Quarkonia at $T>0$ and lattice QCD

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References:
With Y. Burnier  PRL 111 (2013) 182003,
PLB753 (2016) 232-236
With Y. Burnier and O. Kaczmarek PRL 114 (2015) 082001,
Physics Motivation

- From run1 and ongoing run2 at LHC: unprecedented amount of precision data

**Bound states of c\bar{c} or b\bar{b}:**  Heavy quarkonium  \( M_Q \gg T_{\text{med}} \)

- **Theory goal:** 1\textsuperscript{st} principles insight into in-medium \( \bar{Q}Q \) in heavy-ion collisions
I. Direct reconstruction of lattice meson spectra in NRQCD (limited resolution)

II. Via QQ potential from the lattice QCD Wilson loop (currently static potential only)

Assume full kinetic thermalization of QQ & Static medium from lattice QCD

Observables e.g. $\psi' / J/\psi$ ratio

In-medium meson spectra

In - medium meson spectra

$\omega$ [GeV]

$\sigma / \omega^2$

$T>0$ spectra

plot adapted from ALICE Collaboration JHEP05(2016)179
A common challenge

- Dynamical information e.g. spectral functions not directly accessible on the lattice
  \[ D_i = \sum_{l=1}^{N_\omega} \exp[-\omega_l \tau_i] \rho_l \Delta \omega_l \]
  1. \( N_\omega \) parameters \( \rho_l \gg N_T \) datapoints
  2. simulated \( D_i \) has finite precision

- Bayes theorem: Regularize the naïve \( \chi^2 \) functional \( P[D|\rho] \) through a prior \( P[\rho|I] \)

\[
P[\rho|D, I] \propto P[D|\rho] \frac{P[\rho|I]}{\delta \rho_l}
\]

Two Bayesian approaches on the market: Maximum Entropy and BR method

- Differ in the regulator functional \( P[\rho|I] \) and how to find the most probable spectrum

- Recent BR method provides higher resolution of peaks compared to MEM

- Systematic errors different: MEM extra smoothing, BR prone to ringing artifacts

- In Bayesian continuum limit \( N_T \to \infty, \Delta D/D \to 0 \) both methods will agree

References:
- Asakawa, Hatsuda, Nakahara, Prog.Part.Nucl.Phys. 46 (2001) 459
- Y. Burnier, A.R. PRL 111 (2013) 182003
In-medium quarkonium spectral functions from lattice QCD
I. Direct determination: NRQCD

- Relativistic treatment of light and heavy d.o.f.

- Kin. eq. non-relativistic $QQ$ in a background of light medium d.o.f.

- Lattice Non-Relativistic QCD (NRQCD) well established at $T=0$, applicable at $T>0$
  - no modeling, systematic expansion of QCD action in $1/m_Qa$, includes $v\neq 0$ contributions

- State-of-the-art: realistic simulations of the QCD medium by the HotQCD collab.
  - $48^3\times12$ $N_f=2+1$ HISQ action $m_\pi=161$ MeV $T= [140 - 407]$ MeV $m_ba= [2.759 - 0.954]$

- Full Lattice QCD simulation incl. $QQ$ (still too costly)

- Lattice QCD simulation without $QQ$
Correlation functions in NRQCD

Non-rel. propagator of a single heavy quark $G$


QQ propagator projected to a certain channel

"correlator of QQ wavefct. $D_{J/\psi}(\tau) \triangleq \langle \psi_{J/\psi}(\tau)\psi_{J/\psi}^{\dagger}(0) \rangle"$

Brambilla et. al. Rev.Mod.Phys. 77 (2005) 1423

Ratio of $T>0$ and $T\approx 0$ correlators: estimate of overall in-medium effects

$\chi_{b1}$ Correlator ratio

max 6.5% for $\chi_{b1}$

max 1.75% for $\chi_{b1}$

$\gamma$ Correlator ratio $T>0$ vs $T=0$

$\tau \ [\text{fm}]$

T=140MeV T=249MeV T=407MeV
T=160MeV T=273MeV
T=184MeV T=333MeV

$\chi_{b1}$ Correlator ratio

max 6.5% for $\chi_{b1}$
**Bottomonium NRQCD S-wave spectra**

- **$^3S_1$ Channel @ $T>0$**

- **PRELIMINARY**
  - $T_C = 159$ MeV

- **$^3S_1$ Channel @ $T>0$**

- **PRELIMINARY**
  - no jackknife errorbands

- **BR method shows ground state feature at all temperatures $T \leq 407$ MeV**

- **MEM: around $T = 333$ MeV only washed out bump visible**

- **Systematics: MEM over smoothing, BR ringing – use both methods to bracket**

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**Uncertainties in the Bayesian reconstruction: only ground state reliably captured**

**BR method shows ground state feature at all temperatures $T \leq 407$ MeV**

**MEM: around $T = 333$ MeV only washed out bump visible**

**Systematics: MEM over smoothing, BR ringing – use both methods to bracket**
BR Method: Lowest lying wiggly feature up to $T=407 \text{MeV}$

MEM consistent with previous studies: at $T\sim 210 \text{ MeV}$ no more remnant visible

BR: Can we disentangle numerical ringing from actual physics at $T=407 \text{MeV}$
How to identify BR method artifacts?

