Study of lattice QCD at finite baryon density using the canonical approach

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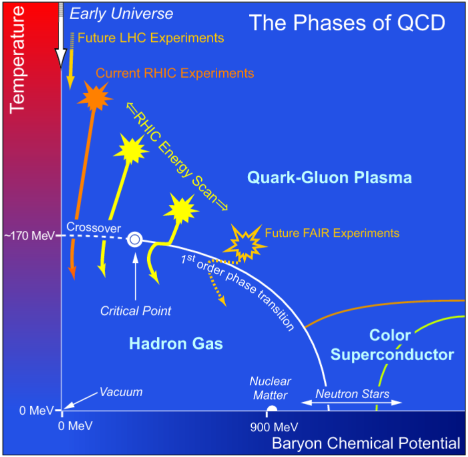
Confinement XII, 02.09.2016

Outline

- Introduction
- Canonical approach formulation
- Results
- Conclusions

Study of the QCD phase diagram



Experiments:

- RHIC
- LHC
- J-PARC
- FAIR
- NICA

Ab initio theoretical calculations are required.

LQCD with non-zero chemical potential

In LQCD we have the following relation for the Dirac operator:

$$det\left[M(\mu_B)\right]^* = det\left[M(-\mu_B^*)\right]$$

μ_B is real $\rightarrow$ $det\left[M(\mu_B)\right]$ is complex $\rightarrow$ no importance sampling

Techniques, which are now being used in LQCD at $\mu_B \neq 0$:

- Taylor expansion;
- analytic continuation;
- reweighting;
- complex Langevin;
- density of states;
- ...

Possible alternative: canonical approach

Suppose that we have conserved charge: $\left[\hat{H}, \hat{Q}_X\right] = 0$. In this case

$$Z_{GC}(\mu_X, T) = Tr\left[e^{-(\hat{H}-\mu_X\hat{Q}_X)/T}\right] = Tr\left[e^{-\hat{H}/T}e^{(\mu_X\hat{Q}_X)/T}\right] =$$

$$= \sum_{Q_X=-\infty}^{\infty} \mathrm{Tr}\left[e^{-\hat{H}/T}\right]\left(e^{\mu_X/T}\right)^{Q_X} = \sum_{Q_X=-\infty}^{\infty} Z_{Q_X}(T)\left(e^{\mu_X/T}\right)^{Q_X},$$

where $X = B,\ Q,\ S\ etc.$

- $Z_{Q_X}(T)$ are calculable in LQCD up to some norm. constant
- once Z_{GC} is known, susceptibilities may be studied

$$\lambda_k^X = \frac{\partial^k}{\partial(\mu_X/T)^k} log\, Z_{GC}(\mu_X, T)$$

- another conserved charges may be added in the same way

Canonical partition functions

Obtaining canonical partition function [first suggested by A. Hasenfratz and D. Toussaint, Nucl. Phys. **B 371** (1992)]:

$$Z_{n_B}(T) = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\theta \, Z_{GC}(\mu_B = i\theta T, T) e^{-i n_B \theta} \, ,$$

where for degenerate quarks

$$Z_{GC}(\mu_B = i\theta T, T) = \int DU \left(det\Delta(\mu_B = i\theta T, U) \right)^{N_f} e^{-S_G[U]}$$

We need to calculate $Z_{GC}(\mu_B^{Im.}, T)$ to perform the Fourier transformation. Possible ways:

1. calculation of the $det\,\Delta(\mu_B, U)$ with reweighting;
2. integration of the n_B (new method).

