Temperature dependence of shear viscosity in SU(3)-gluodynamics

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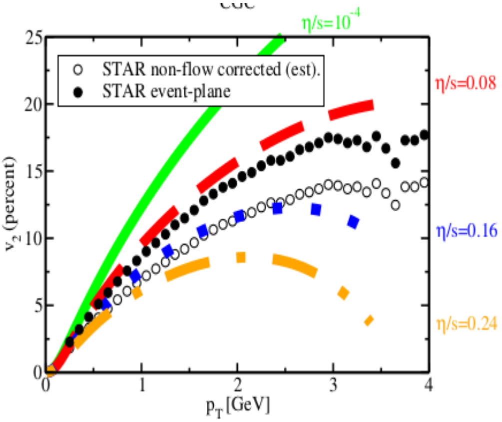
Outline:

- Introduction
- Details of the calculation
- Fitting of the data
- Backus-Gilbert method
- Conclusion



Elliptic flow from STAR experiment (Nucl. Phys. A 757, 102 (2005))

$$\frac{dN}{d\varphi} \sim (1 + 2v_1 \cos\varphi + 2v_2 \cos^2\varphi), \ \ \varphi\text{–scattering angle}$$

Quark-gluon plasma is close to ideal liquid,

$$\eta/s = (1-3)/4\pi$$

M. Luzum and P. Romatschke, Phys. Rev. C 78, 034915 (2008)

Other works (SU(3) gluodynamics):

- Karsch, F. et al. Phys.Rev. D35 (1987)
- A. Nakamura, S. Sakai Phys. Rev. Lett. 94, 072305 (2005)
- H. B. Meyer, Phys.Rev. D76 (2007) 101701
- H. B. Meyer, Nucl.Phys. A830 (2009) 641C-648C

Results:

- $\eta/s = 0.134 \pm 0.033$ $(T/T_c = 1.65, 8 \times 28^3)$
- $\eta/s = 0.102 \pm 0.056$ $(T/T_c = 1.24, 8 \times 28^3)$
- $\eta/s = 0.20 \pm 0.03$ $(T/T_c = 1.58, 16 \times 48^3)$
- $\eta/s = 0.26 \pm 0.03$ $(T/T_c = 2.32, 16 \times 48^3)$

SU(2) gluodynamics:

- $\eta/s = 0.134 \pm 0.057$ $(T/T_c = 1.2, 16 \times 32^3)$

  N. Yu. Astrakhantsev, V. V. Braguta, A. Yu. Kotov, JHEP 1509 (2015) 082

Indicates that small viscosity is a general feature of non-abelian gauge theories?

Lattice calculation of shear viscocity

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Measurement of the correlation function:

$$C(t) = \langle T_{12}(t) T_{12}(0) \rangle$$

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The second step:

Calculation of the spectral function $\rho(\omega)$:

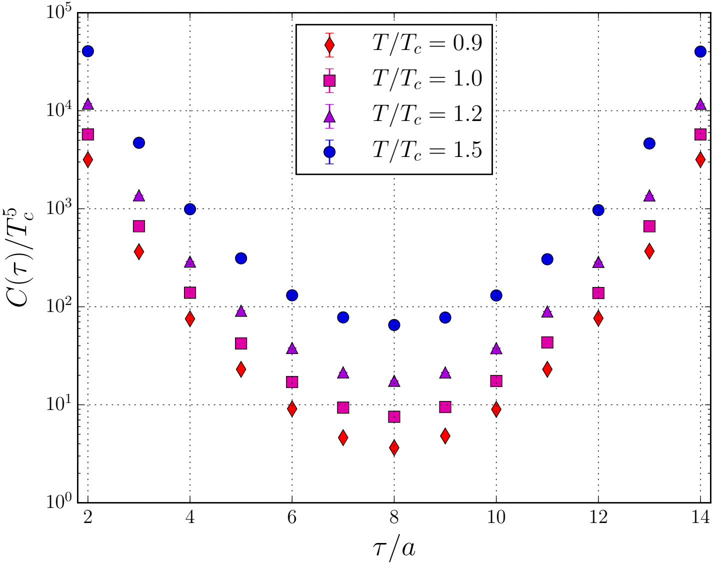
$$C(t) = \int_0^\infty d\omega \rho(\omega) \frac{\cosh\left(\frac{\omega}{2T} - \omega t\right)}{\sinh\left(\frac{\omega}{2T}\right)},$$

