leftrightarrow}{D}^{\mu_n\}} \frac{\tau^a}{2} \psi(x)$$

Special cases:

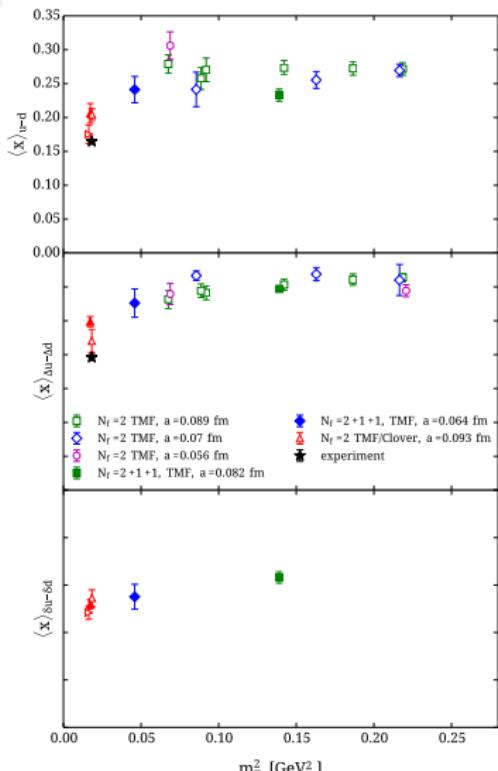
- no-derivative → nucleon form factors
- For  $Q^2 = 0$  → parton distribution functions  
one-derivative → first moments e.g. average momentum fraction  $\langle x \rangle$   
Generalized form factor decomposition:

$$\langle N(p', s') | \mathcal{O}_{V^3}^{\mu\nu} | N(p, s) \rangle = \bar{u}_N(p', s') \left[ A_{20}(q^2) \gamma^{\{\mu} P^{\nu\}} + B_{20}(q^2) \frac{i \sigma^{\{\mu\alpha} q_\alpha P^{\nu\}}}{2m} + C_{20}(q^2) \frac{q^{\{\mu} q^{\nu\}}}{m} \right] \frac{1}{2} u_N(p, s)$$

$$\text{Nucleon spin } J^q = \frac{1}{2} \left[ A_{20}(0) + B_{20}(0) \right] \text{ and } \langle x \rangle_q = A_{20}(0)$$

## Momentum fraction $\langle x \rangle_{u-d}$ , helicity $\langle x \rangle_{u+d}$ and transversity $\langle x \rangle_{u-d}$

Updated results using  $N_f = 2$  twisted mass fermions with a clover term at a physical value of the pion mass,  $48^3 \times 96$  and  $a = 0.093(1)$  fm with  $\sim 9260$  statistics for  $t_s/a = 10, 12, 14$ ,  $\sim 48000$  for  $t_s/a = 16$  and  $\sim 70000$  for  $t_s/a = 18$ .



At the physical point we find in the  $\overline{\text{MS}}$  at 2 GeV from the plateau method:

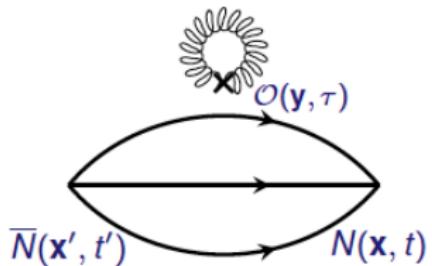
- $\langle x \rangle_{u-d} = 0.206(14)(5)$  and
- $\langle x \rangle_{u+d+s} = 0.78(10)$ .
- $\langle x \rangle_{u+d+s}$  is perturbatively renormalized to one-loop due to its mixing with the gluon operator.
- $\langle x \rangle_{\Delta u - \Delta d} = 0.259(9)(10)$
- $\langle x \rangle_{\delta u - \delta d} = 0.273(17)(18)$

The first error is statistical and the second systematic determined by the difference between the values from the plateau and two-state fits.

A. Abdel-Rehim *et al.* (ETMC):1507.04936, 1507.05068, 1411.6842, 1311.4522

## Gluon content of the nucleon

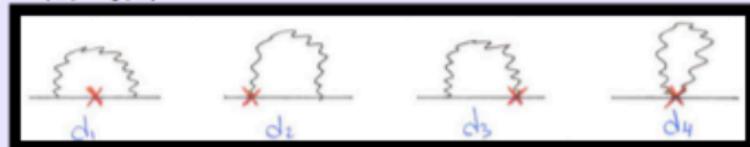
- Gluons carry a significant amount of momentum and spin in the nucleon
  - ▶ Compute gluon momentum fraction :  $\langle x \rangle_g = A_{20}^g$
  - ▶ Compute gluon spin:  $J_g = \frac{1}{2}(A_{20}^g + B_{20}^g)$
- Nucleon matrix of the gluon operator:  $O_{\mu\nu} = -G_{\mu\rho} G_{\nu\rho}$   
→ gluon momentum fraction extracted from  
 $\langle N(0) | O_{44} - \frac{1}{3} O_{jj} | N(0) \rangle = m_N < x >_g$
- Disconnected correlation function, known to be very noisy  
⇒ we employ several steps of **stout smearing** in order to remove fluctuations in the gauge field
- Results are computed on the  $N_f = 2$  ensemble at the physical point,  $m_\pi = 131$  MeV,  $a = 0.093$  fm,  
 $V = 48^3 \times 96$ , [A. Abdel-Rehim et al. \(ETMC\):1507.04936](#)
- The methodology was tested for  $N_f = 2 + 1 + 1$  twisted mass at  $m_\pi = 373$  MeV, [C. Alexandrou, V. Drach, K. Hadjyiannakou, K. Jansen, B. Kostrzewa, C. Wiese, PoS LATTICE2013 \(2014\) 289](#)