Fermionic determinant calculation (1)

$$Z_{GC}(\mu_B^{Im.}, T) = \int DU\, det\Delta^{N_f}(\mu_B^0, U)\, \frac{det\Delta^{N_f}(\mu_B^{Im.}, U)}{det\Delta^{N_f}(\mu_B^0, U)}\, e^{-S_G[U]}$$

It is possible to calculate not Z_{GC} itself but its ratio:

$$\frac{Z_{GC}(i\theta T, T)}{Z_{GC}(\mu_B^0, T)} = \left\langle \frac{det\Delta^{N_f}(i\theta T, U)}{det\Delta^{N_f}(\mu_B^0, U)} \right\rangle_{\mu_B^0}$$

- $\mu_B^0 = 0$ or $\mu_B^0 = i\mu_{Im.}$
- calculation of $det\Delta(\mu_B, U)$ is required

Fermionic determinant calculation (2)

We consider clover improved Wilson fermions:

$$D_W(\mu_B) = I - \kappa \left[\sum_{i=1}^{3} \left(Q_i^+ + Q_i^- \right) + T \right] - \kappa e^{\mu_B a} Q_4^+ - \kappa e^{-\mu_B a} Q_4^-,$$

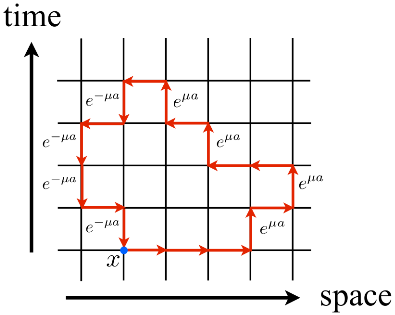
where

$$\kappa = \frac{1}{2\,(ma+4)}$$

$$\left(Q_\mu^+\right)_{nm} = (1-\gamma_\mu)\, U_{n,\mu} \delta_{m,n+\hat{\mu}}$$

$$\left(Q_\mu^-\right)_{nm} = (1+\gamma_\mu)\, U^\dagger_{m,\mu} \delta_{m,n-\hat{\mu}}$$

$$T_{nm} = \frac{1}{2} c_{SW} \sum_{\mu<\nu} \sigma_{\mu\nu} F_n^{\mu\nu} \delta_{nm}$$



$$Tr\left[\ln D_W\right] = Tr\left[\ln\left(I - \kappa Q\right)\right] = -\sum_{n=1}^{\infty} \frac{\kappa^n}{n} Tr\left[Q^n\right] = \sum_{n=-\infty}^{\infty} W_n \left(e^{\mu_B a N_\tau}\right)^n$$

Fermionic determinant calculation (3)

We use hopping parameter expansion (HPE) to evaluate the determinant ($\xi = e^{\mu_B a N_f} = e^{\mu_B/T}$):

$$detD_W(U) = e^{Tr[lnD_W]} = exp\left[\sum_{n=-N_{cut}}^{N_{cut}} W_n[U]\xi^n\right]$$

$W_n[U]$ may be calculated using stochastic estimators for $Tr\left[Q^n\right]$, the property $W_n^* = W_{-n}$ is satisfied for large enough N_{stoch}.

For physical quark masses one has to use reduction formula [1411.5133, 1009.2149]:

$$detD_W(U) = \sum_{n=-2N_xN_yN_zN_c}^{2N_xN_yN_zN_c} z_n[U]\xi^n$$

Integration of the baryon number density

$$\langle n_B \rangle = \frac{1}{V} \frac{\partial (ln\, Z_{GC})}{\partial (\mu_B / T)}$$

Let us consider imaginary chemical potential $\mu_B = i\mu_B^{Im.}$

$$ln\, Z_{GC}(\theta) - ln\, Z_{GC}(0) = V \int_0^{\theta} i d\bar{\theta}^{Im.} i \mathrm{Im} \left[\langle n_B(\bar{\theta}) \rangle \right]$$

$$\frac{Z_{GC}(\theta)}{Z_{GC}(0)} = exp \left[-V \int_0^{\theta} d\bar{\theta}^{Im.} \mathrm{Im} \langle n_B(\bar{\theta}) \rangle \right]$$

Integration is performed through the parametrization of $\langle n_B \rangle$:

- in deconfinement $\langle n_B(\theta_{Im.}) \rangle = a\, \theta_{Im.} + b\, \theta^3_{Im.} + O(\theta^5_{Im.})$
  [see also A. Roberge, N. Weiss, Nucl. Phys. **B275** (1986)]
- in confinement $\langle n_B(\theta_{Im.}) \rangle = \alpha \sin(\theta_{Im.}) + O(\sin(2\theta_{Im.}))$
  [see also J. Takahashi *et. al.*, Phys. Rev. **D91**, 014501 (2015)]

