Quark flavour anomalies of the SM

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Quark Confinement and Hadron Spectrum,
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Why electroweak penguin decays?

- In the SM allows only the charged interactions to change flavour.
  - Other interactions are flavour conserving.
- One can escape this constrain and produce $b \rightarrow s$ and $b \rightarrow d$ at loop level.
  - This kind of processes are suppressed in the SM $\rightarrow$ Rare decays.
  - New Physics can enter in the loops.
\[ H_{\text{eff}} = -\frac{4G_f}{\sqrt{2}} VV'^* \sum_i \left[ C_i(\mu)O_i(\mu) + C'_i(\mu)O'_i(\mu) \right], \]

where \( C_i \) are the Wilson coefficients and \( O_i \) are the corresponding effective operators.
• Excellent Impact Parameter (IP) resolution (20 $\mu$m).
  $\Rightarrow$ Identify secondary vertices from heavy flavour decays
• Proper time resolution $\sim 40 - 50$ fs.
  $\Rightarrow$ Good separation of primary and secondary vertices.
• Excellent momentum ($\delta p/p \sim 0.5 - 1.0\%$) and inv. mass resolution.
  $\Rightarrow$ Low combinatorial background.
• Excellent Muon identification $\epsilon_{\mu \rightarrow \mu} \sim 97\%$, $\epsilon_{\pi \rightarrow \mu} \sim 1 - 3\%$
• Good $K - \pi$ separation via RICH detectors, $\epsilon_{K \rightarrow K} \sim 95\%$, $\epsilon_{\pi \rightarrow K} \sim 5\%$.
  $\Rightarrow$ Reject peaking backgrounds.
• High trigger efficiencies, low momentum thresholds.
$B \rightarrow J/\psi X$: Trigger $\sim 90\%$. 
Recent measurements of $b \rightarrow s \ell \ell$

⇒ Branching fractions:

\[ B \rightarrow K \mu^- \mu^+ \quad 1606.04731 \]
\[ B^0_s \rightarrow \phi \mu^- \mu^+ \quad \text{JHEP 09 (2015) 179} \]
\[ B^{\pm} \rightarrow \pi^{\pm} \mu^- \mu^+ \quad \text{JHEP 12 (2012) 125} \]
\[ \Lambda_b \rightarrow \Lambda \mu^- \mu^+ \quad \text{JHEP 06 (2015) 115} \]
\[ B \rightarrow \mu^- \mu^+ \quad \text{Nature 15} \]

⇒ CP asymmetry:

\[ B^{\pm} \rightarrow \pi^{\pm} \mu^- \mu^+ \quad \text{JHEP 10 (2015) 034} \]

⇒ Isospin asymmetry:

\[ B \rightarrow K \mu^- \mu^+ \quad \text{JHEP 06 (2014) 133} \]

⇒ Lepton Universality:

\[ B^{\pm} \rightarrow K^{\pm} \ell \ell \quad \text{PRL 113, (2014)} \]

⇒ Angular:

\[ B^0 \rightarrow K^* \ell \ell \quad \text{JHEP 02 (2016) 104} \]
\[ B^{0,\pm} \rightarrow K^{*,\pm} \ell \ell \quad \text{PRD 86 032012} \]
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$B \to K \mu^- \mu^+$ \quad JHEP 06 (2014) 133

⇒ This talk is not possible to cover all flavour anomalies. See T.Blake talk tmr for more of them!
Observables in $B \to K^* \mu \mu$

⇒ The kinematics of $B^0 \to K^* \mu^- \mu^+$ decay is described by three angles $\theta_l$, $\theta_k$, $\phi$ and invariant mass of the dimuon system ($q^2$).
⇒ The angular distribution can be written as:

