Event-by-event picture for the medium-induced jet evolution

Miguel A. Escobedo

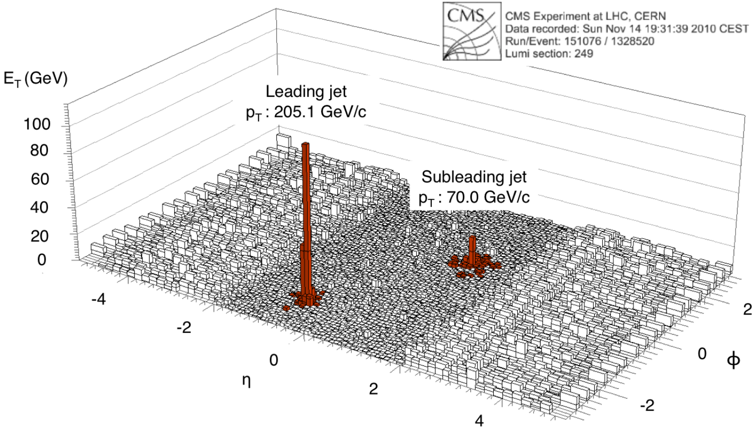
IPhT Saclay

1st September, 2016

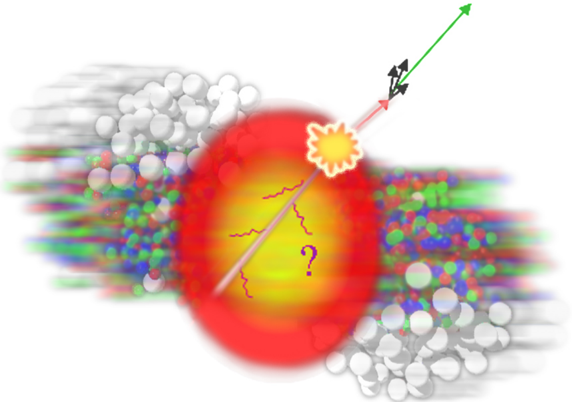
JHEP 1605 (2016) 008 and work in preparation. Collaboration with E. Iancu.

Table of Contents

Jet quenching in heavy-ion collisions



Jet quenching, the generally expected picture



Mediums of different sizes are seen by the two jets $\rightarrow$ asymmetry, but that might be naive.

Jet quenching, problems of the naive picture

There is an implicit assumption

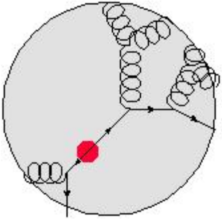
- The energy loss is always the same at fixed medium size. Fluctuations are small.

Is it true?

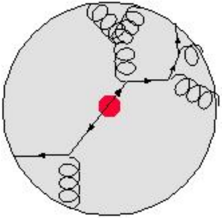
- A recent Monte Carlo computation (Milhano and Zapp, EPJC 76:288 (2016)) shows that fluctuations could be the dominant mechanism explaining dijet asymmetry.
- In JHEP 1605 (2016) 008 we perform an analytic computation that shows that fluctuations in the energy loss are of the order of the average value.

Fluctuations in size vs fluctuations in energy loss

Asymmetry due to different path length.



Asymmetry due to fluctuations in the energy loss.



To estimate the size of the fluctuations in energy is important to understand the physics behind the dijet asymmetry.

Where does the energy go?

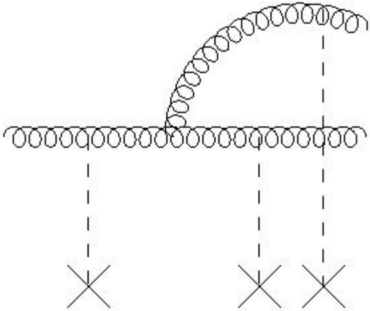
The energy loss of the jet produces a lot of soft particles

- How many of them?
- What are the statistical properties of these particles and how are they different/similar to what we can find in a jet in the vacuum?