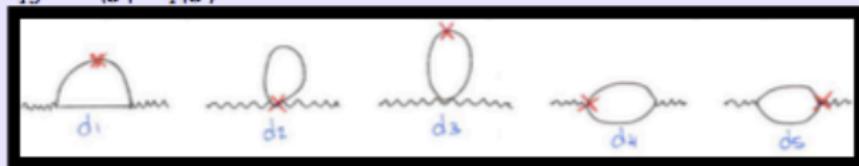
## Nucleon gluon moment-Renormalization

Mixing with  $\langle x \rangle_{u+d+s} \Rightarrow$  Perturbation theory - M. Constantinou and H. Panagopoulos

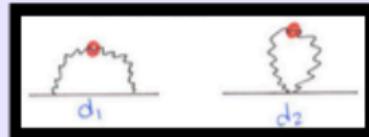
$$\times Z_{qq} : \quad \Lambda_{qq} = \langle q | \mathcal{O}_q | q \rangle$$



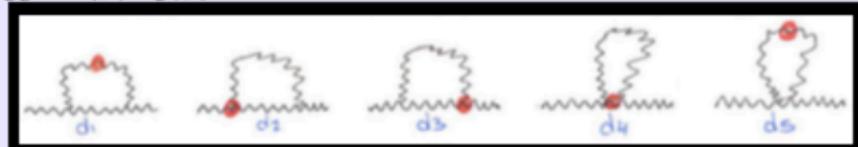
$$\times Z_{qg} : \quad \Lambda_{qg} = \langle g | \mathcal{O}_q | g \rangle$$



$$\bullet Z_{gq} : \quad \Lambda_{gq} = \langle q | \mathcal{O}_g | q \rangle$$



$$\bullet Z_{gg} : \quad \Lambda_{gg} = \langle g | \mathcal{O}_g | g \rangle$$



## Nucleon gluon moment-Renormalization

Mixing with  $\langle x \rangle_{u+d+s} \implies$  Perturbation theory - M. Constantinou and H. Panagopoulos

$\times Z_{qq} : \quad \Lambda_{qq} = \langle q | \mathcal{O}_q | q \rangle$

$$Z_{gg} = 1 + \frac{g^2}{16\pi^2} \left( 1.0574 N_f + \frac{-13.5627}{N_c} - \frac{2 N_f}{3} \log(a^2 \bar{\mu}^2) \right)$$

$\times Z_{qg} : \quad \Lambda_{qg} = \langle g | \mathcal{O}_q | g \rangle$

$$Z_{qg} = 0 + \frac{g^2 C_f}{16\pi^2} (0.8114 + 0.4434 c_{SW} - 0.2074 c_{SW}^2 + \frac{4}{3} \log(a^2 \bar{\mu}^2))$$

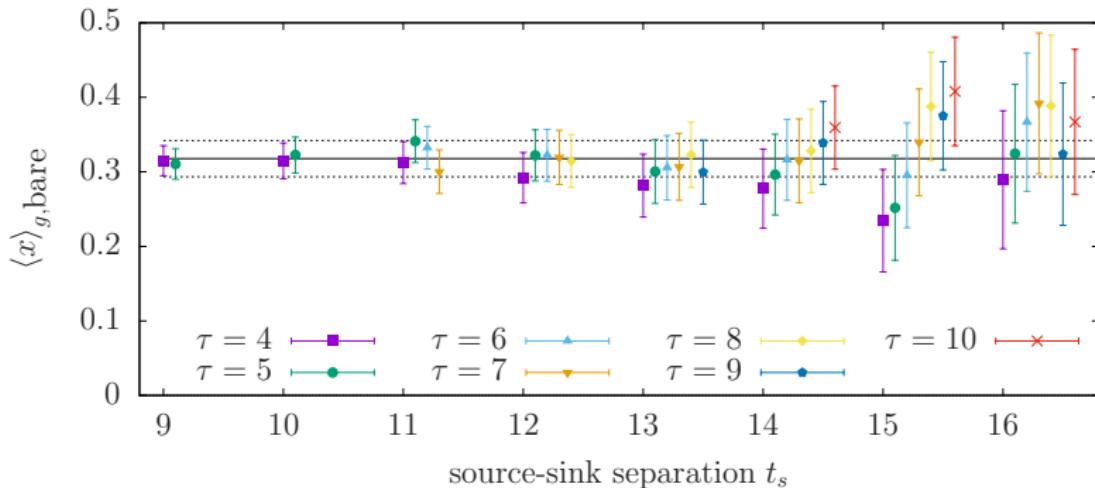
•  $Z_{gq} : \quad \Lambda_{gq} = \langle q | \mathcal{O}_g | q \rangle$

$$Z_{gq} = 1 + \frac{g^2}{16\pi^2} (-1.8557 + 2.9582 c_{SW} + 0.3984 c_{SW}^2 - \frac{8}{3} \log(a^2 \bar{\mu}^2))$$

•  $Z_{gg} : \quad \Lambda_{gg} = \langle g | \mathcal{O}_g | g \rangle$

$$Z_{gg} = 0 + \frac{g^2 N_f}{16\pi^2} (0.2164 + 0.4511 c_{SW} + 1.4917 c_{SW}^2 - \frac{4}{3} \log(a^2 \bar{\mu}^2))$$

