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Exact Sum Rules for Vector Channel at Finite Temperature and its Applications in Lattice QCD Analysis

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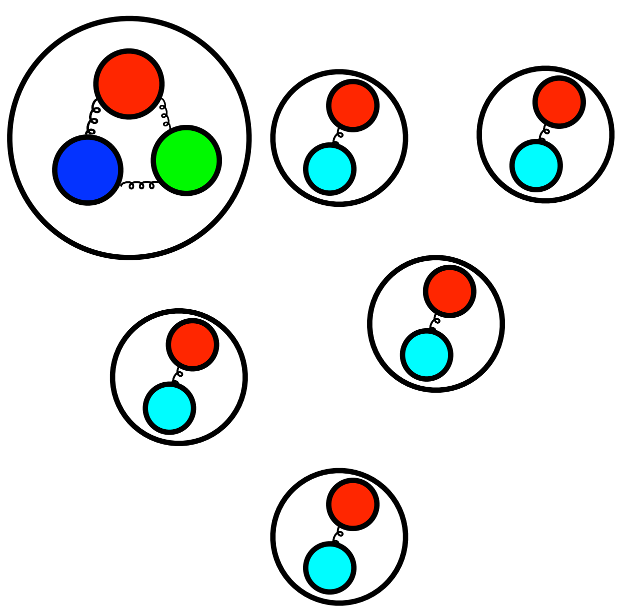
P. Gubler and **D. S.**, arXiv:1602.08265 [hep-ph].

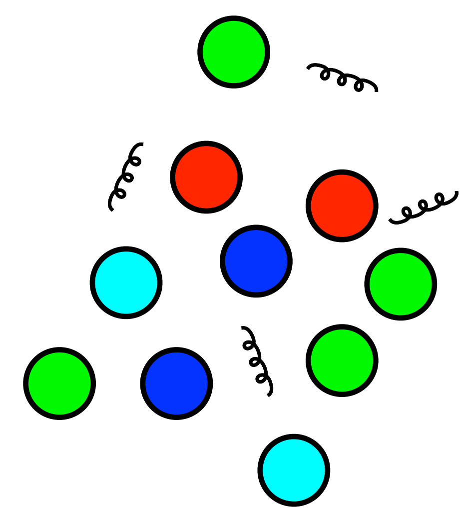
Introduction

Deconfined phase

Hadron gas



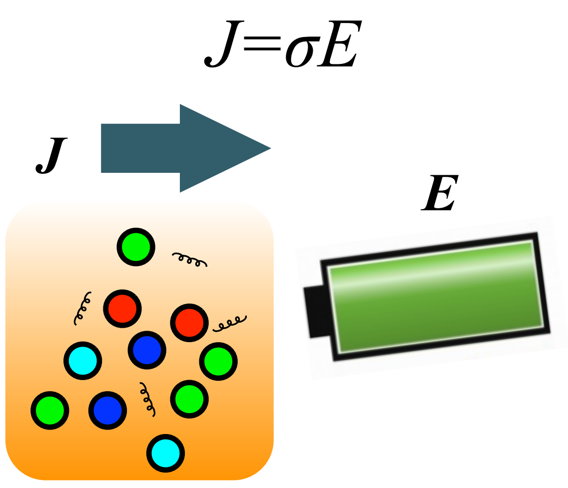
Quark-Gluon Plasma



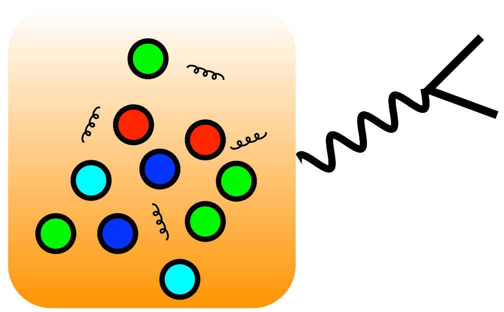
$T_c \sim 160\,\mathrm{MeV}$

Introduction

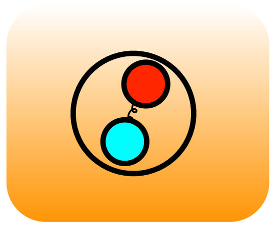
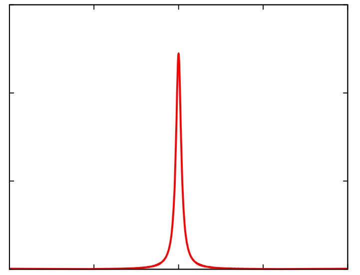
- Electrical conductivity



- Dilepton production rate

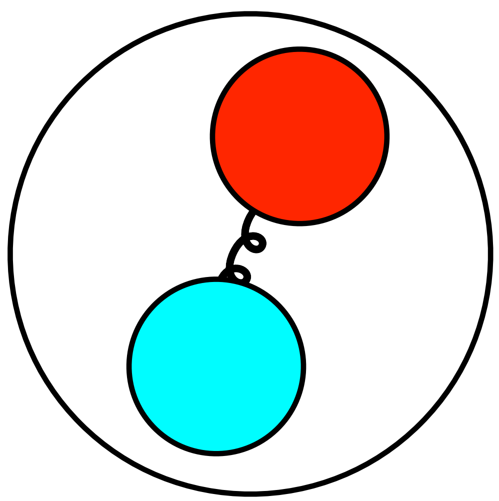


- Vector meson spectrum at finite T

Introduction

Vector spectral function contains all information of them.