\[
\frac{1}{d(\Gamma + \bar{\Gamma})/dq^2} \frac{d(\Gamma + \bar{\Gamma})}{d\cos \theta_l \, d\cos \theta_k \, d\phi} \bigg|_P = \frac{9}{32\pi} \left[ \frac{3}{4} (1 - F_L) \sin^2 \theta_k \\
+ F_L \cos^2 \theta_k + \frac{1}{4} (1 - F_L) \sin^2 \theta_k \cos 2\theta_l \\
- F_L \cos^2 \theta_k \cos 2\theta_l + S_3 \sin^2 \theta_k \sin^2 \theta_l \cos 2\phi \\
+ S_4 \sin 2\theta_k \sin 2\theta_l \cos \phi + S_5 \sin 2\theta_k \sin \theta_l \cos \phi \\
+ \frac{4}{3} A_{FB} \sin^2 \theta_k \cos \theta_l + S_7 \sin 2\theta_k \sin \theta_l \sin \phi \\
+ S_8 \sin 2\theta_k \sin 2\theta_l \sin \phi + S_9 \sin^2 \theta_k \sin^2 \theta_l \sin 2\phi \right].
\]

⇒ This equation is valid in the SM for massless leptons!
The observables $S_i$ are bilinear combinations of transversity amplitudes: $A_{\perp}^{L,R}$, $A_{\parallel}^{L,R}$, $A_0^{L,R}$.

So here is where the magic happens. At leading order the amplitudes can be written as:

$$A_{\perp}^{L,R} = \sqrt{2}N m_B (1 - \hat{s}) \left[ (C_{9}^{\text{eff}} + C_{9}'^{\text{eff}}) \mp (C_{10} + C_{10}') + \frac{2\hat{m}_b}{\hat{s}} (C_7^{\text{eff}} + C_7'^{\text{eff}}) \right] \xi_{\perp} (E_{K^*})$$

$$A_{\parallel}^{L,R} = -\sqrt{2}N m_B (1 - \hat{s}) \left[ (C_{9}^{\text{eff}} - C_{9}'^{\text{eff}}) \mp (C_{10} - C_{10}') + \frac{2\hat{m}_b}{\hat{s}} (C_7^{\text{eff}} - C_7'^{\text{eff}}) \right] \xi_{\perp} (E_{K^*})$$

$$A_0^{L,R} = -\frac{N m_B (1 - \hat{s})^2}{2\hat{m}_{K^*} \sqrt{\hat{s}}} \left[ (C_{9}^{\text{eff}} - C_{9}'^{\text{eff}}) \mp (C_{10} - C_{10}') + 2\hat{m}_b (C_7^{\text{eff}} - C_7'^{\text{eff}}) \right] \xi_{\parallel} (E_{K^*}),$$

where $\hat{s} = q^2 / m_B^2$, $\hat{m}_i = m_i / m_B$. The $\xi_{\parallel,\perp}$ are the soft form factors.
The observables $S_i$ are bilinear combinations of transversity amplitudes: $A_{L,R}^L, A_{L,R}^R, A_0^{L,R}$.

So here is where the magic happens. At leading order the amplitudes can be written as:

$$A_{L,R}^L = \sqrt{2N m_B} (1 - \hat{s}) \left[ (C_9^{\text{eff}} + C_9^{\text{eff}'}) \mp (C_{10} + C_{10}') + \frac{2 \hat{m}_b}{\hat{s}} (C_7^{\text{eff}} + C_7^{\text{eff}'}) \right] \xi_{\perp} (E_K^*)$$

$$A_{L,R}^R = -\sqrt{2N m_B} (1 - \hat{s}) \left[ (C_9^{\text{eff}} - C_9^{\text{eff}'}) \mp (C_{10} - C_{10}') + \frac{2 \hat{m}_b}{\hat{s}} (C_7^{\text{eff}} - C_7^{\text{eff}'}) \right] \xi_{\perp} (E_K^*)$$

$$A_0^{L,R} = -\frac{N m_B (1 - \hat{s})}{2 \hat{m}_K^* \sqrt{\hat{s}}} \left[ (C_9^{\text{eff}} - C_9^{\text{eff}'}) \mp (C_{10} - C_{10}') + 2 \hat{m}_b (C_7^{\text{eff}} - C_7^{\text{eff}'}) \right] \xi_{\parallel} (E_K^*),$$

where $\hat{s} = q^2/m_B^2$, $\hat{m}_i = m_i/m_B$. The $\xi_{\parallel,\perp}$ are the soft form factors.