## Results for the gluon content

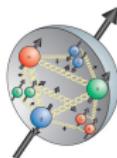


- 2094 gauge configurations with 100 different source positions each → more than 200 000 measurements
- Due to mixing with the quark singlet operator, the renormalization and mixing coefficients had to be extracted from a one-loop perturbative lattice calculation, M. Constantinou and H. Panagopoulos
- $\langle x \rangle_{g,\text{bare}} = 0.318(24) \xrightarrow{\text{Renormalization}} \langle x \rangle_g^R = Z_{gg} \langle x \rangle_g + Z_{gq} \langle x \rangle_{u+d+s} = 0.321(23)(16)$ . The renormalization is perturbatively done using two-levels of stout smearing. The systematic error is the difference between using one- and two-levels of stout smearing.
- Momentum sum is satisfied:  $\sum_q \langle x \rangle_q + \langle x \rangle_g = \langle x \rangle_{u+d}^{CI} + \langle x \rangle_{u+d+s}^{DI} + \langle x \rangle_g = 1.10(10)(2)$

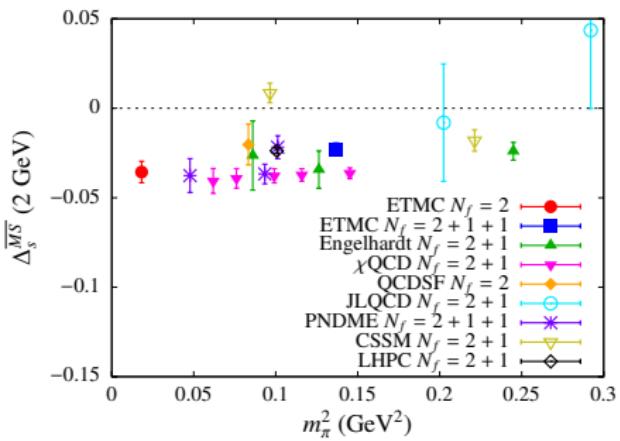
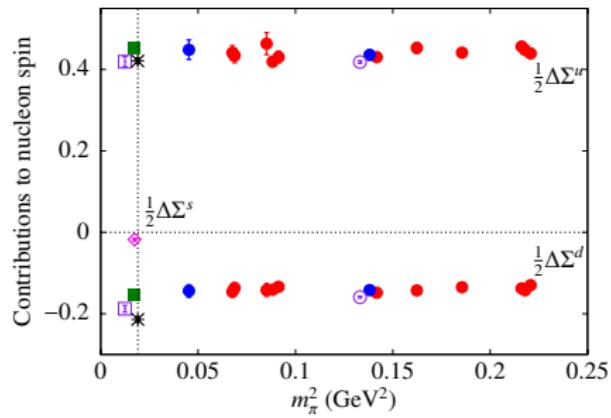
# Nucleon spin?

$$\text{Spin sum: } \frac{1}{2} = \sum_q \underbrace{\left( \frac{1}{2} \Delta \Sigma^q + L^q \right)}_{J^q} + J^G$$

$$J^q = A_{20}^q(0) + B_{20}^q(0) \text{ and } \Delta \Sigma^q = g_A^q$$



Disconnected contribution using  $\mathcal{O}(860000)$  statistics

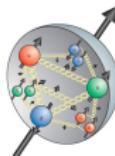


- $\Delta \Sigma^{u,d}$  consistent with experimental values after disconnected contributions are included

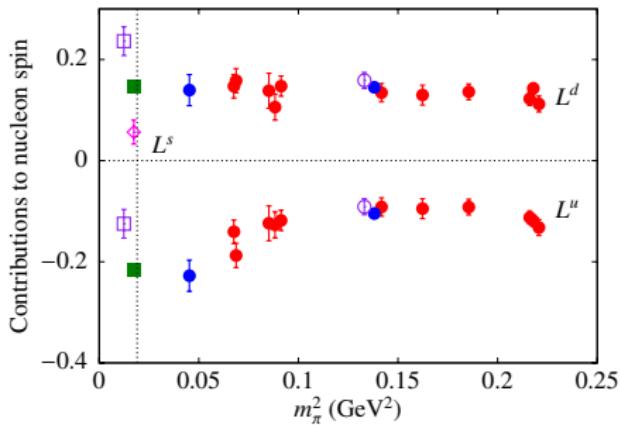
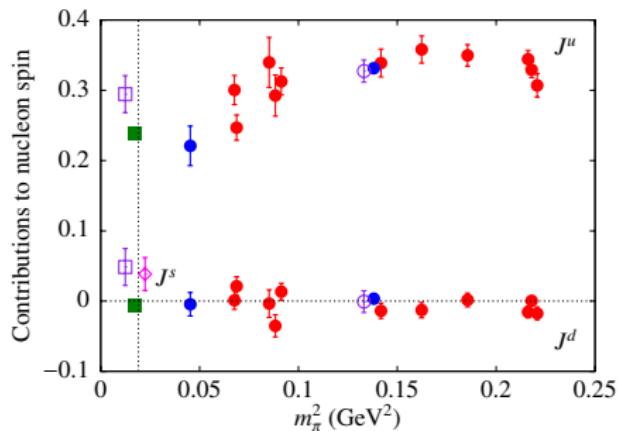
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$$\text{Spin sum: } \frac{1}{2} = \sum_q \underbrace{\left( \frac{1}{2} \Delta \Sigma^q + L^q \right)}_{J^q} + J^G$$

$$J^q = A_{20}^q(0) + B_{20}^q(0) \text{ and } \Delta \Sigma^q = g_A^q$$



Disconnected contribution using  $\mathcal{O}(860000)$  statistics



⇒ Total spin for u-quarks  $J^u < 0.3$  and for d-quark  $J^d \sim 0$

- Diconnected contributions affect the value of  $L^q$
- Preliminary results:  $L^u$  and  $L^d$  increase if disconnected are included