Non-trivial consistency check of the energy loss mechanism.

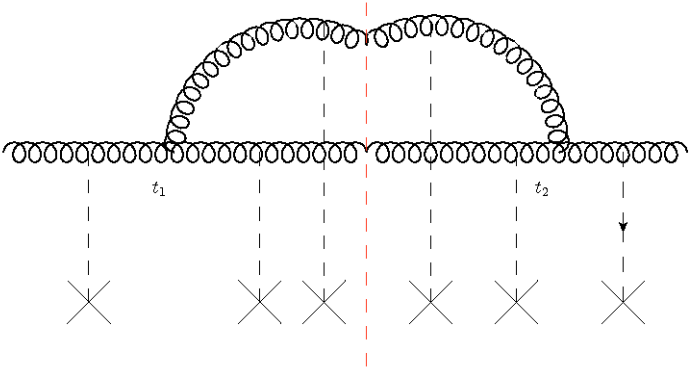
Table of Contents

General picture



- All the needed information from the medium is encoded in $\hat{q}$ and the length L.
- Emission probability given by the BDMPS-Z theory.

Medium-induced gluon emission: formation time



- $\tau_f = t_1 - t_2$ can not be infinitely large. By uncertainty relation $\frac{1}{\tau_f} \sim \frac{k_\perp^2}{2w}$.
- In a medium, during a time τ_f, the acquired transverse momentum is $k_\perp^2 \sim \hat{q}\tau_f$.
- $\tau_f \sim \sqrt{\frac{2w}{\hat{q}}}$

Branching time

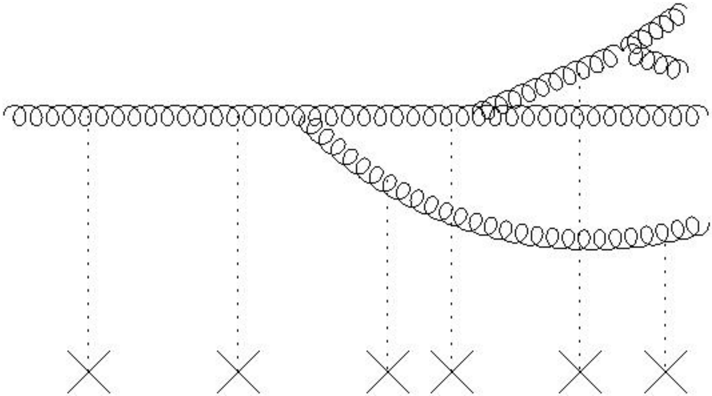
The probability of emitting a gluon with energy w during a time Δt goes like

$$P(w, \Delta t) \propto \alpha_s \sqrt{\frac{\hat{q}}{w}} \Delta t \,,$$

the time after which it is almost sure that a gluon with energy w will have been emitted is called the branching time $\tau_{br}(w) = \frac{\pi}{N_c \alpha_s} \tau_f(w)$.

- Soft gluons with $\tau_{br} \ll L$ that will be emitted in abundantly.
- Gluons with $\tau_{br} \sim L$ whose energy goes like $w \sim \alpha_s^2 \hat{q} L^2$. The emission of these gluons by the leading particle will dominate the energy loss of the jet. These gluons will lose energy very quickly so that most of the energy is lost at large angles at the end of the day.

Multiple branching



Evolution equation of this type of cascade. Blaizot, Dominguez, Iancu and Mehtar-Tani JHEP06(2014)075.

Democratic branching

When a parton branches in a way in which the two resulting partons have a similar energy.

- This will be a rare event for leading particles with energy higher than $\alpha_s^2 \hat{q} L^2$.
- For the typical gluons emitted by the leading particles (primary gluons) and the subsequent gluons in the cascade democratic branching is a very efficient mechanism of energy loss.