$$\eta = \pi \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}$$

Details of the calculation

- SU(3) gluodynamics
- Two-level algorithm
- Lattice size $32^3 \times 16$
- Temperatures $T/T_c = 0.9, 0.925, 0.95, 1.0, 1.1, 1.2, 1.35, 1.425, 1.5$
- Accuracy $\sim 2 - 3\%$ at $t = \frac{1}{2T}$
- $\langle T_{12}(x) T_{12}(y) \rangle \sim (\langle T_{11}(x) T_{11}(y) \rangle - \langle T_{11}(x) T_{22}(y) \rangle)$
- Clover discretization for the $\hat{F}_{\mu\nu}$
- Renormalization of EMT: F. Karsch, Nucl.Phys. B205 (1982) 285-300

Correlation functions



Spectral function

$$C(t) = \int_0^\infty d\omega \rho(\omega) \frac{\cosh\left(\frac{\omega}{2T} - \omega t\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

Properties of the spectral function:

- $\rho(\omega) \geq 0,\ \rho(-\omega) = -\rho(\omega)$
- Asymptotic freedom: $\rho(\omega)|^{NLO}_{\omega\to\infty} = \frac{1}{10}\frac{d_A}{(4\pi)^2}\omega^4\left(1 - \frac{5N_c\alpha_s}{9\pi}\right)$

  $\sim 90\%$ of the total contribution $t = 1/2T$
- Hydrodynamics: $\rho(\omega)|_{\omega\to 0} = \frac{\eta}{\pi}\omega$

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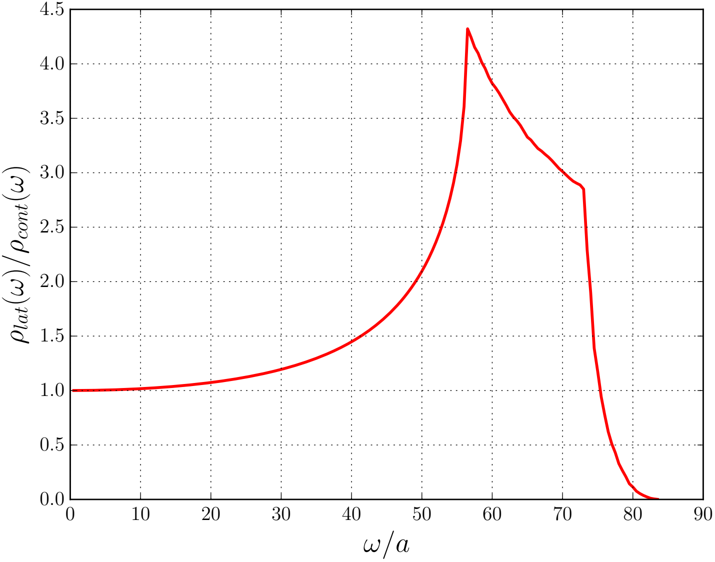
Ansatz for the spectral function (QCD sum rules motivation)

$$\rho(\omega) = \frac{\eta}{\pi}\omega\theta(\omega_0 - \omega) + A\rho_{lat}(\omega)\theta(\omega - \omega_0)$$

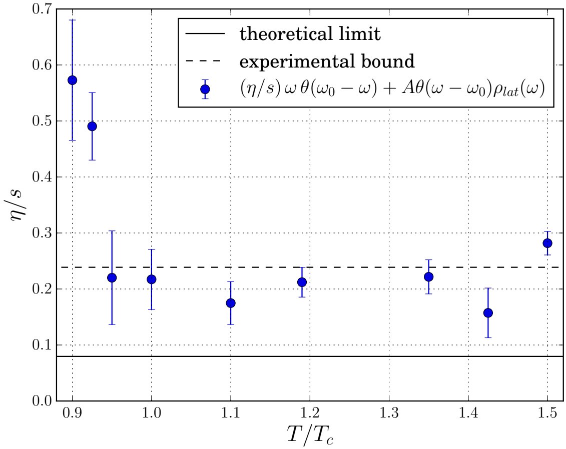
Lattice spectral function ρ_{lat}



Spectral function

$$\rho(\omega) = \frac{\eta}{\pi}\omega\theta(\omega_0 - \omega) + A\rho_{lat}(\omega)\theta(\omega - \omega_0)$$

$$\chi^2/\text{dof} \sim 1.0,\ A \sim 1,\ \omega_0/T \sim 7-8$$



Properties of the spectral function

- Hydrodynamical approximation works well up to $\omega < \pi T \sim 1$ GeV (H. B. Meyer, arXiv:0809.5202).