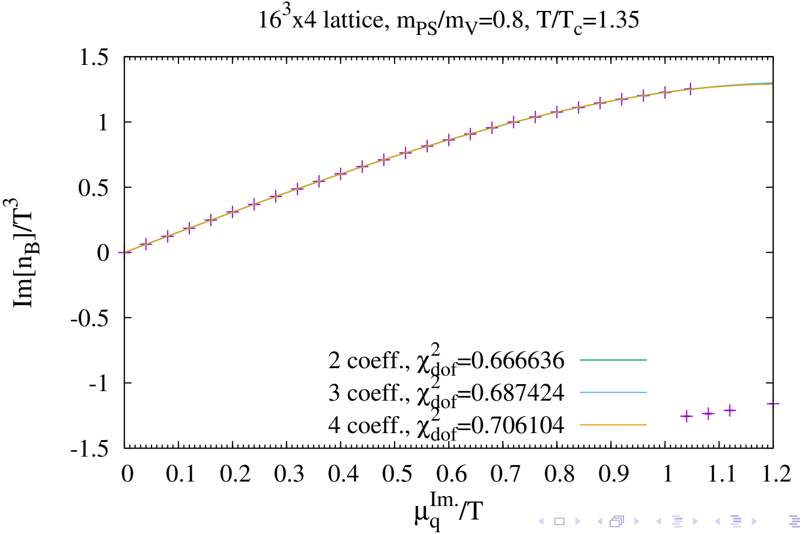
Simulation details

- clover improved Wilson fermion action
- Iwasaki gauge action
- $16^3 \times 4$ lattices, $\frac{m_\pi}{m_\rho} \approx 0.8$

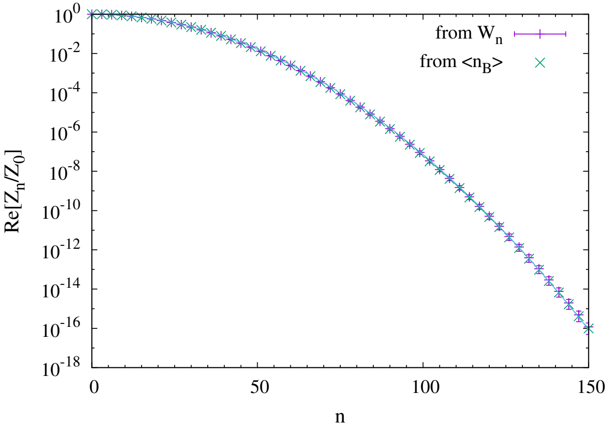
β	T/T_c
1.80	**0.93(5)**
1.85	0.99(5)
1.90	1.08(5)
2.00	**1.35(7)**

LCP parameters are taken from
S. Ejiri *et. al.*, PRD **82**, 014508 (2010).

