Hadronic light-by-light contribution to muon $g-2$: dispersive approach

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JHEP **09** (2014) 091 [arXiv:1402.7081 [hep-ph]]

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XIIth Quark Confinement and the Hadron Spectrum, Thessaloniki

Outline

Overview

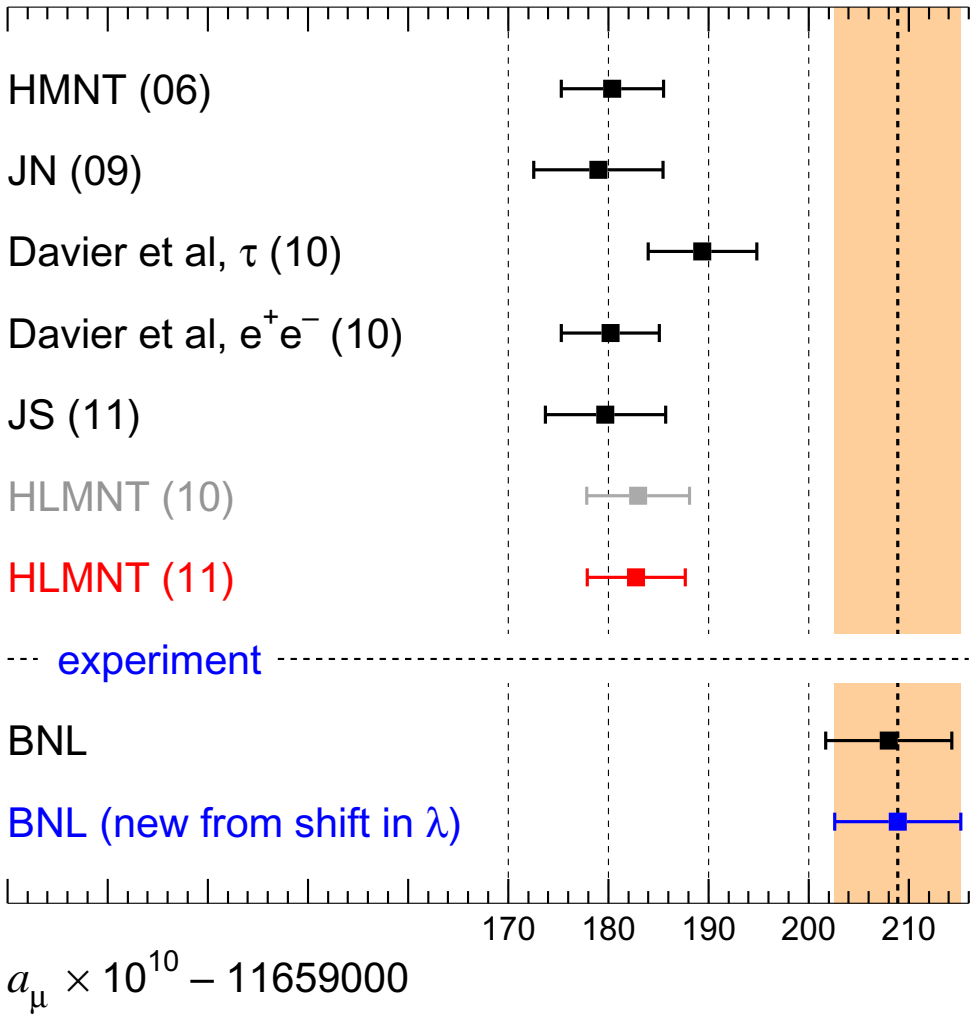
Magnetic moment

- relation of spin and magnetic moment of a lepton:

$$\vec{\mu}_\ell = g_\ell \frac{e}{2m_\ell} \vec{s}$$

  g_ℓ: Landé factor, gyromagnetic ratio
- Dirac's prediction: $g_e = 2$
- anomalous magnetic moment: $a_\ell = (g_\ell - 2)/2$
- helped to establish QED and QFT as the framework for elementary particle physics
- today: probing not only QED but entire SM