# Direct evaluation of parton distribution functions - an exploratory study

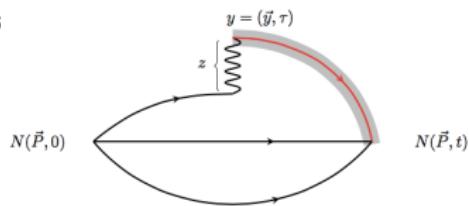
$$\tilde{a}_n(x, \Lambda, P_3) = \int_{-\infty}^{+\infty} dx x^{n-1} \tilde{q}(x, \Lambda, P_3),$$

$$\tilde{q}(x, \Lambda, P_3) = \int_{-\infty}^{+\infty} \frac{dz}{4\pi} e^{-izxP_3} \underbrace{\langle P | \bar{\psi}(z, 0) \rangle \gamma_3 W(z) \psi(0, 0) | P \rangle}_{h(P_3, z) \rightarrow \text{can be computed in LQCD}}$$

is the quasi-distribution defined by X. Ji Phys.Rev.Lett. 110 (2013) 262002, arXiv:1305.1539

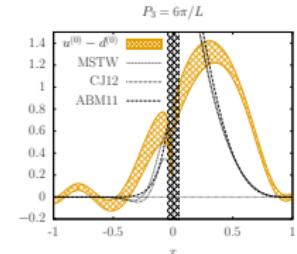
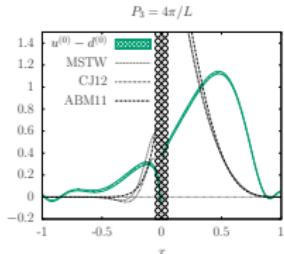
Exploratory calculations:

- Huey-Wen Lin *et al.* Phys. Rev. D91 (2015) 054510, Clover on  $N_f = 2 + 1 + 1$  HISQ,  $m_\pi = 310$  MeV and Jiunn-Wei Chen *et al.*, arXiv:1603.06664
- C.A., K. Cichy, E. G. Ramos, V. Drach, K. Hadjiyiannakou, K. Jansen, F. Steffens, C. Wiese, Phys.Rev. D92 (2015) 014502
- $N_f = 2 + 1 + 1$ ,  $V = 32^3 \times 64$ ,  $m_\pi = 373$  MeV,  $a \approx 0.082$  fm
- 1000 gauge configurations with 15 source positions each and 2 sets stochastic samples
- 30 000 measurements
- 5 steps of HYP smearing for the gauge links in the operator
- Stochastic method for the three-point functions
- Matching and mass corrections are included
- Currently under study for future applications:
  - ▶ Renormalization
  - ▶ A new smearing method indicates an improvement of errors that can enable us to reach larger momentum

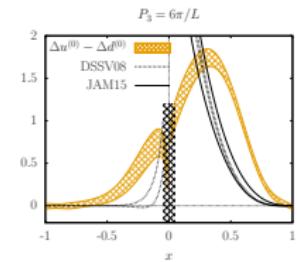
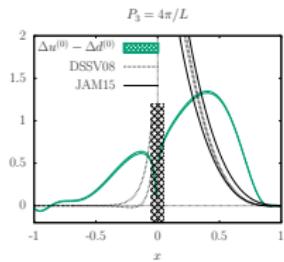


# The momentum, helicity and transversity parton distributions

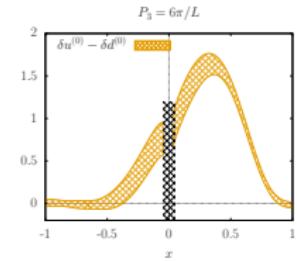
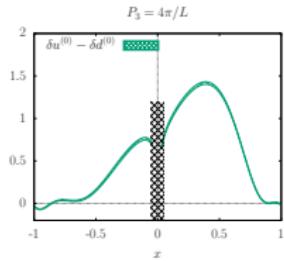
- $\tilde{q}(x, P_3) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-izk_3} \langle P | \bar{\psi}(z) \gamma_3 W_3(z, 0) \psi(0) | P \rangle$
- crossing relation:  $\bar{q}(x) = -q(-x)$
- negative  $x$  region  $\Rightarrow \bar{d} - \bar{u}$



- $\Delta q(x) = q^\uparrow(x) - q^\downarrow(x)$
- $\Delta \tilde{q}(x, P_3) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-izk_3} \langle P | \bar{\psi}(z) \gamma_5 \gamma_3 W_3(z, 0) \psi(0) | P \rangle$
- crossing relation:  $\Delta \bar{q}(x) = \Delta q(-x)$
- negative  $x$  region  $\Rightarrow \Delta \bar{u} - \Delta \bar{d}$



