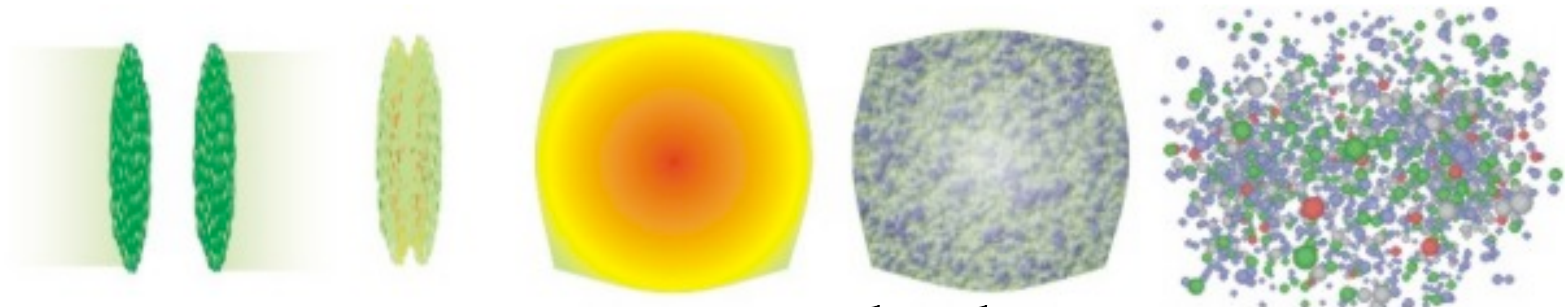
Electromagnetic probes of the quark-gluon plasma: perturbation theory meets the lattice

Jacopo Ghiglieri, AEC ITP, University of Bern

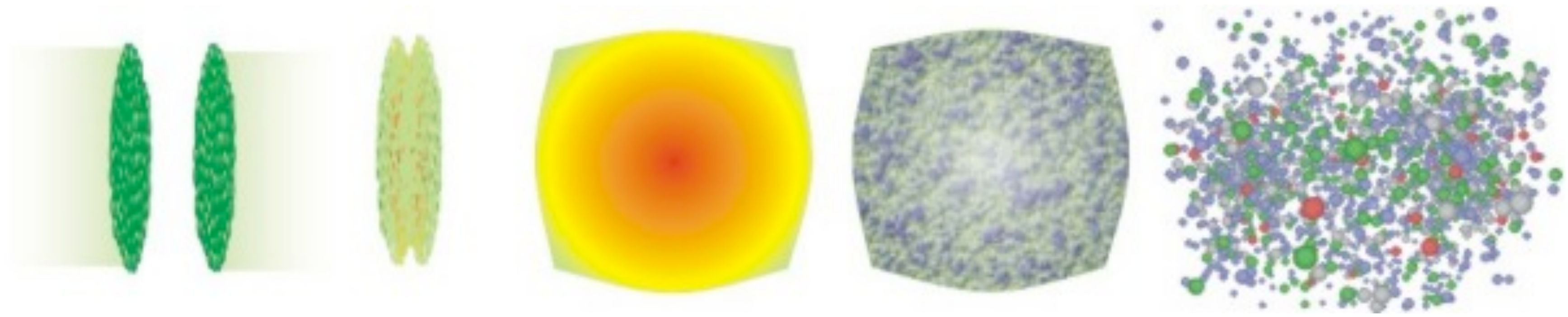
QCHS12, Θεσσαλονίκη, September 2 2016

How photons are made



- The hard partonic processes in the heavy ion collision produce quarks, gluons and *prompt/primary photons*
- At a later stage, quarks and gluons form a plasma.
  - Scatterings of thermal partons produce *partonic thermal photons*
  - A jet traveling can radiate *jet-thermal photons*
- Later on, hadronization. *hadron gas photons*
- (Some) hadrons decay into *decay photons*

How photons are made



- Theoretical description: convolution of microscopic rates over the macroscopic evolution of the medium

In this talk

- In this talk:
  - the *thermal* photon and dilepton rates at **NLO** in an infinite, equilibrated medium in different kinematical regimes
    - at zero virtuality JG Hong Kurkela Lu Moore Teaney **JHEP1305** (2013)
    - at small virtuality JG Moore **JHEP1412** (2014)
    - at larger virtuality Ghisoiu Laine **JHEP1410** (2014), JG Moore
  - a comparison with lattice data JG Kaczmarek Laine Meyer **PRD94** (2016)
  - This symbol:  $\Rightarrow$ interesting, but no time for it. look for details in the backup slides (or just come ask me)

