Initial stage of the HIC: thermalization and isotropization

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Keegan, Kurkela, Romatschke, van Der Schee, YZ, JHEP 1604 (2016) 031

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HIC phenomenology is based on two disconnected set of observables.

- Hard probes (jets, electromagnetic, heavy flavor, ... ) based on pert. theory.
- Soft probes (flow) based on relativistic hydrodynamics.
Introduction

- HIC phenomenology is based on two disconnected set of observables.
- Locally thermalized plasma described by relativistic hydrodynamics

\[ \partial_\mu T^{\mu\nu} = 0 \]
HIC phenomenology is based on two disconnected set of observables.

Locally thermalized plasma described by relativistic hydrodynamics

\[ \partial_\mu T^{\mu\nu} = 0 \]

Ideal hydro: \( T^{\mu\nu} = T_0^{\mu\nu} \),
1st order: \( \Delta T_1^{\mu\nu}(\eta, \xi) \), 2nd order: \( \Delta T_2^{\mu\nu}(\tau, \lambda, \kappa, \ldots) \).
Motivation

- Strong anisotropy $P_L/P_T \ll 1$, sign of large corrections
- At early times *pre-equilibrium* evolution
- Hydro simulations start at *initialization time* $\tau_i$
Motivation

If prethermal evolution converges smoothly to hydro, independence of unphysical $\tau_i$

Explicit example: Strong coupling $\mathcal{N} = 4$ SYM


In QCD, only at weak coupling!!!

Bottom-up thermalization at weak coupling


\[ Q_s \tau \ll 1: \text{high gluon density, strongly coherent fields} \rightarrow \text{decoherence} \]

Epelbaum & Gelis, PRL. 111 (2013) 23230

\[ A: f \sim 1/\alpha_s \rightarrow f \sim 1, P_L/P_T \sim (Q_s \tau)^{-1/3} \]


\[ B: f < 1, k_s^2 \sim \alpha Q_s^2, N_s < N_h, \text{no thermal bath.} \]

\[ C: \text{low temperature thermal bath} \rightarrow \text{fully thermalization.} \]

\[ Q_s \tau \gtrsim (\alpha_s)^{-13/5}: \text{hydrodynamics} \]
Bottom-up thermalization at weak coupling

Anisotropy: $P_T/P_L$

Occupancy: $f$

Overoccupied

Underoccupied

Initial

Thermal

$f \sim \alpha$

$f \sim 1$

$f \sim \alpha^{-1}$

占用: $f$

- Color Glass Condensate: Initial condition overoccupied
  

  \[ f(Q_s) \sim 1/\alpha_s, \quad Q_s \sim 2 \text{GeV} \]

- Expansion makes system underoccupied before thermalizing
  

  \[ f(Q_s) \ll 1 \]
Bottom-up thermalization at weak coupling

Anisotropy: $P_T / P_L$

Occupancy: $f$

Thermal

Kinetic theory

Both

Classical

YM

Initial

$f \sim \alpha$

$f \sim 1$

$f \sim \alpha^{-1}$

Occupancy: $f$

 Degrees of freedom:
- $f \gg 1$: Classical Yang-Mills theory (CYM)
- $f \ll 1/\alpha_s$: (Semi-)classical particles, Eff. Kinetic Theory (EKT)

 Transmutation of fields to particles: Field-particle duality


$1 \ll f \ll 1/\alpha_s$
Strategy at weak coupling

Anisotropy: $P_L/P_T$

Time: $\tau$

Hydro

EKT

τ$_{EKT}$~0.1 fm/c

τ$_i$~1 fm/c

Strategy: Switch from CYM to EKT at $\tau_{EKT}$, $1 \ll f \ll 1/\alpha_s$

From EKT to hydro at $\tau_i$, $P_L/P_T \sim 1$
Early times $0 < Q_s \tau \lesssim 1$: classical evolution

- Melting of the coherent boost invariant CGC fields
  - The initial condition from CGC: MV-model, JIMWLK
    \[ T_{CGC,LO}^{\mu\nu} = \text{diag}(\epsilon, \epsilon, \epsilon, -\epsilon) \] vs. \[ T_0^{\mu\nu} = \text{diag}(\epsilon, p, p, p) \]
  - After $\tau \sim 1/Q_s$, fields decohere, $P_L > 0$, $P_L/P_T \approx \text{const.}$

Epelbaum & Gelis, PRL. 111 (2013) 23230
Later times $Q_{sT} > 1$: classical evolution

- Numerical demonstration of classical/overoccupied part of the diagram

- $P_L / P_T$ decreases at later time
Later times $Q_s \tau > 1$: classical evolution

- Numerical demonstration of classical/overoccupied part of the diagram
- $P_L/P_T$ decreases at later time, universal attractor
- Classical theory never thermalises or isotropizes. Before $f \sim 1$, must switch to kinetic theory!

Self-similar scaling in the classical approx.

