Yang-Mills correlation functions at non-zero temperature

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XIIth Quark Confinement and the Hadron Spectrum - Thessaloniki, Greece

Sept. 2, 2016

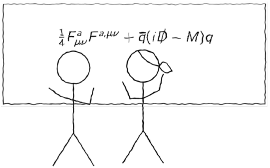
Der Wissenschaftsfonds.

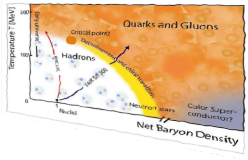
NAWI Graz
Natural Sciences

QCD: A simple theory?

$$\frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu} + \bar{q}(i\not{D} - M)q$$

QCD: A simple theory?





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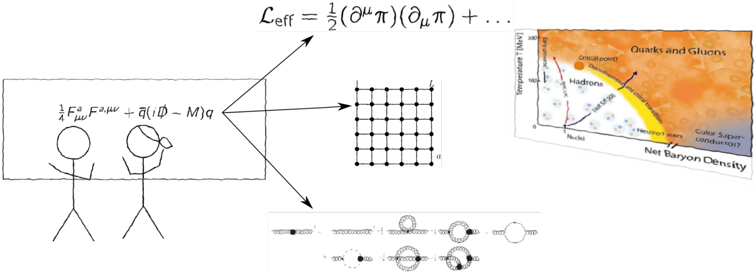

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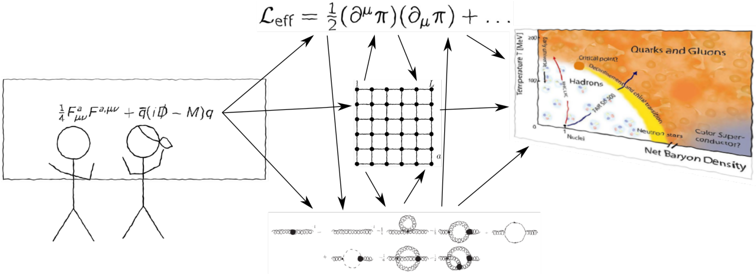

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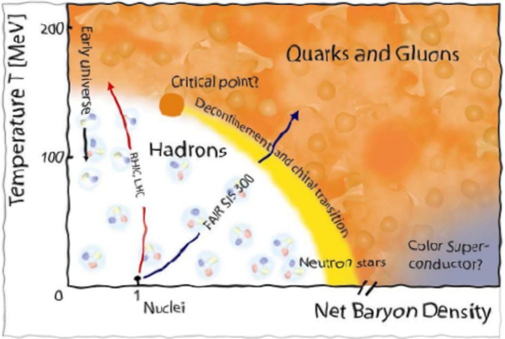
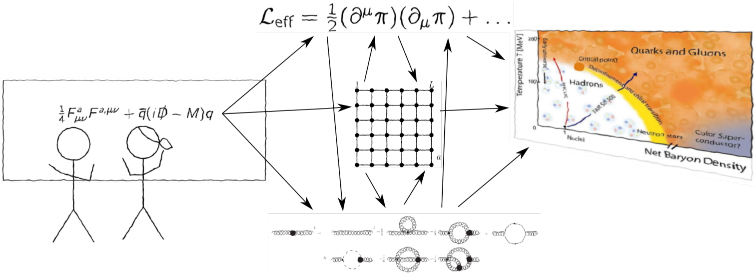


- Phases: hadronic phase, quark-gluon plasma, color superconductor, quarkyonic?
- Transitions: first order line, crossover at $\mu = 0$
- Critical point: existence? position?

QCD: A simple theory?



- Challenges for all methods at $\mu > T$, e.g.
  - Lattice QCD: complex action problem
  - Models: parameters
  - Functional methods: reliability of truncations

Functional methods

Functional equations: Exact equations derived from QCD action.

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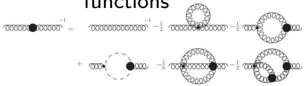
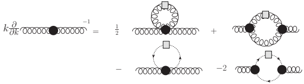
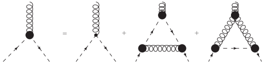
Dyson-Schwinger eqs.: eqs. of motion of corr. functions





funct. renorm. group eqs.



eqs. of motion from 3PI eff. action

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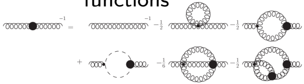
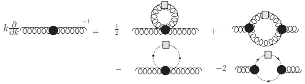


- Chiral limit accessible.
- No sign problem.
- Large scale separations easy.
- Real-time quantities accessible
  $\rightarrow$ [talk by Pawlowski]



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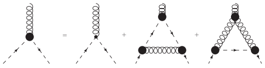

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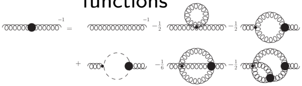

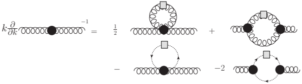

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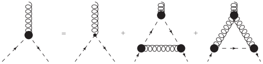

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  $\longrightarrow$ [talk by Pawlowski]

Difficulty

Infinitely large systems of equations without obvious ordering scheme.