Basics of e/m production

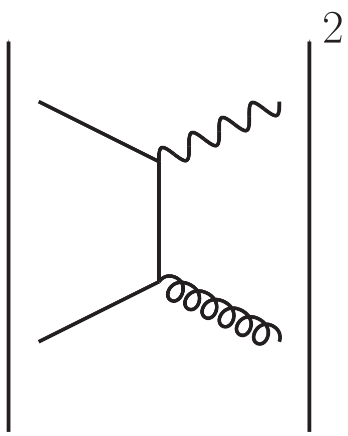
- $\alpha \ll 1$ implies that real/virtual photon production are rare events and that rescatterings and back-reactions are negligible: medium is transparent to/not cooled by EM radation
- At leading order in QED and to all orders in QCD the photon and dilepton rates are given by (in eq.)

$$\frac{d\Gamma_\gamma(k)}{d^3k} = -\frac{\alpha}{4\pi^2 k} n_{\rm B}(k^0)\, \rho_{\rm EM}(k)$$

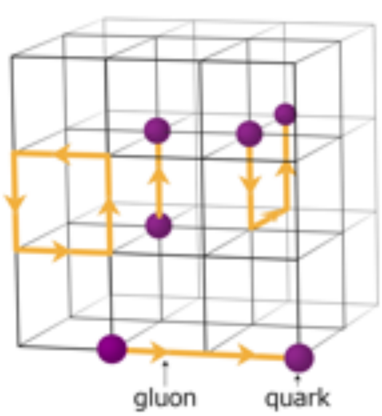
$$\frac{d\Gamma_{l^+l^-}(K)}{dk^0 d^3k} = -\frac{\alpha^2}{6\pi^3 K^2} n_{\rm B}(k^0)\, \rho_{\rm EM}(K)$$

thermal distribution x spectral function of the EM current

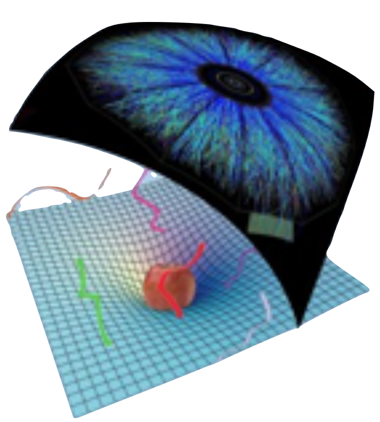
Theory approaches



pQCD: QCD action (and EFTs thereof), thermal average can be generalized to non-equilibrium. Real world: extrapolate from $g \ll 1$ to $\alpha_s \sim 0.3$



lattice QCD: Euclidean QCD action, pure thermal average. Real world: analytically continue to Minkowskian domain



AdS/CFT: $\mathcal{N}=4$ action, in and out of equilibrium, weak and strong coupling. Real world: extrapolate to QCD

