Recent Progress in Neutron-Antineutron Oscillation Theory

Michael Wagman (UW/INT)

Quark Confinement and the Hadron Spectrum XII

with Michael Buchoff, Enrico Rinaldi, Chris Schroeder, and Joe Wasem (LLNL), and Sergey Syritsyn (Jefferson Lab/Stony Brook)
Neutron-Antineutron Oscillations

$n\bar{n}$ violates fundamental symmetries of baryon number and $B - L$, sensitive to different physics than proton decay

Testable signature of possible BSM baryogenesis mechanisms explaining matter-antimatter asymmetry
Neutron-Antineutron Phenomenology

Similarities to kaon, neutrino oscillations

\[ P_{n\bar{n}}(t) = \sin^2 \left( \frac{t}{\tau_{n\bar{n}}} \right) e^{-\Gamma_n t} \]

\[ \frac{1}{\tau_{n\bar{n}}} = \langle \bar{n} | H_{n\bar{n}} | n \rangle \]

Magnetic fields, nuclear interactions modify transition rate
**Experimental Constraints**

**ILL:** $\tau_{n\bar{n}} > 2.7$ years

**SNO:** $\tau_{n\bar{n}} > 5.7$ years (preliminary)

**Super K:** $\tau_{n\bar{n}} > 11$ years
Experimental Outlook

European Spallation Source could have 1000 times ILL sensitivity, probe 30 times higher $\tau_{n\bar{n}}$ within next decade
Neutron-Antineutron Theory: The Standard Model and Beyond

Theory must make robust predictions for \( \tau_{nn} \) to reliably interpret the constraints from these experiments.
Baryogenesis

Baryon asymmetry and $n\bar{n}$ produced by same interactions in several BSM theories

Post-sphaleron baryogenesis in e.g. left-right symmetric theories predicts there is a theoretical upper bound on $\tau_{n\bar{n}}$

Babu, Dev, Fortes, and Mohapatra (2013)
Six-Quark Operators

SM effective theory describes $n\bar{n}$ with six-quark operators

$$\mathcal{H}_{n\bar{n}} = \sum_I C_I^{\overline{MS}}(M) Q_I^{\overline{MS}}(M) \quad Q_I \sim uudddd$$

BSM theories predict coefficients at high scales

Early estimates of six-quark matrix elements in bag model  
Rao and Shrock (1984)

Lattice QCD needed for reliable connection between BSM predictions and experimental constraints on $\tau_{n\bar{n}}$
\[ \psi = \begin{pmatrix} u \\ d \end{pmatrix} \]

**Chiral Operator Basis**

\[ Q_1 = (\psi C P_R i \tau^2 \psi)(\psi C P_R i \tau^2 \psi)(\psi C P_R i \tau^2 \tau^+ \psi) T^{AAS} = \mathcal{D}_R \mathcal{D}_R \mathcal{D}_R^+ T^{AAS} \]