Now we can construct observables that cancel the $\xi$ soft form factors at leading order:

$$P_5' = \frac{S_5 + \bar{S}_5}{2 \sqrt{F_L(1 - F_L)}}$$
$B^0 \rightarrow K^* \mu^- \mu^+$, Selection

- Signal modelled by a sum of two Crystal-Ball functions.
- Shape is defined using $B \rightarrow J/\psi K^*$ and corrected for $q^2$ dependency.
- Combinatorial background modelled by exponent.

$K\pi$ system:
- Beside the $K^*$ resonance there might might a tail from other higher mass states.
- We modelled it in the analysis.
- Reduced the systematic compared to previous analysis.

- In total we found $2398 \pm 57$ candidates in the $(0.1, 19) \text{ GeV}^2/c^4$ $q^2$ region.
- $624 \pm 30$ candidates in the theoretically the most interesting $(1.1 - 6.0) \text{ GeV}^2/c^4$ region.
Detector acceptance

- Detector distorts our angular distribution.
- We need to model this effect.
- Full 4D function is used:

\[
\epsilon(\cos \theta_l, \cos \theta_k, \phi, q^2) = \sum_{ijkl} c_{ijkl} P_i(\cos \theta_l) P_j(\cos \theta_k) P_k(\phi) P_l(q^2),
\]

where \( P_i \) is the Legendre polynomial of order \( i \).
- We use up to 4th, 5th, 6th, 5th order for the \( \cos \theta_l, \cos \theta_k, \phi, q^2 \).
- 600 terms in total!
• We tested our unfolding procedure on $B \to J/\psi K^*$. 
• The result is in perfect agreement with other experiments and our different analysis of this decay.
$B^0 \rightarrow K^*\mu\mu$ results

LHCb

- SM from ABSZ
- Likelihood fit
- Method of moments

$q^2 [\text{GeV}^2/c^4]$
$B^0 \rightarrow K^* \mu\mu$ results

LHCb

- SM from ABSZ
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- Method of moments

$q^2$ [GeV$^2/c^4$]

$A_{FB}$

$S_8$

$S_7$

$S_9$

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Quark flavour anomalies of the SM
Results in $B \rightarrow K^* \mu\mu$

- Tension gets confirmed!
- The two bins deviate by 2.8 and 3.0 $\sigma$ from the SM prediction.
- Result compatible with previous result.
BF measurements of $B_S^0 \rightarrow \phi \mu \mu$

- Suppressed by $\frac{f_s}{f_d}$.
- Cleaner because of narrow $\phi$ resonance.
- $3.3\,\sigma$ deviation in the SM in the $1-6\,\text{GeV}^2/\text{c}^4$ bin.
- Angular part in agreement with the SM ($S_5$ is not accessible).
Measurements of $\Lambda_b \rightarrow \Lambda\mu\mu$

- In total $\sim 300$ candidates in data set.
- Decay not present in the low $q^2$.
- For the bins in which we have $> 3\sigma$ significance the forward backward asymmetry is measured for the hadronic and leptonic system.
Lepton universality test

- Challenging analysis due to bremsstrahlung.
- Migration of events modeled by MC.
- Correct for bremsstrahlung.
- Take double ratio with $B^+ \rightarrow J/\psi K^+$ to cancel systematics.
- In $3\text{fb}^{-1}$, LHCb measures $R_K = 0.745^{+0.090}_{-0.074} (\text{stat.}) +0.036 -0.036 (\text{syst.})$.
- Consistent with the SM at $2.6\sigma$. 

$R_K = \frac{\int q^2=6 \text{GeV}^2/c^4 (dB[B^+ \rightarrow K^+\mu^+\mu^-]/dq^2) dq^2}{\int q^2=1 \text{GeV}^2/c^4 (dB[B^+ \rightarrow K^+e^+e^-]/dq^2) dq^2} = 1 \pm \mathcal{O}(10^{-3})$
Theory implications

• A fit prepared by S. Descotes-Genon, L. Hofer, J. Matias, J. Virto.