$j^\mu \equiv e\sum_f q_f \overline{\psi}_f \gamma^\mu \psi_f$

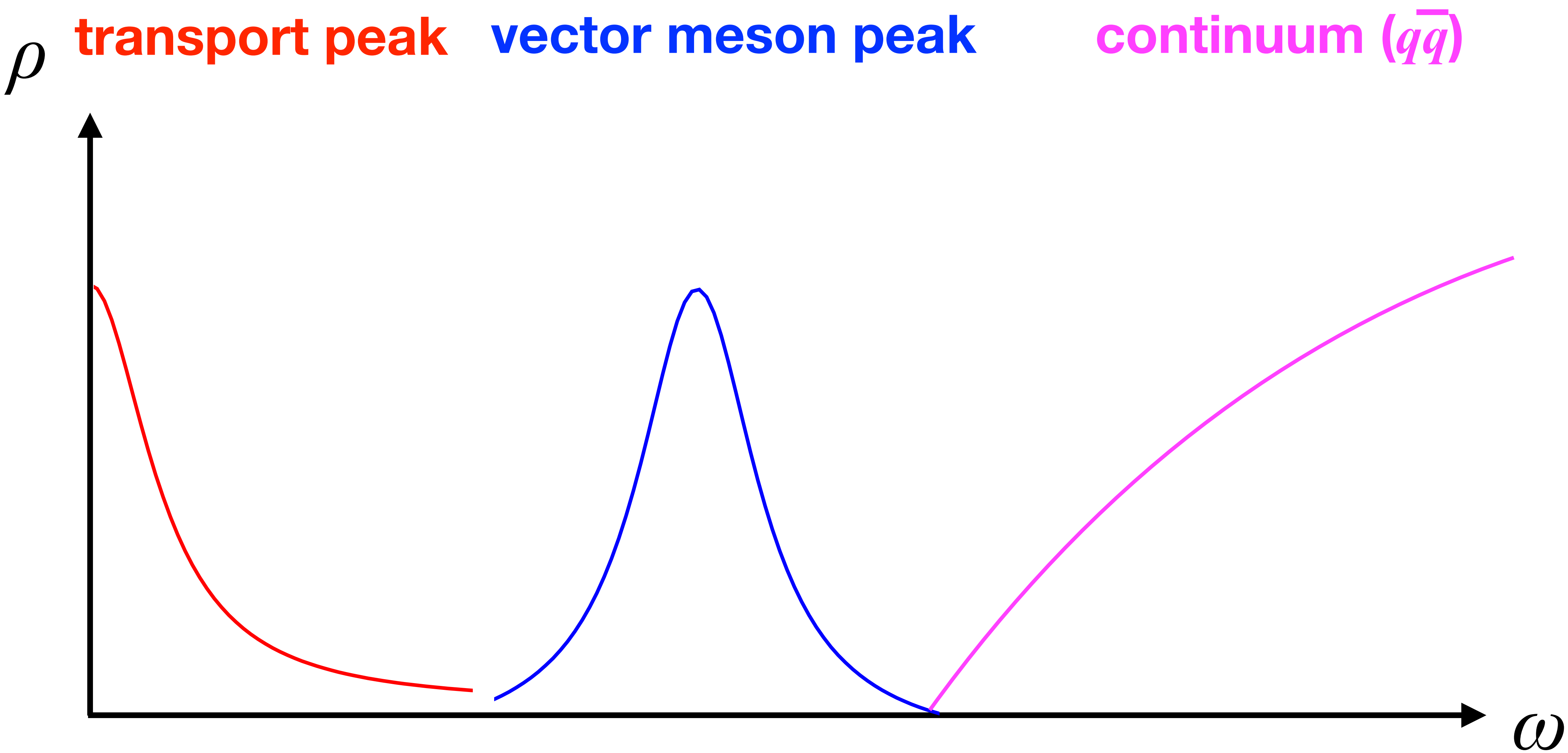
f: flavor index, q_{f}: electric charge

$$G^{R\mu\nu}(t,\mathbf{x}) \equiv i\theta(t)\langle[j^\mu(t,\mathbf{x}), j^\nu(0,\mathbf{0})]\rangle$$

$$\rho^{\mu\nu}(p) = \mathrm{Im}G^{R\mu\nu}(p)$$

Introduction

Possible form of vector spectral function



Rich and complicated structure.

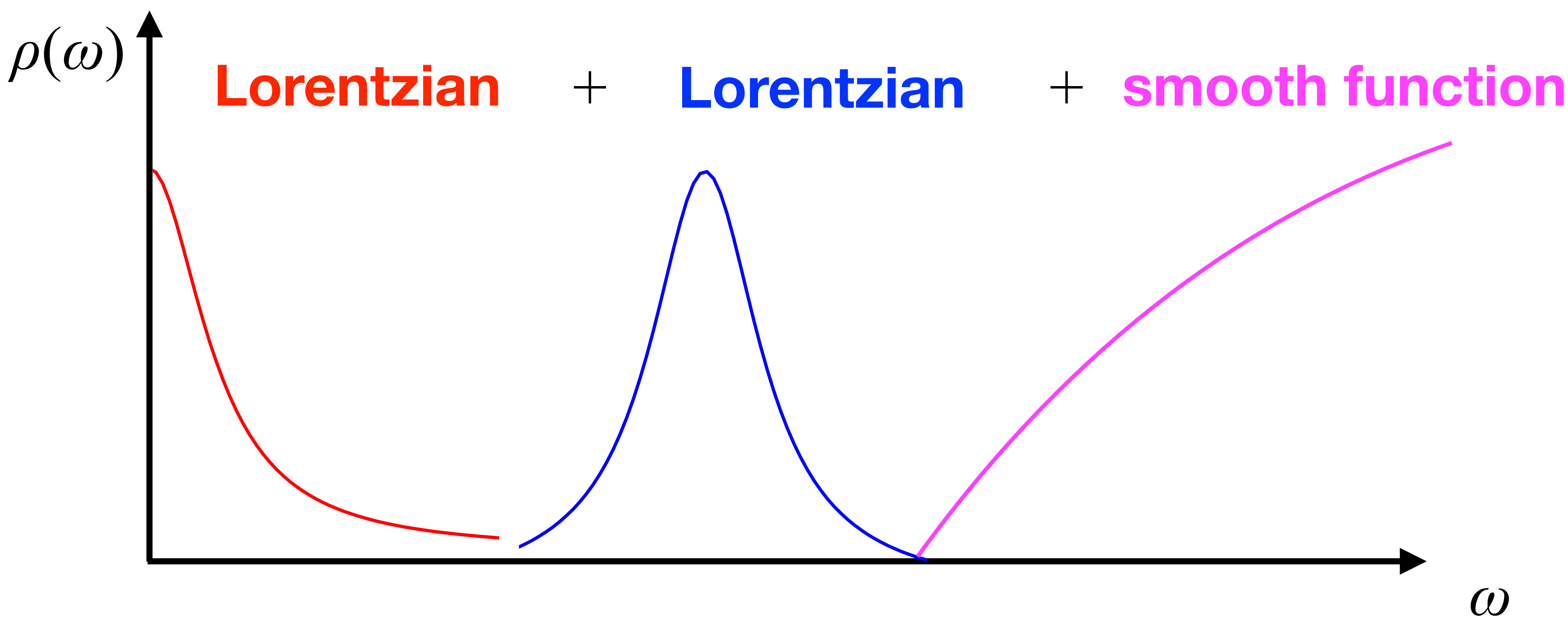
Introduction

pQCD: E. Braaten and R. D. Pisarski, Nucl. Phys. B **339** 310 (1990).
AdS/CFT: S. Caron-Huot, P. Kovtun, G. D. Moore, A. Starinets and L. G. Yaffe, JHEP **0612**, 015 (2006).
PNJL model: C. A. Islam, S. Majumder, N. Haque and M. G. Mustafa, JHEP **1502**, 011 (2015).
Semi-QGP: **D. S.** and W. Weise, Phys. Rev. D **92** 056001 (2015).
Sum rule: P. Gubler, W. Weise, Phys. Lett. B **751** 396 (2015).
Lattice QCD: H.-T. Ding et. al., Phys.Rev. D **83** 034504 (2011).