Blaizot, Iancu and Mehtar-Tani *Phys.Rev.Lett* 111, 052001

Table of Contents

The gluon spectrum

Energy density per unit x

$$D(x,t) = x\langle \sum_i \delta(x_i - x)\rangle$$

$$\frac{\partial}{\partial\tau} D(x,\tau) = \int \mathrm{d}z\, \mathcal{K}(z) \left[\sqrt{\frac{z}{x}} D\left(\frac{x}{z},\tau\right) - \frac{z}{\sqrt{x}} D(x,\tau) \right]$$

where $\tau = \frac{\alpha_s N_c}{\pi}\sqrt{\frac{\hat{q}}{E}}t = \frac{t}{\tau_{br}(E)}$.
Two cases:

- $\tau_{br}(E) \gg L$. Interesting for LHC physics.
- $\tau_{br}(E) \sim L$. Interesting to study the case in which the jet is completely absorbed by the medium.

Baier, Mueller Schiff and Son *Phys.Lett.*B502(2001) 51-58.

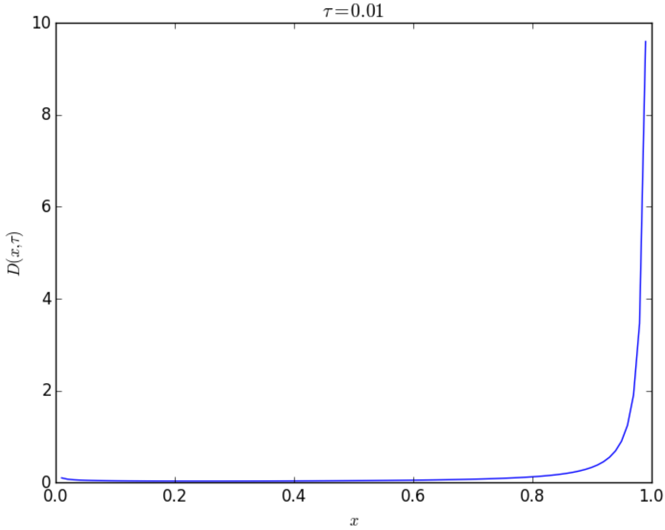
The gluon spectrum

With the initial condition $D(x,0)=\delta(x-1)$ there is an analytic solution with an approximate kernel $\mathcal{K}\to\mathcal{K}_0$.

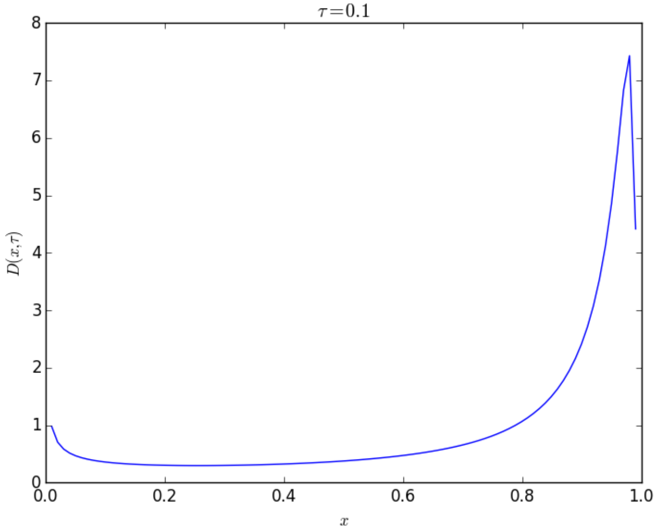
$$D(x,\tau)=\frac{\tau}{\sqrt{x}(1-x)^{3/2}}\exp\{-\frac{\pi\tau^2}{1-x}\}$$