• The data can be explained by modifying the $C_9$ Wilson coefficient.

• Overall there is $> 4 \sigma$ discrepancy wrt. the SM prediction.
If not NP?

- We are not there yet!
- There might be something not taken into account in the theory.
- Resonances ($J/\psi$, $\psi(2S)$) tails can mimic NP effects.
- There might be some non factorizable QCD corrections.

"However, the central value of this effect would have to be significantly larger than expected on the basis of existing estimates" D. Straub, arXiv:1503.06199.
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There is more!

- There is one other Lepton Universality Violation decay recently measured by LHCb.
  
  \[ R(D^*) = \frac{\mathcal{B}(B \to D^*\tau\nu)}{\mathcal{B}(B \to D^*\mu\nu)} \]

- Clean SM prediction: \( R(D^*) = 0.252(3), \text{ PRD 85 094025 (2012)} \)
- LHCb result: \( R(D^*) = 0.336 \pm 0.027 \pm 0.030 \)
- HFAG average: \( R(D^*) = 0.322 \pm 0.022 \)
- \( 4.0 \sigma \) discrepancy wrt. SM.
Conclusions

• Clear tensions wrt. SM predictions!
• Measurements cluster in the same direction.
• We are not opening the champagne yet!
• Still need improvement both on theory and experimental side.
• More data will shade a light on the matter.
• Time will tell if this is QCD+fluctuations or new Physics:

... when you have eliminated all the Standard Model explanations, whatever remains, however improbable, must be New Physics.

Prof. Joaquim Matias
Conclusions

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Thank you for the attention!
## Theory implications

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Best fit</th>
<th>$1\sigma$</th>
<th>$3\sigma$</th>
<th>Pull$_{\text{SM}}$</th>
<th>p-value (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_7^{\text{NP}}$</td>
<td>-0.02</td>
<td>[-0.04, -0.00]</td>
<td>[-0.07, 0.04]</td>
<td>1.1</td>
<td>16.0</td>
</tr>
<tr>
<td>$C_9^{\text{NP}}$</td>
<td>-1.11</td>
<td>[-1.32, -0.89]</td>
<td>[-1.71, -0.40]</td>
<td>4.5</td>
<td>62.0</td>
</tr>
<tr>
<td>$C_{10}^{\text{NP}}$</td>
<td>0.58</td>
<td>[0.34, 0.84]</td>
<td>[-0.11, 1.41]</td>
<td>2.5</td>
<td>25.0</td>
</tr>
<tr>
<td>$C_7^{\text{NP}}$</td>
<td>0.02</td>
<td>[-0.01, 0.04]</td>
<td>[-0.05, 0.09]</td>
<td>0.7</td>
<td>15.0</td>
</tr>
<tr>
<td>$C_9^{\text{NP}}$</td>
<td>0.49</td>
<td>[0.21, 0.77]</td>
<td>[-0.33, 1.35]</td>
<td>1.8</td>
<td>19.0</td>
</tr>
<tr>
<td>$C_{10}^{\text{NP}}$</td>
<td>-0.27</td>
<td>[-0.46, -0.08]</td>
<td>[-0.84, 0.28]</td>
<td>1.4</td>
<td>17.0</td>
</tr>
<tr>
<td>$C_9^{\text{NP}} = C_{10}^{\text{NP}}$</td>
<td>-0.21</td>
<td>[-0.40, 0.00]</td>
<td>[-0.74, 0.55]</td>
<td>1.0</td>
<td>16.0</td>
</tr>
<tr>
<td>$C_9^{\text{NP}} = -C_{10}^{\text{NP}}$</td>
<td>-0.69</td>
<td>[-0.88, -0.51]</td>
<td>[-1.27, -0.18]</td>
<td>4.1</td>
<td>55.0</td>
</tr>
<tr>
<td>$C_9^{\text{NP}} = C_{10}^{\text{NP}}$</td>
<td>-0.09</td>
<td>[-0.35, 0.17]</td>
<td>[-0.88, 0.66]</td>
<td>0.3</td>
<td>14.0</td>
</tr>
<tr>
<td>$C_9^{\text{NP}} = -C_{10}^{\text{NP}}$</td>
<td>0.20</td>
<td>[0.08, 0.32]</td>
<td>[-0.15, 0.56]</td>
<td>1.7</td>
<td>19.0</td>
</tr>
<tr>
<td>$C_9^{\text{NP}} = -C_9^{\text{NP}}$</td>
<td>-1.09</td>
<td>[-1.28, -0.88]</td>
<td>[-1.62, -0.42]</td>
<td>4.8</td>
<td>72.0</td>
</tr>
<tr>
<td>$C_9^{\text{NP}} = -C_{10}^{\text{NP}}$</td>
<td>-0.68</td>
<td>[-0.49, -0.49]</td>
<td>[-1.36, -0.15]</td>
<td>3.9</td>
<td>50.0</td>
</tr>
<tr>
<td>$C_9^{\text{NP}} = -C_9^{\text{NP}} = -C_{10}^{\text{NP}}$</td>
<td>-0.17</td>
<td>[-0.29, -0.06]</td>
<td>[-0.54, 0.18]</td>
<td>1.5</td>
<td>18.0</td>
</tr>
</tbody>
</table>