Motivation

- Test the reliability of the perturbative rates

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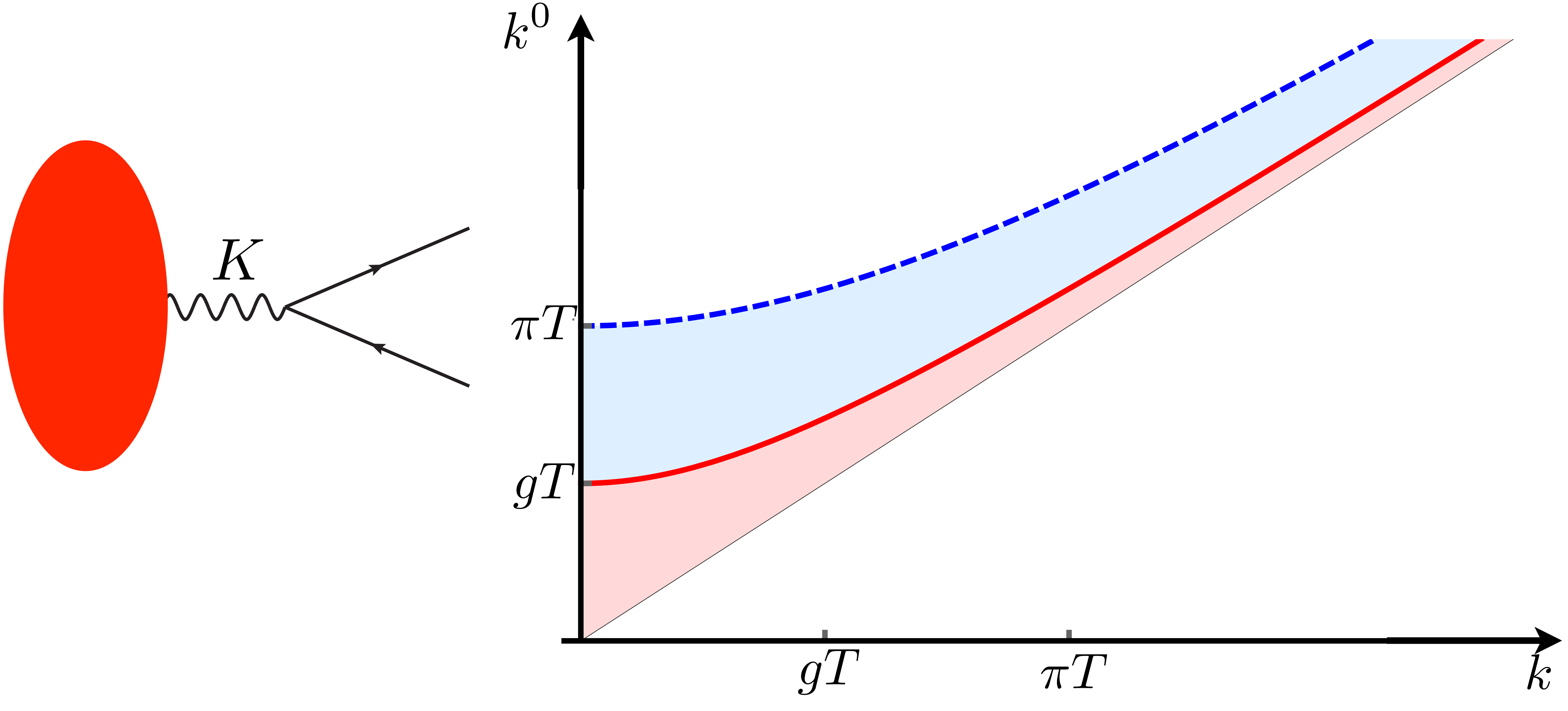
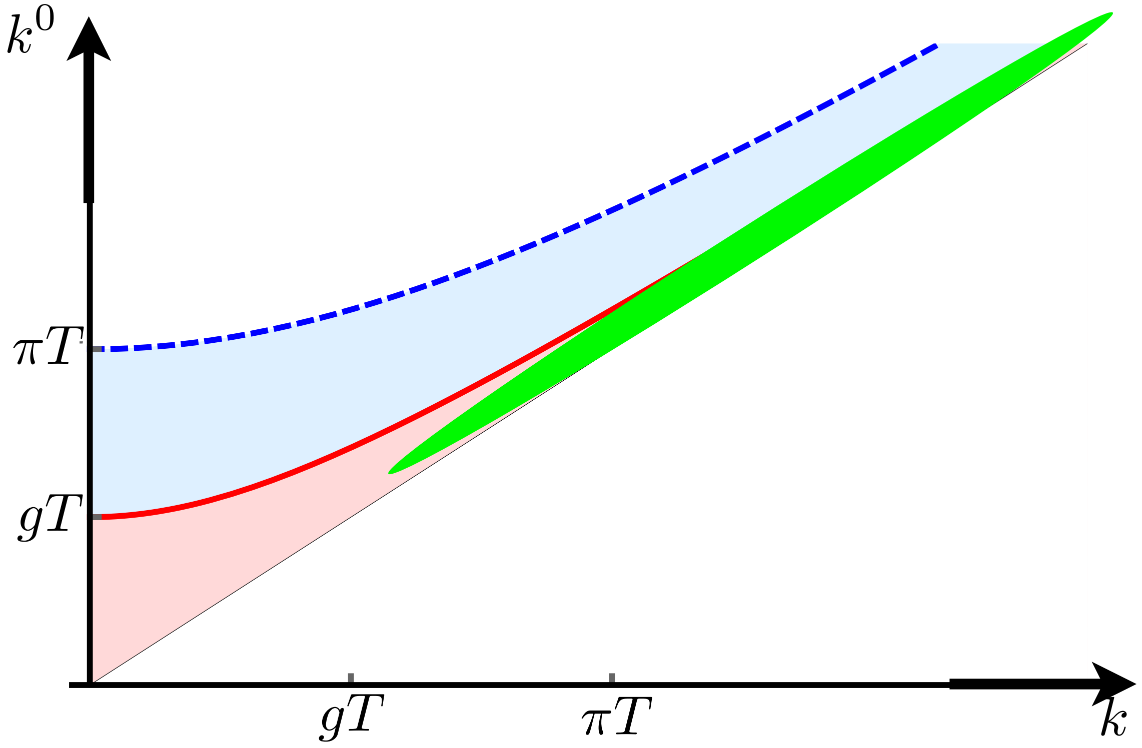
- Phenomenological motivation clear
- More theoretical motivation: lots of knowledge about perturbative thermodynamics to high orders, not so much about dynamical quantities. Is convergence better/worse?