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- Hydrodynamical approximation works well up to $\omega < \pi T \sim 1$ GeV (H. B. Meyer, arXiv:0809.5202).
- Asymptotic freedom works well from $\omega > 3$ GeV.
- Poor knowledge of the spectral function in the region $\omega \in (1, 3)$ GeV
  $\Rightarrow$ Main source of uncertainty in the parametrical estimation $\Longrightarrow$ need to apply non-parametrical estimation procedure.

Backus-Gilbert method for the spectral function

- Problem: find $\rho(\omega)$ from the integral equation

$$C(x_i) = \int_0^\infty d\omega \rho(\omega) K(x_i, \omega), \quad K(x_i, \omega) = \frac{\cosh\left(\frac{\omega}{2T} - \omega x_i\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

- Define an estimator $\tilde{\rho}(\bar{\omega})$ ($\delta(\bar{\omega}, \omega)$ — resolution function):

$$\tilde{\rho}(\bar{\omega}) = \int_0^\infty d\omega \hat{\delta}(\bar{\omega}, \omega) \rho(\omega)$$

- Represent $\delta(\bar{\omega}, \omega)$ in the form

$$\delta(\bar{\omega}, \omega) = \sum_i b_i(\bar{\omega}) K(x_i, \omega) \implies \tilde{\rho}(\bar{\omega}) = \sum_i b_i(\bar{\omega}) C(x_i)$$

- Goal: minimize the Backus-Gilbert functional

$$\mathcal{H} = (1-\lambda)\mathcal{A} + \lambda\mathcal{B} = \lambda \int (\omega' - \omega)^2 \delta^2(\omega', \omega) d\omega' + (1-\lambda)\mathrm{Var}(\tilde{\rho}(\omega)).$$

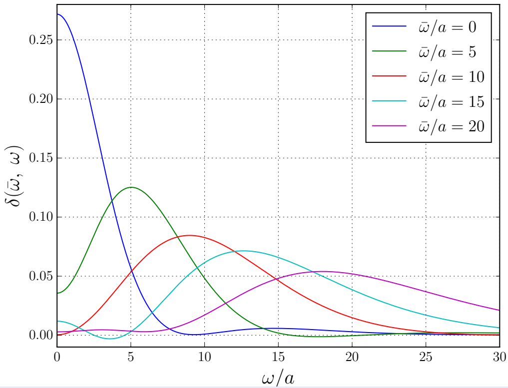
- First term tends to minimize the resolution function width, second term ensures "stability" — $\tilde{\rho}(\omega)$ should not vary much when the data is changed within errors.

$$b_i(\bar{\omega}) = \frac{\sum_j W_{ij}^{-1} R_j}{\sum_{ij} R_i W_{ij}^{-1} R_j},$$
$$W_{ij} = \int d\omega K(x_i, \omega)(\omega - \bar{\omega})^2 K(x_j, \omega), R_i = \int d\omega K(x_i, \omega)$$

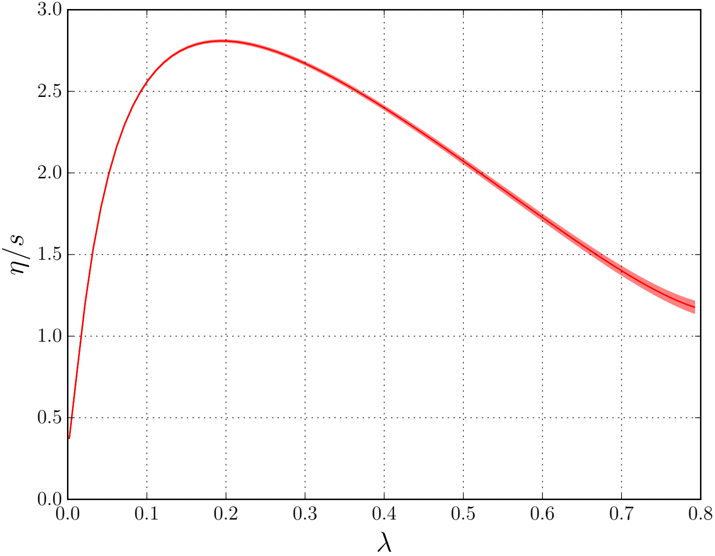
- Regularization by the covariance matrix S_{ij}:

$$W_{ij} \to \lambda W_{ij} + (1-\lambda) S_{ij}, \quad 0 < \lambda < 1$$

Resolution functions for $T/T_c = 1.2,\ \lambda = 0.9$



- Width of the resolution function $\omega/T \sim 4$.
- Hydrodynamical approximation works up to $\omega/T < \pi$.
- Problem: large contribution from ultraviolet tail ($\sim 50\%$) because of finite δ width.
- The result is λ-dependent, the ratio η/s significantly grows as λ increases.