There are several approaches:
perturbation, holography, model, lattice…

But no conclusive results are obtained.

Even lattice QCD analysis needs an ansatz for the spectral function because it can not directly treat dynamical quantity.



Motivation

Is there any exact relation we can use for improvement?

$\Rightarrow$ **QCD sum rule**

Sum rule

P. Romatschke, D. T. Son, Phys.Rev. D **80** 065021 (2009).

Retarded Green function: $G^{R\mu\nu}(\omega, \mathbf{p})$

analyticity in upper ω plane

$$\underbrace{\delta G^R(0, \mathbf{p})}_{\text{IR}} - \underbrace{\delta G^R_\infty(\mathbf{p})}_{\text{UV}} = \frac{2}{\pi}\int_0^\infty d\omega \frac{\delta\rho(\omega, \mathbf{p})}{\omega}$$

$$\delta G^R(\omega) \equiv G^R(\omega) - G^R_{T=0}(\omega)$$

remove UV divergence

Sum of spectral function is constrained by the UV/IR behaviors! (sum rule)

Zero momentum case

For simplicity, we consider p=$\mathbf{0}$ case.

isotropy: $$G^R_{ij}(\omega, \mathbf{0}) = \delta_{ij} G^R(\omega)$$

Only one independent component.

Asymptotic behavior—UV (OPE)

UV behavior: operator product expansion (OPE)

separation of scale: $T, \Lambda_{\mathrm{QCD}} \ll 1/x \sim \omega$

$$\langle j^\mu(x) j^\nu(0)\rangle = \sum_i C^i(x)\langle \mathcal{O}_i(x=0)\rangle_T$$

factorization:

High-energy information (ω dependent)

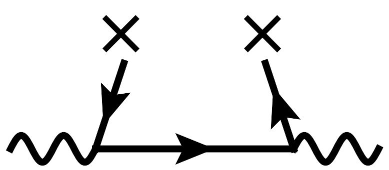
Low-energy information (ω independent, static)

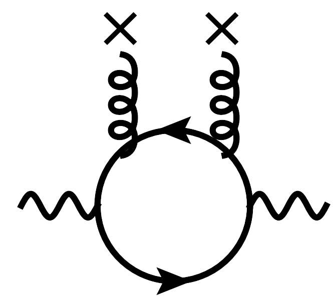
It can be **computed perturbatively**.

It contains **all nonperturbative information**.

Asymptotic behavior — UV (OPE)

Only T-dependent term was retained.





$$G^R(\omega) = e^2 \sum_f q_f^2 \frac{1}{\omega^2}\left[2m_f \delta\left\langle \overline{\psi}_f \psi_f \right\rangle + \frac{1}{12}\delta\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle + \frac{8}{3}\langle T_f^{00}\rangle\right] + \mathcal{O}(\omega^{-4})$$

calculated from lattice.

$\omega \longrightarrow \infty$

operator mixing

$$\frac{8}{3}\frac{1}{4C_F + N_f}\langle T_f^{00} + T_g^{00}\rangle$$

$C_F = (N_c^2 - 1)/(2N_c)$

$\omega \longrightarrow \infty$

Asymptotic freedom

No α_s correction in the coefficients!

P. Romatschke, D. T. Son, Phys.Rev. D **80** 065021 (2009).

Asymptotic behavior—IR (Hydro)

IR behavior: hydrodynamics

Assume the locality of the current:

$$\mathbf{j}(t,\mathbf{x}) = \mathbf{j}[\mathbf{E}(t,\mathbf{x}),\mathbf{B}(t,\mathbf{x})]$$

(Small frequency/wavelength of $\boldsymbol{E}$, $\boldsymbol{B}$ justifies this assumption.)

Ohmic

$$\mathbf{j} = \sigma\mathbf{E} - \sigma\tau_J\partial_t\mathbf{E} + O(\partial^2 E)$$

σ: Electric conductivity

Asymptotic behavior—IR (Hydro)

$$\mathbf{j} = \sigma\mathbf{E} - \sigma\tau_J\partial_t\mathbf{E}$$

linear response:

$$\mathbf{j}(\omega) = -G^R(\omega)\mathbf{A}(\omega)$$

$$\mathbf{E} = -\partial_t\mathbf{A}$$

$$G^R(\omega) = i\omega\sigma\,(1 + i\tau_J\omega) + \mathcal{O}(\omega^3)$$

Sum rule 1

UV

$$\delta G^R(\omega) = e^2 \sum_f q_f^2 \frac{1}{\omega^2}\left[2m_f\delta\langle\overline{\psi}_f\psi_f\rangle + \frac{1}{12}\delta\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle + \frac{8}{3}\frac{\delta\langle T^{00}\rangle}{4C_F+N_f}\right].$$

$\xrightarrow{\omega\to\infty}$ $\delta G^R(\omega)\to 0$ irrelevant.