Kinematics of e/m production

$$\frac{d\Gamma_{l^+l^-}(K)}{dk^0 d^3k} = -\frac{\alpha^2}{6\pi^3 K^2}\, n_{\rm B}(k^0)\, \rho_{\rm EM}(K)$$

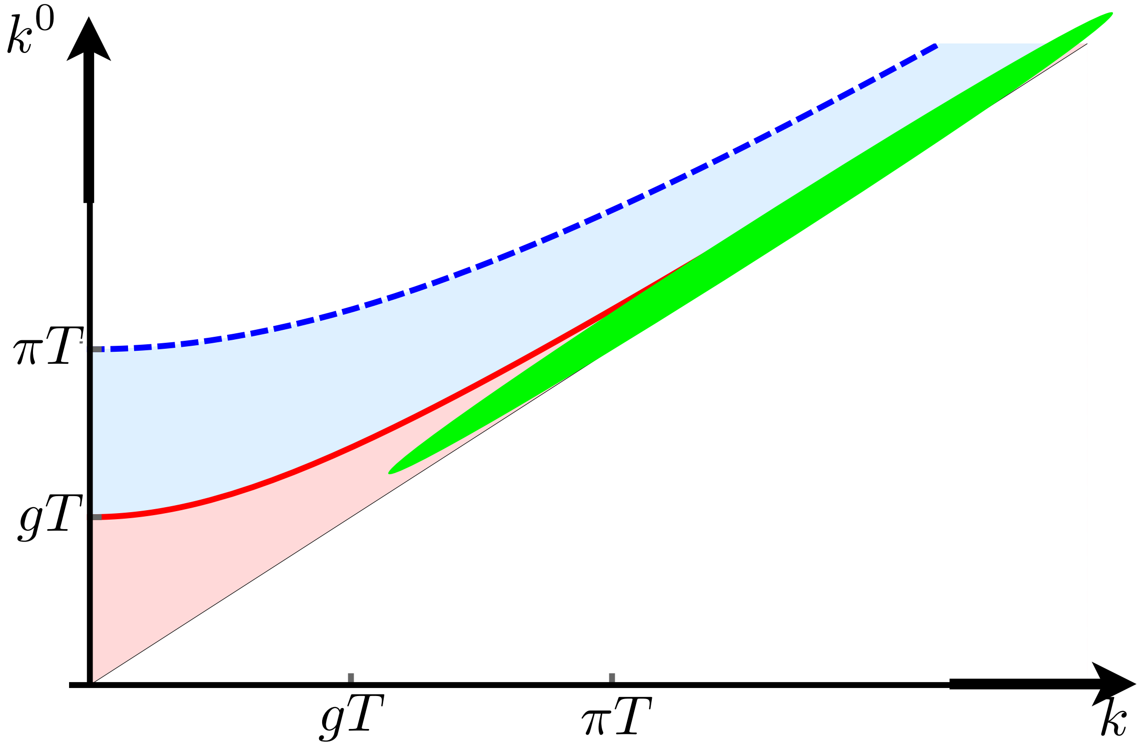


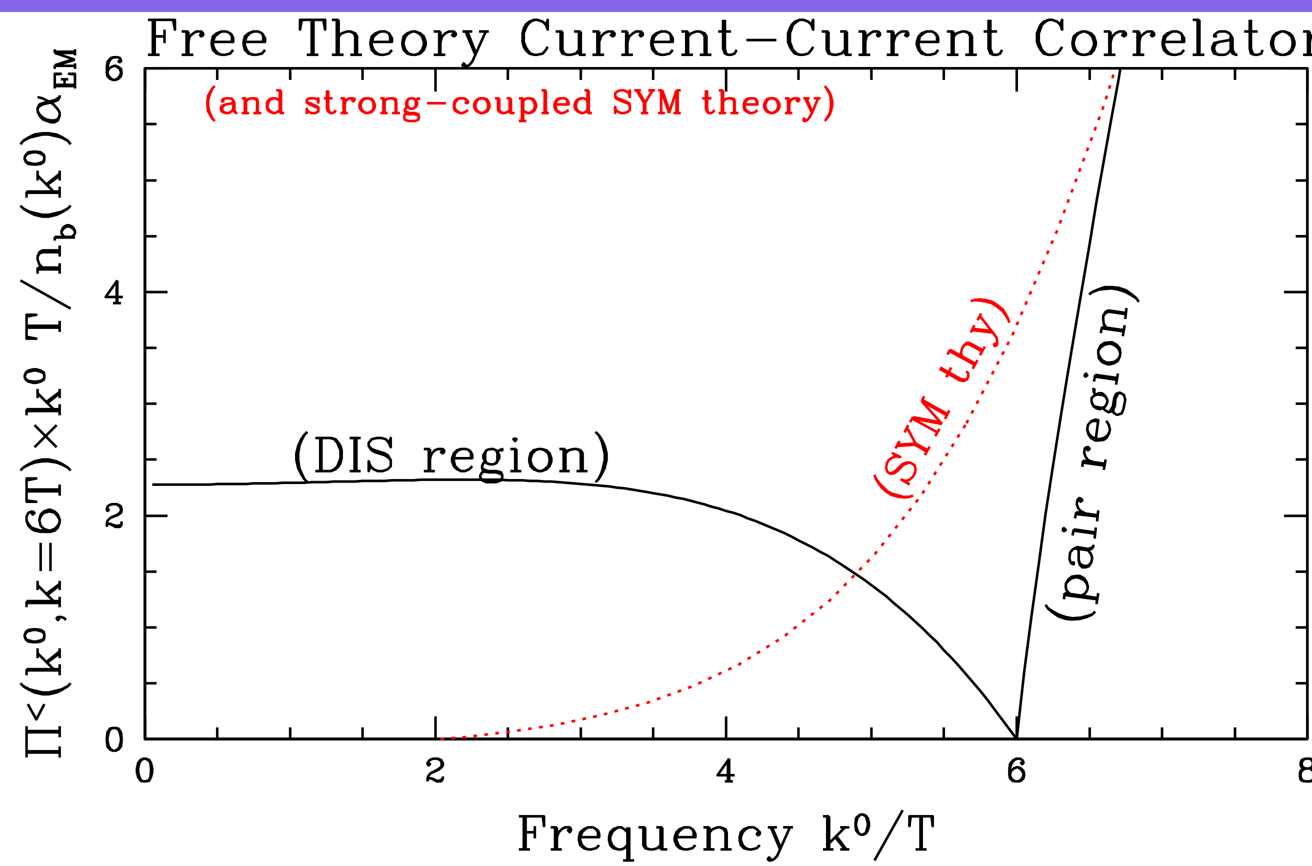
NLO at small K^2



- Consider $k^0 + k \sim T \;\; k^0 - k \sim g^2 T$

NLO at small K^2

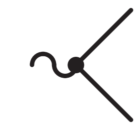




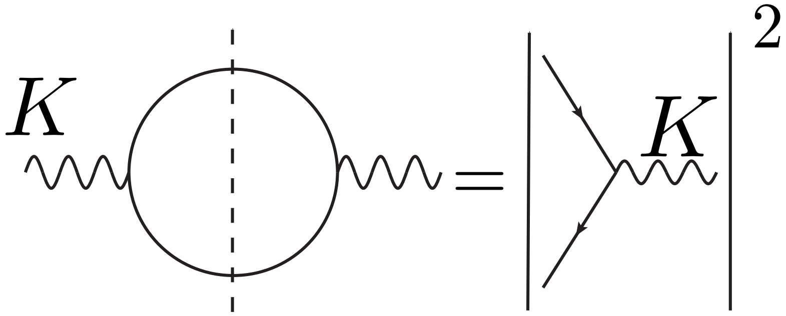
- Consider $k^0+k \sim T$, $k^0-k \sim g^2T$. Includes real photons
- A phenomenological motivation: low-mass dileptons as an ersatz real photon measurement (see for instance PHENIX). Is the spectral function smooth approaching the light cone?

