Initial stage of the HIC: thermalization and isotropization

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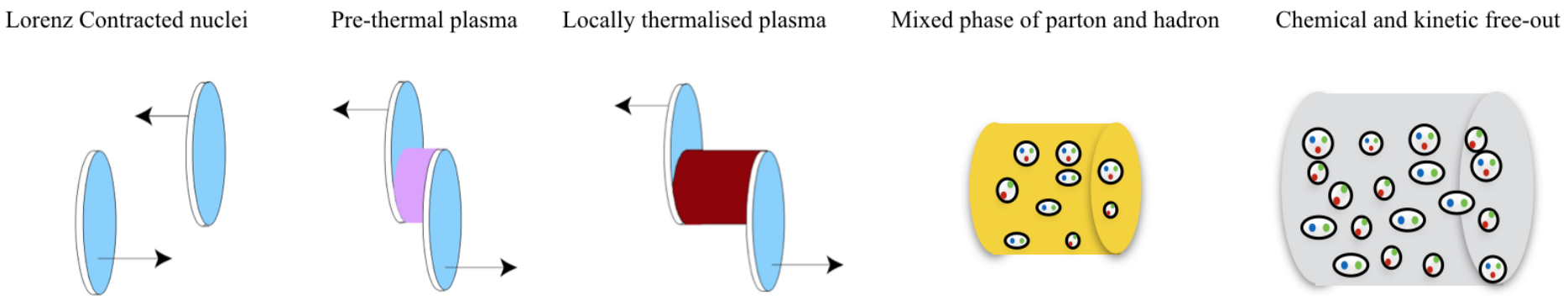
Kurkela, YZ, Phys.Rev.Lett. 115 (2015) 182301
Keegan, Kurkela, Romatschke, van Der Schee, YZ, JHEP 1604 (2016) 031

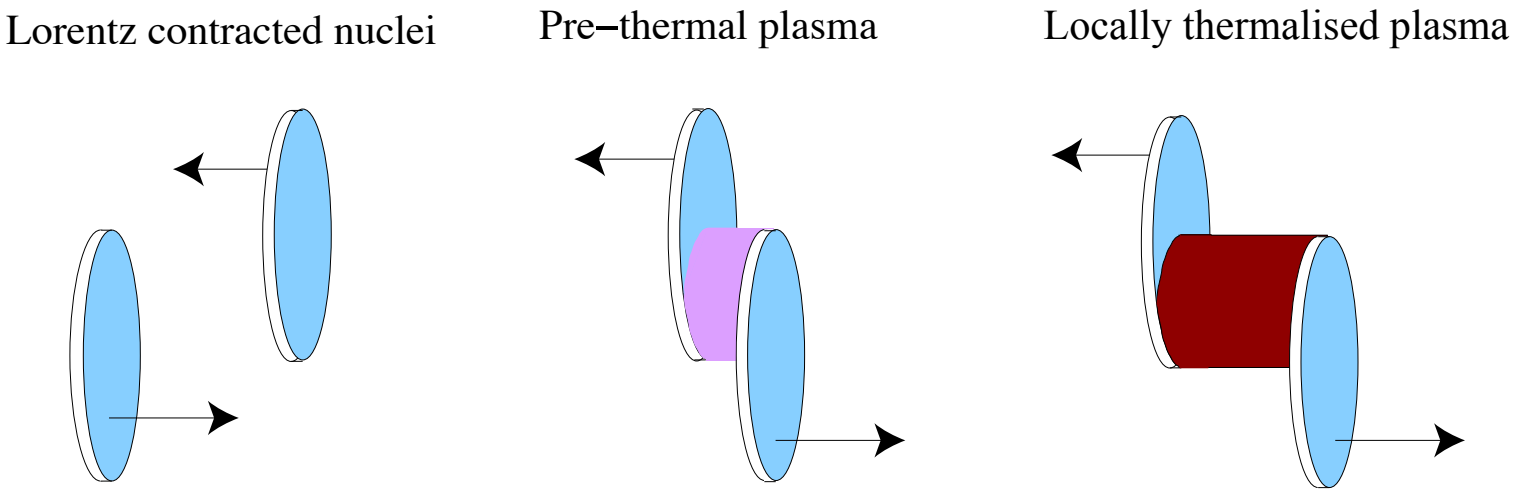
29 Aug 2016, Thessaloniki

Introduction



- HIC phenomenology is based on two disconnected set of observables.
  - Hard probes (jets, electromangetic, heavy flavor,...) based on pert. theory.
  - Soft probes (flow) based on relativistic hydrodynamics.

Introduction



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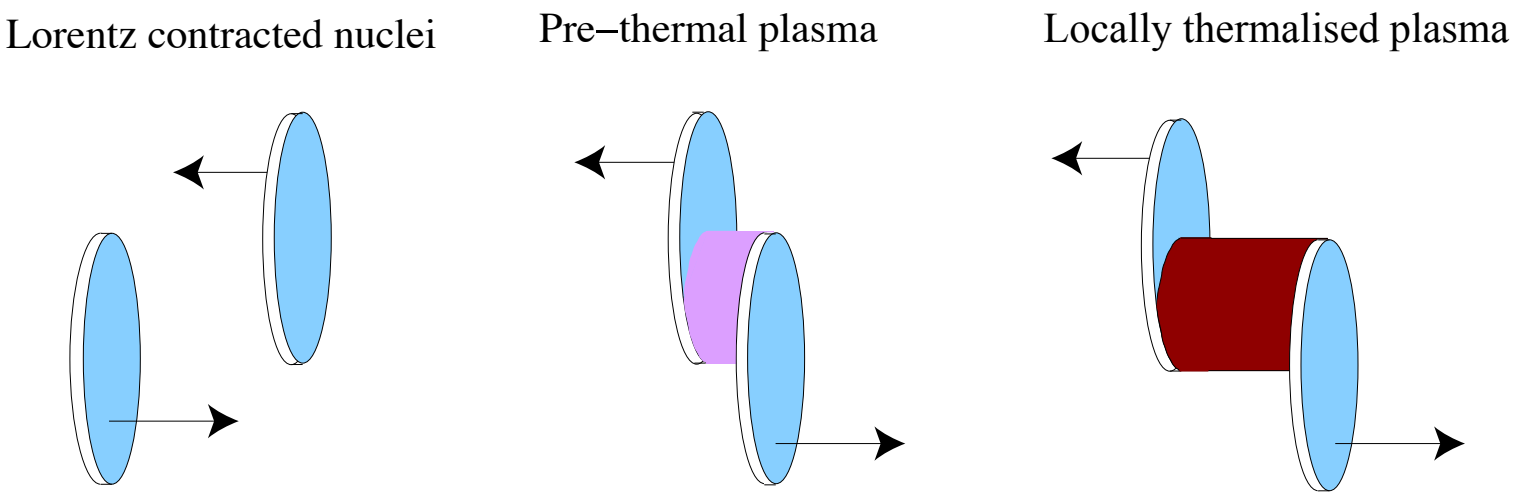
$$\partial_\mu T^{\mu\nu} = 0$$

Introduction

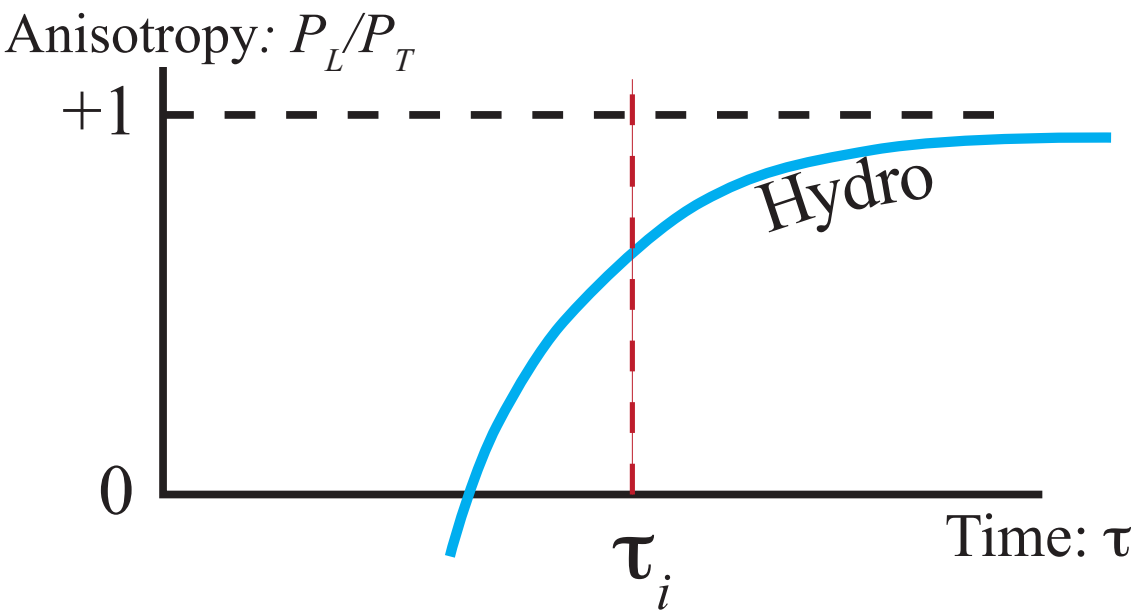


- HIC phenomenology is based on two disconnected set of observables.
- Locally thermalized plasma described by relativistic hydrodynamics