$(g-2)_\mu$: comparison of theory and experiment

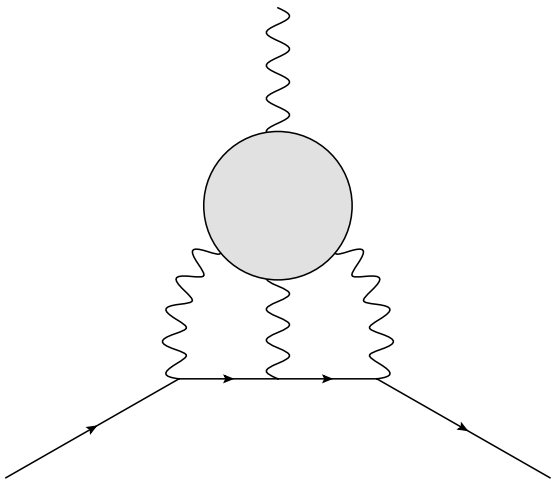


→ Hagiwara et al. 2012

$(g-2)_\mu$: theory vs. experiment

- discrepancy between SM and experiment $\sim 3\sigma$
- hint to new physics?
- new experiments (FNAL, J-PARC) aim at reducing the experimental error by a factor of 4
- theory error completely dominated by hadronic effects
- hadronic vacuum polarisation responsible for largest uncertainty, but will be systematically improved with better data input

Hadronic light-by-light (HLbL) scattering



- up to now only model calculations
- uncertainty estimate based rather on consensus than on a systematic method
- will dominate theory error in a few years

Model calculations of HLbL

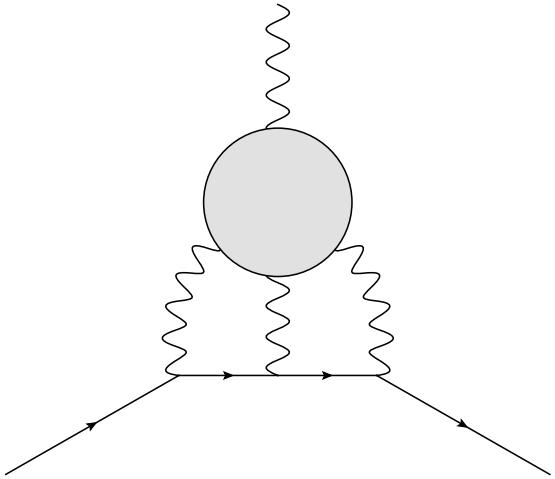
Table 13
Summary of the most recent results for the various contributions to $a_\mu^{\mathrm{LbL;had}} \times 10^{11}$. The last column is our estimate based on our new evaluation for the pseudoscalars and some of the other results.

Contribution	BPP	HKS	KN	MV	BP	PdRV	N/JN
π^0, η, η'	85±13	82.7±6.4	83±12	114±10	–	114±13	99±16
π, K loops	−19±13	−4.5±8.1	–	–	–	−19±19	−19±13
π, K loops + other subleading in N_c	–	–	–	0±10	–	–	–
axial vectors	2.5±1.0	1.7±1.7	–	22± 5	–	15±10	22± 5
scalars	−6.8±2.0	–	–	–	–	−7± 7	−7± 2
quark loops	21± 3	9.7±11.1	–	–	–	2.3	21± 3
total	83±32	89.6±15.4	80±40	136±25	110±40	105±26	116±39

→ Jegerlehner, Nyffeler 2009

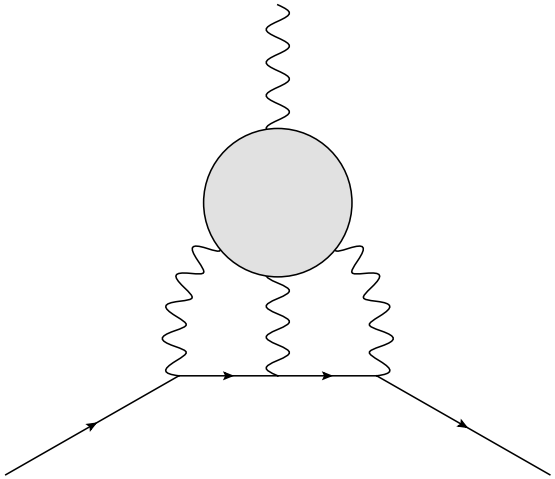
- pseudoscalar pole contribution most important
- pion-loop second most important
- differences between models, large uncertainties

How to improve HLbL calculation?