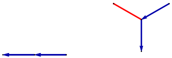
Outline

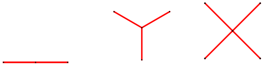
1. Introduction
2. Dyson-Schwinger equations and truncations
3. Testing truncations in $d = 3$
4. Non-vanishing temperature results

Landau gauge QCD

$$\mathcal{L} = \bar{q}(-\not{D} + m)q + \frac{1}{2}F^2 + \mathcal{L}_{gf} + \mathcal{L}_{gh}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i\,g\,[A_\mu, A_\nu]$$





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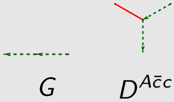
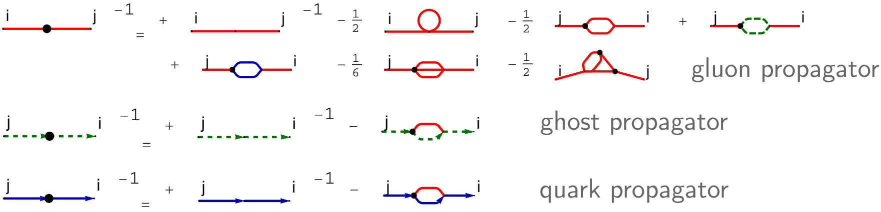
Landau gauge

- simplest one for functional equations
- $\partial_\mu A_\mu = 0$: $\quad \mathcal{L}_{gf} = \frac{1}{2\xi}(\partial_\mu A_\mu)^2, \quad \xi \to 0$
- requires ghost fields: $\quad \mathcal{L}_{gh} = \bar{c}\,(-\Box + g\,A\,\times)\,c$



The tower of DSEs



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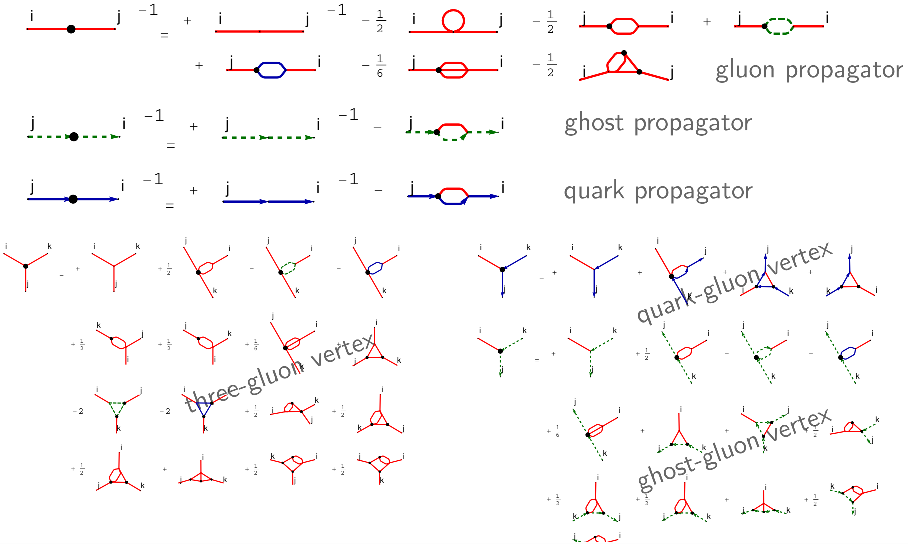

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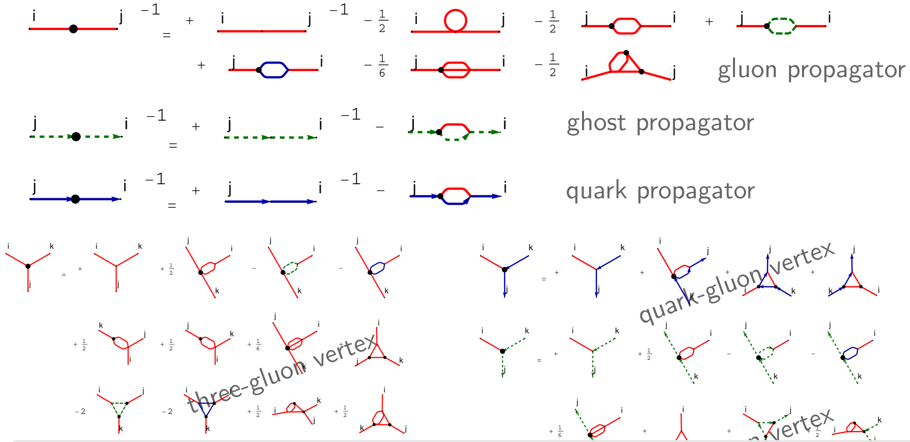