Blaizot, Iancu and Mehtar-Tani *Phys.Rev.Lett* 111, 052001

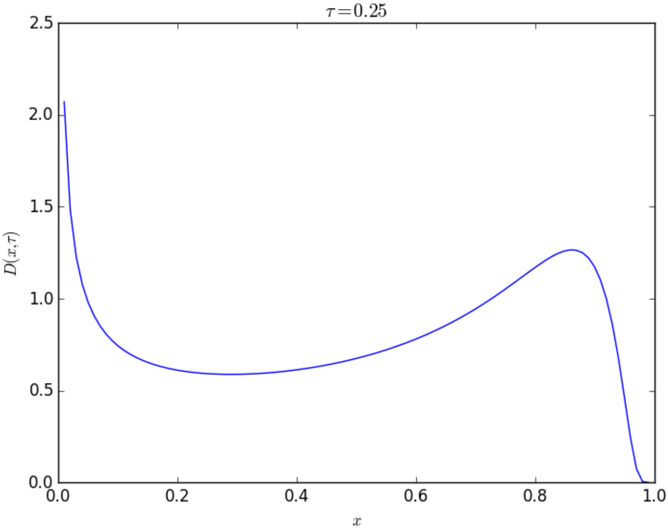
The gluon spectrum



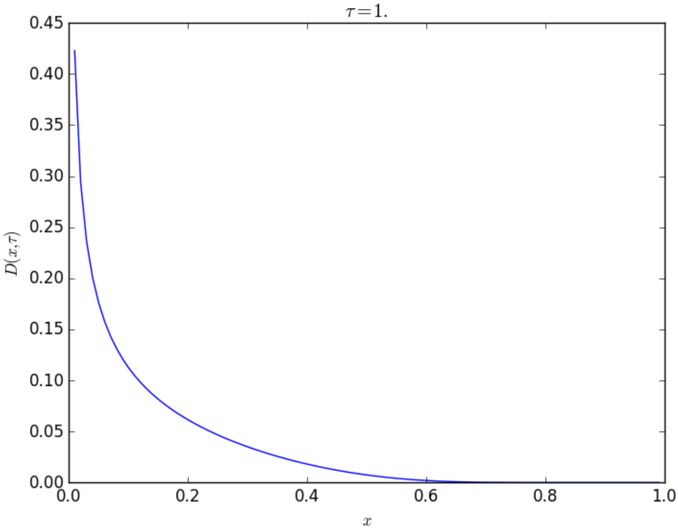
The gluon spectrum



The gluon spectrum



The gluon spectrum



Average energy loss

Average energy inside the jet

$$\langle X(\tau)\rangle = \int_0^1 dx D(x,\tau) = e^{-\pi\tau^2}\,,$$

Blaizot, Iancu and Mehtar-Tani, *Phys.Rev.Lett* 111(2013)052001

Where does the energy go?

There is a lower cut-off x_0, below this energy the jet evolution equations are not accurate due to thermalization effects.

Energy loss is the one that goes to modes with $x < x_0$. Remarkably we can compute it setting $x_0 = 0$.

$$\mathcal{E}(\tau) = E\left(1 - e^{-\pi\tau^2}\right)\,.$$

Table of Contents

Fluctuations in the energy loss

$$\langle \mathcal{E}^2(t)\rangle - \langle \mathcal{E}(t)\rangle^2 = E^2(\langle X^2(t)\rangle - \langle X(t)\rangle^2)\,,$$

We need a new ingredient, the average energy squared given by a pair of gluons with energy fractions equal to x and x'.

$$D^{(2)}(x,x',t) = xx'\langle \sum_{i\neq j} \delta(x_i - x)\delta(x_j - x')\rangle\,,$$

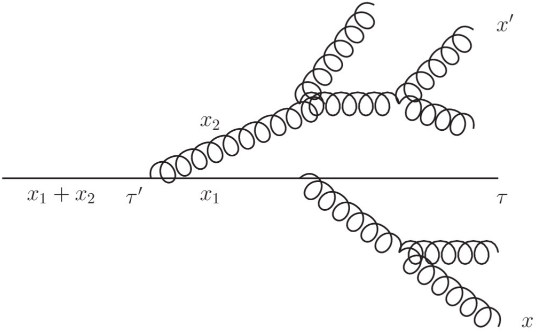
with this we can compute $\langle X^2(t)\rangle$

$$\langle X^2(t)\rangle = \int_0^1 dx x D(x,t) + \int_0^1 dx \int_0^1 dx' D^{(2)}(x,x',t)\,.$$

Escobedo and Iancu, JHEP 1605 (2016) 008

$D^{(2)}$

For the initial condition $D^{(2)}(x, x', 0) = 0$



Intuitively

- Compute all the branchings that happen at any τ' between 0 and τ.
- Evolve independently the results of these branching.
- Integrate for all τ'.