- lattice QCD making progress
- dispersive approach

Dispersive approach to HLbL



- make use of fundamental principles:
  - gauge invariance, crossing symmetry
  - unitarity, analyticity
- relate HLbL to experimentally accessible quantities

Overview

The HLbL tensor: definitions

- hadronic four-point function:

$$\Pi^{\mu\nu\lambda\sigma}(q_1, q_2, q_3)$$
$$= -i \int dxdydze^{-i(q_1 x + q_2 y + q_3 z)} \langle 0|T j^{\mu}_{\rm em}(x) j^{\nu}_{\rm em}(y) j^{\lambda}_{\rm em}(z) j^{\sigma}_{\rm em}(0)|0\rangle$$

- EM current:

$$j^{\mu}_{\rm em} = \sum_{i=u,d,s} Q_i \bar{q}_i \gamma^{\mu} q_i$$

- Mandelstam variables:

$s = (q_1 + q_2)^2$, $t = (q_1 + q_3)^2$, $u = (q_2 + q_3)^2$

- Ward identities: $\{q_1^{\mu}, q_2^{\nu}, q_3^{\lambda}, q_4^{\sigma}\}\Pi_{\mu\nu\lambda\sigma} = 0$

HLbL tensor: Lorentz decomposition

Solution for the Lorentz decomposition, following a recipe by Bardeen, Tung (1968) and Tarrach (1975):

$$\Pi^{\mu\nu\lambda\sigma}(q_1, q_2, q_3) = \sum_{i=1}^{54} T_i^{\mu\nu\lambda\sigma} \Pi_i(s, t, u; q_j^2)$$

- Lorentz structures manifestly gauge invariant
- crossing symmetry manifest: only 7 distinct structures, 47 follow from crossing
- scalar functions Π_i free of kinematic singularities $\Rightarrow$ ideal quantities for a dispersive treatment

Overview

Master formula: contribution to $(g-2)_\mu$

- from gauge invariance:

$$\Pi_{\mu\nu\lambda\rho} = -q_4^\sigma \frac{\partial}{\partial q_4^\rho} \Pi_{\mu\nu\lambda\sigma}$$

- for $(g-2)_\mu$: afterwards take $q_4 \to 0$
- no kinematic singularities in scalar functions: perform these steps with the derived Lorentz decomposition
- only 12 linear combinations of the scalar functions Π_i contribute to $(g-2)_\mu$

Master formula: contribution to $(g-2)_\mu$

$$a_\mu^{\mathrm{HLbL}} = e^6 \int \frac{d^4q_1}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}\frac{\sum_{i=1}^{12}\hat{T}_i(q_1,q_2;p)\hat{\Pi}_i(q_1,q_2,-q_1-q_2)}{q_1^2q_2^2(q_1+q_2)^2[(p+q_1)^2-m_\mu^2][(p-q_2)^2-m_\mu^2]}$$

- $\hat{T}_i$: known integration kernel functions
- five loop integrals can be performed with Gegenbauer polynomial techniques
- Wick rotation possible even in the presence of anomalous thresholds

Master formula: contribution to $(g-2)_\mu$

$$a_\mu^{\mathrm{HLbL}} = \frac{2\alpha^3}{3\pi^2}\int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^{1} d\tau \sqrt{1-\tau^2} Q_1^3 Q_2^3 \times \sum_{i=1}^{12} T_i(Q_1, Q_2, \tau)\bar{\Pi}_i(Q_1, Q_2, \tau),$$

- T_i: known integration kernels
- $\bar{\Pi}_i$: linear combinations of the scalar functions Π_i
- Euclidean momenta: $Q_i^2 = -q_i^2$
- $Q_3^2 = Q_1^2 + Q_2^2 + 2Q_1Q_2\tau$

Overview

Analytic properties of scalar functions

- right- and left-hand cuts in each Mandelstam variable
- double-spectral regions (box topologies)
- anomalous thresholds for large photon virtualities

Mandelstam representation

- we limit ourselves to intermediate states of at most two pions
- writing down a double-spectral (Mandelstam) representation allows us to split up the HLbL tensor:

$$\Pi_{\mu\nu\lambda\sigma} = \Pi^{\pi^0\text{-pole}}_{\mu\nu\lambda\sigma} + \Pi^{\text{box}}_{\mu\nu\lambda\sigma} + \bar{\Pi}_{\mu\nu\lambda\sigma} + \dots$$