- Distinguish ringing and actual bound state signal via free spectral functions
- Hints at disappearance of genuine bound state feature above $T=333\text{MeV}$
- Better understanding of Bayesian systematics (MEM,BR) but remains a challenge:
  - increasing statistics
  - or increasing $N_T$

PRELIMINARY

$\rho(\omega)$

$\rho(\omega)$

$^{3}P_1$ Channel @ $T>0$, $n=4$

$^{3}P_1$ Channel comparison to rec. free

$T_C=159\text{MeV}$

$T=407\text{MeV}$ interacting

shifted rec. free

$\omega$ [GeV]

$\omega$ [GeV]
II. Indirect determination: pNRQCD

- pNRQCD Effective field theory: \[ \frac{\Lambda_{\text{QCD}}}{m_Q} \ll 1, \quad \frac{T}{m_Q} \ll 1, \quad \frac{p}{m_Q} \ll 1 \]

- Describes QQ as singlet and octet wavefunctions: \( \psi_S(R, t), \psi_O(R, t) \)
  \[ i\partial_t \psi_S = \left( V^{\text{QCD}}(R) + \mathcal{O}(m_Q^{-1}) \right) \psi_S \]

- Derived from QCD: \( V^{\text{QCD}} \) as Wilson coefficient determined via matching at \( m=\infty \)
  \[ V^{\text{QCD}}(R) = \lim_{t \to \infty} \frac{i\partial_t W_\square(R, t)}{W_\square(R, t)} \in \mathbb{C} \]

- Challenge: real-time definition not directly evaluable in lattice QCD simulations

- Spectral functions as bridge between the Euclidean and real-time Wilson loop
  \[ W_\square(R, t) = \int_{-\infty}^{\infty} d\omega \ e^{-i\omega t} \rho_\square(R, \omega) \quad \leftrightarrow \quad W_\square(R, \tau) = \int_{-\infty}^{\infty} d\omega \ e^{-\omega \tau} \rho_\square(R, \omega) \]

Quarkonia at $T>0$ and lattice QCD

$T>0$ static potential from the lattice

- Robust lattice determination of $\text{Re}[V] & \text{Im}[V]$
- At $T\sim 0$ $\text{Re}[V]$ on the lattice well described by naïve Cornell ansatz: $V = -\alpha/r + \sigma r + c$
- Analytic parametrization from a generalized gauss law with a single T-dep. parameter $m_D$

Y. Burnier, A.R. PLB753 (2016) 232

$N_f=2+1$, $48^3 \times 12$, asqtad action, $m_\pi \sim 300$MeV


continuum corrected interpolation
S-wave spectral functions

- In absence of a true continuum extrapolation of the potential, combine: phenomenological T=0 Cornell potential & T>0 lattice QCD $m_D$

- Zero angular momentum radial Schrödinger equation (not for the WF!)

$$i\frac{\partial}{\partial t}D^>(t, r) = \left(2m_Q - \frac{1}{2m_Q} \frac{d^2}{dr^2} + V_{Q\bar{Q}}(r)\right)D^>(t, r) \quad \Rightarrow \quad \lim_{r \to 0} \int d\omega e^{-i\omega t}D^>(t, r) = \rho(\omega)$$

- Hierarchical modification of states according to their vacuum binding energy

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Melting Temperatures: S-wave

- In-medium modified $\text{Re}[V]$: threshold from confinement moves to lower energies
- Meaningful definition of melting in the presence of $\text{Im}[V]$: use $\Gamma = E_{\text{bind}}$

<table>
<thead>
<tr>
<th>state</th>
<th>$J/\Psi(1S)$</th>
<th>$\Psi'(2S)$</th>
<th>$\Upsilon(1S)$</th>
<th>$\Upsilon(2S)$</th>
<th>$\Upsilon(3S)$</th>
<th>$\Upsilon(4S)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{\text{melt}}$</td>
<td>$213^{+13}_{-11}$</td>
<td>$&lt; 147$</td>
<td>$412^{+76}_{-22}$</td>
<td>$193^{+26}_{-8}$</td>
<td>$157^{+5}_{-4}$</td>
<td>$&lt; 147$</td>
</tr>
</tbody>
</table>
P-wave spectral functions

- Finite angular momentum radial Schrödinger equation: \( V_{QQ} + l(l+1)/mr^2 \)

\[ 3P_1 \text{ Bottomonium } \chi_b \text{ channel} \]

\[ 3P_1 \text{ Charmonium } \chi_c \text{ channel} \]