$$\partial_\mu T^{\mu\nu} = 0$$

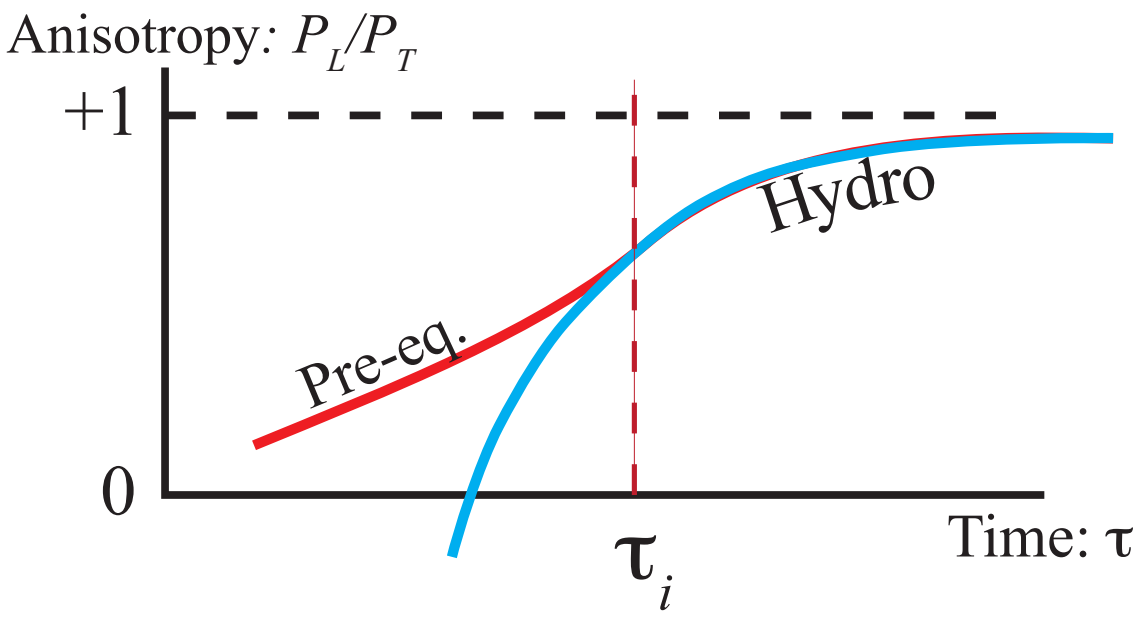
- Ideal hydro: $T^{\mu\nu} = T_0^{\mu\nu}$,
  1st order: $\Delta T_1^{\mu\nu}(\eta, \xi)$, 2nd order: $\Delta T_2^{\mu\nu}(\tau_\pi, \lambda_i, \kappa, \dots)$.

Motivation



- Strong anisotropy $P_L/P_T \ll 1$, sign of large corrections
- At early times *pre-equilibrium* evolution
- Hydro simulations start at *intialization time* τ_i

Motivation

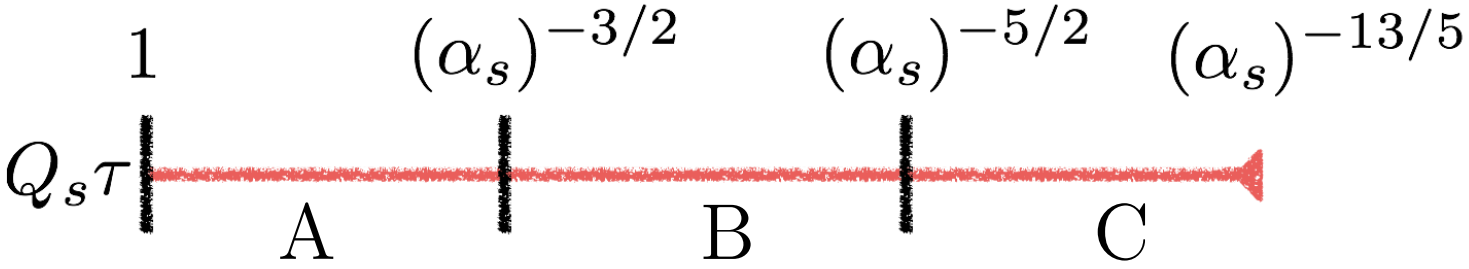


- If prethermal evolution converges smoothly to hydro, independence of unphysical τ_i
- Explicit example: Strong coupling $\mathcal{N} = 4$ SYM
  Chesler, Yaffe PRL 106 (2011) 021601; van der Schee et al. PRL 111 (2013) 22, 222302, arXiv:1507.08195; Keegan et al arXiv:1512.05347

In QCD, only at weak coupling!!!
Kurkela and YZ arXiv:1506.06647; Keegan et al arXiv:1512.05347

Bottom-up thermalization at weak coupling

Baier et al Phys.Lett. B502 (2001) 51-58



- $Q_s\tau \ll 1$: high gloun density, stongly coherent fields $\rightarrow$ decoherence

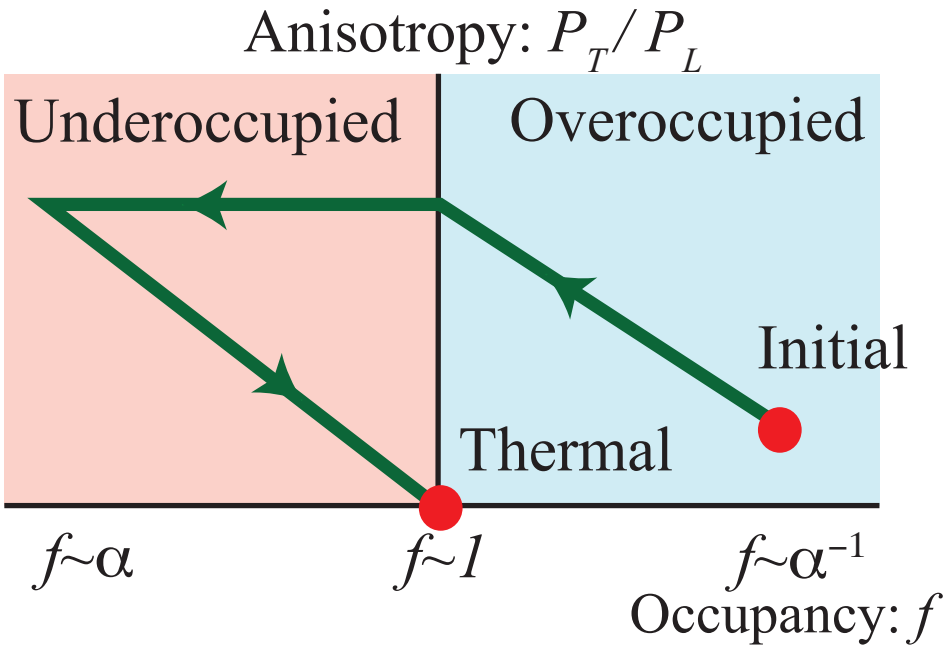
  Epelbaum & Gelis, PRL. 111 (2013) 23230

- A: $f \sim 1/\alpha_s \rightarrow f \sim 1, P_L/P_T \sim (Q_s\tau)^{-1/3}$

  Berges et al. Phys.Rev. D89 (2014) 7, 074011

- B: $f < 1, k_s^2 \sim \alpha Q_s^2$, $N_s < N_h$, no thermal bath.
- C: low temperature thermal bath $\rightarrow$ fully thermalization.
- $Q_s\tau \gtrsim (\alpha_s)^{-13/5}$: hydrodynamics