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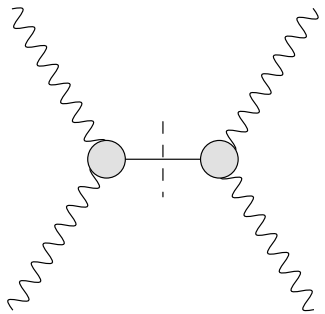
one-pion intermediate state:



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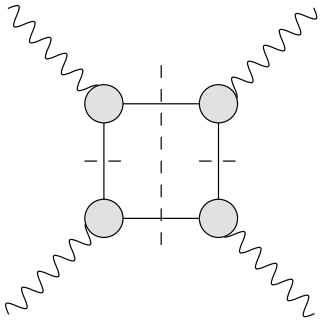
two-pion intermediate state in both channels:



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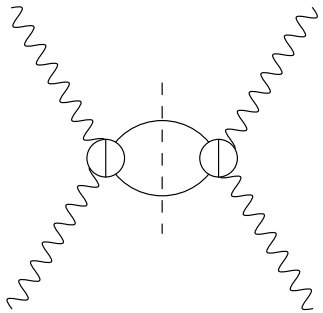
two-pion intermediate state in first channel:



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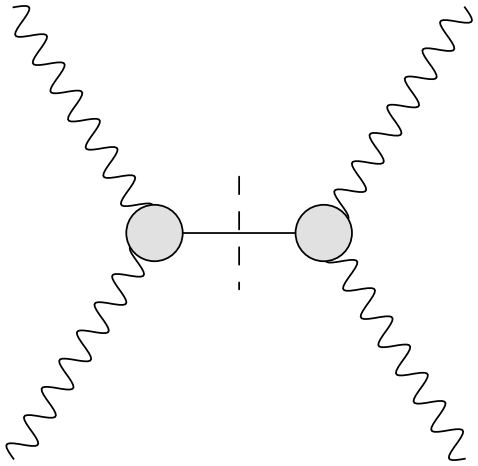
neglected so far: higher intermediate states

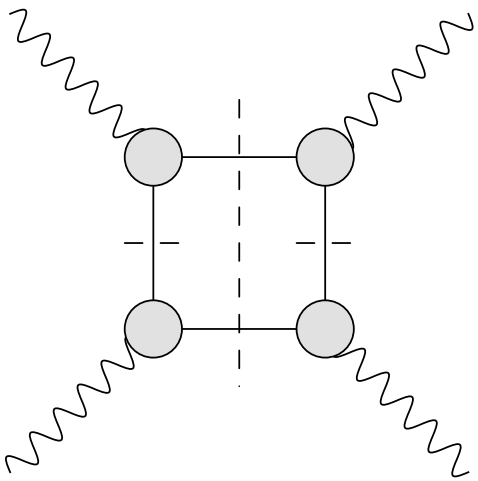
Pion pole



- input: doubly-virtual and singly-virtual pion transition form factors $\mathcal{F}_{\gamma^*\gamma^*\pi^0}$ and $\mathcal{F}_{\gamma^*\gamma\pi^0}$
- dispersive analysis of transition form factor:

  → Hoferichter et al., EPJC **74** (2014) 3180

Pion box



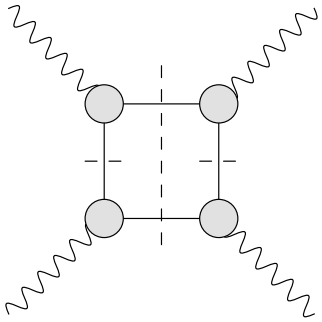
- simultaneous two-pion cuts in two channels
- Mandelstam representation explicitly constructed

$$\Pi_i = \frac{1}{\pi^2} \int ds' dt' \frac{\rho_i^{st}(s', t')}{(s' - s)(t' - t)} + (t \leftrightarrow u) + (s \leftrightarrow u)$$

- q^2-dependence: pion vector form factors $F_\pi^V(q_i^2)$ for each off-shell photon factor out

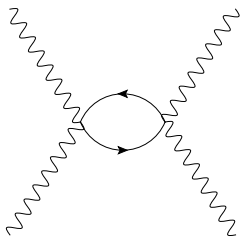
Pion box

- sQED loop projected on BTT basis fulfils the same Mandelstam representation
- only difference are factors of F_π^V
- $\Rightarrow$ box topologies are identical to FsQED:



$$\equiv F_\pi^V(q_1^2)F_\pi^V(q_2^2)F_\pi^V(q_3^2)$$

$$\times \Bigg[$$



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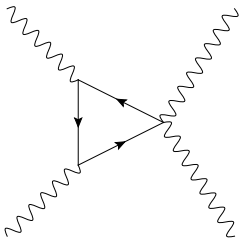