IR

$$G^R(\omega) = i\omega\sigma(1+i\tau_J\omega)$$

$\xrightarrow{\omega\to 0}$ $\delta G^R(\omega)\to 0$ irrelevant.

$$\delta G^R(0,\mathbf{p}) - \delta G^R_\infty(\mathbf{p}) = \frac{2}{\pi}\int_0^\infty d\omega \frac{\delta\rho(\omega,\mathbf{p})}{\omega}$$

$\Downarrow$

$$0 = \int_0^\infty \frac{d\omega}{\omega}\delta\rho(\omega)$$ **sum rule 1**

$\delta\rho(\omega)=\rho(\omega)-\rho(\omega)_{T=0}$

(Also obtained by current conservation: D. Bernecker and H. B. Meyer, Eur. Phys. J. A **47**, 148 (2011))

Ansatz in lattice calculation

H.-T. Ding, A. Francis, O. Kaczmarek, F. Karsch, E. Laermann, W. Soeldner, Phys.Rev. D **83** 034504 (2011).

ansatz:

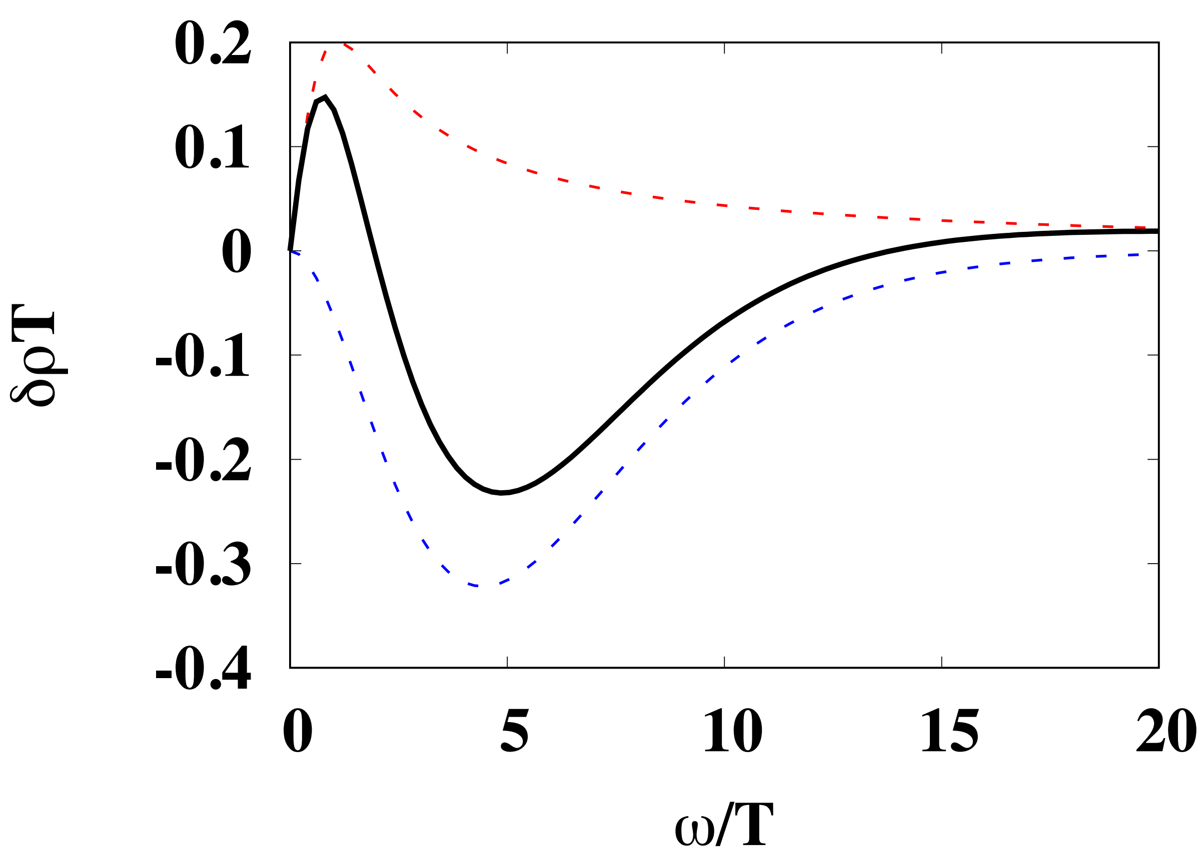
transport peak **continuum**

$$\rho(\omega) = C_{\mathrm{em}}\left[3\chi c_{BW}\frac{\omega\Gamma/2}{\omega^2+(\Gamma/2)^2} + \frac{9}{4\pi}(1+k)\omega^2\left(\cancel{1} - 2n_F\left(\frac{\omega}{2}\right)\right)\right]$$

T=0 contribution is subtracted.

3 parameters.

n_F: Fermi distribution



Ansatz in lattice calculation

Sum rule 1 $$0 = \int_0^\infty \frac{d\omega}{\omega}\delta\rho(\omega)$$

Transport peak and continuum should cancel.
(We confirm it in the weak coupling case)

⇒ **constraint:** $\chi c_{BW} = (1+k)T^2$

We can reduce independent parameters.

Also done in
B. B. Brandt, A. Francis, B. Jäger and H. B. Meyer, Phys. Rev. D **93**, 054510 (2016).

Sum rule 2

$$\frac{\delta\rho}{\omega}\times\omega^2$$ ➡ **Sum rule for $\omega\rho$, not ρ/ω**

UV

$$\omega^2\times\ \delta G^R(\omega) = e^2\sum_f q_f^2\frac{1}{\omega^2}\Big[2m_f\delta\langle\bar\psi_f\psi_f\rangle + \frac{1}{12}\delta\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle + \frac{8}{3}\frac{\delta\langle T^{00}\rangle}{4C_F+N_f}\Big]\cdot\times\omega^2$$

relevant.

IR

irrelevant.

➡

$$\frac{2}{\pi}\int_0^\infty d\omega\omega\delta\rho(\omega) = -e^2\sum_f q_f^2\Big[2m_f\delta\langle\bar\psi_f\psi_f\rangle + \frac{1}{12}\delta\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle + \frac{8}{3(4C_F+N_f)}\delta\langle T^{00}\rangle\Big].$$

sum rule 2

Expectation values of operators appear.

Sum rule 3

$\frac{\delta\rho}{\omega} \quad /\omega^2$ ➡ **Sum rule for ρ/ω^3, not ρ/ω**

UV irrelevant.

IR

$$G^R(\omega)/\omega^2 = i\omega\sigma(1 + i\tau_J\omega)/\omega^2$$

➡

$$-\sigma\tau_J = \frac{2}{\pi}\int_0^\infty \frac{d\omega}{\omega^3}\left[\delta\rho(\omega) - \sigma\omega\right]$$ **sum rule 3**

Transport coefficients appear.