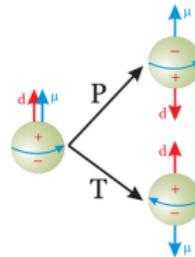
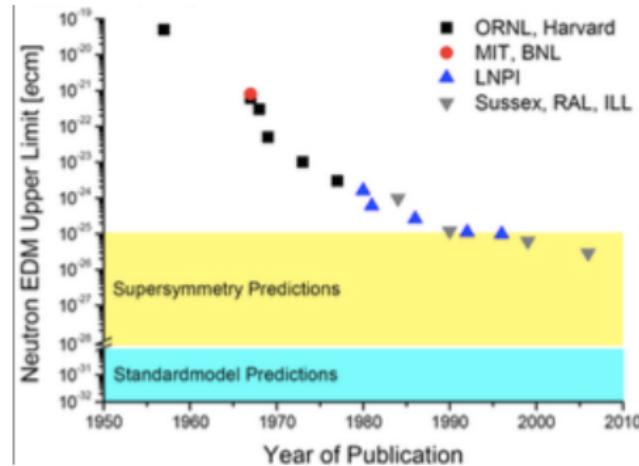
- $\delta q(x) = q^\top(x) - q^\perp(x)$
- $\delta \tilde{q}(x, P_3) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-izk_3} \langle P | \bar{\psi}(z) \gamma_j \gamma_3 W_3(z, 0) \psi(0) | P \rangle$
- crossing relation:  $\delta \bar{q}(x) = -\delta q(-x)$
- negative  $x$  region  $\Rightarrow \delta \bar{d} - \delta \bar{u}$



# Neutron Electric Dipole Moment (nEDM)

# Neutron Electric Dipole Moment (nEDM)

Probe for beyond the standard model physics



Current best upper limit :  $|d_n| < 2.9 \times 10^{-26} \text{ e cm}$  (90% C.L. from ILL Grenoble)

# Lattice determination of Neutron Electric Dipole Moment (nEDM)

Add  $\theta$ -term to the Langragian → complex action

- Measure neutron energy in an external electric field
- Simulate with imaginary  $\theta$ , see e.g. QCDSF, Guo *et al.* 2015
- Assume  $\theta$  is small and expand to first order: Compute the CP-violating form factor  $F_3(0) \rightarrow$

$$|d_n| = \lim_{q^2 \rightarrow 0} \frac{F_3(q^2)}{2m_N}$$

But  $F_3(0)$  cannot be determined directly → use:

- Fit the  $q^2$ -dependence
- Use space methods to extract it

# How to extract nEDM in lattice QCD

- Presence of a non-zero  $\theta$ -term  $\sim \frac{g^2 \theta}{32\pi^2} G\tilde{G} \equiv \theta\omega$  allows for  $CP$  in QCD

$$\langle \mathcal{O}(x_1, \dots, x_n) \rangle_\theta = \frac{1}{Z_0} \int d[G] d[\psi] d[\bar{\psi}] \mathcal{O}(x_1, \dots, x_n) e^{-S_{QCD} - i\theta \int \omega(x) d^4x}.$$

⇒ Sign problem due to  $S_\theta = i\theta Q_{\text{top}} = -i\theta \int \omega(x) d^4x$

- Treat  $\theta$  as a small perturbation instead:

$$\langle \mathcal{O}(x_1, \dots, x_n) \rangle_\theta = \langle \mathcal{O}(x_1, \dots, x_n) \rangle_{\theta=0} + \langle \mathcal{O}(x_1, \dots, x_n) (-i\theta \int \omega(x) d^4x) \rangle_{\theta=0} + \mathcal{O}(\theta^2).$$

- On the lattice we need to calculate the correlation between topological charge  $Q_{\text{top}} = \int d^4x \frac{g^2}{32\pi^2} G\tilde{G}$  and 2,3-pt functions
- Several different methods on the lattice for nEDM; skip many technical details ... See talk by A. Athenodorou, Tuesday, Section B

JHEP 0304 (2003) 019

Phys.Rev. D72 (2005) 014504

Phys.Rev. D75 (2007) 034507

Phys.Rept. 470 (2009) 93-150

## Nucleon matrix element for nEDM

- Form factor decomposition for the 3pt function reads

$$\langle N^\theta(\vec{p}_f, s) | J_\mu^{\text{EM}} | N^\theta(\vec{p}_i, s') \rangle \sim \bar{u}_N^\theta(\vec{p}_f, s) \Lambda_\mu^\theta(q) u_N^\theta(\vec{p}_i, s')$$

where  $\Lambda_\mu^\theta(q) = \Lambda_\mu^{\text{even}}(q) + i\theta \Lambda_\mu^{\text{odd}}(q) + \mathcal{O}(\theta^2)$  contains a (standard)  $\mathcal{CP}$ -even and a  $\mathcal{CP}$ -odd part

$$\begin{aligned}\Lambda_\mu^{\text{even}}(q) &= \gamma_\mu F_1(q^2) + \frac{F_2(q^2)}{2m_N} q_\nu \sigma_{\mu\nu}, \\ \Lambda_\mu^{\text{odd}}(q) &= \frac{F_3(q^2)}{2m_N} q_\nu \sigma_{\mu\nu} \gamma_5 + F_A(q^2) (q_\mu \gamma \cdot q - \gamma_\mu q^2) \gamma_5.\end{aligned}$$

- The 3pt function can be calculated from

$$C_{3pt}^{\theta, \mu}(t_s t, \vec{q}, \Gamma_\nu) = \langle N(\vec{p}_f, t_s) J_\mu^{\text{EM}}(\vec{q}, t) \bar{N}(\vec{p}_i, 0) e^{i\theta Q} \rangle.$$

## CP-odd form factor $F_3(0)$

- For the same projector  $\Gamma_k$  as used for  $G_M^{\text{iso}}$ , we obtain