3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
η/s
λ

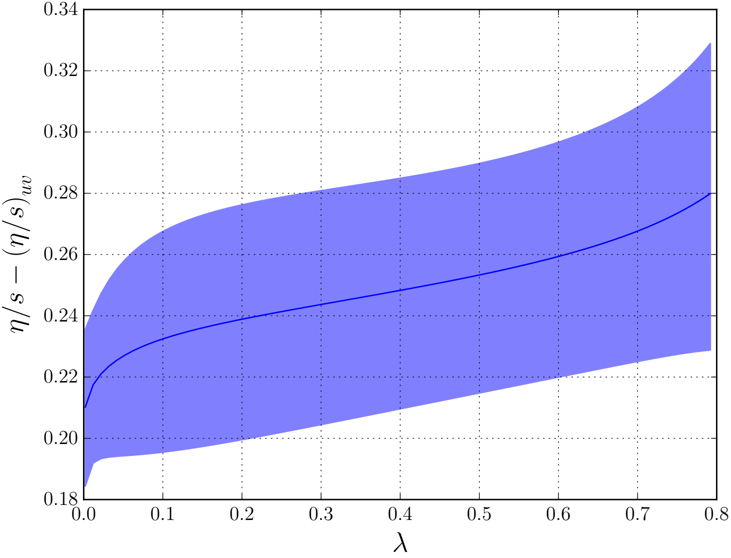
Solution:

- Determine ultraviolet contribution to spectral function of the form

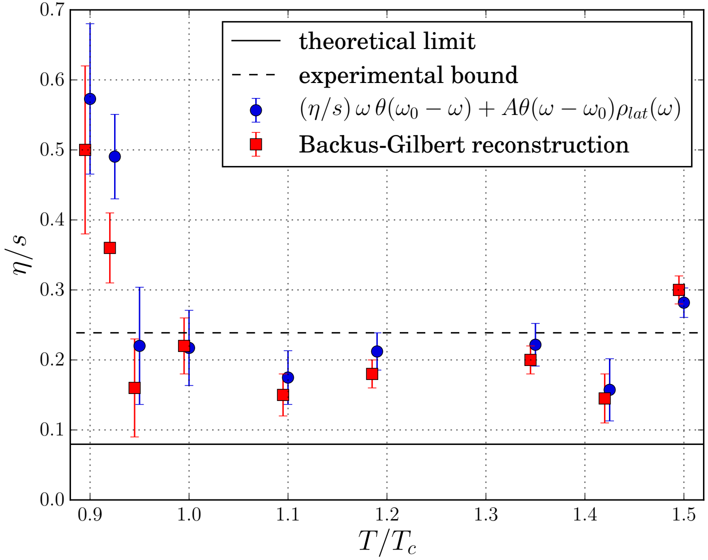
$$\rho_{uv} = A\rho_{lat}(\omega)\theta(\omega - \omega_0) \ .$$

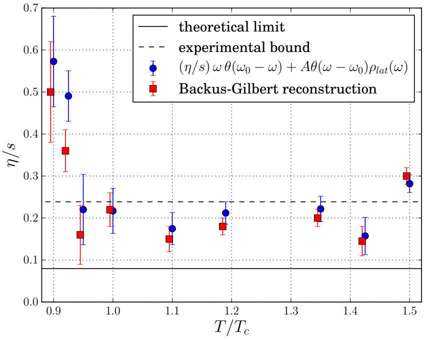
- Contract this with the resolution function and subtract the result from η/s.
- This will not make assumptions about intermediate ω region, thus the use of method is still sensible.
- The result now only mildly depends on $\lambda \implies$ two problems cancel each other.
- The result of reconstruction mostly coincides with the usual fit, but now we can be sure that assumptions about medium frequencies don't change the answer dramatically.

Ratio η/s without ultraviolet contribution, λ dependence



Preliminary results





Conclusion:

- Ratio η/s calculated for the set of temperatures $T/T_c \in [0.9, 1.5]$
- Applied fitting procedure and Backus-Gilbert method for the SF
- η/s is close to $N=4$ SYM and in agreement with the experiment