Results for $T/T_c = 1.35$: power law fit

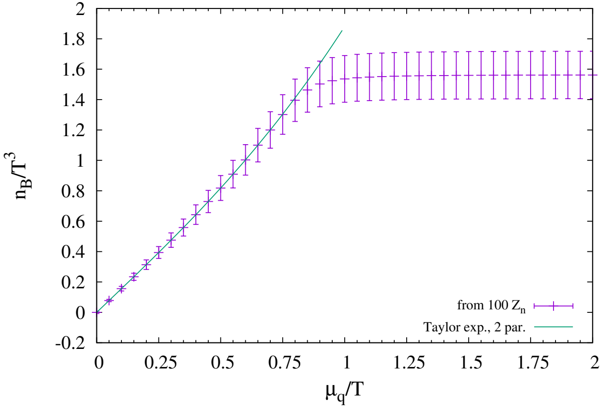


Results for $T/T_c = 1.35$: Z_n



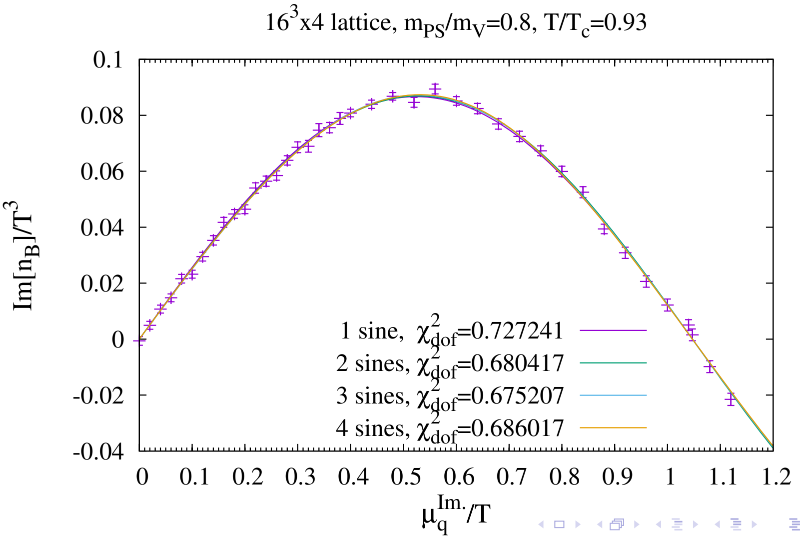
Asymptotics $Z_n = Z_0 e^{-an^2} \rightarrow$ fugacity expansion convergence

Results for $T/T_c = 1.35$: extrapolation of n_B

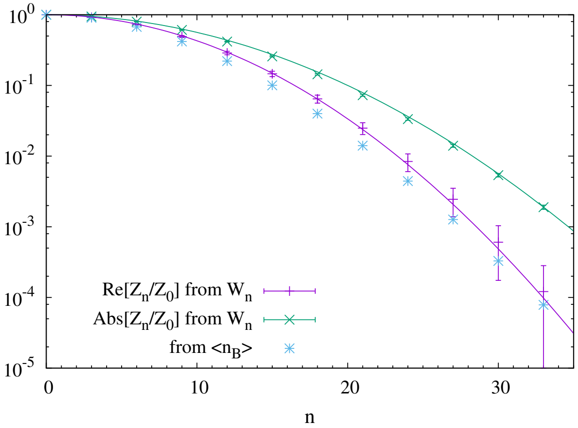


Taylor coefficients by WHOT-QCD Collaboration [S. Ejiri *et. al.*, PRD **82**, 014508 (2010)]

Results for $T/T_c = 0.93$: sine fit

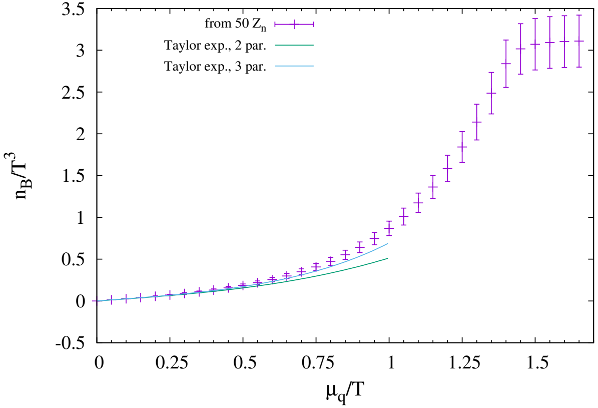


Results for $T/T_c = 0.93$: Z_n



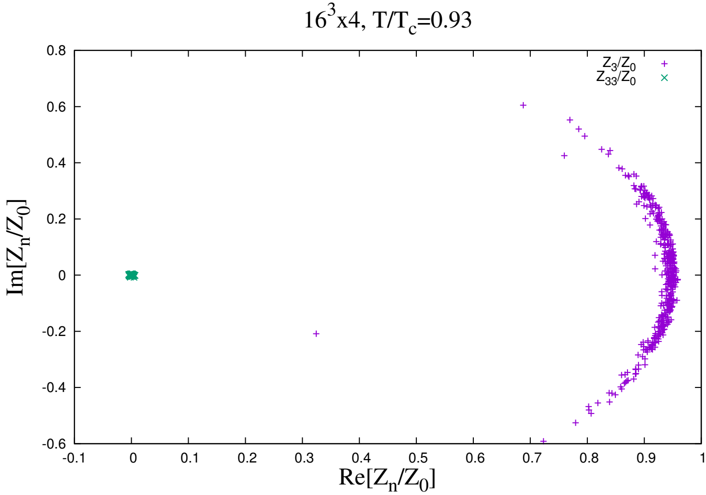
Solid lines are $Z_n = Z_0 e^{-an^2}$ fits