Ansatz in lattice calculation

ansatz: $$\rho(\omega) = C_{\text{em}}\left[c_{BW}\rho_{\text{peak}}(\omega) + (1+k)\underbrace{\rho_{\text{cont}}(\omega)}^{\sim\omega^2}\right]$$

H.-T. Ding, A. Francis, O. Kaczmarek, F. Karsch, E. Laermann, W. Soeldner, Phys.Rev. D **83** 034504 (2011).

sum rule 3

$$-\sigma\tau_J = \frac{2}{\pi}\int_0^\infty \frac{d\omega}{\omega^3}\left[\delta\rho(\omega) - \sigma\omega\right]$$

Contribution from the continuum contains IR divergence.
More sophisticated ansatz is necessary.

Ansatz in lattice calculation

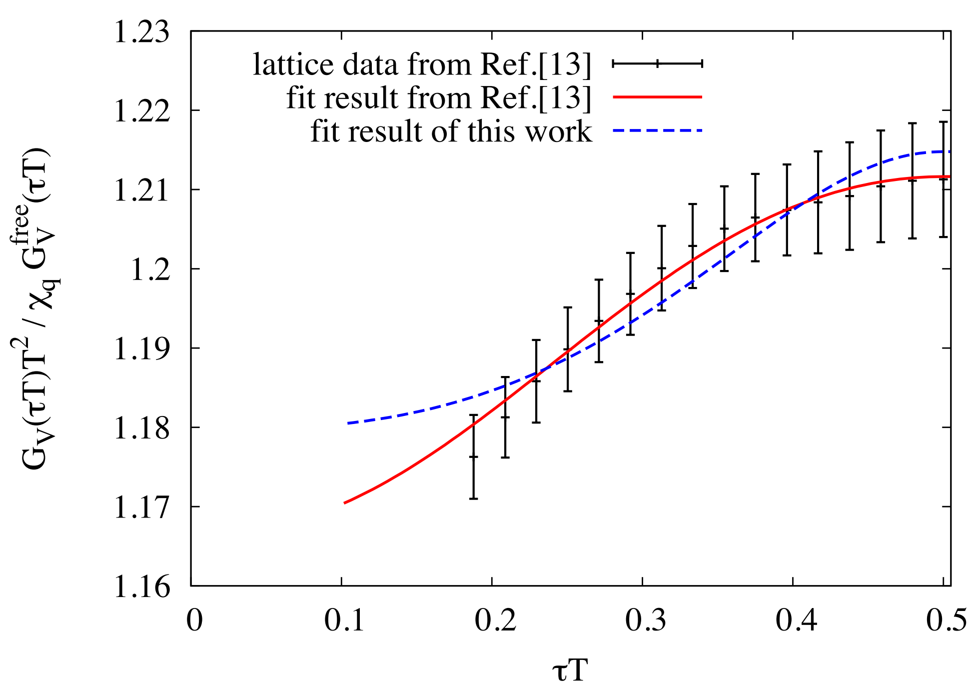
suggestion for improved ansatz:

$$\rho(\omega) = C_{\rm em}\left[c_{BW}\rho_{\rm peak}(\omega)[1 - A(\omega)] + A(\omega)(1 + k)\rho_{\rm cont}(\omega)\right]$$

$$A(\omega) \equiv \tanh(\omega^2/\Delta^2).$$

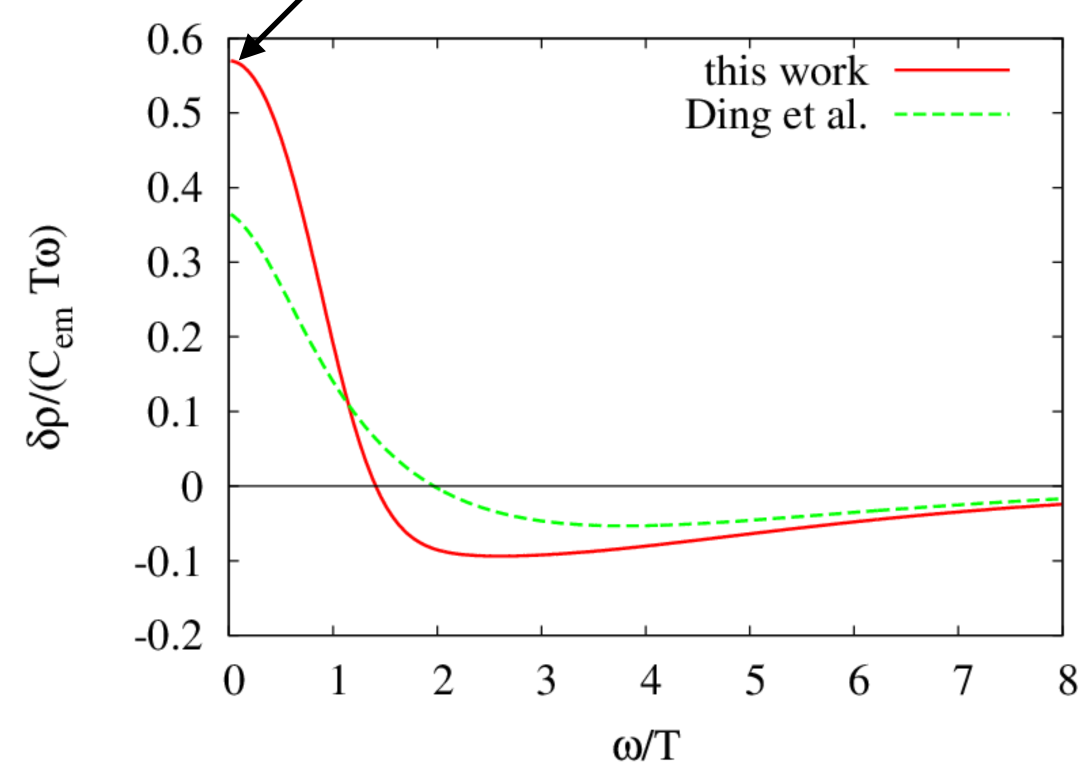
Connect the two regions smoothly.

H.-T. Ding, A. Francis, O. Kaczmarek, F. Karsch, E. Laermann, W. Soeldner, Phys.Rev. D **83** 034504 (2011).



fit

σ becomes 50% larger.