Bottom-up thermalization at weak coupling



- Color Glass Condensate: Initial condition overoccupied

  McLerran, Venugopalan PRD49 (1994) 2233-2241 , PRD49 (1994) 3352-3355 ; Gelis et. al Int.J.Mod.Phys. E16 (2007) 2595-2637 , Ann.Rev.Nucl.Part.Sci. 60 (2010) 463-489

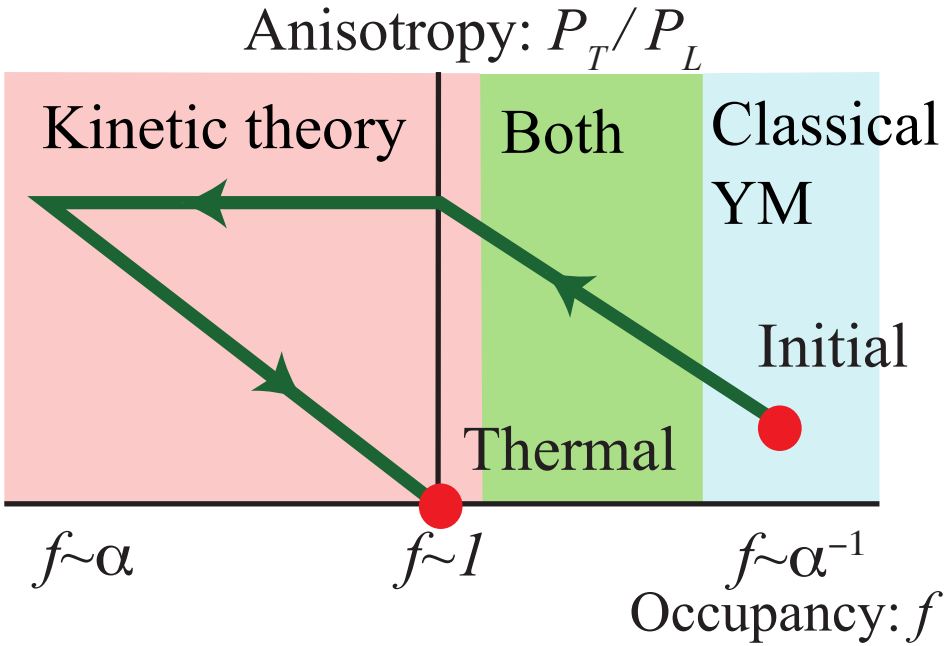
$$f(Q_s) \sim 1/\alpha_s, \qquad Q_s \sim 2\text{GeV}$$

- Expansion makes system underoccupied before thermalizing

  Baier et al Phys.Lett. B502 (2001) 51-58; Kurkela, Moore JHEP 1111 (2011) 120

$$f(Q_s) \ll 1$$

Bottom-up thermalization at weak coupling

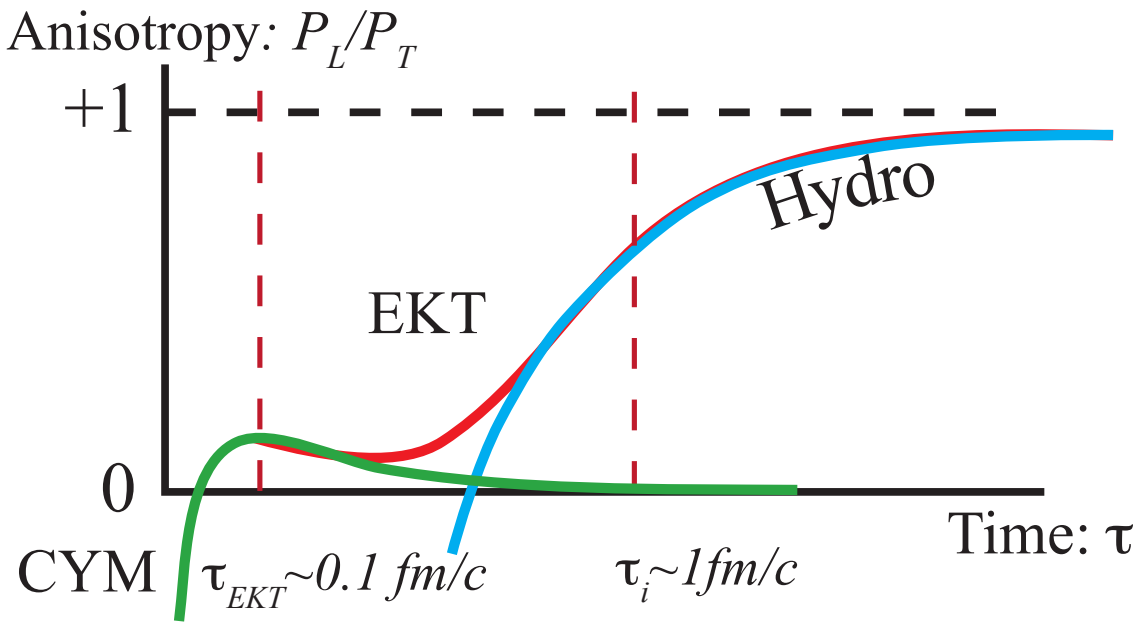


- Degrees of freedom:
  - $f \gg 1$: Classical Yang-Mills theory (CYM)
  - $f \ll 1/\alpha_s$: (Semi-)classical particles, Eff. Kinetic Theory (EKT)

- Transmutation of fields to particles: Field-particle duality
  Son, Mueller PLB582 (2004) 279-287; Jeon PRC72 (2005) 014907; Mathieu et al EPJ. C74 (2014) 2873 ; Kurkela, Moore PRD89 (2014) 7, 074036

$$1 \ll f \ll 1/\alpha_s$$

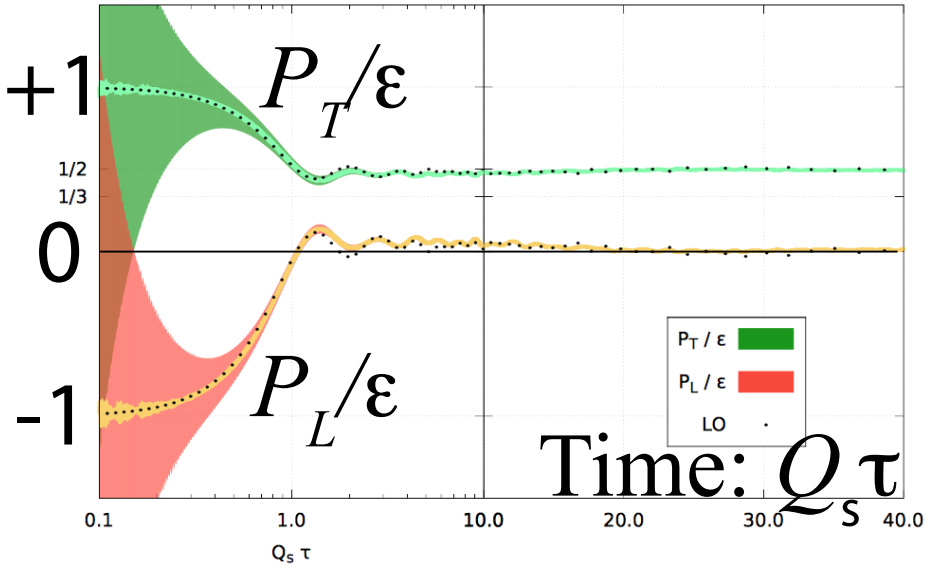
Strategy at weak coupling



Strategy: Switch from CYM to EKT at τ_{EKT}, $1 \ll f \ll 1/\alpha_s$

From EKT to hydro at τ_i, $P_L/P_T \sim 1$

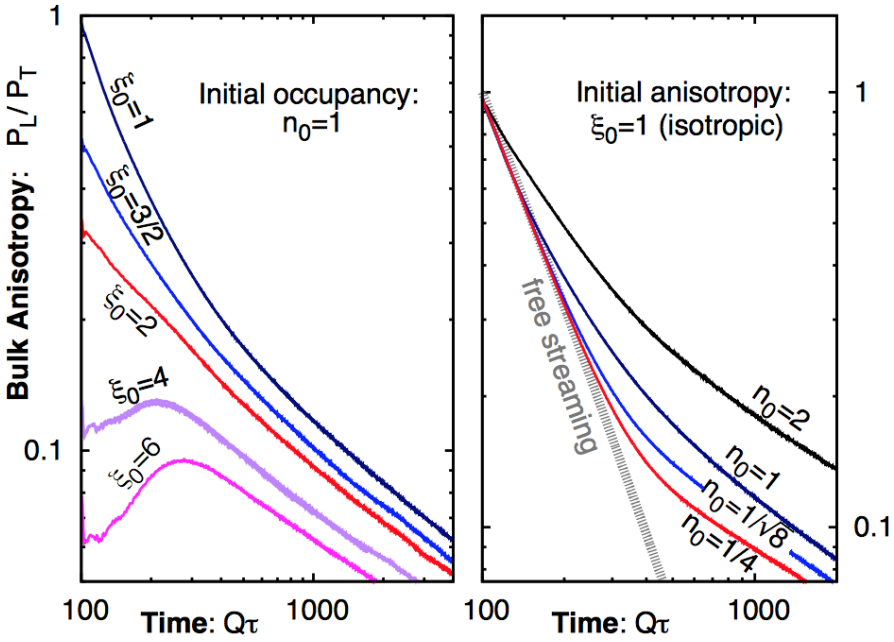
Early times $0 < Q_s\tau \lesssim 1$: classical evolution