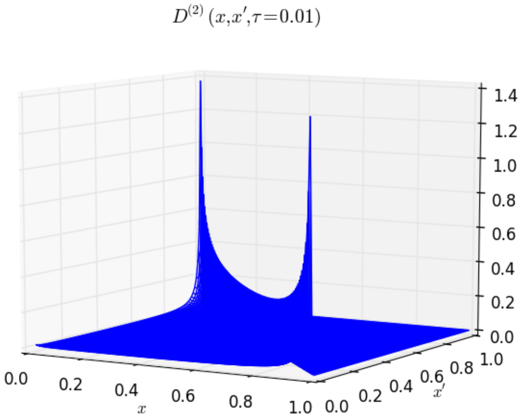
Analytic solution of $D^{(2)}$

$$D^{(2)}(x, x', \tau) = \int_0^{\tau} \mathrm{d}\tau' (2\tau - \tau') \frac{\mathrm{e}^{-\frac{\pi(2\tau-\tau')^2}{1-x-x'}}}{\sqrt{xx'}(1-x-x')^{3/2}} \,,$$

The integrand is the contribution to $D^{(2)}$ of all the branchings that happened at τ'.

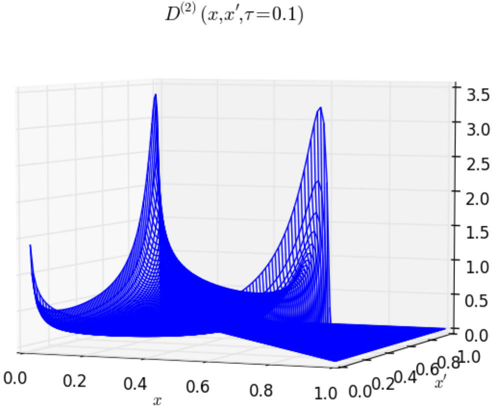
$$D^{(2)}(x, x', \tau) \;=\; \frac{1}{2\pi} \frac{1}{\sqrt{xx'(1-x-x')}} \left[\mathrm{e}^{-\frac{\pi\tau^2}{1-x-x'}} - \mathrm{e}^{-\frac{4\pi\tau^2}{1-x-x'}} \right].$$

Plot of $D^{(2)}$



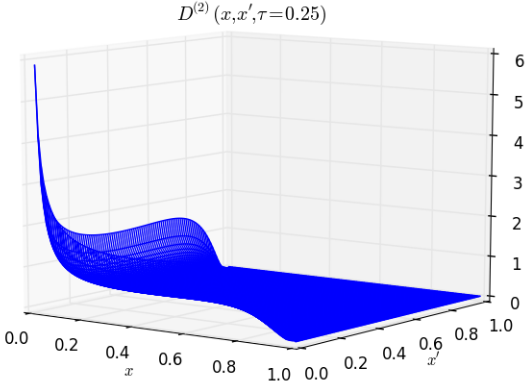
$D^{(2)}\,(x,x',\tau=0.01)$
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0 0.2 0.4 0.6 0.8 1.0
x
0.0 0.2 0.4 0.6 0.8 1.0
x'