Ansatz in lattice calculation

Spectral function obtained by fit of lattice data

$$\rho(\omega) = C_{\rm em}\left[c_{BW}\rho_{\rm peak}(\omega)[1 - A(\omega)] + A(\omega)(1 + k)\rho_{\rm cont}(\omega)\right]$$

Sum rule 3

$$-\sigma\tau_J = \frac{2}{\pi}\int_0^\infty \frac{d\omega}{\omega^3}\left[\delta\rho(\omega) - \sigma\omega\right]$$

$$\tau_J = 0.067 C_{\rm em}/T$$

$$C_{\rm em} \equiv e^2 \textstyle\sum_f q_f^2$$

τ_J is evaluated for the first time.

Summary

- We **derived three exact sum rules** in vector channel **at finite temperature** by using **OPE (UV)** and **hydrodynamics (IR).**

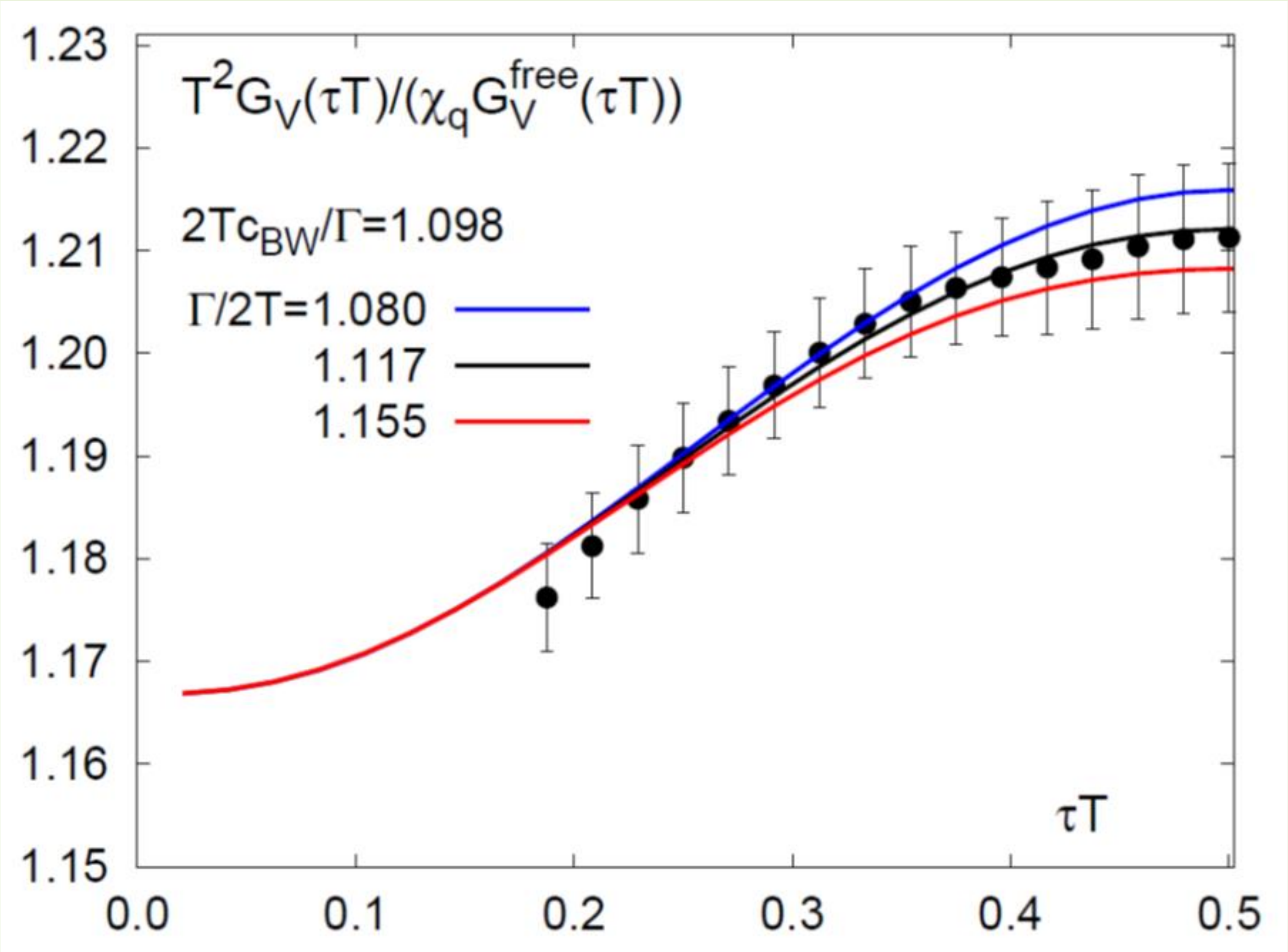
1: $$0 = \int_0^\infty \frac{d\omega}{\omega}\delta\rho(\omega)$$

2: $$\frac{2}{\pi}\int_0^\infty d\omega\omega\delta\rho(\omega) = -e^2\sum_f q_f^2\Big[2m_f\delta\langle\bar{\psi}_f\psi_f\rangle + \frac{1}{12}\delta\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle + \frac{8}{3(4C_F+N_f)}\delta\langle T^{00}\rangle\Big].$$

3: $$-\sigma\tau_J = \frac{2}{\pi}\int_0^\infty \frac{d\omega}{\omega^3}\left[\delta\rho(\omega) - \sigma\omega\right]$$

- We used our sum rules to **improve the ansatz used in the lattice calculation, and evaluate τ_J.**

Back Up



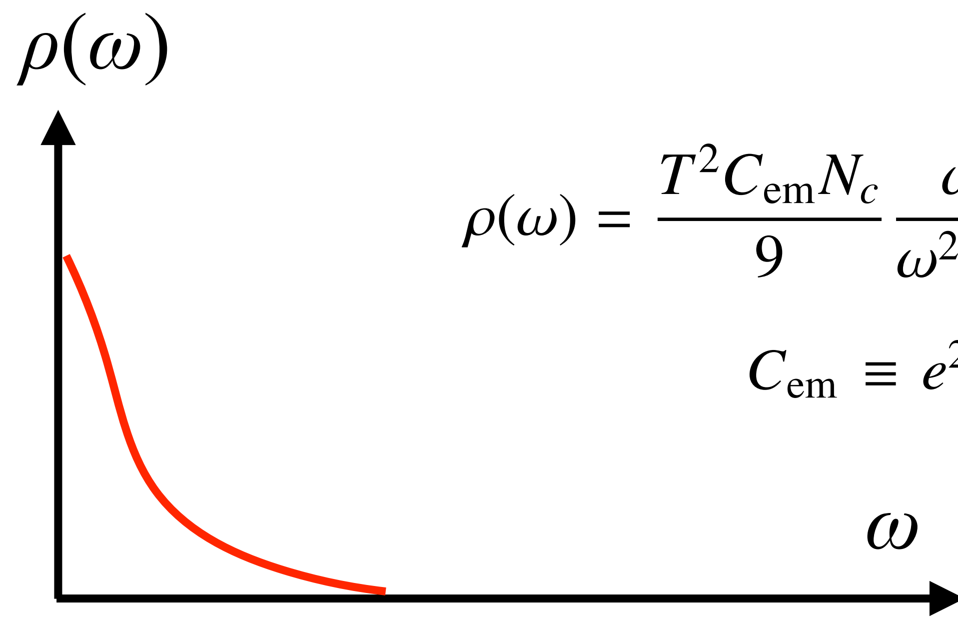
H.T. Ding *et al*., Phys. Rev. D **83**, 034504 (2011).