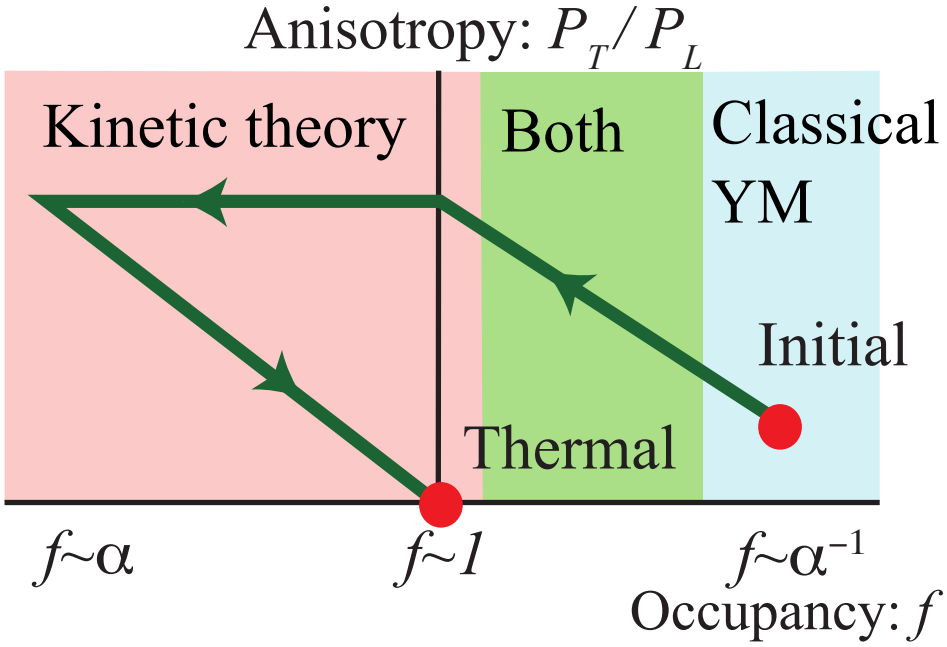


Epelbaum & Gelis, PRL. 111 (2013) 23230

- Melting of the coherent boost invariant CGC fields
  - The initial condition from CGC: MV-model, JIMWLK $T^{\mu\nu}_{\mathrm{CGC,LO}} = \mathrm{diag}(\epsilon, \epsilon, \epsilon, -\epsilon)$ vs. $T^{\mu\nu}_0 = \mathrm{diag}(\epsilon, p, p, p)$
- After $\tau \sim 1/Q_s$, fields decohere, $P_L > 0$, $P_L/P_T \simeq const.$

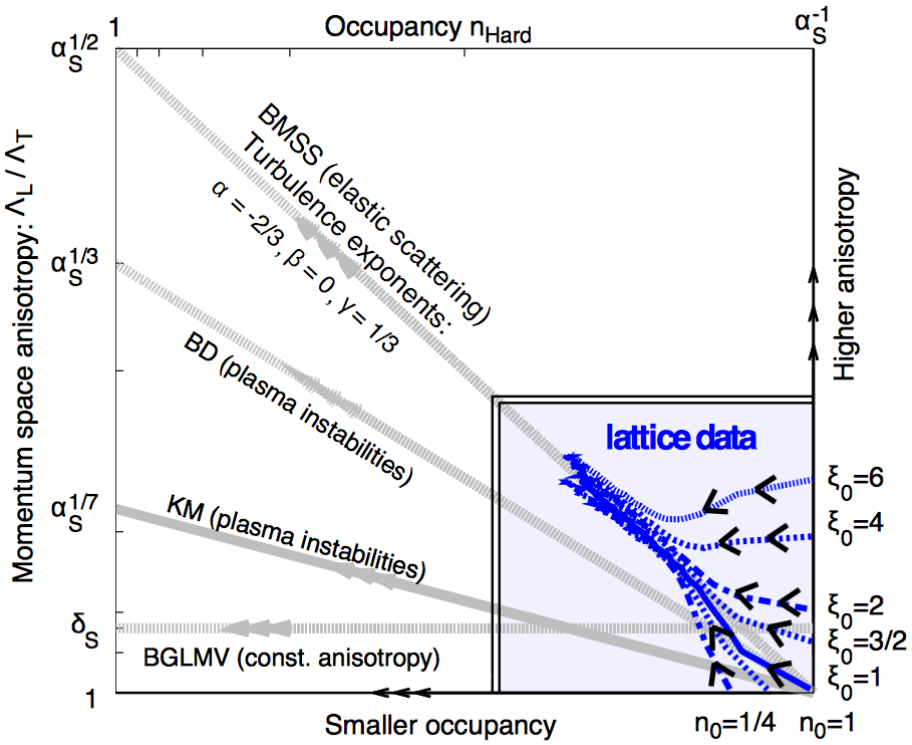
Later times $Q_s\tau > 1$: classical evolution

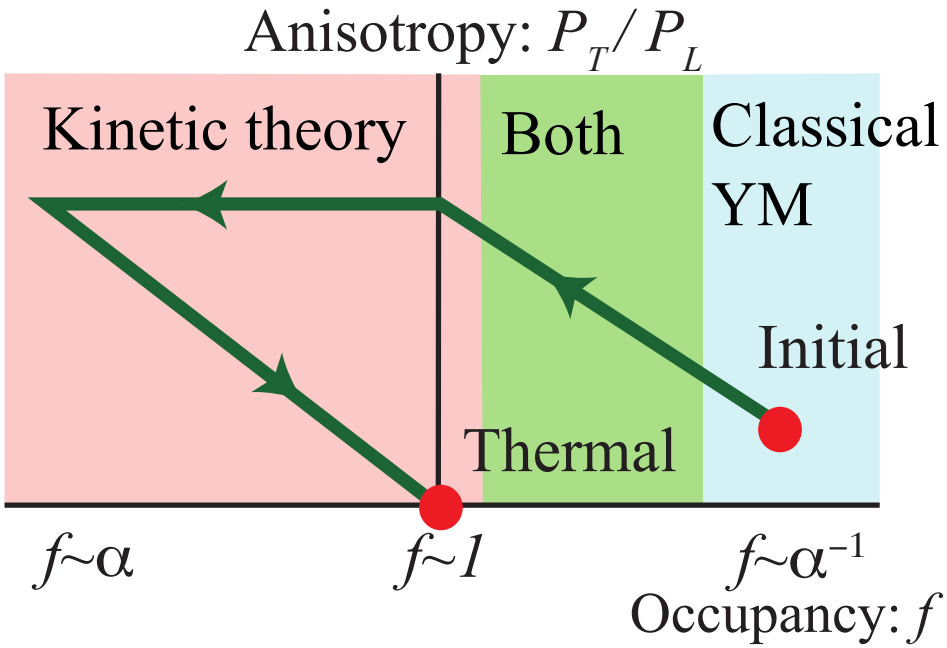




- Numerical demonstration of classical/overoccupied part of the diagram
- P_L/P_T decreases at later time