Table 2: Best-fit points, confidence intervals, pulls for the SM hypothesis and p-values for different one-dimensional NP scenarios.
If not NP?

- How about our clean $P_i$ observables?
- The QCD cancel as mentioned only at leading order.
- Comparison to normal observables with the optimised ones.
Transversity amplitudes

⇒ One can link the angular observables to transversity amplitudes

\[ J_{1s} = \frac{(2 + \beta_{\ell}^2)}{4} \left[ |A_L|^2 + |A_L^\perp|^2 + |A_R|^2 + |A_R^\perp|^2 \right] + \frac{4m_{\ell}^2}{q^2} \Re \left( A_L^\perp A_R^\perp + A_L A_R^* \right), \]

\[ J_{1c} = |A_0^L|^2 + |A_0^R|^2 + \frac{4m_{\ell}^2}{q^2} \left[ |A_t|^2 + 2\Re(A_0^L A_0^R*) \right] + \beta_{\ell}^2 |A_S|^2, \]

\[ J_{2s} = \frac{\beta_{\ell}^2}{4} \left[ |A_L|^2 + |A_L^\perp|^2 + |A_R|^2 + |A_R^\perp|^2 \right] + J_{2c} = -\beta_{\ell}^2 \left[ |A_0^L|^2 + |A_0^R|^2 \right], \]

\[ J_3 = \frac{1}{2} \beta_{\ell}^2 \left[ |A_L|^2 - |A_L^\perp|^2 + |A_R|^2 - |A_R^\perp|^2 \right] + J_4 = \frac{1}{\sqrt{2}} \beta_{\ell}^2 \left[ \Re(A_0^L A_L^* + A_0 A_R^*) \right], \]

\[ J_5 = \sqrt{2} \beta_{\ell} \left[ \Re(A_0^L A_L^* - A_0^R A_R^*) - \frac{m_{\ell}}{\sqrt{q^2}} \Re(A_L^* A_S + A_R^* A_S) \right], \]

\[ J_6_{s} = 2 \beta_{\ell} \left[ \Re(A_L^\perp A_L^* - A_R^\perp A_R^*) \right] + J_{6c} = 4 \beta_{\ell} \frac{m_{\ell}}{\sqrt{q^2}} \Re(A_0^L A_S^* + A_0^R A_S), \]

\[ J_7 = \sqrt{2} \beta_{\ell} \left[ \Im(A_0^L A_L^* - A_0^R A_R^*) + \frac{m_{\ell}}{\sqrt{q^2}} \Im(A_L^* A_S^* - A_L A_S) \right], \]

\[ J_8 = \frac{1}{\sqrt{2}} \beta_{\ell}^2 \left[ \Im(A_0^L A_L^* + A_0^R A_R^*) \right] + J_9 = \beta_{\ell}^2 \left[ \Im(A_L^* A_L^\perp + A_R^* A_R^\perp) \right], \]
So here is where the magic happens. At leading order the amplitudes can be written as (soft form factors):