Check at weak coupling

Check Transport peak with Boltzmann eq.

relaxation time approximation

$\tau \sim (g^4 T \ln(1/g))^{-1}$: relaxation time



$$\rho(\omega) = \frac{T^2 C_{\rm em} N_c}{9} \frac{\omega \tau^{-1}}{\omega^2 + \tau^{-2}}.$$

$$C_{\rm em} \equiv e^2 \sum_f q_f^2$$

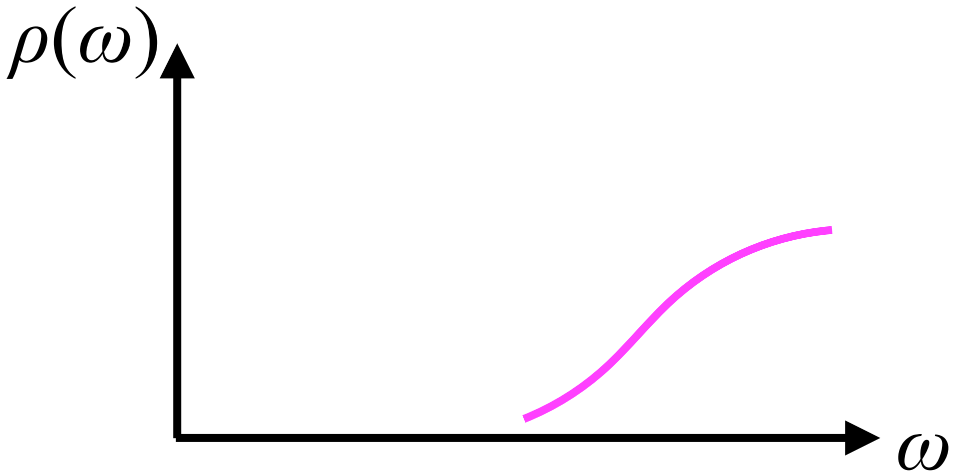
Transport peak. (Lorentzian)

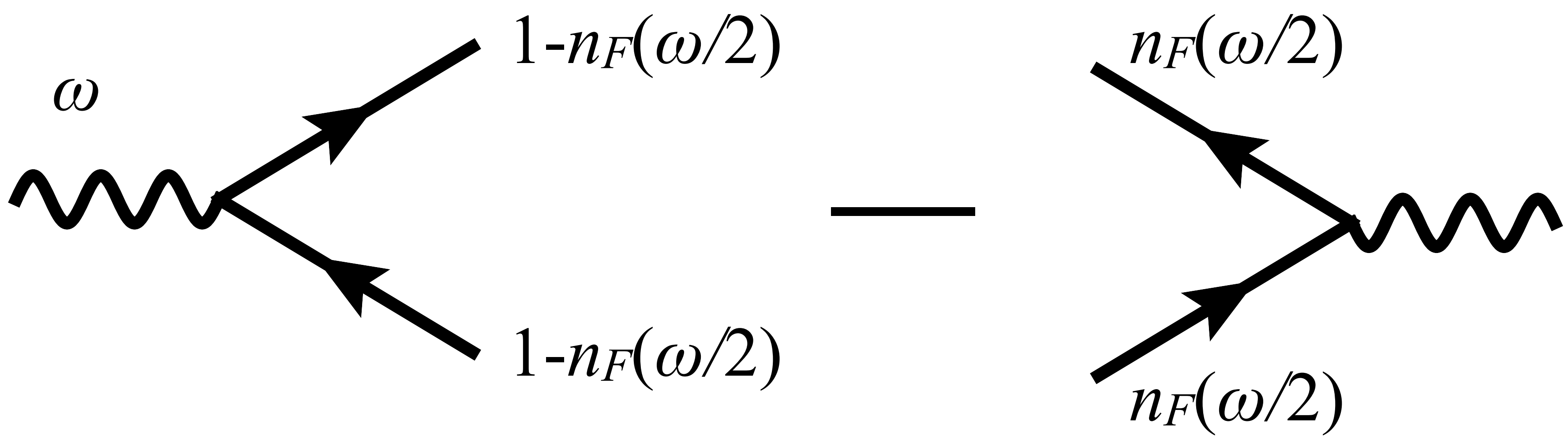
$$\int_0^\infty \frac{d\omega}{\omega} \delta\rho(\omega) = \frac{\pi T^2 C_{\rm em} N_c}{18}$$

Check at weak coupling

Continuum:

$$\rho(\omega) = \frac{N_c C_{\rm em}}{12\pi}\omega^2\left(1 - 2n_F\left(\frac{\omega}{2}\right)\right).$$