<table>
<thead>
<tr>
<th>Chiral Basis</th>
<th>Fixed-Flavor Basis</th>
<th>Chiral Tensor Structure</th>
<th>Chiral Irrep</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q_1 )</td>
<td>( \mathcal{O}^3_{RRR} )</td>
<td>( \mathcal{D}_R \mathcal{D}_R \mathcal{D}_R^+ T^{AAS} )</td>
<td>((1_L, 3_R))</td>
</tr>
<tr>
<td>( Q_2 )</td>
<td>( \mathcal{O}^3_{LRR} )</td>
<td>( \mathcal{D}_L \mathcal{D}_R \mathcal{D}_R^+ T^{AAS} )</td>
<td>((1_L, 3_R))</td>
</tr>
<tr>
<td>( Q_3 )</td>
<td>( \mathcal{O}^3_{LLR} )</td>
<td>( \mathcal{D}_L \mathcal{D}_L \mathcal{D}_R^+ T^{AAS} )</td>
<td>((1_L, 3_R))</td>
</tr>
<tr>
<td>( Q_4 )</td>
<td>( 4/5 \mathcal{O}^2_{RRR} + 1/5 \mathcal{O}^1_{RRR} )</td>
<td>( \mathcal{D}_R^{33+} T^{SSS} )</td>
<td>((1_L, 7_R))</td>
</tr>
<tr>
<td>( Q_5 )</td>
<td>( \mathcal{O}^1_{RLL} )</td>
<td>( \mathcal{D}_R^{-} \mathcal{D}_L^{++} T^{SSS} )</td>
<td>((5_L, 3_R))</td>
</tr>
<tr>
<td>( Q_6 )</td>
<td>( \mathcal{O}^2_{RLL} )</td>
<td>( \mathcal{D}_L^3 \mathcal{D}_L^{3+} T^{SSS} )</td>
<td>((5_L, 3_R))</td>
</tr>
<tr>
<td>( Q_7 )</td>
<td>( 2/3 \mathcal{O}^2_{LLR} + 1/3 \mathcal{O}^1_{LLR} )</td>
<td>( \mathcal{D}_R^{33} T^{SSS} )</td>
<td>((5_L, 3_R))</td>
</tr>
<tr>
<td>( \tilde{Q}_1 )</td>
<td>( 1/3 \mathcal{O}^2_{RRR} - 1/3 \mathcal{O}^1_{RRR} )</td>
<td>( \mathcal{D}_R \mathcal{D}_R \mathcal{D}_R^+ T^{SSS} )</td>
<td>((1_L, 3_R))</td>
</tr>
<tr>
<td>( \tilde{Q}_3 )</td>
<td>( 1/3 \mathcal{O}^2_{LLR} - 1/3 \mathcal{O}^1_{LLR} )</td>
<td>( \mathcal{D}_L \mathcal{D}_L \mathcal{D}_R^+ T^{SSS} )</td>
<td>((1_L, 3_R))</td>
</tr>
</tbody>
</table>

**Chiral symmetry provides a basis with no operator mixing**

Tiburzi (unpublished)
Regularization-Independent Renormalization

RI-MOM scheme: hold vertex functions $\Lambda_I$ fixed to tree-level values at chosen reference scale $\mu$ (momentum subtraction)


$$\mathcal{P}_I \text{ projects contribution from tree-level } \Lambda_I^{(0)} \Rightarrow \left[ \mathcal{P}_I \Lambda_J (\mu) \right] = \delta_{IJ}$$

Vertex function should preserve chiral symmetry

$$[\Lambda_I]_{ijklmn}^{\alpha\beta\gamma\delta\eta\zeta} (p) = \frac{1}{5} \left. \left\langle Q_I(0) \bar{u}_i^\alpha (p) \bar{u}_j^\beta (p) d_k^\gamma (p) d_l^\delta (-p) \bar{d}_m^\eta (-p) \bar{d}_n^\zeta (-p) \right\rangle \right|_{\text{amp}}$$

$$+ \frac{3}{5} \left. \left\langle Q_I(0) \bar{u}_i^\alpha (p) \bar{u}_j^\beta (-p) d_k^\gamma (p) d_l^\delta (p) \bar{d}_m^\eta (-p) \bar{d}_n^\zeta (-p) \right\rangle \right|_{\text{amp}}$$

$$+ \frac{1}{5} \left. \left\langle Q_I(0) \bar{u}_i^\alpha (-p) \bar{u}_j^\beta (-p) d_k^\gamma (p) d_l^\delta (p) \bar{d}_m^\eta (p) \bar{d}_n^\zeta (-p) \right\rangle \right|_{\text{amp}}$$

Syritsyn (2015)
Perturbative Renormalization

\[ \mathcal{H}_{n\bar{n}} = \sum I C_I^{\overline{\text{MS}}}(M) Q_I^{\overline{\text{MS}}}(M) = \sum I C_I^{\overline{\text{MS}}}(M) U_I(M, \mu) Q_I^{\text{RI}}(\mu) \]

One-loop running:
Caswell, Milutinovic, and Senjanovic (1983)

One-loop matching
Buchoff, MW (2015)

Two-loop running,
needed for \( \alpha_s(\mu) \) accuracy
Buchoff, MW (2015)

Negligible
One-Loop Matching

15 one-loop diagrams in 3 topologies

Same topologies appear in four-quark weak matrix elements and proton decay
Two-Loop Running

350 two-loop diagrams. Evanescent operators introduce complications.

