Temperature dependence of shear viscosity in SU(3)-gluodynamics

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Outline:

- Introduction
- Details of the calculation
- Fitting of the data
- Backus-Gilbert method
- Conclusion
Elliptic flow from STAR experiment (Nucl. Phys. A 757, 102 (2005))

\[ \frac{dN}{d\varphi} \sim (1 + 2v_1 \cos \varphi + 2v_2 \cos^2 \varphi), \quad \varphi-\text{scattering angle} \]

**Quark-gluon plasma is close to ideal liquid,**

\[ \eta/s = (1 - 3)/4\pi \]

Other works (SU(3) gluodynamics):


Results:

- $\eta/s = 0.134 \pm 0.033$ ($T/T_c = 1.65, 8 \times 28^3$)
- $\eta/s = 0.102 \pm 0.056$ ($T/T_c = 1.24, 8 \times 28^3$)
- $\eta/s = 0.20 \pm 0.03$ ($T/T_c = 1.58, 16 \times 48^3$)
- $\eta/s = 0.26 \pm 0.03$ ($T/T_c = 2.32, 16 \times 48^3$)

SU(2) gluodynamics:

- $\eta/s = 0.134 \pm 0.057$ ($T/T_c = 1.2, 16 \times 32^3$)


Indicates that small viscosity is a general feature of non-abelian gauge theories?
Lattice calculation of shear viscosity

The first step:

Measure the correlation function:

\[ C(t) = \langle T_{12}(t) T_{12}(0) \rangle \]

The second step:

Calculation of the spectral function \( \rho(\omega) \):

\[ C(t) = \int_0^\infty d\omega \rho(\omega) \cosh(\omega T - \omega t) \sinh(\omega T) \]

\[ \eta = \pi \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega} \]
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Details of the calculation

- SU(3) gluodynamics
- Two-level algorithm
- Lattice size $32^3 \times 16$
- Temperatures
  $T/T_c = 0.9, 0.925, 0.95, 1.0, 1.1, 1.2, 1.35, 1.425, 1.5$
- Accuracy $\sim 2 - 3\%$ at $t = \frac{1}{2T}$
- $\langle T_{12}(x)T_{12}(y) \rangle \sim (\langle T_{11}(x)T_{11}(y) \rangle - \langle T_{11}(x)T_{22}(y) \rangle)$
- Clover discretization for the $\hat{F}_{\mu\nu}$
Correlation functions

\[ C(\tau)/T^5_c \]

\( T/T_c = 0.9 \)
\( T/T_c = 1.0 \)
\( T/T_c = 1.2 \)
\( T/T_c = 1.5 \)

\( \tau/a \)

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Spectral function

\[ C(t) = \int_0^\infty d\omega \rho(\omega) \frac{\cosh(\frac{\omega}{2T} - \omega t)}{\sinh(\frac{\omega}{2T})} \]

Properties of the spectral function:

- \( \rho(\omega) \geq 0, \rho(-\omega) = -\rho(\omega) \)
- Asymptotic freedom: \( \rho(\omega)|_{\omega \to \infty}^{NLO} = \frac{1}{10} \frac{d_A}{(4\pi)^2} \omega^4 \left( 1 - \frac{5N_c\alpha_s}{9\pi} \right) \)
  \( \sim 90\% \) of the total contribution \( t = 1/2T \)
- Hydrodynamics: \( \rho(\omega)|_{\omega \to 0} = \frac{\eta}{\pi} \omega \)
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Ansatz for the spectral function (QCD sum rules motivation)

\[ \rho(\omega) = \frac{\eta}{\pi} \omega \theta(\omega_0 - \omega) + A\rho_{lat}(\omega) \theta(\omega - \omega_0) \]
Lattice spectral function $\rho_{\text{lat}}$
Spectral function

\[ \rho(\omega) = \frac{\eta}{\pi} \omega \theta(\omega_0 - \omega) + A \rho_{\text{lat}}(\omega) \theta(\omega - \omega_0) \]

\[ \chi^2/\text{dof} \sim 1.0, \ A \sim 1, \ \omega_0/T \sim 7 - 8 \]
Properties of the spectral function

- Hydrodynamical approximation works well up to $\omega < \pi T \sim 1$ GeV (H. B. Meyer, arXiv:0809.5202).

Asymptotic freedom works well from $\omega > 3$ GeV.