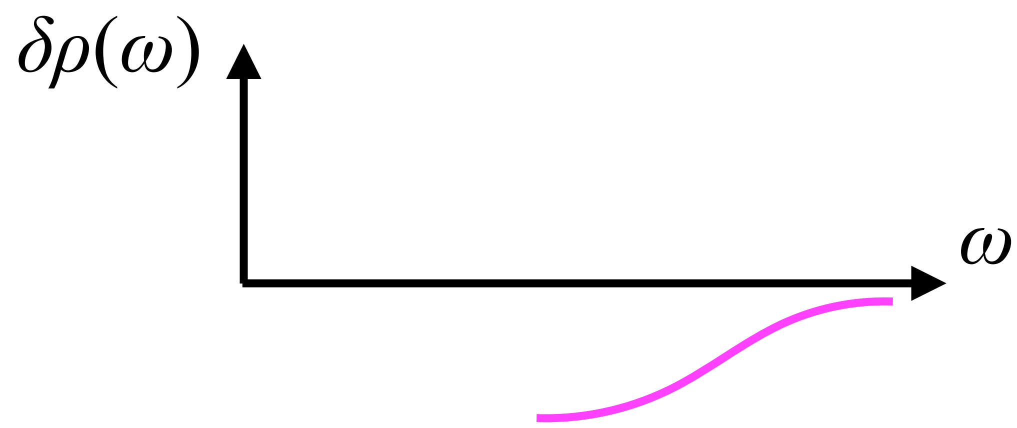


$[1\text{-}n_F]^2\text{-}[n_F]^2\text{=}1\text{-}2n_F$

Check at weak coupling

$\cancel{1}\text{-}2n_F$

T=0 contribution is subtracted.



$$\int_0^\infty \frac{d\omega}{\omega}\delta\rho(\omega) = -\frac{\pi T^2 C_{\mathrm{em}} N_c}{18}$$

It cancels the transport peak, so that the sum becomes zero!

The sum rule 1 is satisfied.

Sum rule

P. Romatschke, D. T. Son, Phys.Rev. D **80** 065021 (2009).

$$G^{R\mu\nu}(\omega,\mathbf{p}) = i\int dt \int d^3\mathbf{x} e^{i\omega t - i\mathbf{p}\cdot\mathbf{x}}\theta(t)\langle[j^\mu(t,\mathbf{x}), j^\nu(0,\mathbf{0})]\rangle.$$

analytic in upper ω plane

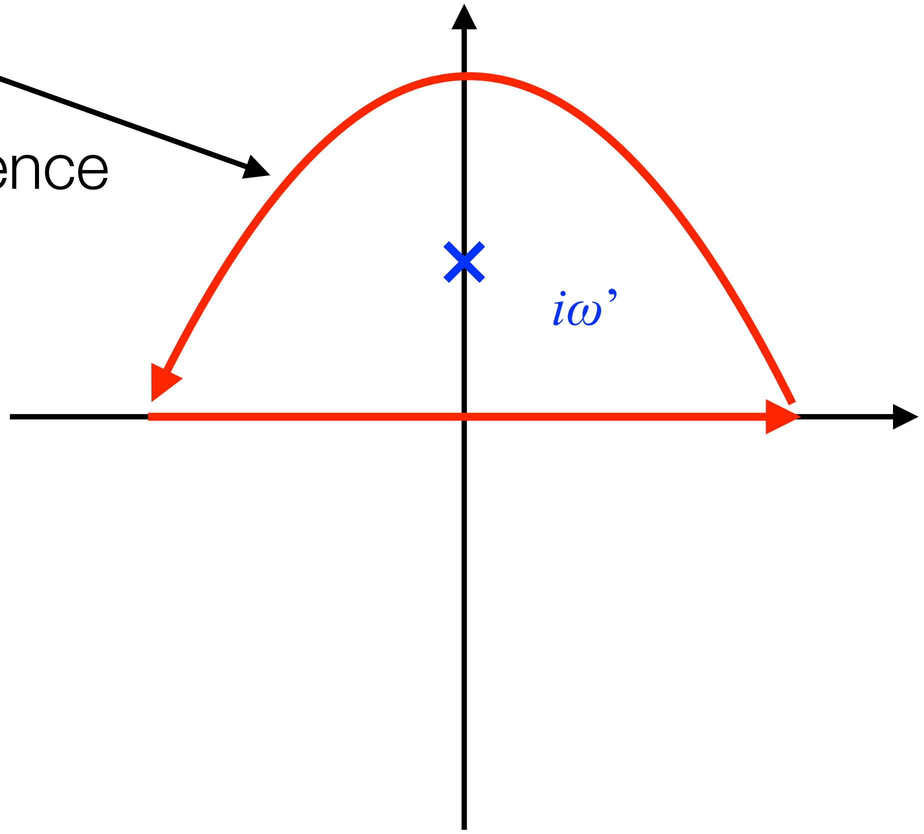
$$f(i\omega') = \frac{1}{2\pi i}\int_{-\infty}^{\infty} d\omega \frac{f(\omega)}{\omega - i\omega'}$$

$$f(\omega) = \delta G^R(\omega,\mathbf{p}) - \delta G^R_\infty(\mathbf{p})$$

$$\delta G^R(\omega) \equiv G^R(\omega) - G^R_{T=0}(\omega)$$

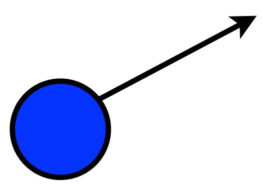
remove UV divergence

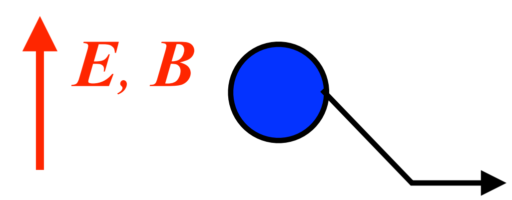
remove the possible divergence
coming from the arc

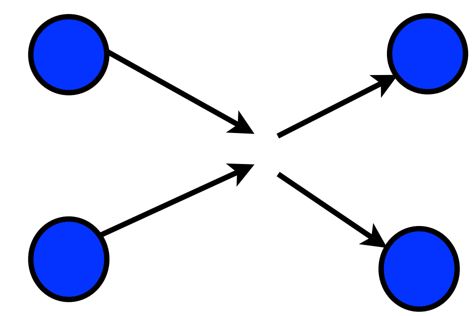


Check at weak coupling

Check Transport peak with Boltzmann eq.







drift term

external force term

collision term

$$v \cdot \partial_X n_\pm(\mathbf{k}, X) \pm e(\mathbf{E} + \mathbf{v} \times \mathbf{B})(X) \cdot \nabla_k n_\pm(\mathbf{k}, X) = C[f]$$

$n_\pm(\boldsymbol{k}, X)$: (anti-) quark distribution function

$v=(1, \boldsymbol{k}/|\boldsymbol{k}|)$: 4-velocity

relaxation time approximation

$$C[f]=\tau^{-1}(n_\pm - n^{(\mathrm{eq})}_\pm)$$

$\tau \sim (g^4 T \ln(1/g))^{-1}$: relaxation time