Results for $T/T_c = 0.93$: extrapolation of n_B

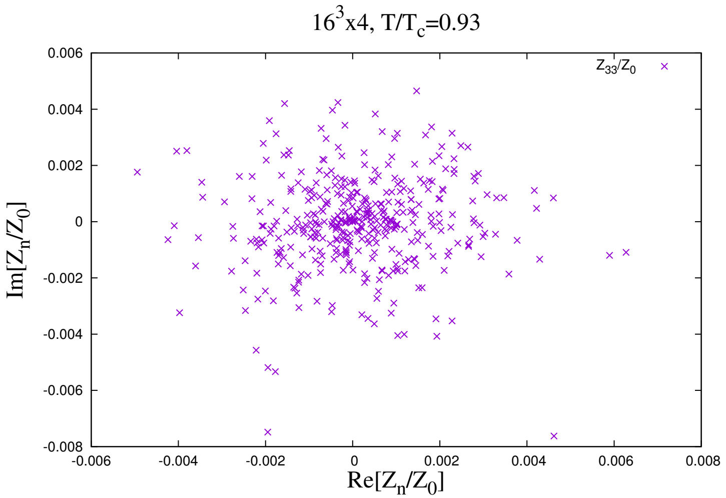


Taylor coefficients by WHOT-QCD Collaboration [S. Ejiri *et. al.*, PRD **82**, 014508 (2010)]

Results for $T/T_c = 0.93$: complexity of Z_n



Results for $T/T_c = 0.93$: complexity of Z_n



Conclusions

Results:

- canonical approach seems to be a viable method
- new approach for Z_n calculation has been tested
- baryon density extrapolation agrees with the Taylor expansion

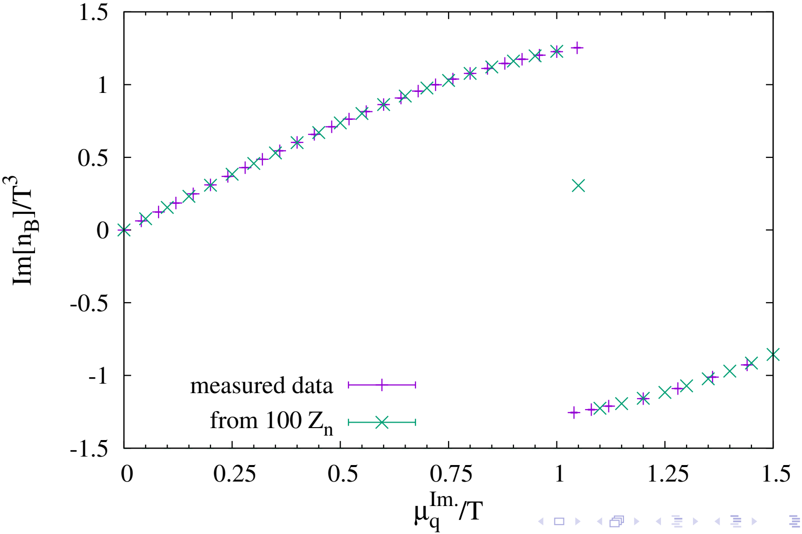
Problems:

- Z_n are complex at $T < T_c$ (see also the talk by Asobu Suzuki on Lattice 2016)
- possible statistical overlap problem

Thank you for attention

The end

$T/T_c = 1.35$


1.5
1
0.5
0
-0.5
-1
-1.5
Im[n_B]/T^3
measured data
from 100 Z_n
0
0.25
0.5
0.75
1
1.25
1.5
μ_q^Im./T