GOETHE UNIVERSITÄT FRANKFURT AM MAIN

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Complex Langevin simulations of a finite density Matrix Model for QCD

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XIIth Quark Confinement and the Hadron Spectrum
Thessaloniki, Greece

Work in collaboration with J. Glesaaen (Frankfurt U.), O. Philipsen (Frankfurt U.), J. Verbaarschot (Stony Brook U.)

Many approaches to attack the sign problem

- Conventional/Monte Carlo based methods
  - Reweighting
  - Taylor expansion
  - Imaginary μ
  - Strong Coupling Expansion
  - Mean Field analyses
- Alternative methods
  - Stochastic Quantization-Complex Langevin
  - Lefschetz Thimble
  - Canonical ensembles
  - Dual variables
  - Density of States

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Stochastic quantization as an alternative

- consider the trivial "QFT" given by the partition function
- $\mathcal{Z} = \int e^{-S(x)} dx$
- in the real Langevin formulation
- $x(t+\delta t) = x(t) - \partial_x S(x(t))\delta t + \delta\xi$
- stochastic variable $\delta\xi$ with zero mean and variance given by $2\delta t$
- generalization to complex actions Parisi(1983), Klauder (1983)
- $x \rightarrow z = x + iy$
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- one can study gauge theories with complex actions Aarts, James, Seiler, Sexty, Stamatescu, ...

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- proof relating Langevin dynamics to the path integral quantization-no longer holds
- simulations are not guaranteed to converge to "the correct solution"
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RMT

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- same flavor symmetries with QCD which uniquely determine (in the ϵ-regime)
- mass dependence of the chiral condensate $\langle \bar{\eta}\eta \rangle = \partial_m \log Z$
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  Stephanov (1996)

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The phase transition

- in the thermodynamic limit evaluate $\mathcal{Z}$ via a saddle point approximation
- there is a phase transition separating a phase with zero and non-zero baryon density
- In the chiral limit $\mu_c = 0.527$ for $\mu \in \mathbb{R}$
- $\mu_c = i$ for $\mu \in \mathbb{I}$
- we can compute $\Sigma(m, \mu)$ and $n_B(m, \mu)$ and compare it with the Complex Langevin simulation
- first attempts in the Osborn model Mollgaard and Splittorff(2013-2014), Nagata, Nishimura, Shimasaki (2015-2016)

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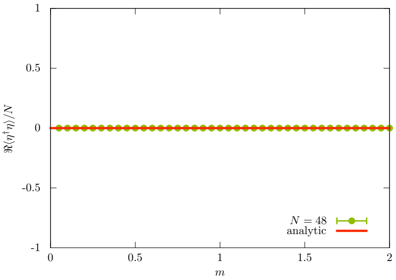
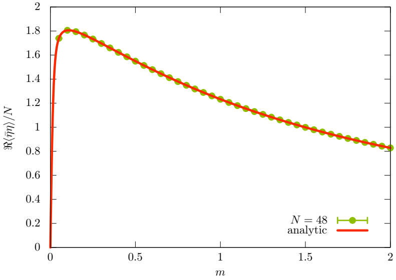


$\langle \eta^\dagger \eta \rangle$ for $\mu = 0$



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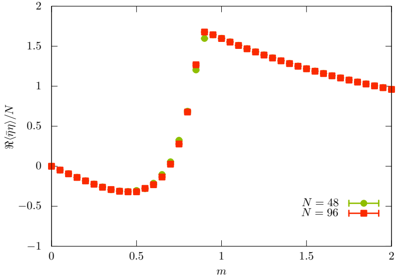
Numerical Validity-Matrix Size



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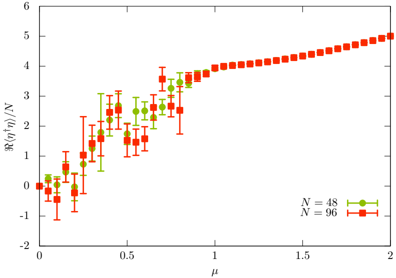

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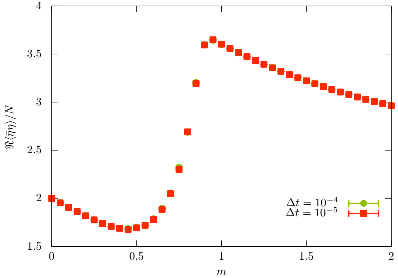
Numerical Validity-Step Size