Infinitely many equations. In QCD, every n-point function depends on $(n+1)$- and possibly $(n+2)$-point functions.

Truncating the equations

Guides

- Perturbation theory
- Symmetries
- Lattice
- Analytic results

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Truncation

- Drop quantities (unimportant?)
- Use fits
- Model quantities (good models available? 'true' or 'effective'?)

Ideally: Find a truncation that has (I) no parameters and yields (II) quantitative results.

But...

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- Vacuum

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- $T > 0$
- Three-point functions (restricted kinematics)

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$\rightarrow$ Comparison with lattice is helpful, but finally self-consistent checks are required.

Two words of caution:

- One cannot assume naturally that the hierarchy is the same for all T and μ.
- In simple truncations, the effect of a single correlation function is difficult to estimate.

Top-down for Yang-Mills theory

Top-down vs. bottom up

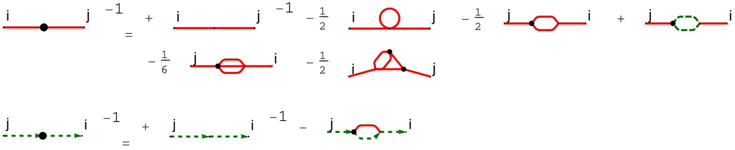
$\updownarrow$

Green functions from QCD action vs. effective models

Top-down for Yang-Mills theory

Neglect all non-primitively divergent Green functions. $\rightarrow$ Self-contained.

Full propagator equations (two-loop diagrams!):



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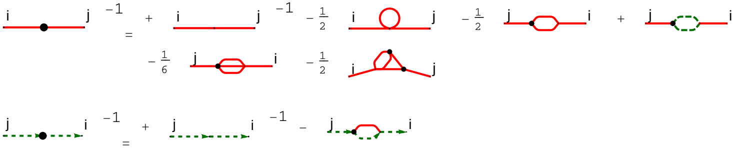
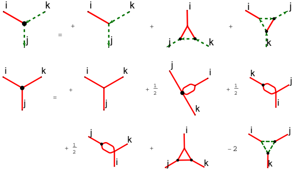
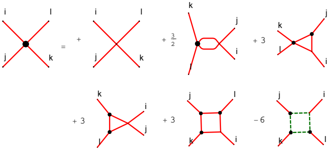


Truncated three-point functions:



Truncated four-gluon vertex:



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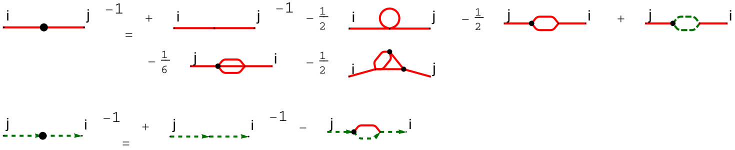

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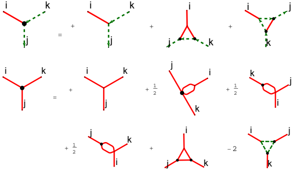

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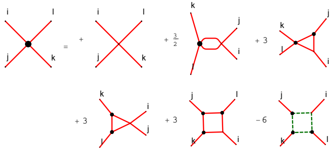
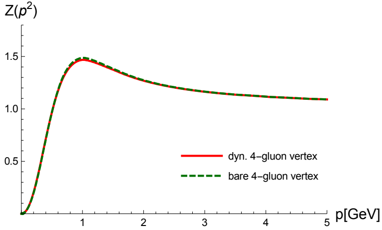


Testing top-down for Yang-Mills in $d = 3$

[MQH '16]

Quantitative study of truncation effects possible

- Varying the four-gluon vertex: bare vs. dynamic



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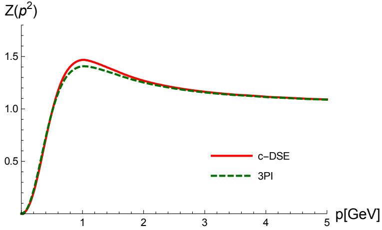