\[
A_{L,R}^{\perp} = \sqrt{2} N m_B (1 - \hat{s}) \left[ (C_9^{\text{eff}} + C_9^{\text{eff}'}) \mp (C_{10} + C_{10}') + \frac{2 \hat{m}_b}{\hat{s}} (C_7^{\text{eff}} + C_7^{\text{eff}'}) \right] \xi_{\perp} (E_{K^*})
\]

\[
A_{L,R}^{\parallel} = -\sqrt{2} N m_B (1 - \hat{s}) \left[ (C_9^{\text{eff}} - C_9^{\text{eff}'}) \mp (C_{10} - C_{10}') + \frac{2 \hat{m}_b}{\hat{s}} (C_7^{\text{eff}} - C_7^{\text{eff}'}) \right] \xi_{\perp} (E_{K^*})
\]

\[
A_{L,R}^{0} = -\frac{N m_B (1 - \hat{s})^2}{2 \hat{m}_{K^*} \sqrt{\hat{s}}} \left[ (C_9^{\text{eff}} - C_9^{\text{eff}'}) \mp (C_{10} - C_{10}') + 2 \hat{m}_b (C_7^{\text{eff}} - C_7^{\text{eff}'}) \right] \xi_{\parallel} (E_{K^*}),
\]

where \( \hat{s} = q^2 / m_B^2 \), \( \hat{m}_i = m_i / m_B \). The \( \xi_{\parallel,\perp} \) are the form factors.
So here is where the magic happens. At leading order the amplitudes can be written as (soft form factors):

\[
\begin{align*}
A_{L,R}^\perp &= \sqrt{2}N m_B (1 - \hat{s}) \left[ (C_9^{\text{eff}} + C_9^{\text{eff}'} \mp (C_{10} + C_{10}') + \frac{2\hat{m}_b}{\hat{s}} (C_7^{\text{eff}} + C_7^{\text{eff}'}) \right] \xi_{\perp} (E_{K^*}) \\
A_{L,R}^\parallel &= -\sqrt{2}N m_B (1 - \hat{s}) \left[ (C_9^{\text{eff}} - C_9^{\text{eff}'} \mp (C_{10} - C_{10}') + \frac{2\hat{m}_b}{\hat{s}} (C_7^{\text{eff}} - C_7^{\text{eff}'}) \right] \xi_{\perp} (E_{K^*}) \\
A_{L,R}^0 &= -\frac{N m_B (1 - \hat{s})^2}{2\hat{m}_K^* \sqrt{\hat{s}}} \left[ (C_9^{\text{eff}} - C_9^{\text{eff}'} \mp (C_{10} - C_{10}') + 2\hat{m}_b (C_7^{\text{eff}} - C_7^{\text{eff}'}) \right] \xi_{\parallel} (E_{K^*}),
\end{align*}
\]

where \( \hat{s} = q^2 / m_B^2 \), \( \hat{m}_i = m_i / m_B \). The \( \xi_{\parallel,\perp} \) are the form factors.

Now we can construct observables that cancel the \( \xi \) form factors at leading order:

\[
P'_5 = \frac{J_5 + \bar{J}_5}{2\sqrt{-(J_2^c + \bar{J}_2^c)(J_2^s + \bar{J}_2^s)}}
\]
\(B^0 \rightarrow K^*\mu^-\mu^+\) kinematics

⇒ The kinematics of \(B^0 \rightarrow K^*\mu^-\mu^+\) decay is described by three angles \(\theta_l, \theta_k, \phi\) and invariant mass of the dimuon system \((q^2)\).

⇒ \(\cos \theta_k\): the angle between the direction of the kaon in the \(K^* (\bar{K}^*)\) rest frame and the direction of the \(K^* (\bar{K}^*)\) in the \(B^0 (\bar{B}^0)\) rest frame.

⇒ \(\cos \theta_l\): the angle between the direction of the \(\mu^- (\mu^+)\) in the dimuon rest frame and the direction of the dimuon in the \(B^0 (\bar{B}^0)\) rest frame.