Plot of $D^{(2)}$

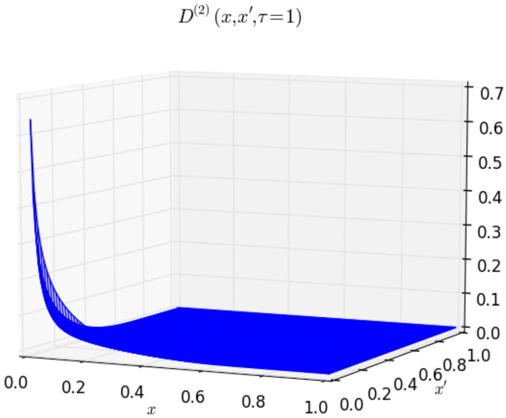


$D^{(2)}(x, x', \tau = 0.1)$
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
1.0
x
0.0
0.2
0.4
0.6
0.8
1.0
x'

Plot of $D^{(2)}$



Plot of $D^{(2)}$



$D^{(2)}(x,x',\tau=1)$
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0
0.2
0.4
0.6
0.8
1.0
x
0.0
0.2
0.4
0.6
0.8
1.0
x'

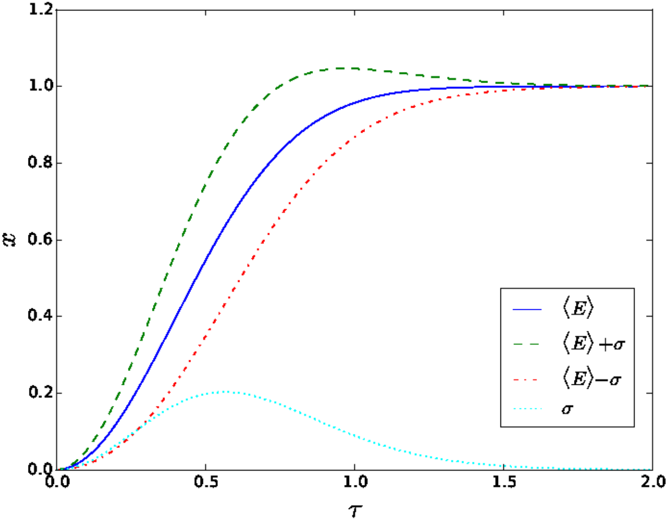
Fluctuations in energy

$$\langle \mathcal{E}^2(t) \rangle - \langle \mathcal{E}(t) \rangle^2 = E^2 \sigma_\varepsilon^2(t)$$

$$\begin{aligned}
\sigma_\varepsilon^2(\tau) &= 2\pi\tau \left[\mathrm{erf}(\sqrt{\pi}\tau) - \mathrm{erf}(2\sqrt{\pi}\tau)\right] + 2\mathrm{e}^{-\pi\tau^2} - \mathrm{e}^{-4\pi\tau^2} - \mathrm{e}^{-2\pi\tau^2} \\
&= \frac{1}{3}\pi^2\tau^4 - \frac{11}{15}\pi^3\tau^6 + \mathcal{O}(\tau^8)\,.
\end{aligned}$$

There are terms that go like τ and τ^2 in the intermediate steps but they cancel out when computing the variance.

Fluctuations in energy



Fluctuations in energy and fluctuations in size

Average of all back-to-back jets created in a heavy-ion collision with initial energy E taking into account both fluctuations in energy and in size

$$\langle E_1 - E_2\rangle^2 = (N_c\alpha_s\hat{q})^2(\langle L_1^2\rangle - \langle L_2^2\rangle)^2$$

$$\sigma^2_{E_1-E_2} = \langle (E_1-E_2)^2\rangle - \langle E_1-E_2\rangle^2 = (N_c\alpha_s\hat{q})^2\left[\frac{1}{3}(\langle L_1^4\rangle + \langle L_2^4\rangle) + \sigma^2_{L_1^2} + \sigma^2_{L_2^2}\right]$$

The dijet asymmetry is produced

- Fluctuations of the energy loss that are present even if the size is fixed.
- Fluctuations in the size of the medium seen by the jet.