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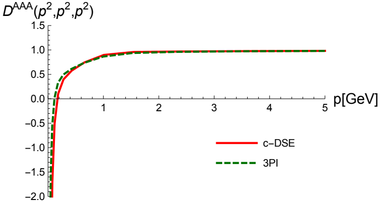

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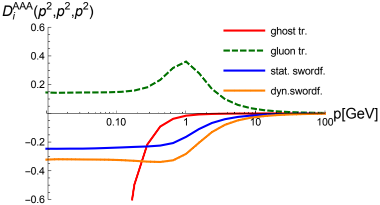

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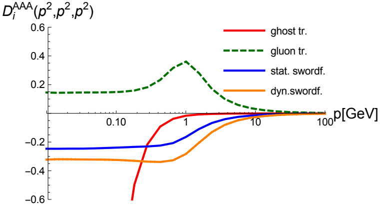

Similar truncation in FRG ($d = 4$):
[Cyrol, Mitter, Strodthoff, Pawlowski '16; talks by Cyrol, Mitter]

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What do we need to go beyond modern QCD phase diagram calculations?

Beyond effective interaction approximation: ✓ [Fischer, Lücker, Welzbacher '14]

Input for DSEs:

- model for quark-gluon vertex
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- fully dynamical propagators
- fully dynamical quark-gluon vertex

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- fully dynamical quark-gluon vertex $\rightarrow$ requires propagators & other vertices

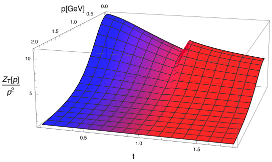
Ultimately, full control over Yang-Mills part required!

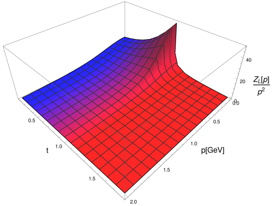
Non-vanishing temperature

Elementary Green functions → (dual) quark condensate, Polyakov loop potential, ...

Propagators

Lattice results → input to calculate other quantities [Fischer, Maas, Müller '10].





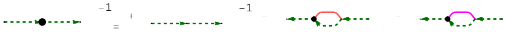
Fits based on [Maas, Pawlowski, von Smekal, Spielmann '12].

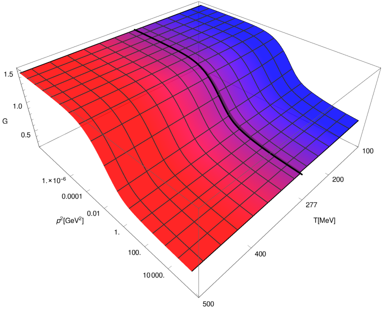
Note

Gluon propagator from lattice has no truncation artifacts.

Example: Ghost DSE

Ghost dressing $G(p^2)$ from DSE [MQH, von Smekal '13]:



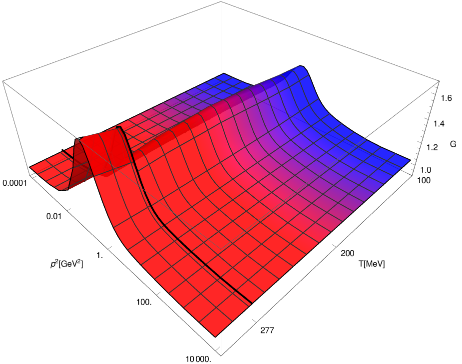


- Ghost insensitive to phase transition.

Ghost-gluon vertex

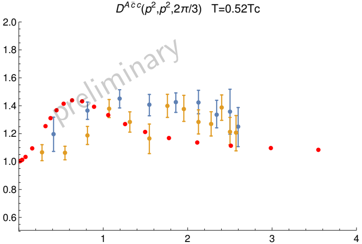
- Vertices on lattice more difficult than propagators.
- Full momentum dependence from functional equations.

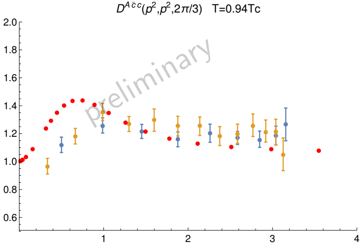
Self-consistent solution, zeroth Matsubara only

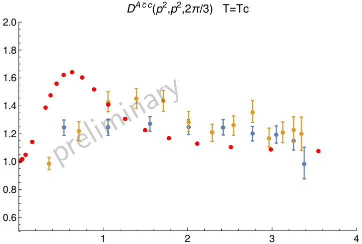


Vertex from FRG: [Fister, Pawlowski '11; talk by Cyrol]