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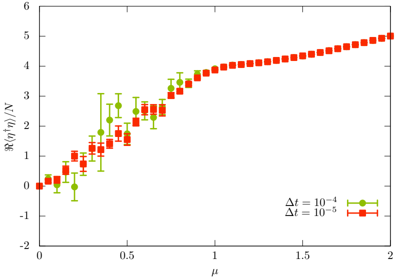

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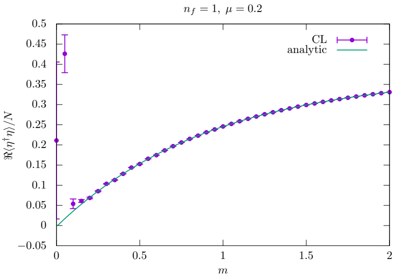

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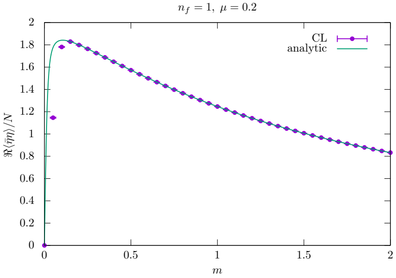

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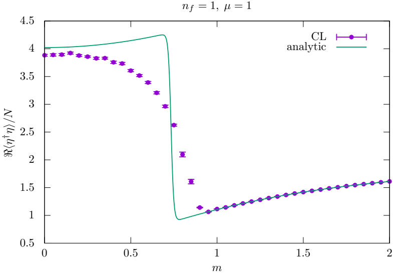

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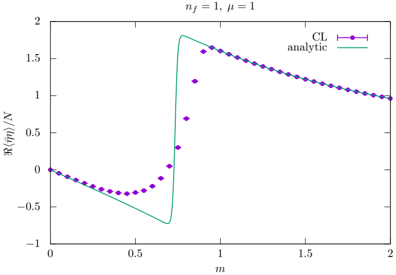

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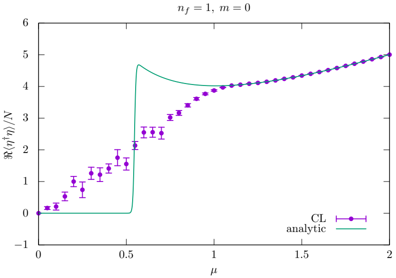

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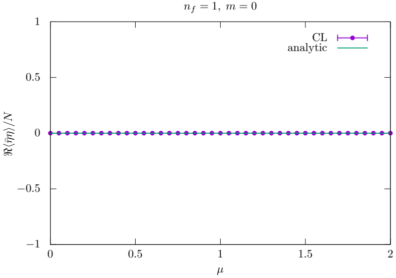

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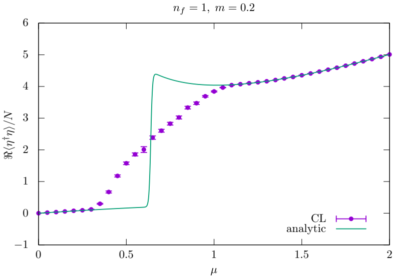

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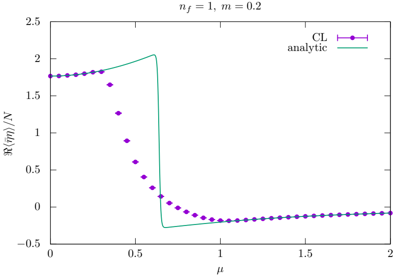

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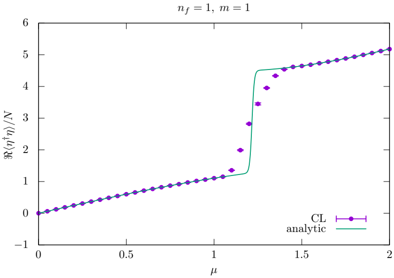

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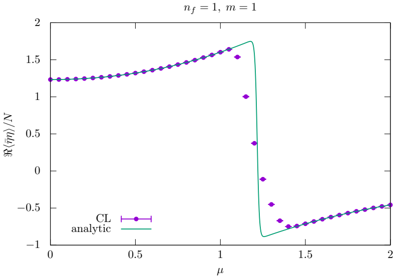