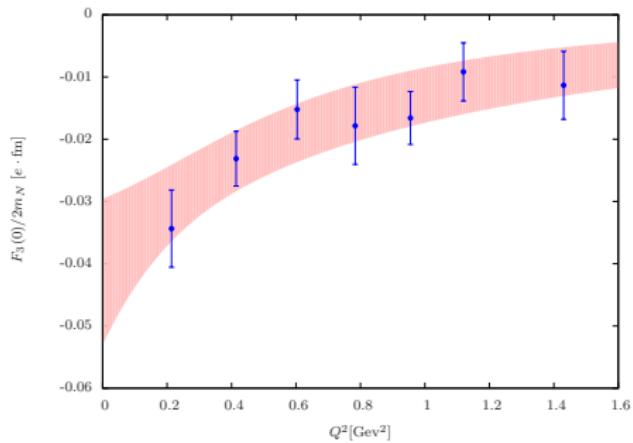
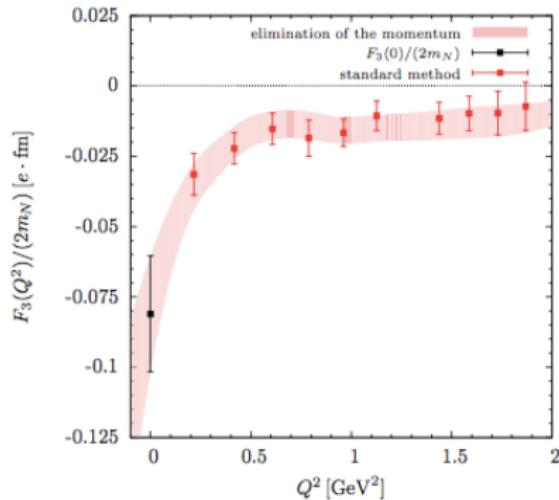
$$\Pi_0^\theta(\vec{q}, \Gamma_k) = \theta C \frac{i}{4m_N} \left[ \alpha^1 q_k F_1(Q^2) + \frac{q_k(E+3m_N)\alpha^1 F_2(Q^2)}{2m_N} + \frac{q_k(E+m_N)F_3(Q^2)}{2m_N} \right].$$

- Need  $\alpha^1$  as input
- Build linear combination of CP-even ( $\Pi_i(\vec{q}, \Gamma_0)$ ,  $\Pi_i(\vec{q}, \Gamma_k)$ ) and CP-odd ratio  $\Pi_0^\theta(\vec{q}, \Gamma_k)$  to isolate  $F_3(Q^2)$ 
  - ⇒ Can apply “derivative” to remove  $q_k$  for  $F_3$  in the same way as for  $G_M^{\text{iso}}$
  - ... or use standard method, i.e. fit ansatz to extract  $F_3(0)/(2m_N)$
- $\alpha^1$  can be determined from ratios suitably projected of 2pt functions at large  $t$

$$\frac{C_{2\text{pt}}^\theta(t, \gamma_5)}{C_{2\text{pt}}(t, 1 + \gamma_0)} \rightarrow 2i\alpha^1\theta$$

## Results for nEDM

- We find a non-zero signal for the nEDM
- All definitions of  $Q_{\text{top}}$  give signal
- Momentum elimination method and dipole fit yield results that are compatible within the errors



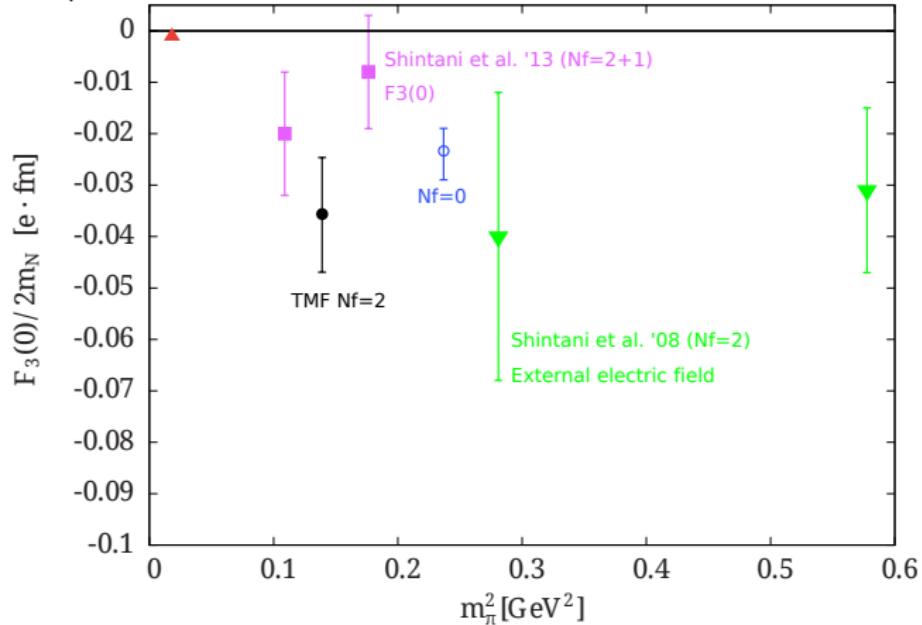
$\mathcal{O}(4700)$  gauge confs of  $B55.32$  using improved gluonic  $Q_{\text{top}}$  and gradient flow to define  $Q_{\text{top}}$

## Results on nEDM

- $N_f = 2 + 1 + 1$  twisted mass,  $a = 0.082$  fm,  $m_\pi = 373$  MeV

Use gradient flow to define the topological charge

Comparison of results



ETMC, C. Alexandrou *et al.*, arXiv:1510.05823

Computation at the physical point under study

# Conclusions

## Conclusions and Future Perspectives

- Confirm  $g_A$ ,  $\langle x \rangle_{u-d}$ , etc, at the physical point
- Provide predictions for  $g_s$ ,  $g_T$ , tensor moment, sigma-terms, etc.
- Increase statistics on the proton radius using e.g. position methods
- Compute gluonic observables
- Nucleon excited states and resonance properties |item Compute scattering lengths
- ...