Ghost-gluon vertex: Continuum and lattice







Lattice: [Fister, Maas '14]

Three-gluon vertex: Continuum and lattice

Note: IR suppression observed at $T = 0$. Zero crossing? [talk by Sternbeck]

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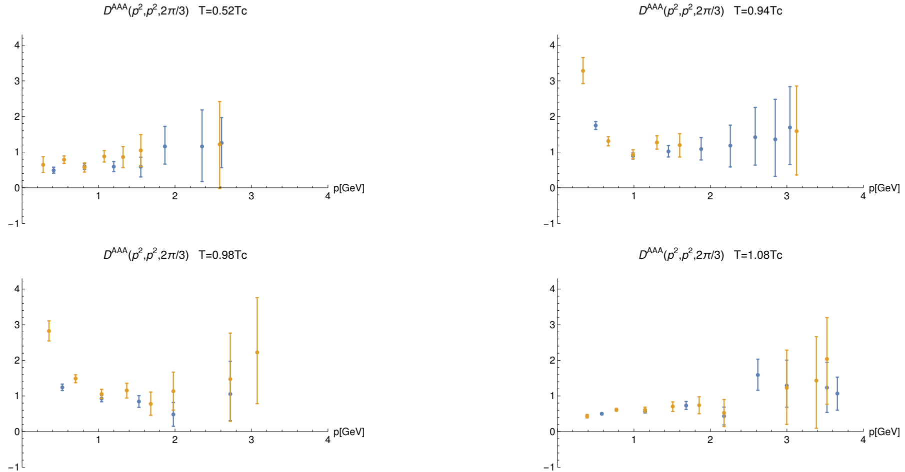

Lattice: [Fister, Maas '14]

$\rightarrow$ Enhancement around T_c, but still zero crossing.

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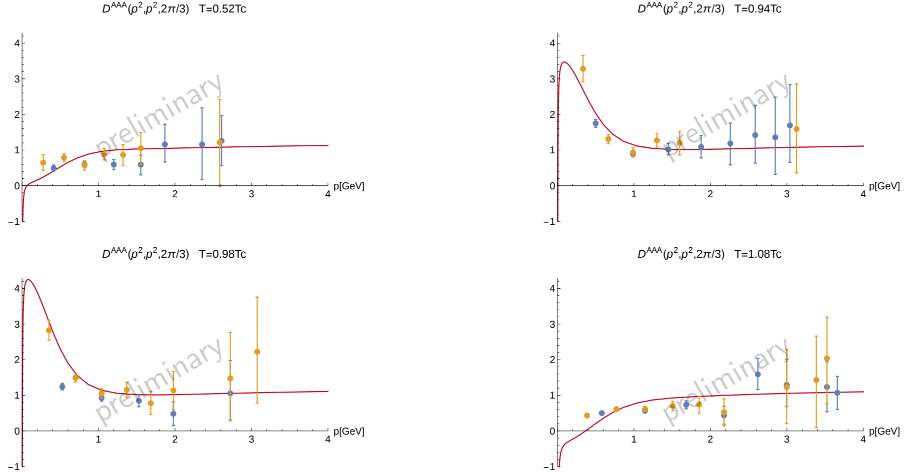

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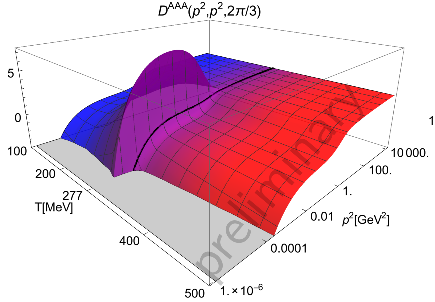
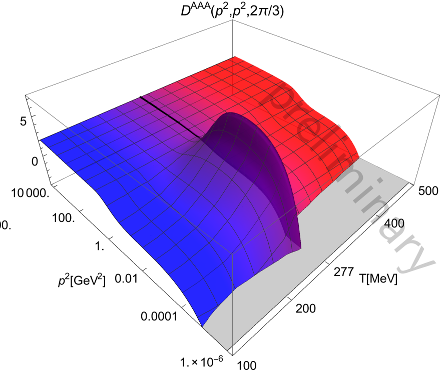
Three-gluon vertex

DSE calculation: semi-perturbative approximation (first iteration only)





Summary and conclusions

- Functional equations: Non-perturbative approach to QCD.
- Calculations of propagators, vertices and partially mixed systems show a coherent picture at $T = 0$.
- Top-down approach provides a self-contained description.
- Results for ghost and gluonic three-point functions at $T > 0$.

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- Coupling the equations.
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Thank you for your attention.