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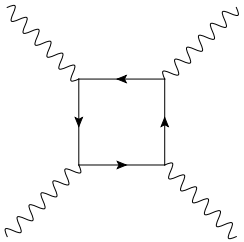
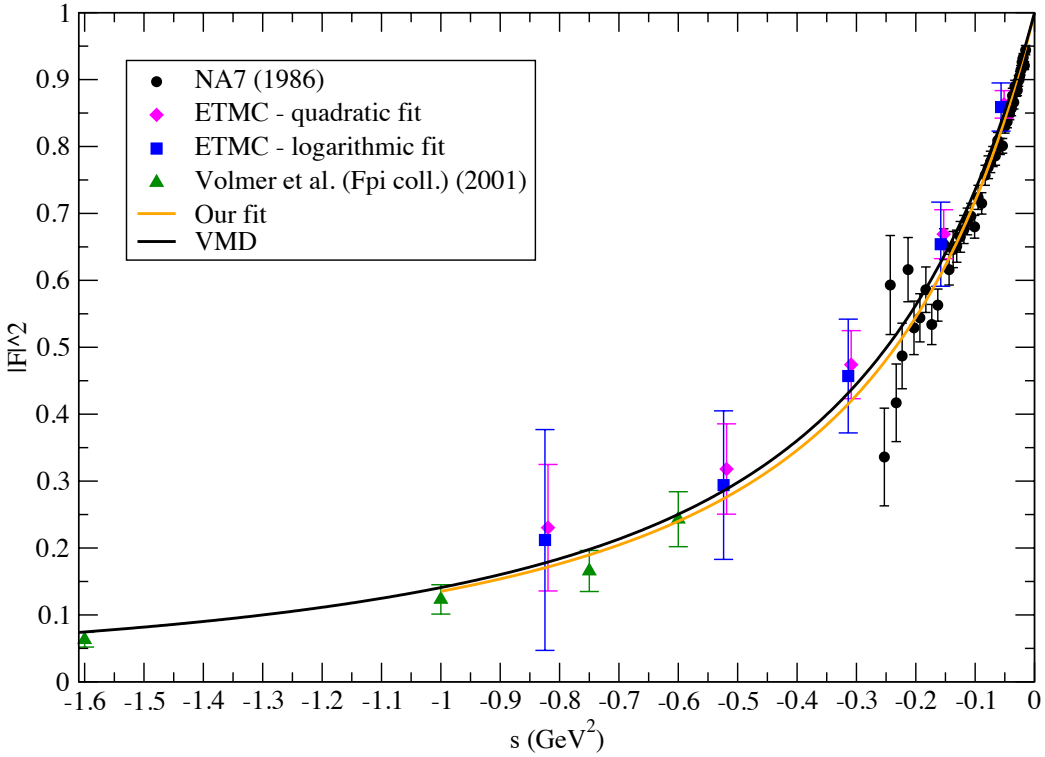