# European Twisted Mass Collaboration

European Twisted Mass Collaboration (ETMC)



Cyprus (Univ. of Cyprus, Cyprus Inst.), France (Orsay, Grenoble), Germany (Berlin/Zeuthen, Bonn, Frankfurt, Hamburg, Münster), Italy (Rome I, II, III, Trento), Netherlands (Groningen), Poland (Poznan), Spain (Valencia), Switzerland (Bern), UK (Liverpool)

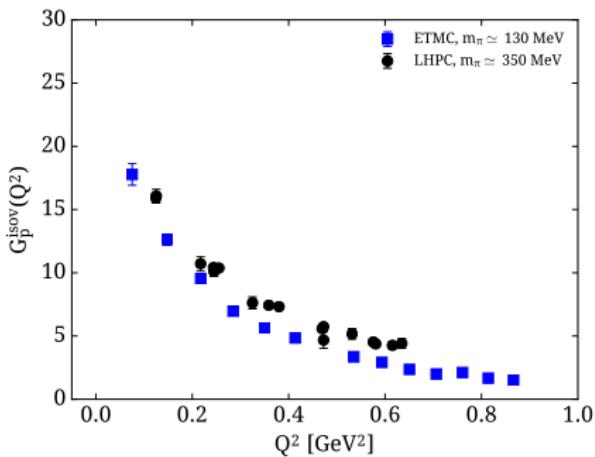
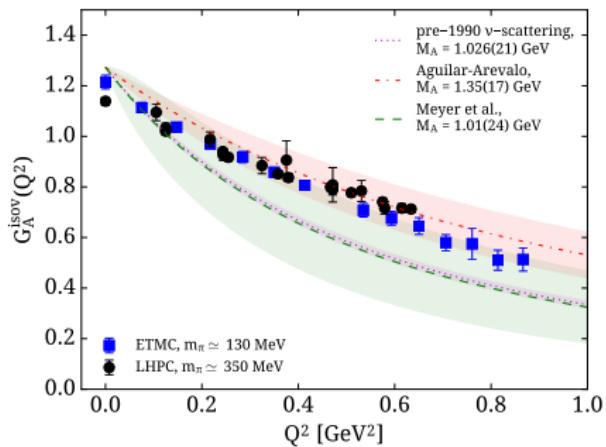
Collaborators:

A. Abdel-Rehim, S. Bacchio, K. Cichy, M. Constantinou, V. Drach, E. Garcia Ramos, J. Finkenrath, K. Hadjyiannakou, K.Jansen, Ch. Kallidonis, G. Koutsou, K. Ott nad, M. Petschlies, F. Steffens, A. Vaquero, C. Wiese

## Backup slides

# Recent results on nucleon axial form factors

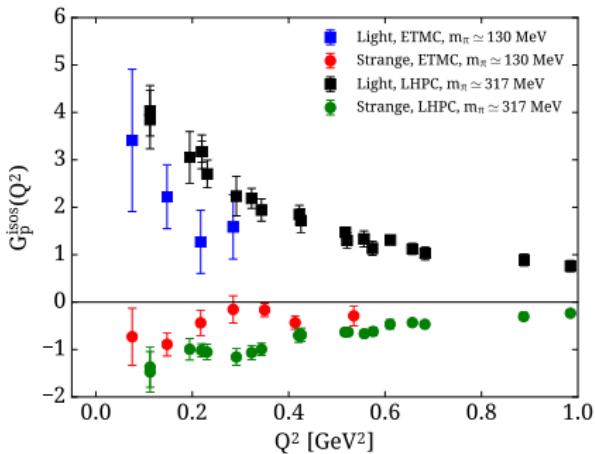
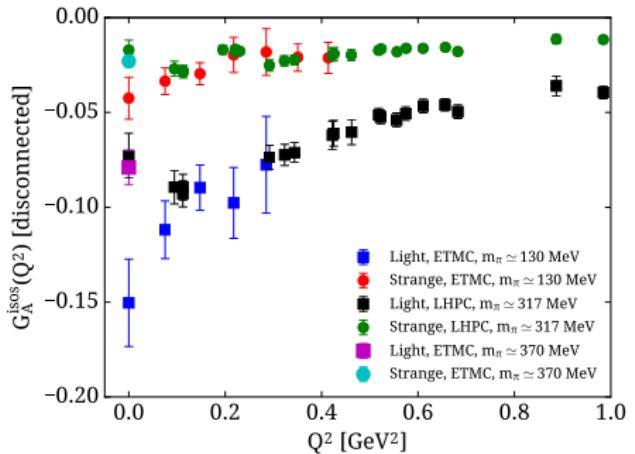
## Isovector form factors



- ETMC using  $N_f = 2$  twisted mass fermions (TMF),  $a = 0.093 \text{ fm}$ ,  $48^3 \times 96$   $G_E$  with  $t_s = 1.7 \text{ fm}$  and 66,000 statistics,  $G_M$  with  $t_s = 1.3 \text{ fm}$  and 9,300 statistics
- LHPC using  $N_f = 2 + 1$  clover fermions,  $a = 0.116 \text{ fm}$ ,  $48^4$ , summation method with 3 values of  $t_s$  from 0.9 fm to 1.4 fm and  $\sim 7,800$  statistics, 1404.4029