Later times $Q_s\tau > 1$: classical evolution





Berges et al. Phys.Rev. D89 (2014) 7, 074011

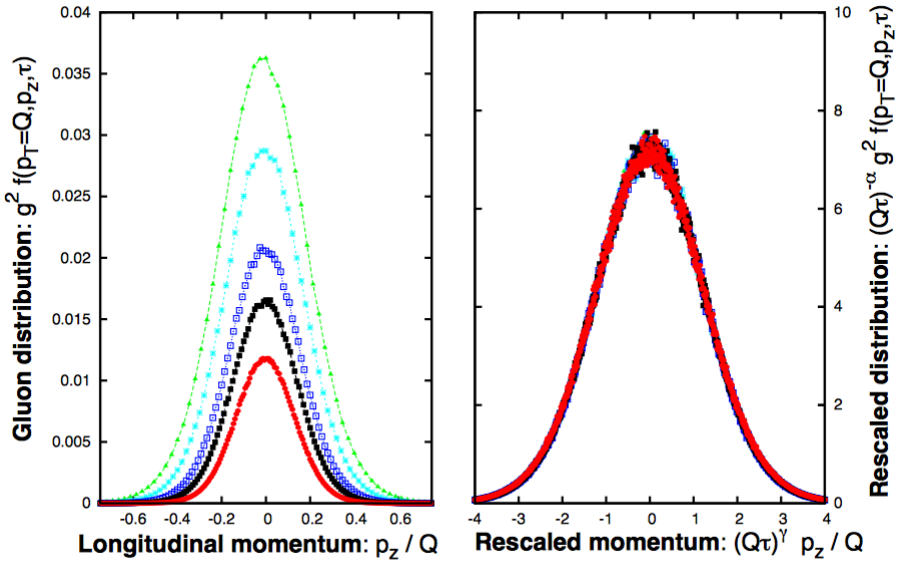
- Numerical demonstration of classical/overoccupied part of the diagram
- P_L/P_T decreases at later time, universal attractor
- Classical theory never thermalises or isotropizes. Before $f \sim 1$, must switch to kinetic theory!

Self-similar scaling in the classical approx.

In non-pert classical regime $1 \ll f \ll 1/\alpha_s$

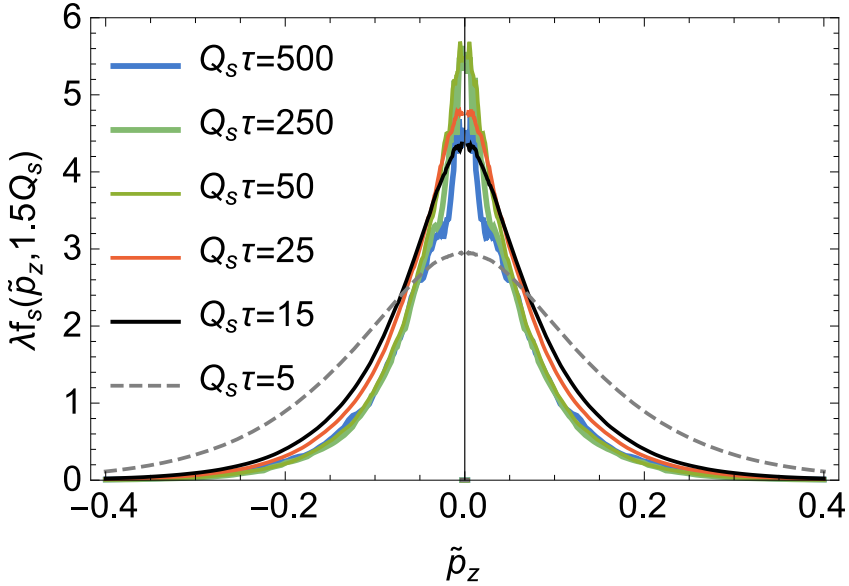
$$f(p_z, p_\perp, \tau) = (Q_s\tau)^{-2/3} f_S((Q_s\tau)^{1/3} p_z, p_\perp),$$

CYM $\alpha_s f \ll 1$ limit but $f \gg 1$



Berges et al. Phys.Rev. D89 (2014) 7, 074011

EKT $f \gg 1$ limit but $f \ll 1/\alpha_s$



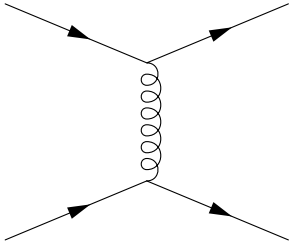
Kurkela, YZ arXiv:1506.06647

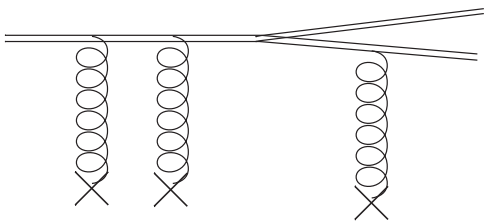
- The quantitative connection allows for smooth matching between classical Yang-Mills and kinetic theory

Effective kinetic theory of AMY

Arnold, Moore, Yaffe, JHEP 0301 (2003) 030

$$\frac{df}{dt} = -C_{2\leftrightarrow 2}[f] - C_{1\leftrightarrow 2}[f]$$





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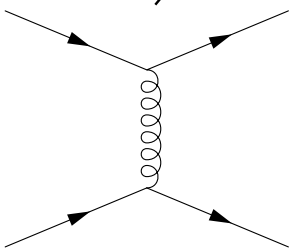

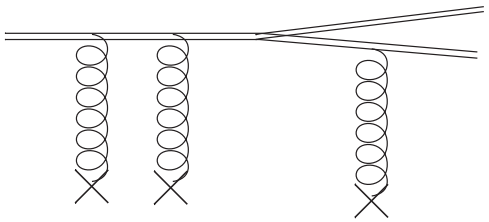

$$C_{2\leftrightarrow 2}[f] = \int_{k,p',k'} |M|^2 \left[f_p f_k (1+f_{p'})(1+f_{k'}) - f_{p'} f_{k'} (1+f_p)(1+f_k) \right]$$

$|M|^2$ diverges as $1/q^4$. Dynamically regulated by screening

$$\frac{1}{q^4} \Rightarrow \frac{1}{(q^2 + \Pi(\omega, q, m_D))^2} \Rightarrow \frac{1}{(q^2 + \tilde{m}^2)^2}$$

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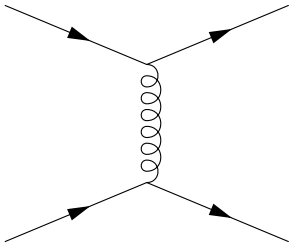
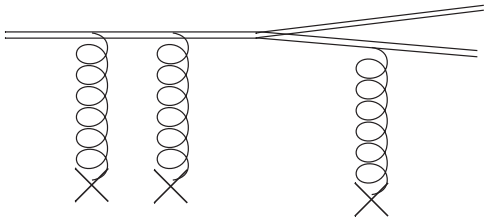




$$C_{1\leftrightarrow 2} \sim \int dp\, \gamma^{p}_{k,p-k}\, [f_p(1+f_k)(1+f_{p-k}) - f_k f_{p-k}(1+f_p)]$$

$$\Gamma_{\rm hard} \sim \sigma n(1+f) \sim p_{\rm max} f^2 \qquad \Gamma_{\rm soft} \sim p_{\rm max} f^2 \frac{p^2_{\rm max}}{m^2},$$

Each time a particle undergoes a soft scattering, has g^2 chance to split.

$\Gamma_{\rm split} \sim \alpha_s \Gamma_{\rm soft}(1+f_{\rm final}) \gtrsim \Gamma_{\rm hard}$

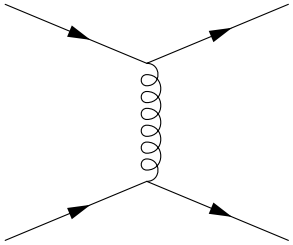
Formation time of radiated gluon $t_{\rm form} \sim \frac{k}{(k_\perp)^2} = \frac{k}{\hat{q}t} > l_{\rm mfp} \Rightarrow t_{\rm form} \sim \sqrt{\frac{k}{\hat{q}}}$,

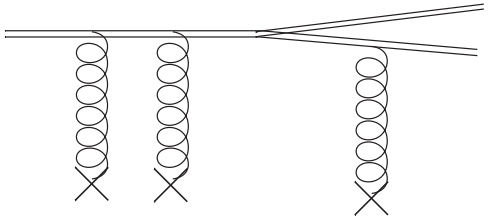
$\Gamma^{\rm LPM}_{\rm split} \sim \alpha_s / t_{\rm form} = \alpha_s \sqrt{\frac{\hat{q}}{k}}$

Effective kinetic theory of AMY

Arnold, Moore, Yaffe, JHEP 0301 (2003) 030

$$\frac{df}{dt} = -C_{2\leftrightarrow 2}[f] - C_{1\leftrightarrow 2}[f]$$



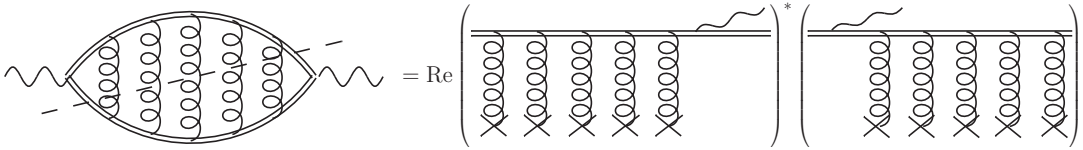


- Soft and collinear divergences lead to nontrivial matrix elements
  - Coulombic divergence in t, u-channels regulated by screening:

    Hard-loop resummed matrix element

  - Collinear divergence regulated by LPM-suppression:

    Ladder resummed effective $1 \leftrightarrow 2$ matrix element



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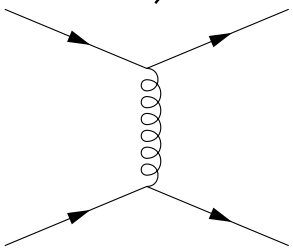
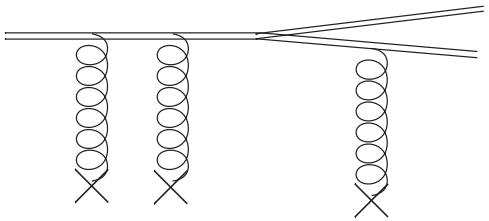
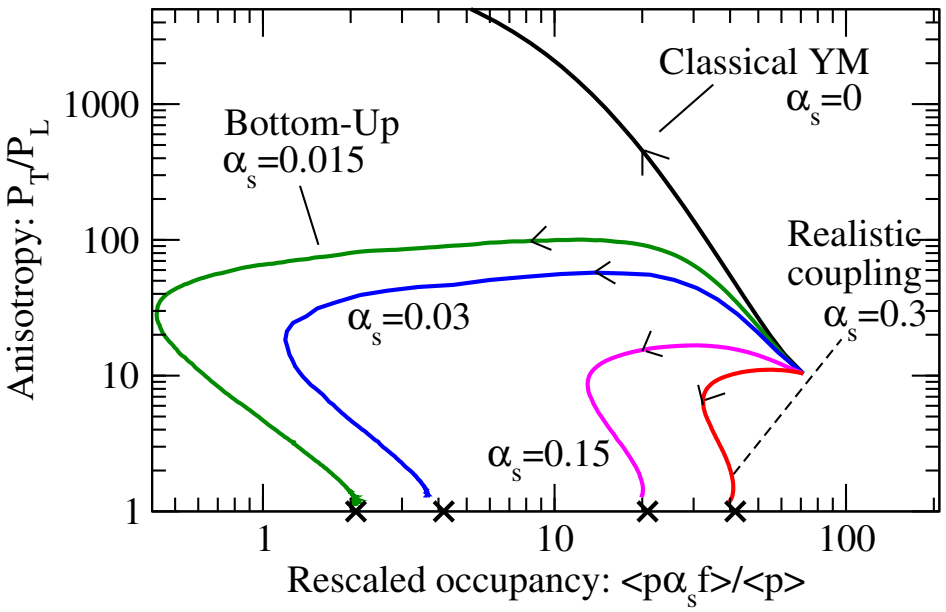
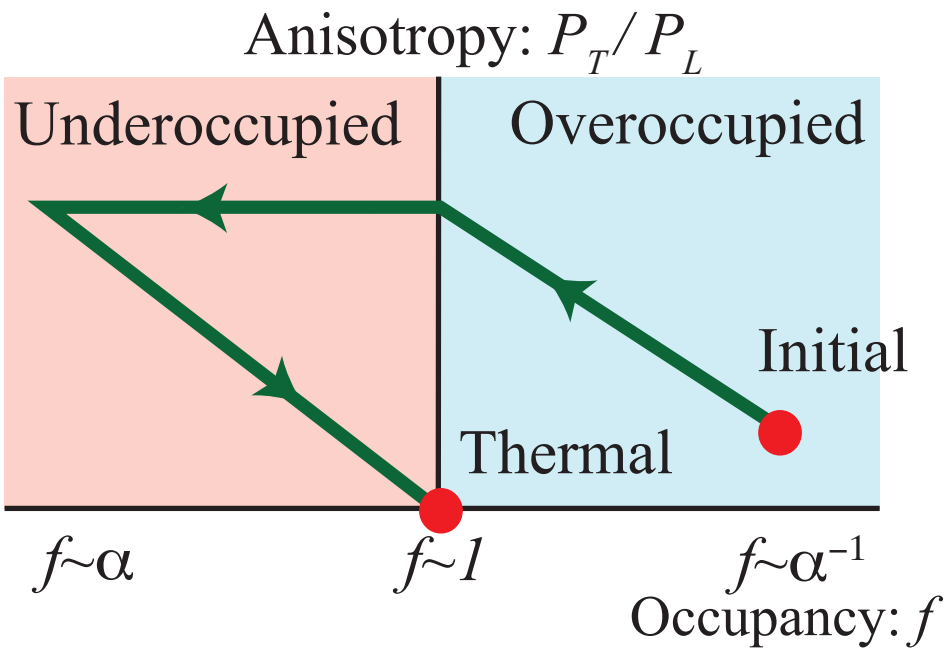




- Soft and collinear divergences lead to nontrivial matrix elements
- Momentum of quasiparticle $p^2 > m^2 = 2N_c g^2 \int_{\mathbf{p}} f(p)/p$
- No free parameters; LO accurate in the $\alpha_s \to 0$, $\alpha_s f \to 0$ limit.
- Used for LO transport coefficients in QCD, jet energy loss
  Arnold et al. JHEP 0305 (2003) 051; Moore, York PRD79 (2009) 054011; Ghiglieri, Teaney 1502.03730; Kurkela, Wiedemann PLB740 (2015) 172-178; Iancu, Wu 1506.07871
- Now also available in NLO $\mathcal{O}(\sqrt{\alpha_s})$ Ghiglieri 1509.07773

Route to equilibrium in EKT

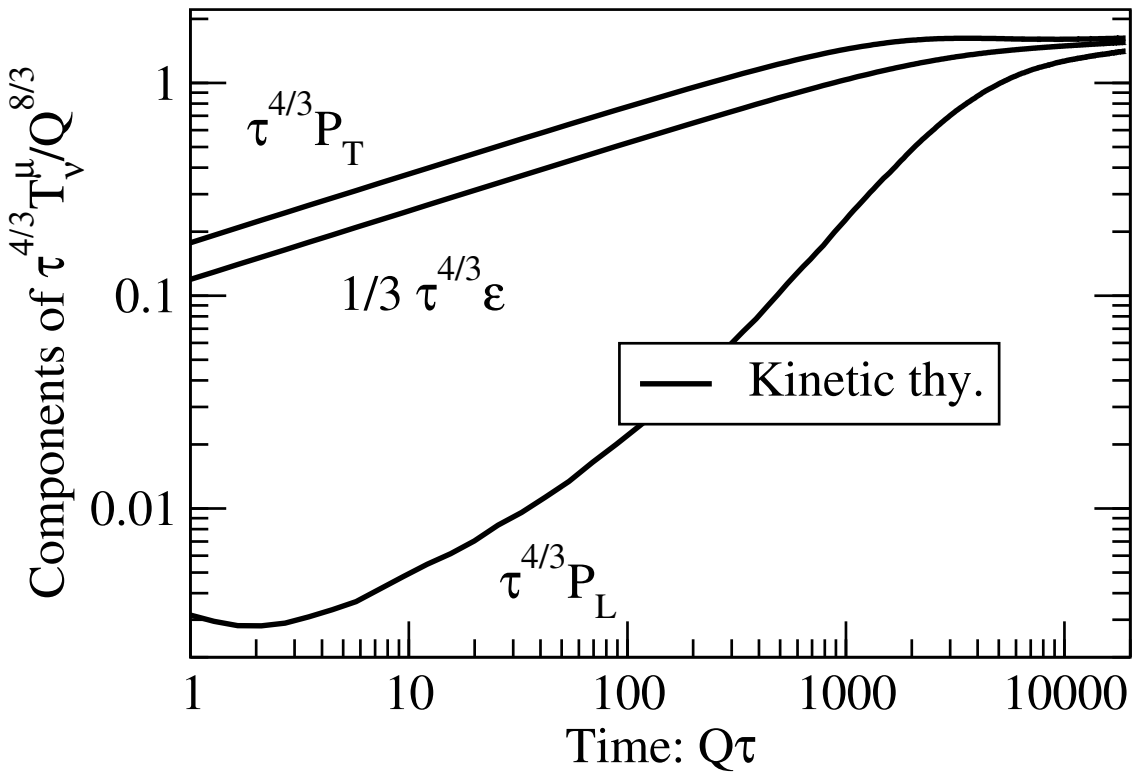




- Initial condition ($f \sim 1/\alpha_s$) from classical field theory calculation
  Lappi PLB703 (2011) 325-330
- In the classical limit ($\alpha_s \to 0, \alpha_s f$ fixed), no thermalization
- At small values of couplings, clear Bottom-Up behaviour
- Features become less defined as α_s grows