Includes all diagram topologies needed for two-loop running of any operator built from spin singlet diquarks.
Perturbative Renormalization

Two-loop corrections $< 26\%$ at $\mu = 2$ GeV, perturbative matching under control

Operator renormalization effects significant

$$\frac{C_{\text{RI}}(2 \text{ GeV})}{C_{\text{MS}}(M)}$$

Fierz-conjugate operators differ in $\overline{\text{MS}}$ by $O(\alpha_s(M))$
Non-Perturbative Renormalization

NPR complete, lattice artifacts small for most important operations

From Sergey Syritsyn, Lattice 2015

\[ Z_{\text{SI}}(\mu) = Z_{\text{lat}}(\mu) \left( \frac{Z(\mu)}{Z(2\text{GeV})} \right)_{\text{MS}} \]

Fit \( \sim Z + \alpha a^2 \mu^2 \)

Dominant electroweak singlet operators
Lattice QCD Matrix Elements

LQCD matrix elements calculated from ratios of three-point to two-point correlation functions at large Euclidean time separations

\[ \langle N_\uparrow^+(t_2) Q_I(0) N_\downarrow^-(t_1) \rangle \rightarrow Z_n Z_{\bar{n}} e^{-M_n(t_1+t_2)} \langle n | Q_I | \bar{n} \rangle \]

Operator insertions at all time separations calculable from single point-to-all propagator

No disconnected diagrams
Exploratory LQCD Studies

Exploratory anisotropic Wilson calculation:
Buchoff, Schroeder, Wasem (2012)

\[ R \]

\[ t_1 = 5 \quad t_1 = 10 \quad t_1 = 15 \quad t_1 = 20 \quad t_1 = 25 \]

\[ a \sim 0.125 \text{ fm} \quad m_\pi \sim 390 \text{ MeV} \]

\[ V = 20^3 \times 256 \]
Physical Point LQCD

Domain wall fermion calculation with RBC/UKQCD configurations
Syritsyn, Buchoff, Schroeder, Wasem (2015)

\[ ([RRR]_1) = 3O_{(RR)R}^3 \]
\[ ([LL]_0) = 3O_{(LL)R}^3 \]
\[ ([RR]_1L_0) = 3O_{(LR)R}^3 \]

\[ ([RR])_{L1}^{(1)} = O_{(LR)R}^1 \]
\[ ([RR])_{L1}^{(2)} = O_{(LR)R}^2 \]
\[ ([RR])_{L1}^{(3)} = O_{(RL)}^1 + 2O_{(RR)R}^2 \]

From Sergey Syritsyn, Lattice 2015

\[ a \sim 0.123 \text{ fm} \quad m_\pi \sim 140 \text{ MeV} \quad V = 48^3 \times 96 \]
Physical Point LQCD Results

<table>
<thead>
<tr>
<th></th>
<th>$Z(\text{lat} \rightarrow \overline{MS})$</th>
<th>$\phi^{MS}(2\text{GeV})[10^{-5}\text{GeV}^6]$</th>
<th>Bag “A”</th>
<th>LQCD Bag “A”</th>
<th>Bag “B”</th>
<th>LQCD Bag “B”</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[(RRR)_3]$</td>
<td>0.62(12)</td>
<td>0</td>
<td>0</td>
<td>–</td>
<td>0</td>
<td>–</td>
</tr>
<tr>
<td>$[(RRR)_1]$</td>
<td>0.454(33)</td>
<td>45.4(5.6)</td>
<td>8.190</td>
<td>5.5</td>
<td>6.660</td>
<td>6.8</td>
</tr>
<tr>
<td>$[R_1(LL)_0]$</td>
<td>0.435(26)</td>
<td>44.0(4.1)</td>
<td>7.230</td>
<td>6.1</td>
<td>6.090</td>
<td>7.2</td>
</tr>
<tr>
<td>$[(RR)_1L_0]$</td>
<td>0.396(31)</td>
<td>-66.6(7.7)</td>
<td>-9.540</td>
<td>7.0</td>
<td>-8.160</td>
<td>8.1</td>
</tr>
<tr>
<td>$[(RR)_2L_1]^{(1)}$</td>
<td>0.537(52)</td>
<td>-2.12(26)</td>
<td>1.260</td>
<td>-1.7</td>
<td>-0.666</td>
<td>3.2</td>
</tr>
<tr>
<td>$[(RR)_2L_1]^{(2)}$</td>
<td>0.537(52)</td>
<td>0.531(64)</td>
<td>-0.314</td>
<td>-1.7</td>
<td>0.167</td>
<td>3.2</td>
</tr>
<tr>
<td>$[(RR)_2L_1]^{(3)}$</td>
<td>0.537(52)</td>
<td>-1.06(13)</td>
<td>0.630</td>
<td>-1.7</td>
<td>-0.330</td>
<td>3.2</td>
</tr>
</tbody>
</table>