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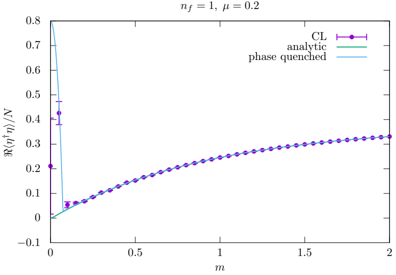

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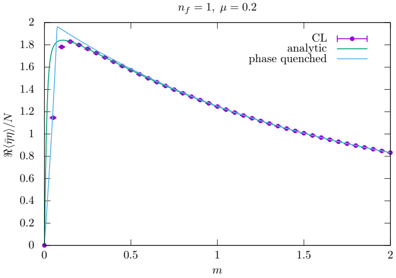

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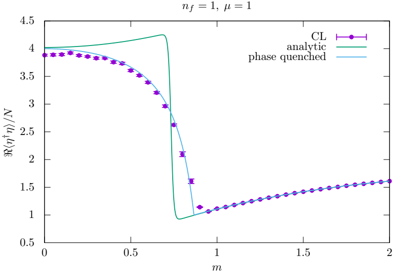

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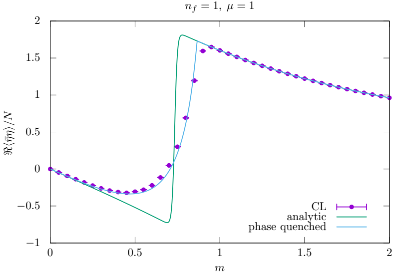

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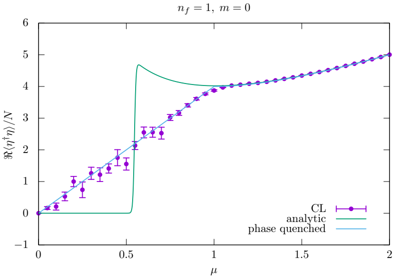

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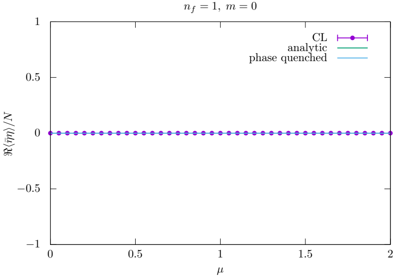

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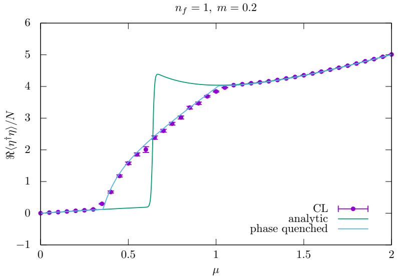

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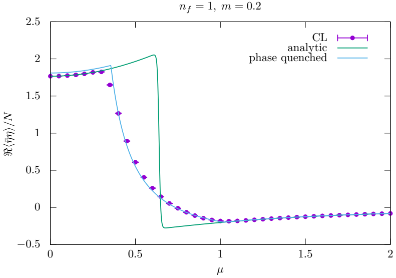

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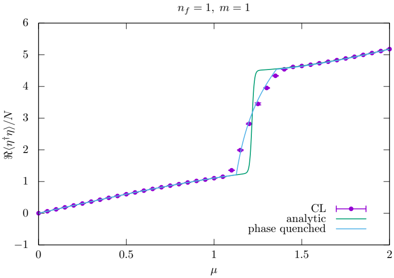

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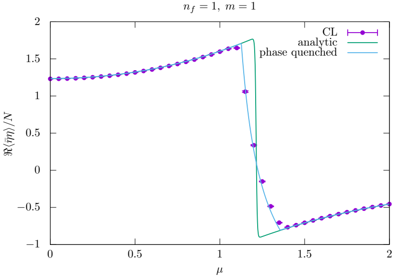

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Conclusions and outlook

- studied the Complex Langevin algorithm for an RMT model for QCD
- can compare with exact analytical results for all the range of parameters$(m,\ \mu)$
- compared to previous similar studies this model posseses a phase transition to a phase with non-zero baryon density
- fails to converge to QCD and it converges to $|QCD|$
- standard ways to fix it $\rightarrow$ gauge cooling Seiler, Sexty and Stamatescu(2012), Nagata, Nishimura, Shimasaki (2015)
- work in progress...
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for upcoming results ...