Table of Contents

Computation of $D^{(n)}$

$\langle E^n \rangle$ and $\langle N^n \rangle$ can be computed if you know $D^{(n)}$.
Recently we have found a exact expression for these quantities

$$D^{(n)}(x_1, \cdots, x_n|\tau) = \frac{(n!)^2}{2^{n-1} n} \frac{\left(1 - \sum_{i=1}^n x_i\right)^{\frac{n-3}{2}}}{\sqrt{x_1 \cdots x_n}} h_n\left(\frac{\tau}{\sqrt{1 - \sum_{j=1}^n x_j}}\right) ,$$

where

$$h_n(l) = \int_0^l dl_{n-1} \cdots \int_0^{l_2} dl_1 (nl - \sum_{i=1}^{n-1} l_i) \mathrm{e}^{-\pi(nl - \sum_{j=1}^{n-1} l_j)^2} .$$

Time dependence enters only through the combination $l = \frac{\tau}{\sqrt{1 - \sum_{j=1}^n x_j}}$.

The small τ limit

For all $l^2 \ll \frac{1}{n^2\pi}$

$$D^{(n)}(x_1, \cdots, x_n|\tau) \sim \frac{(n+1)!}{2^n(1-\sum_{i=1}^n x_i)^{\frac{3}{2}}\sqrt{x_1\cdots x_n}}\tau^n$$

Using that at $x_0 \ll \tau^2$ the number of particles is dominated by low x

$$C_p = \frac{\langle N^p\rangle}{\langle N\rangle^p} = \frac{(p+1)!}{2^p}$$

the property that C_p is a constant is called KNO scaling, moreover these values of C_p correspond to a negative binomial with parameter $k = 2$.

KNO scaling, the negative binomial distribution and jet physics

- Probability of having n successful attempts in a Bernoulli trial before having k failures.
- A jet in the vacuum also fulfils KNO scaling and can be approximately described by a negative binomial with $k = 3$. (Dokshitzer, Khoze, Mueller and Troian *Basics of perturbative QCD*).
- At small times the distribution of gluons generated by medium jet radiations is significantly more correlated and with more fluctuations than that of a vacuum jet cascade.

The large τ limit

$D^{(n)}$ are dominated by branchings that happen at times very close to τ because the effect of the other branchings are already exponentially suppressed.

$$D^{(n)}(x_1, \cdots, x_n|\tau) = \frac{n! \mathrm{e}^{-\frac{\pi\tau^2}{1-\sum_{i=1}^n x_i}} (1 - \sum_{i=1}^n x_i)^{n-5/2}}{(4\pi)^{n-1}\tau^{n-2}\sqrt{x_1 \cdots x_n}}$$

Example, $D^{(3)}$

$$D^{(3)}(x_1, x_2, x_3|\tau) = \frac{3}{\sqrt{x_1 x_2 x_3}} h_3\left(\frac{\tau}{\sqrt{1 - x_1 - x_2 - x_3}}\right)$$

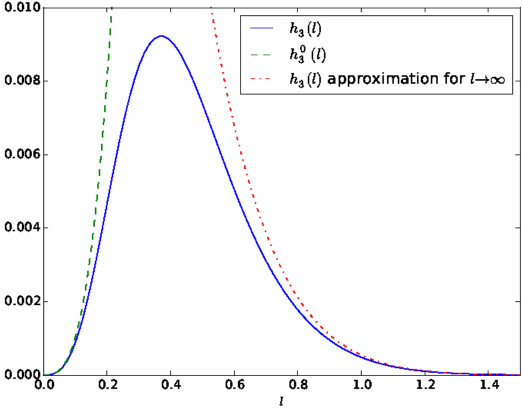


Table of Contents

Conclusions

- Fluctuations are large.
- The standard deviation of the energy loss is of the order of its average.
- At extremely small times the gluons emitted by the jet can be described by a negative binomial distribution and fulfils KNO scaling.
- Correlations and fluctuations in the number of particles are large and bigger that what is found in a jet vacuum cascade.