In non-pert classical regime $1 \ll f \ll 1/\alpha_s$

$$f(p_z, p_\perp, \tau) = (Q_s \tau)^{-2/3} f_S((Q_s \tau)^{1/3} p_z, p_\perp),$$

CYM $\alpha_s f \ll 1$ limit but $f \gg 1$

EKT $f \gg 1$ limit but $f \ll 1/\alpha_s$


Kurkela, YZ arXiv:1506.06647

- The quantitative connection allows for smooth matching between classical Yang-Mills and kinetic theory
Effective kinetic theory of AMY

Arnold, Moore, Yaffe, JHEP 0301 (2003) 030

\[ \frac{df}{dt} = -C_{2\leftrightarrow2}[f] - C_{1\leftrightarrow2}[f] \]
Effective kinetic theory of AMY

Arnold, Moore, Yaffe, JHEP 0301 (2003) 030

\[
\frac{df}{dt} = -C_{2\leftrightarrow 2}[f] - C_{1\leftrightarrow 2}[f]
\]

\[
C_{2\leftrightarrow 2}[f] = \int_{k, p', k'} |M|^2 [f_p f_k (1 + f_{p'}) (1 + f_{k'}) - f_{p'} f_{k'} (1 + f_p) (1 + f_k)]
\]

$|M|^2$ diverges as $1/q^4$. Dynamically regulated by screening

\[
\frac{1}{q^4} \Rightarrow \frac{1}{(q^2 + \Pi(\omega, q, m_D))^2} \Rightarrow \frac{1}{(q^2 + \tilde{m}^2)^2}
\]
Effective kinetic theory of AMY

Arnold, Moore, Yaffe, JHEP 0301 (2003) 030

\[
\frac{df}{dt} = -C_{2\leftrightarrow 2}[f] - C_{1\leftrightarrow 2}[f]
\]

\[
C_{1\leftrightarrow 2} \sim \int dp \, \gamma_{k,p-\hat{k}}^p \left[ f_p (1 + f_k) (1 + f_{p-\hat{k}}) - f_k f_{p-\hat{k}} (1 + f_p) \right]
\]

\[
\Gamma_{\text{hard}} \sim \sigma n (1 + f) \sim p_{\text{max}} f^2 \quad \Gamma_{\text{soft}} \sim p_{\text{max}} f^2 \frac{p_{\text{max}}^2}{m^2},
\]

Each time a particle undergoes a soft scattering, has $g^2$ chance to split.

\[
\Gamma_{\text{split}} \sim \alpha_s \Gamma_{\text{soft}} (1 + f_{\text{final}}) \gtrsim \Gamma_{\text{hard}}
\]

Formation time of radiated gluon $t_{\text{form}} \sim \frac{k}{(k_\perp)^2} = \frac{k}{\hat{q}t} > l_{\text{mfp}} \Rightarrow t_{\text{form}} \sim \sqrt{\frac{k}{\hat{q}}},$

\[
\Gamma_{\text{LPM split}} \sim \alpha_s / t_{\text{form}} = \alpha_s \sqrt{\frac{\hat{q}}{k}}
\]
Effectiv e kinetic theory of AMY

Arnold, Moore, Yaffe, JHEP 0301 (2003) 030

\[
\frac{df}{dt} = -C_{2\leftrightarrow2}[f] - C_{1\leftrightarrow2}[f]
\]

- Soft and collinear divergences lead to nontrivial matrix elements
  - Coulombic divergence in $t, u$-channels regulated by screening:
    
    Hard-loop resummed matrix element

  - Collinear divergence regulated by LPM-suppression:
    
    Ladder resummed effective $1 \leftrightarrow 2$ matrix element
Effective kinetic theory of AMY

Arnold, Moore, Yaffe, JHEP 0301 (2003) 030

\[ \frac{df}{dt} = -C_{2\leftrightarrow 2}[f] - C_{1\leftrightarrow 2}[f] \]

- Soft and collinear divergences lead to nontrivial matrix elements
- Momentum of quasiparticle \( p^2 > m^2 = 2N_c g^2 \int_p f(p)/p \)
- No free parameters; LO accurate in the \( \alpha_s \to 0, \alpha_s f \to 0 \) limit.
- Used for LO transport coefficients in QCD, jet energy loss
- Now also available in NLO \( \mathcal{O}(\sqrt{\alpha_s}) \)
Route to equilibrium in EKT

- Initial condition ($f \sim 1/\alpha_s$) from classical field theory calculation
- In the classical limit ($\alpha_s \to 0, \alpha_s f$ fixed), no thermalization
- At small values of couplings, clear Bottom-Up behaviour
- Features become less defined as $\alpha_s$ grows

Lappi PLB703 (2011) 325-330
Smooth approach to hydrodynamics

\[ \alpha_s = 0.03 \]

- Kinetic theory converges to hydro smoothly and automatically
Smooth approach to hydrodynamics

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- Kinetic theory converges to hydro smoothly and automatically
- Hydro prediction fixed by perturbative \( \eta/s \)

Arnold et al. JHEP 0305 (2003) 051
Smooth approach to hydrodynamics

\[ \alpha_s = 0.3 \]

- For realistic couplings, hydrodynamics reached around \( \lesssim 1\text{fm/c} \).
- Hydro seems to give a good description even when \( P_L/P_T \sim 1/5 \).
\( N=4 \) SYM vs. Kinetic thy.

\[ ds^2 = -dt^2 + dx^2 + dy^2 + g(t)dL^2, \]
\[ g(t \to -\infty) = 1, \text{ and } g(t \to \infty) \to t^2. \]

\[ \text{Far from equilibrium at } t_0 T_i \sim 1. \]
N=4 SYM vs. Kinetic thy.

- Almost universal description of isotropization and hydrodynamization time of strongly and weakly coupled system.
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Summary

- Combination of classical Yang-Mills simulations and effective kinetic theory allows to follow the time evolution from highly occupied initial condition to thermal equilibrium.

- Weak coupling thermalization extrapolated to realistic couplings shows agreement with hydro around

  \[ \tau_i \sim 1\text{fm/c} \]

- Unified description of soft and hard physics: hydro, jets, etc.