- Again hierarchical in-medium broadening and shifts to lower masses

<table>
<thead>
<tr>
<th>states</th>
<th>( \chi_c(1P) )</th>
<th>( \chi_c(2P) )</th>
<th>( \chi_b(1P) )</th>
<th>( \chi_b(2P) )</th>
<th>( \chi_b(3P) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T^\Gamma = E_{\text{bind}} )</td>
<td>161(5)</td>
<td>&lt; 147</td>
<td>219(9)</td>
<td>165(5)</td>
<td>152(3)</td>
</tr>
</tbody>
</table>
| \( T^\Gamma = E_{\text{melt}} \)   |                  |                  |                  |                  |                  | [MeV]
Quarkonium phenomenology from in-medium spectral functions

**Caveat I:** We do not measure in-medium di-lepton emission in experiment instead the decay of vacuum states long after the QGP ceased to exist

**Caveat II:** The assumption of full kinetic thermalization is (if at all) only appropriate for Charmonium
Meaningful c\bar{c} observables

- Many models reproduce individual $R_{AA}$, i.e. pinpointing the relevant physics is difficult
- Ratios with excited states or P-wave are more discriminatory
- How can in-medium spectra inform us about these observables? First results on:
  A. $\psi'$ to $J/\psi$ ratio
  B. $\chi_c$ in-medium feed-down
**ISOQUANT**

**ISOQUANT SFB 1225**

**XIIth Quark Confinement and the Hadron Spectrum Conference (CONF XII) – Thessaloniki, Greece – August 30th 2016**

**QUARKONIA at T>0 and Lattice QCD**

**ψ′ to J/ψ ratio from T>0 spectra**

- Assume instantaneous freezeout: T>0 states convert to real vacuum particles at around T_C
- In-medium dilepton emission from area under spectral resonance peaks
  \[ R_{\ell\ell} \propto \int dp_0 \int \frac{d^3p}{(2\pi)^3} \frac{\rho(P)}{P^2} n_B(p_0) \]
  (to leading order \( \rho(P) = \rho(p_0^2 - p^2) \))

- "How many vacuum states do the in-medium peaks correspond to?"

- Number density: divide in-medium by T=0 dimuon emission rate:
  \[ \frac{N_{\psi'}}{N_{J/\psi}} = \frac{R_{\ell\ell}^{\psi'}}{R_{\ell\ell}^{J/\psi}} \frac{M_{\psi'}^2 |\Phi_{J/\psi}(0)|^2}{M_{J/\psi}^2 |\Phi_{\psi'}(0)|^2} \]


[Graph showing the ψ' to J/ψ ratio from T>0 spectra]
**P-wave feed-down from T>0 spectra**

- **Feed-down in vacuum**
  \[
  pp \mathcal{R}_{\psi(nS)}^{\chi(mP)} \equiv \frac{\sigma(pp \rightarrow \chi(mP)X)}{\sigma(pp \rightarrow \psi(nS)X)} B_{\chi \rightarrow \gamma} = \frac{N(pp \rightarrow \chi(mP)X)}{N(pp \rightarrow \psi(nS)X)} B_{\chi \rightarrow \gamma}
  \]

- **Corresponding in-medium expression (at T=T_C)**
  \[
  AA \mathcal{R}_{\psi(nS)}^{\chi(mP)} \equiv \frac{N(AA \rightarrow \chi(mP)X)}{N(AA \rightarrow \psi(nS)X)} B_{\chi \rightarrow \gamma} = \kappa_{\psi(nS)}^{\chi(mP)} \cdot pp \mathcal{R}_{\psi(nS)}^{\chi(mP)}
  \]

  \[
  \kappa_{\psi(nS)}^{\chi(mP)} = \frac{N(AA \rightarrow \chi)}{N(pp \rightarrow \chi)} / \frac{N(AA \rightarrow \psi)}{N(pp \rightarrow \psi)}
  \]

- For P-wave different decay channel relevant: into light hadrons instead of dileptons
  \[
  R_{LH}^{\chi(mP)}(T = 0) \propto \frac{|\Psi_{\chi(mP)}'(0)|^2}{M_{\chi}^4}.
  \]

  \[
  R_{LH}^{\chi(mP)}(T > 0) \propto \int dP_0 d^3 P \frac{\rho(P)}{P^4} n_B(p_0)
  \]

- Use the LHCb data for T=0
  \[
  pp \mathcal{R}_{\psi(nS)}^{\chi(mP)}
  \]
Summary

- Heavy quarkonium matured into a precision probe in heavy-ion collisions

- Direct and indirect lattice QCD approaches to in-medium quarkonium spectra
  - NRQCD: includes finite velocity corrections but still limited by simulation data quality correlation functions show hierarchical in-medium modification spectra challenging but show reasonable disappearance of bound state features
  - pNRQCD: $V_{QQ}$ does not contain velocity corrections yet but spectra not resolution limited hierarchical modification of spectra: states broaden and shift to lower masses meaningful determination of melting temperatures possible

- Assuming full kinetic thermalization at $T_C$ and instantaneous freezeout
  - Estimate of $\psi'$ to $J/\psi$ ratio from $T>0$ spectra larger than statistical hadronization model
  - Estimate of P-wave feed-down hints at significant reduction in heavy-ion-collisions

- Need experimental focus on discriminatory observables: excited states and P-wave
Thank you for your attention
Ευχαριστώ για την προσοχή σας