$$\Bigg]$$

- model-independent definition of pion loop

Pion box

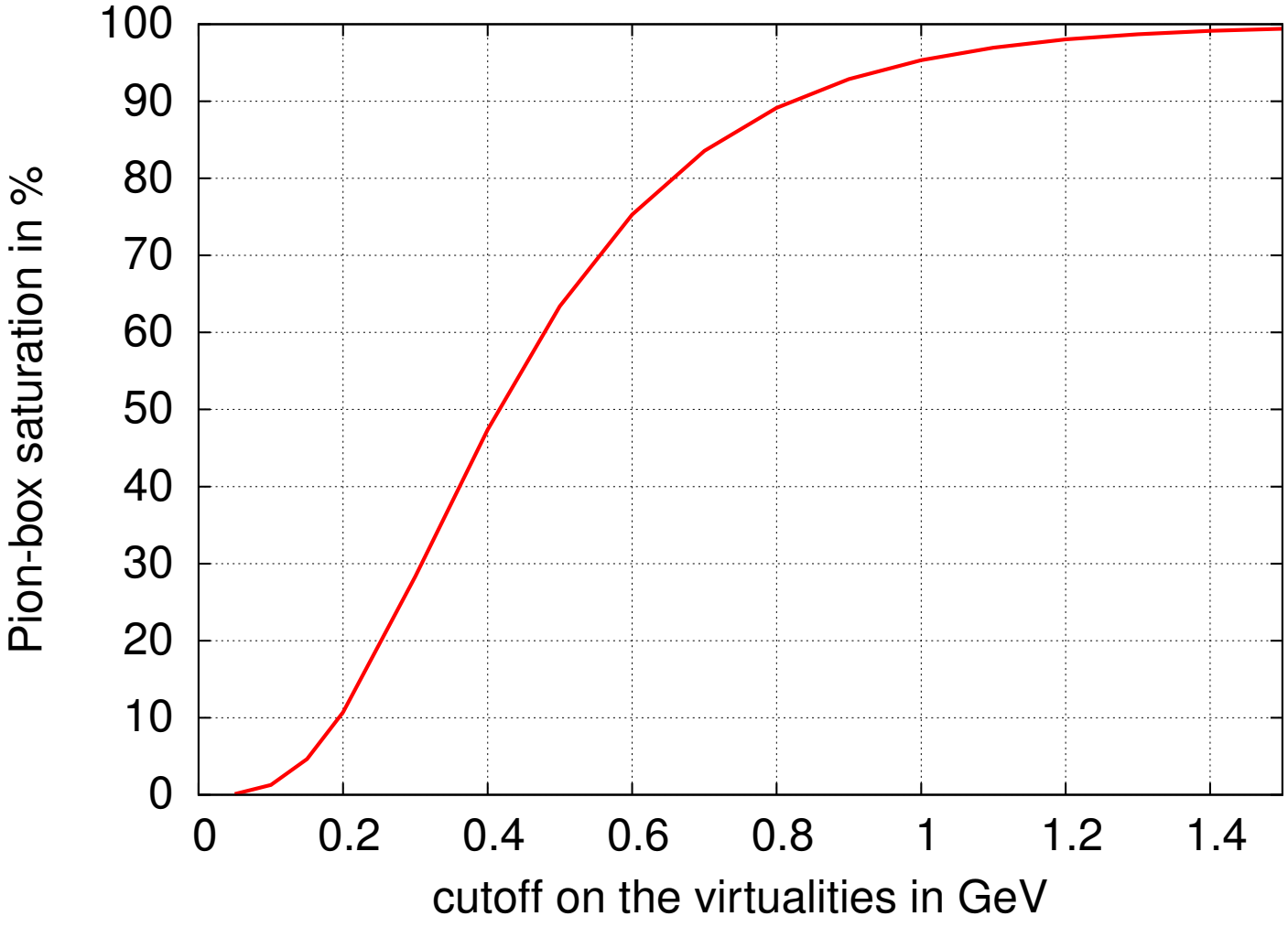
Pion vector form factor in the space-like region:



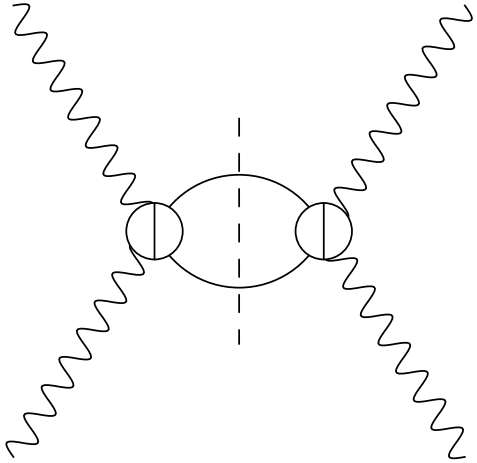
Preliminary results:

$$a_\mu^{\pi\text{-box}} = -15.9 \cdot 10^{-11}, \qquad a_\mu^{\pi\text{-box, VMD}} = -16.4 \cdot 10^{-11}$$