Syritsyn, Buchoff, Schroeder, Wasem (2015)

Preliminary physical point results presented at Lattice 2015, final stages of the calculation underway with results to appear soon

Buchoff, Rinaldi, Schroeder, Syritsyn, MW, Wasem – in preparation
Phenomenological Applications

Phenomenological studies needed to determine constraints of proposed ESS experiments on all BSM models of interest, e.g. Calibbi, et al (2016) for $\mathcal{R}$ SUSY + bag model predictions.

$$\tau_{n\bar{n}}^{224,PSB} \leq (1.5 \pm 0.2) \times 10^{10} \text{ s}$$

<table>
<thead>
<tr>
<th>$(RRR)_3$</th>
<th>$Z_{\text{lat} \rightarrow \tilde{M}S}$</th>
<th>$\phi\overline{MS}(2 \text{ GeV})[10^{-5}\text{GeV}^6]$</th>
<th>Bag “A”</th>
<th>Bag “B”</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.62(12)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$(RRR)_1$</td>
<td>0.454(33)</td>
<td>45.4(5.6)</td>
<td>8.190</td>
<td>6.660</td>
</tr>
<tr>
<td>$[R_1(LL)]_0$</td>
<td>0.435(26)</td>
<td>44.0(4.1)</td>
<td>7.230</td>
<td>6.090</td>
</tr>
<tr>
<td>$[(RR)_1L_0]$</td>
<td>0.396(31)</td>
<td>-66.6(7.7)</td>
<td>-9.540</td>
<td>-8.160</td>
</tr>
<tr>
<td>$(RR)_{2L_1}^{(1)}$</td>
<td>0.537(52)</td>
<td>-2.12(26)</td>
<td>1.260</td>
<td>-0.666</td>
</tr>
<tr>
<td>$(RR)_{2L_1}^{(2)}$</td>
<td>0.537(52)</td>
<td>0.531(64)</td>
<td>-0.314</td>
<td>0.167</td>
</tr>
<tr>
<td>$(RR)_{2L_1}^{(3)}$</td>
<td>0.537(52)</td>
<td>-1.06(13)</td>
<td>0.630</td>
<td>-0.330</td>
</tr>
</tbody>
</table>

Syritsyn, Buchoff, Schroeder, Wasem (2015)

$$\tau_{n\bar{n}}^{ILLL} \geq 0.86 \times 10^8 \text{ s}$$

$$\tau_{n\bar{n}}^{ESS} \sim 2.7 \times 10^9 \text{ s}$$
Summary

Low-energy $n\bar{n}$ Hamiltonian relatively simple
- Three electroweak singlet operators in isospin limit, distinct chiral irreps

Experimental reach higher than expected
- QCD matrix elements 5-10 times larger than bag model estimates for electroweak singlet operators

Operator color-flavor structure matters
- Different sign anomalous dimensions among electroweak singlet operators
- Non-singlet matrix elements 1-2 orders of magnitude smaller than singlets

Concrete BSM predictions including final LQCD results essential to assess reach of proposed ESS experiments
- Can post-sphaleron baryogenesis models be definitively tested at ESS?
Outlook

Exciting days ahead for $n\bar{n}$, stay tuned!

Lattice QCD $\mu$ Renormalization Group $M$ BSM

$\lesssim 1\text{ GeV}$ few GeV $\gtrsim 1\text{ TeV}$