⇒ \(\phi\): the angle between the plane containing the \(\mu^-\) and \(\mu^+\) and the plane containing the kaon and pion from the \(K^*\).
The kinematics of \( B^0 \to K^* \mu^- \mu^+ \) decay is described by three angles \( \theta_L, \theta_K, \phi \) and invariant mass of the dimuon system (\( q^2 \)).

\[
\frac{d^4 \Gamma}{dq^2 \, d\cos \theta_K \, d\cos \theta_L \, d\phi} = \frac{9}{32\pi} \left[ J_{1s} \sin^2 \theta_K + J_{1c} \cos^2 \theta_K + (J_{2s} \sin^2 \theta_K + J_{2c} \cos^2 \theta_K) \cos 2\theta_L \\
+ J_3 \sin^2 \theta_K \sin^2 \theta_L \cos 2\phi + J_4 \sin 2\theta_K \sin 2\theta_L \cos \phi + J_5 \sin 2\theta_K \sin \theta_L \cos \phi \\
+(J_{6s} \sin^2 \theta_K + J_{6c} \cos^2 \theta_K) \cos \theta_L + J_7 \sin 2\theta_K \sin \theta_L \sin \phi + J_8 \sin 2\theta_K \sin 2\theta_L \sin \phi \\
+ J_9 \sin^2 \theta_K \sin^2 \theta_L \sin 2\phi \right],
\]

This is the most general expression of this kind of decay.

The \( CP \) averaged angular observables are defined:

\[
S_i = \frac{J_i + \bar{J}_i}{(d\Gamma + d\bar{\Gamma})/dq^2}
\]
The observables $J_i$ are bilinear combinations of transversity amplitudes: $A_{L,R}^L, A_{L,R}^R, A_{0}^L$. So here is where the magic happens. At leading order the amplitudes can be written as:

$$A_{\perp}^{L,R} = \sqrt{2} N m_B (1 - \hat{s}) \left[ (C_{9}^{\text{eff}} + C_{9}^{\text{eff}'} \mp (C_{10} + C_{10}') + \frac{2\hat{m}_b}{\hat{s}} (C_{7}^{\text{eff}} + C_{7}^{\text{eff}'}) \right] \xi_{\perp} (E_{K*})$$

$$A_{\parallel}^{L,R} = -\sqrt{2} N m_B (1 - \hat{s}) \left[ (C_{9}^{\text{eff}} - C_{9}^{\text{eff}'} \mp (C_{10} - C_{10}') + \frac{2\hat{m}_b}{\hat{s}} (C_{7}^{\text{eff}} - C_{7}^{\text{eff}'}) \right] \xi_{\perp} (E_{K*})$$

$$A_{0}^{L,R} = -\frac{N m_B (1 - \hat{s})^2}{2\hat{m}_B \sqrt{\hat{s}}} \left[ (C_{9}^{\text{eff}} - C_{9}^{\text{eff}'} \mp (C_{10} - C_{10}') + 2\hat{m}_b (C_{7}^{\text{eff}} - C_{7}^{\text{eff}'}) \right] \xi_{\parallel} (E_{K*})$$

where $\hat{s} = q^2/m_B^2$, $\hat{m}_i = m_i/m_B$. The $\xi_{\parallel,\perp}$ are the soft form factors.
The observables \( J_i \) are bilinear combinations of transversity amplitudes: \( A_{L,R}^\perp, A_{L,R}^\parallel, A_{0}^{L,R} \).

So here is where the magic happens. At leading order the amplitudes can be written as:

\[
A_{L,R}^\perp = \sqrt{2N m_B} (1 - \hat{s}) \left[ (C_{9}^{\text{eff}} + C_{9}^{\text{eff}'}) \mp (C_{10} + C_{10}') + \frac{2\hat{m}_b}{\hat{s}} (C_{7}^{\text{eff}} + C_{7}^{\text{eff}'}) \right] \xi (E_{K^*})
\]