Smooth approach to hydrodynamics

$$\alpha_s = 0.03$$



- Kinetic theory converges to hydro smoothly and automatically

Smooth approach to hydrodynamics

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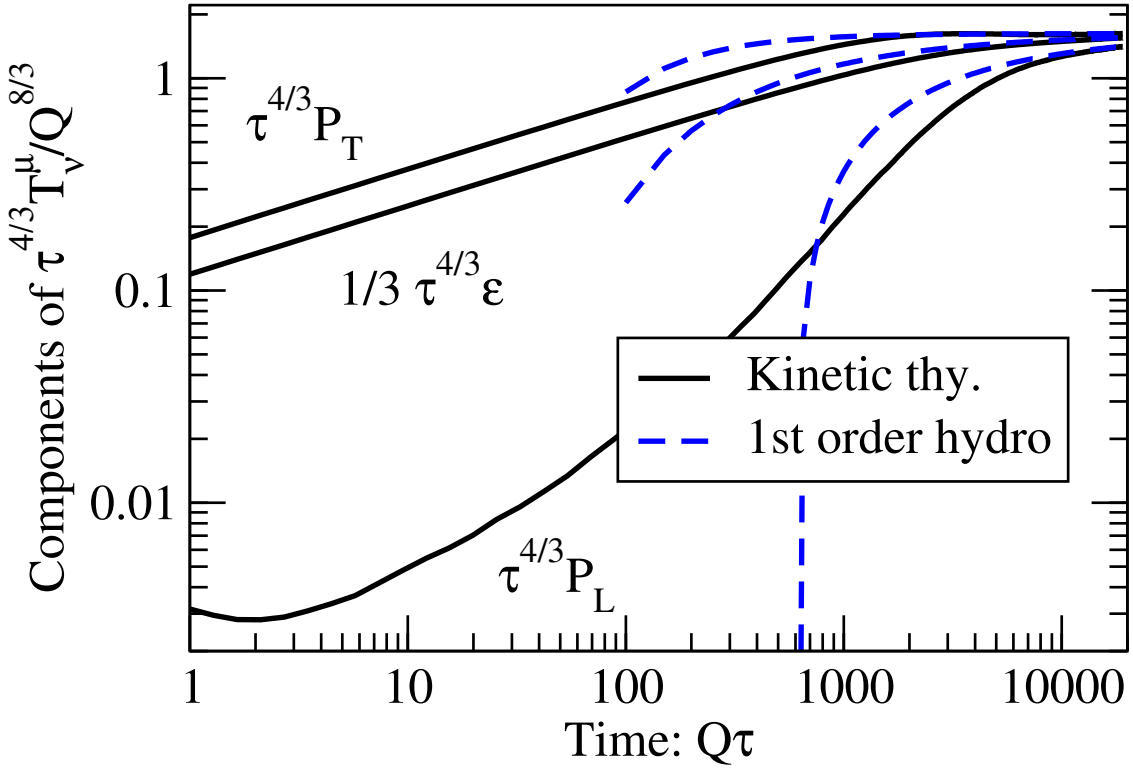
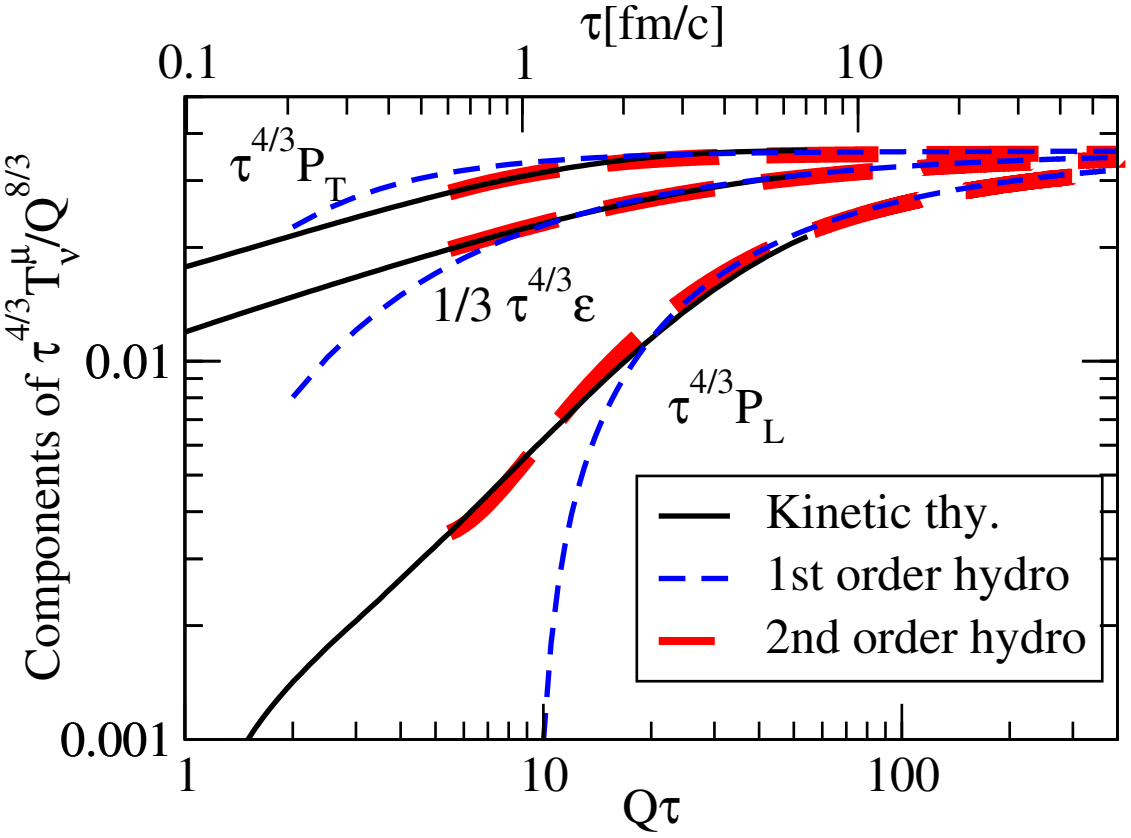


- Kinetic theory converges to hydro smoothly and automatically
- Hydro prediction fixed by perturbative η/s