Poor knowledge of the spectral function in the region $\omega \in (1, 3)$ GeV $\Rightarrow$ Main source of uncertainty in the parametrical estimation $\Rightarrow$ need to apply non-parametrical estimation procedure.
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- Asymptotic freedom works well from \( \omega > 3 \text{ GeV} \).
- Poor knowledge of the spectral function in the region \( \omega \in (1, 3) \text{ GeV} \)  
  \( \Rightarrow \) Main source of uncertainty in the parametrical estimation \( \Rightarrow \) need to apply non-parametrical estimation procedure.
Backus-Gilbert method for the spectral function

- Problem: find $\rho(\omega)$ from the integral equation
  \[ C(x_i) = \int_0^{\infty} d\omega \rho(\omega) K(x_i, \omega), \quad K(x_i, \omega) = \frac{\cosh\left(\frac{\omega - \omega x_i}{2T}\right)}{\sinh\left(\frac{\omega}{2T}\right)} \]

- Define an estimator $\tilde{\rho}(\bar{\omega})$ ($\delta(\bar{\omega}, \omega)$ — resolution function):
  \[ \tilde{\rho}(\bar{\omega}) = \int_0^{\infty} d\omega \delta(\bar{\omega}, \omega) \rho(\omega) \]

- Represent $\delta(\bar{\omega}, \omega)$ in the form
  \[ \delta(\bar{\omega}, \omega) = \sum_i b_i(\bar{\omega}) K(x_i, \omega) \implies \tilde{\rho}(\bar{\omega}) = \sum_i b_i(\bar{\omega}) C(x_i) \]

- Goal: minimize the Backus-Gilbert functional
  \[ \mathcal{H} = (1 - \lambda)A + \lambda B = \lambda \int (\omega' - \omega)^2 \delta^2(\omega', \omega) d\omega' + (1 - \lambda)\text{Var}(\tilde{\rho}(\omega)). \]

  First term tends to minimize the resolution function width, second term ensures ”stability” — $\tilde{\rho}(\omega)$ should not vary much when the data is changed within errors.

  \[ b_i(\bar{\omega}) = \frac{\sum_j W_{ij}^{-1} R_j}{\sum_{ij} R_i W_{ij}^{-1} R_j}, \]

  \[ W_{ij} = \int d\omega K(x_i, \omega)(\omega - \bar{\omega})^2 K(x_j, \omega), \quad R_i = \int d\omega K(x_i, \omega) \]

- Regularization by the covariance matrix $S_{ij}$:
  \[ W_{ij} \rightarrow \lambda W_{ij} + (1 - \lambda)S_{ij}, \quad 0 < \lambda < 1 \]
Resolution functions for $T/T_c = 1.2$, $\lambda = 0.9$

- Width of the resolution function $\omega/T \sim 4$.
- Hydrodynamical approximation works up to $\omega/T < \pi$.
- Problem: large contribution from ultraviolet tail ($\sim 50\%$) because of finite $\delta$ width.
- The result is $\lambda$-dependent, the ratio $\eta/s$ significantly grows as $\lambda$ increases.
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Solution:

- Determine ultraviolet contribution to spectral function of the form
  \[ \rho_{uv} = A \rho_{lat}(\omega) \theta(\omega - \omega_0) . \]

- Contract this with the resolution function and subtract the result from \( \eta/s \).

- This will not make assumptions about intermediate \( \omega \) region, thus the use of method is still sensible.

- The result now only mildly depends on \( \lambda \) \( \Rightarrow \) two problems cancel each other.

- The result of reconstruction mostly coincides with the usual fit, but now we can be sure that assumptions about medium frequencies don’t change the answer dramatically.
Ratio $\eta/s$ without ultraviolet contribution, $\lambda$ dependence

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{ratio_eta_s_without_UV_contribution.png}
\caption{Ratio $\eta/s$ without ultraviolet contribution, $\lambda$ dependence.}
\end{figure}
Preliminary results

\[ \eta/s = (\eta/s) \omega \theta(\omega_0 - \omega) + A(\omega - \omega_0)\rho_{lat}(\omega) \]

Backus-Gilbert reconstruction

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Conclusion:

- Ratio $\eta/s$ calculated for the set of temperatures $T/T_c \in [0.9, 1.5]$
- Applied fitting procedure and Backus-Gilbert method for the SF
- $\eta/s$ is close to $N = 4$ SYM and in agreement with the experiment