# Nucleon axial form factors

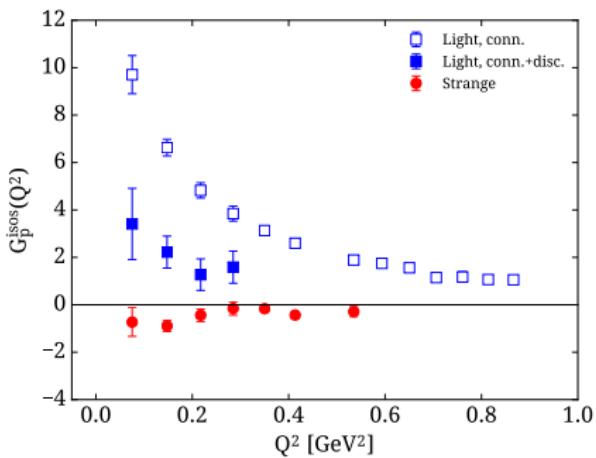
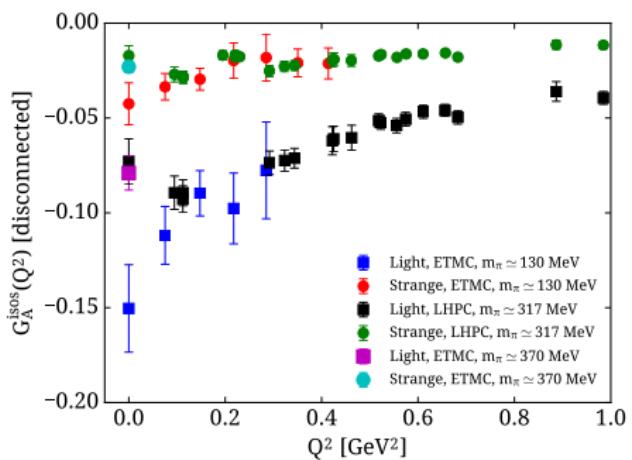
## Disconnected contributions



- ETMC using  $N_f = 2$  twisted mass fermions (TMF),  $a = 0.093$  fm,  $48^3 \times 96$ ,  $a = 0.093$  fm,  $m_\pi = 131$  MeV, 855,000 statistics
- LHPG using  $N_f = 2 + 1$  clover fermions,  $32^3 \times 96$ ,  $a = 0.114$  fm,  $m_\pi = 317$  MeV, 98,700 statistics

# Nucleon axial form factors

## Disconnected contributions

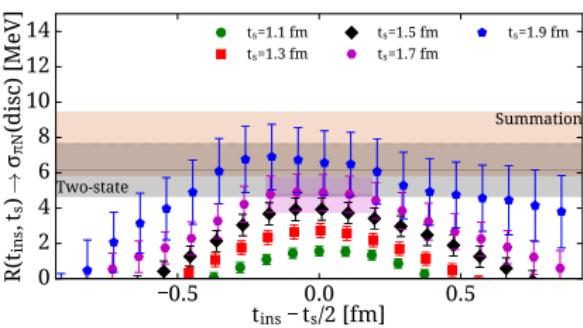
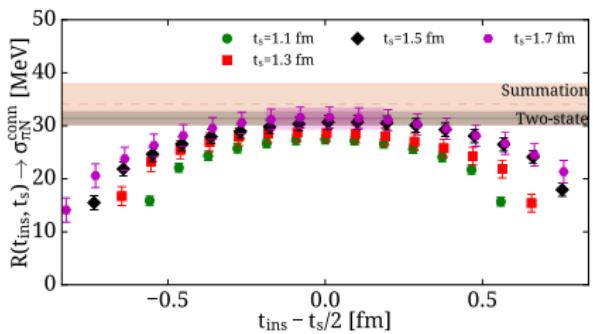


## Large disconnected contributions

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- LHPC using  $N_f = 2 + 1$  clover fermions,  $32^3 \times 96$ ,  $a = 0.114$  fm,  $m_\pi = 317$  MeV, 98,700 statistics

# The quark content of the nucleon or nucleon $\sigma$ -terms

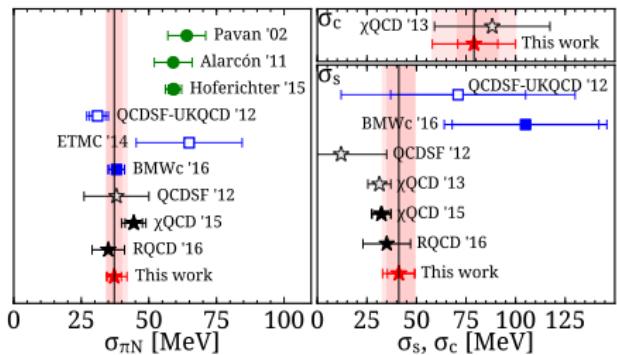
- $\sigma_f \equiv m_f \langle N | \bar{q}_f q_f | N \rangle$ : measures the explicit breaking of chiral symmetry  
Largest uncertainty in interpreting experiments for dark matter searches - Higgs-nucleon coupling depends on  $\sigma$ , J. Ellis, K. Olive, C. Savage, arXiv:0801.3656
- In lattice QCD:
  - ▶ Feynman-Hellmann theorem:  $\sigma_I = m_I \frac{\partial m_N}{\partial m_I}$
  - ▶ Similarly  $\sigma_s = m_s \frac{\partial m_N}{\partial m_s}$
  - ▶ Direct computation of the scalar matrix element, A. Abdel-Rehim et al. arXiv:1601.3656, PRL116 (2016) 252001



With our increased statistics we find  $\sigma_{\pi N} = 36(2)$  MeV,  $\sigma_s = 37(8)$  MeV,  $\sigma_c = 83(17)$  MeV

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