Pion-box saturation with photon virtualities

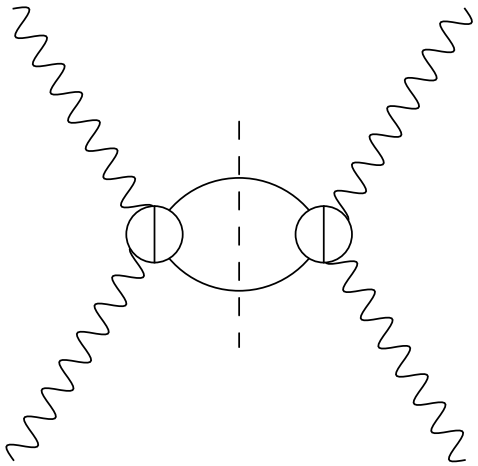


Rescattering contribution



- neglect left-hand cut due to multi-particle intermediate states in crossed channel
- two-pion cut in only one channel
- unitarity relates it to the helicity amplitudes of the subprocess $\gamma^*\gamma^{(*)} \to \pi\pi$
- imaginary part expanded into partial waves

Rescattering contribution



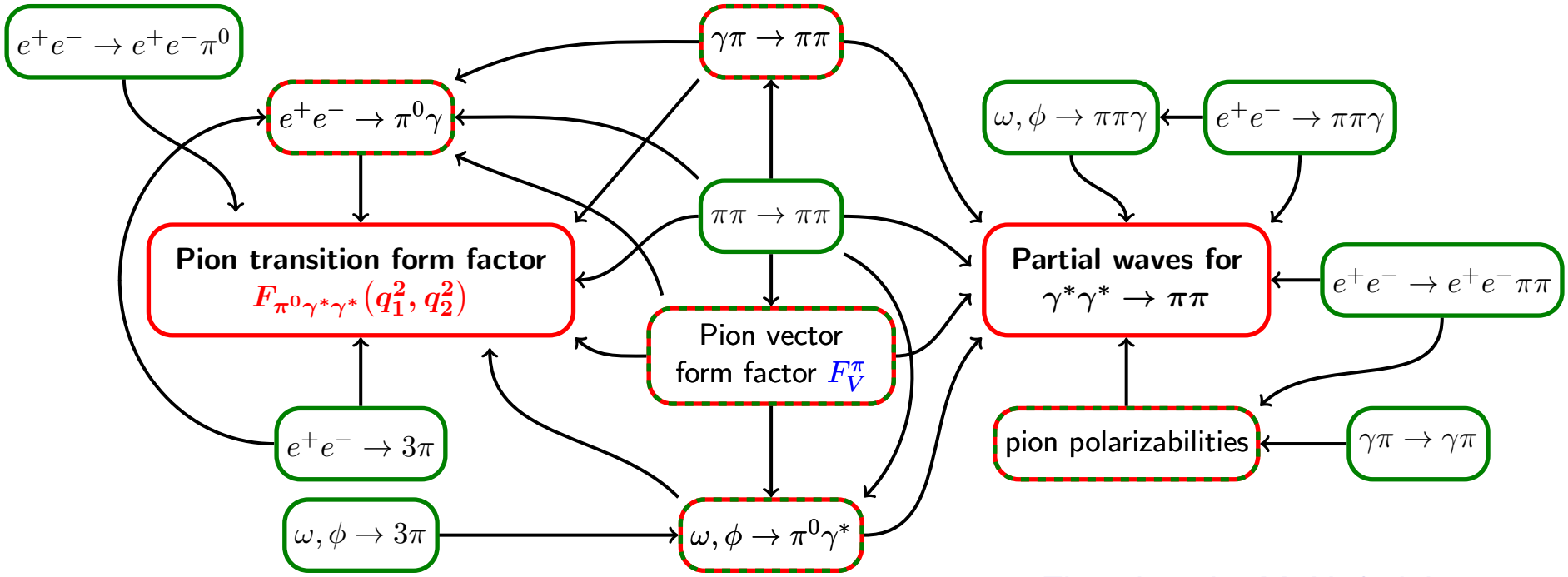
- fixed-$s/t/u$ dispersion relations allow to reconstruct $\bar{\Pi}_{\mu\nu\lambda\sigma}$
- sum rules ensure cancellation of unphysical helicity amplitudes
- S- and D-waves: contain model-independent description of scalar and tensor contributions

Overview

Summary

- our dispersive approach to HLbL scattering is based on fundamental principles:
  - gauge invariance, crossing symmetry
  - unitarity, analyticity
- we take into account the lowest intermediate states: π^0-pole and $\pi\pi$-cuts
- relation to experimentally accessible (or again with data dispersively reconstructed) quantities
- a step towards a model-independent calculation of a_μ

A roadmap for HLbL



$\to$ Flowchart by M. Hoferichter