\[
A_{L,R}^\parallel = -\sqrt{2N m_B} (1 - \hat{s}) \left[ (C_{9}^{\text{eff}} - C_{9}^{\text{eff}'}) \mp (C_{10} - C_{10}') + \frac{2\hat{m}_b}{\hat{s}} (C_{7}^{\text{eff}} - C_{7}^{\text{eff}'} ) \right] \xi (E_{K^*})
\]

\[
A_{0}^{L,R} = -\frac{N m_B (1 - \hat{s})^2}{2\hat{m}_K^* \sqrt{\hat{s}}} \left[ (C_{9}^{\text{eff}} - C_{9}^{\text{eff}'} ) \mp (C_{10} - C_{10}') + 2\hat{m}_b (C_{7}^{\text{eff}} - C_{7}^{\text{eff}'} ) \right] \xi (E_{K^*}),
\]

where \( \hat{s} = q^2/m_B^2 \), \( \hat{m}_i = m_i/m_B \). The \( \xi_{\parallel,\perp} \) are the soft form factors.

Now we can construct observables that cancel the \( \xi \) soft form factors at leading order:

\[
P'_5 = \frac{J_5 + \bar{J}_5}{2 \sqrt{-(J_2^c + \bar{J}_2^c)(J_2^s + \bar{J}_2^s)}}
\]
Symmetries in $B \rightarrow K^* \mu \mu$

⇒ We have 12 angular coefficients ($S_i$).
⇒ There exist 4 symmetry transformations that leave the angular distributions unchanged:

$$n_{\parallel} = \left( \frac{A^L_{\parallel}}{A^R_{\parallel*}} \right), \quad n_{\perp} = \left( \frac{A^L_{\perp}}{-A^R_{\perp*}} \right), \quad n_0 = \left( \frac{A^L_0}{A^R_{0*}} \right).$$

$$n'_i = U n_i = \begin{bmatrix} e^{i\phi_L} & 0 \\ 0 & e^{-i\phi_R} \end{bmatrix} \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \cosh i\tilde{\theta} & -\sinh i\tilde{\theta} \\ -\sinh i\tilde{\theta} & \cosh i\tilde{\theta} \end{bmatrix} n_i.$$

⇒ Using this symmetries one can show that there are 8 independent observables. The pdf can be written as:

$$\frac{1}{d(\Gamma + \bar{\Gamma})/dq^2} \frac{d(\Gamma + \bar{\Gamma})}{d\cos \theta_l \, d\cos \theta_k \, d\phi} \Bigg|_P = \frac{9}{32\pi} \left[ \frac{3}{4} (1 - F_L) \sin^2 \theta_k \\
+ F_L \cos^2 \theta_k + \frac{1}{4} (1 - F_L) \sin^2 \theta_k \cos 2\theta_l \\
- F_L \cos^2 \theta_k \cos 2\theta_l + S_3 \sin^2 \theta_k \sin^2 \theta_l \cos 2\phi \\
+ S_4 \sin 2\theta_k \sin 2\theta_l \cos \phi + S_5 \sin 2\theta_k \sin \theta_l \cos \phi \\
+ \frac{4}{3} A_{FB} \sin^2 \theta_k \cos \theta_l + S_7 \sin 2\theta_k \sin \theta_l \sin \phi \\
+ S_8 \sin 2\theta_k \sin 2\theta_l \sin \phi + S_9 \sin^2 \theta_k \sin^2 \theta_l \sin 2\phi \right].$$
Results in $B \to K^{*}\mu\mu$

- Thanks to Method of Moments there was the possibility to measure a new observable $S_{6c}$.

Measurement is consistent with the SM prediction.
Angular analysis of $B^0 \rightarrow K^*ee$

- With the full data set (3fb$^{-1}$) we performed angular analysis in $0.0004 < q^2 < 1$ GeV$^2$/c$^4$.
- Electrons channels are extremely challenging experimentally:
  - Bremsstrahlung.
  - Trigger efficiencies.
- Determine the angular observables: $F_L$, $A_T^{(2)}$, $A_T^{Re}$, $A_T^{Im}$.
- Results in full agreement with the SM.
- Similar strength on $C_7$ Wilson coefficient as from $b \rightarrow s\gamma$ decays.