Arnold et al. JHEP 0305 (2003) 051

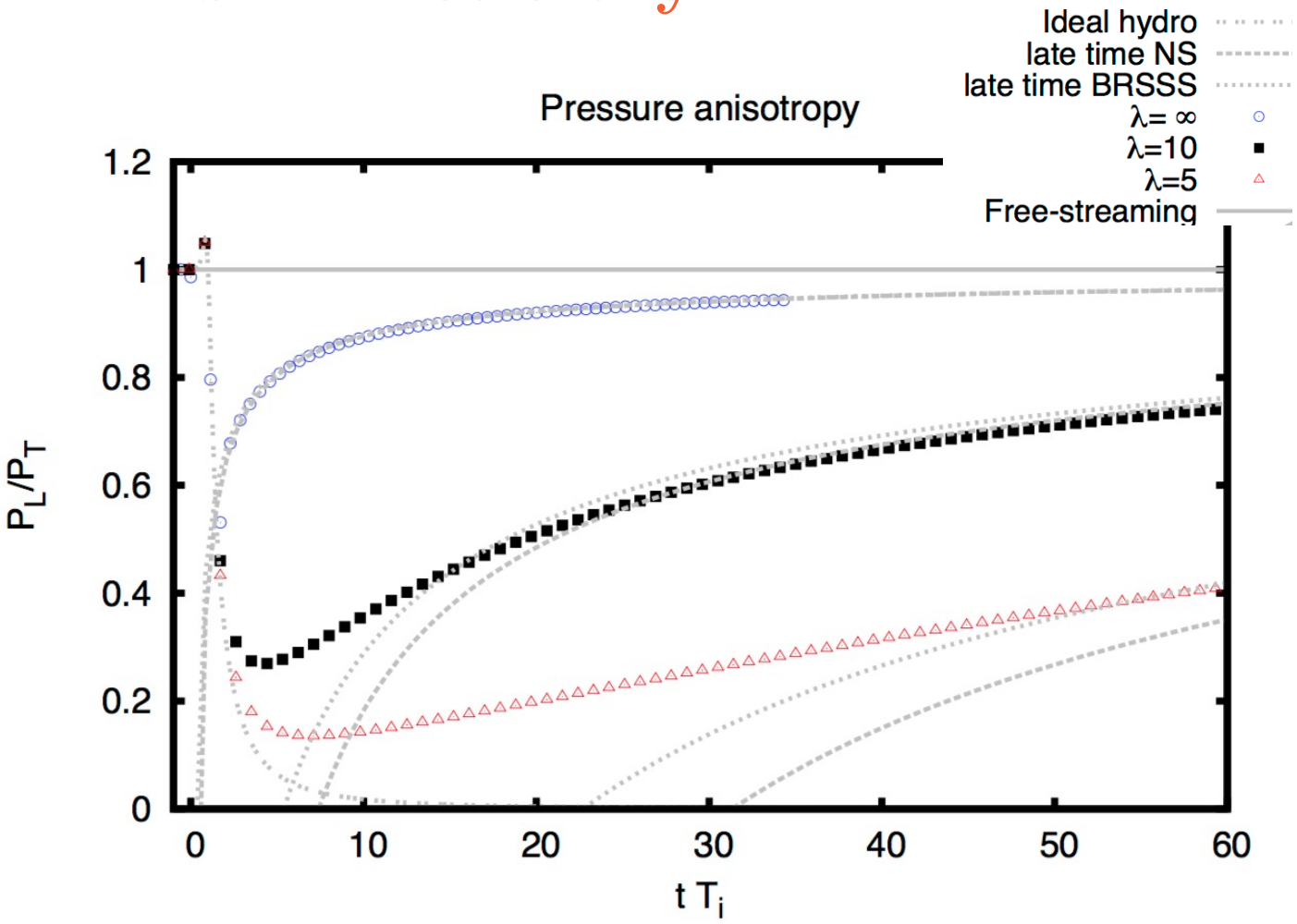
Smooth approach to hydrodynamics

$$\alpha_s = 0.3$$



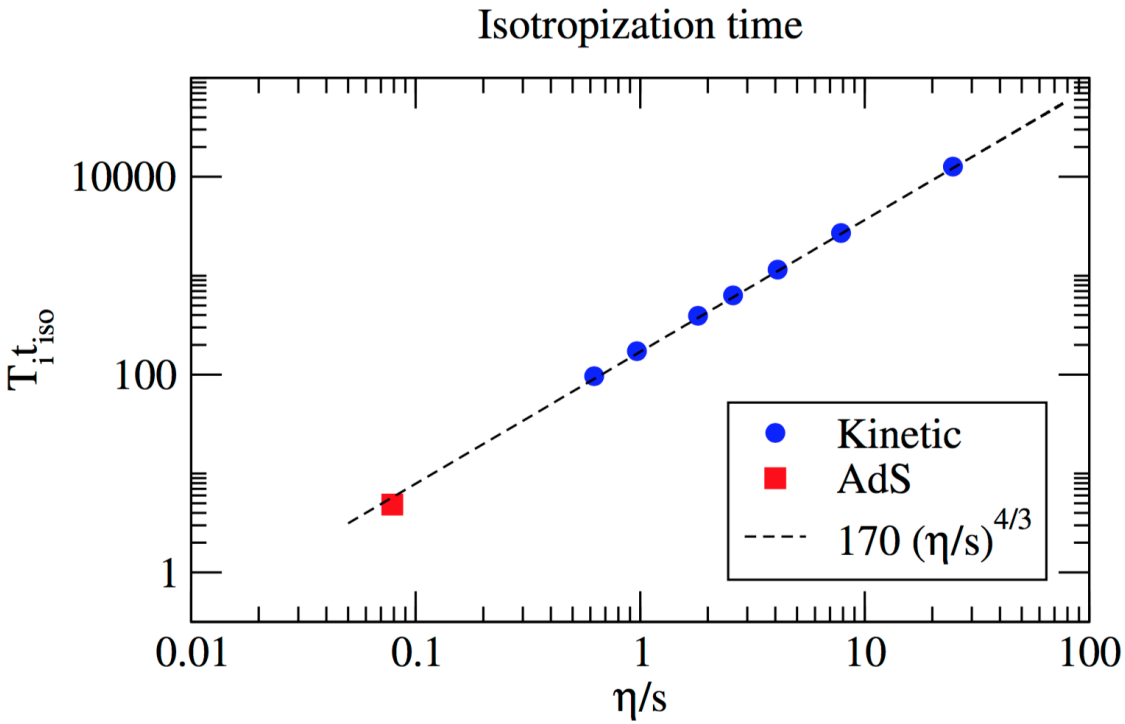
- For realistic couplings, hydrodynamics reached around $\lesssim$ 1fm/c.
- Hydro seems to give a good description even when $P_L/P_T \sim 1/5$

N=4 SYM vs. Kinetic thy.



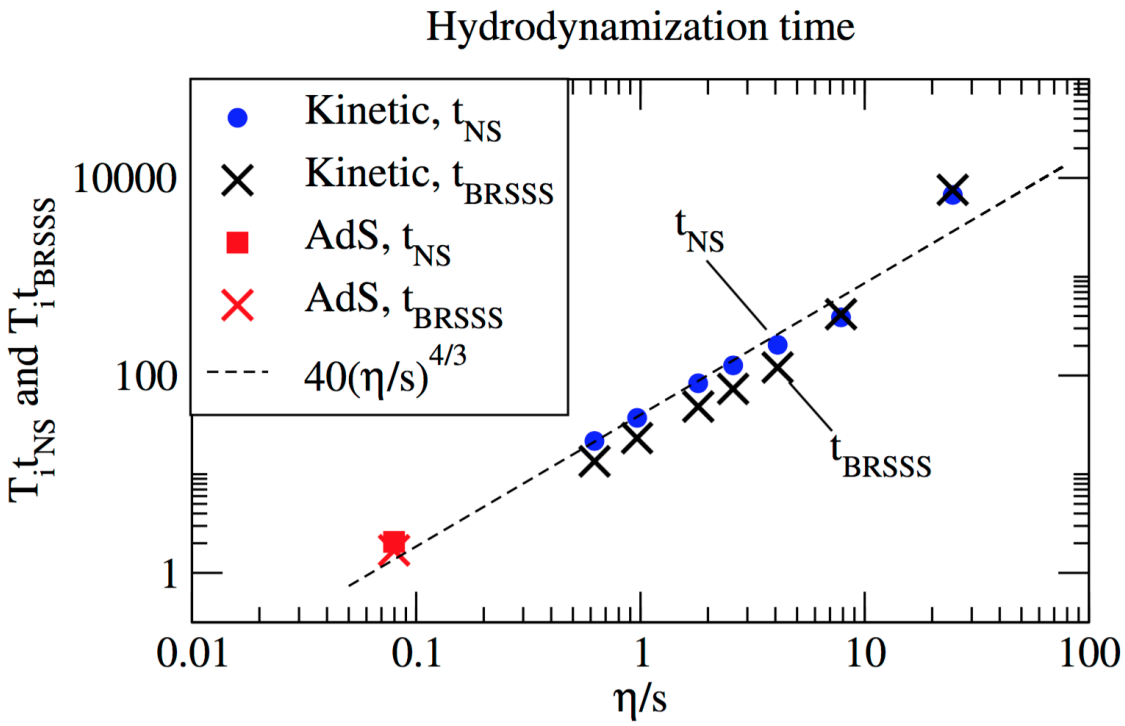
- $ds^2 = -dt^2 + dx^2 + dy^2 + g(t)dL^2$,
  $g(t \to -\infty) = 1$, and $g(t \to \infty) \to t^2$.
- Far from equilibrium at $t_0 T_i \sim 1$.

N=4 SYM vs. Kinetic thy.



- Almost universal description of isotropization and hydrodynamization time of strongly and weakly coupled system

N=4 SYM vs. Kinetic thy.



- Almost universal description of isotropization and hydrodynamization time of strongly and weakly coupled system

Summary

- Combination of classical Yang-Mills simulations and effective kinetic theory allows to follow the time evolution from highly occupied initial condition to thermal equilibrium.

- Weak coupling thermalization extrapolated to realistic couplings shows agreement with hydro around

$$\tau_{\mathrm{i}} \sim 1\mathrm{fm/c}$$

- Unified description of soft and hard physics: hydro, jets, etc.