Small K^2 dileptons

$$\frac{d\Gamma_{l^+l^-}(K)}{dk^0 d^3k} = -\frac{\alpha^2}{6\pi^3 K^2}\, n_B(k^0)\, \rho_{EM}(K) \qquad J^\mu = \sum_{q=uds} e_q \bar{q}\gamma^\mu q :$$

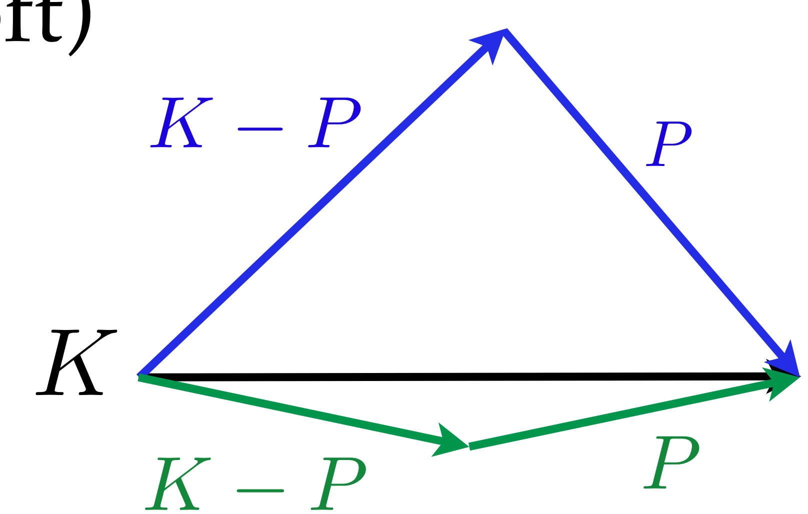


- At zeroth order ($\alpha_{EM}\, g^0$):



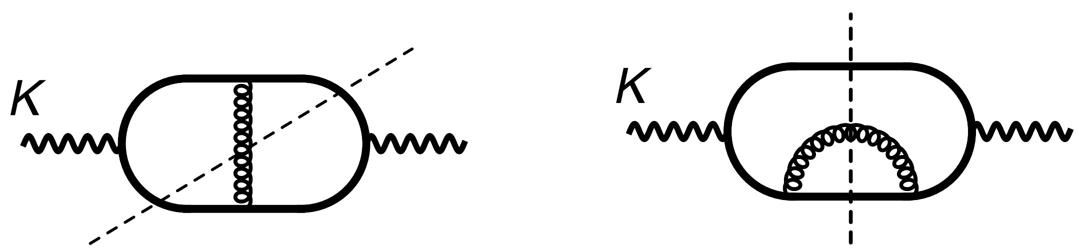
  Apparently LO, but very small phase space, proportional to $K^2 \sim g^2 T^2$. This is a collinear process.

- As in the real photon case, the calculation is split in the distinct 2↔2 processes (hard+soft) and collinear processes. Only collinear processes are modified wrt the photon case

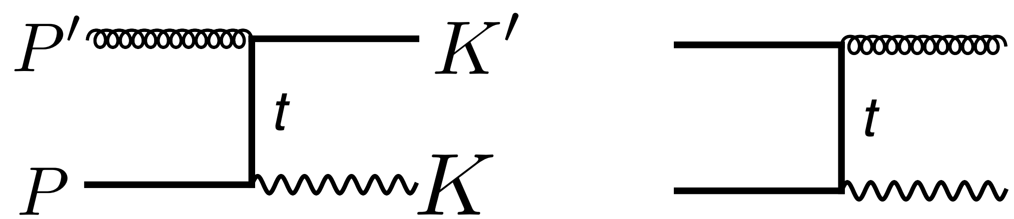


2↔2 processes

- Cut two-loop diagrams ($\alpha_{\rm EM}\, g^2$)



2↔2 processes (with crossings and interferences):



$$\int_{\rm ph.\,space} {\color{red} f(p) f(p')(1 \pm f(k'))} {\color{blue}|\mathcal{M}|^2} \delta^4(P+P'-K-K')$$

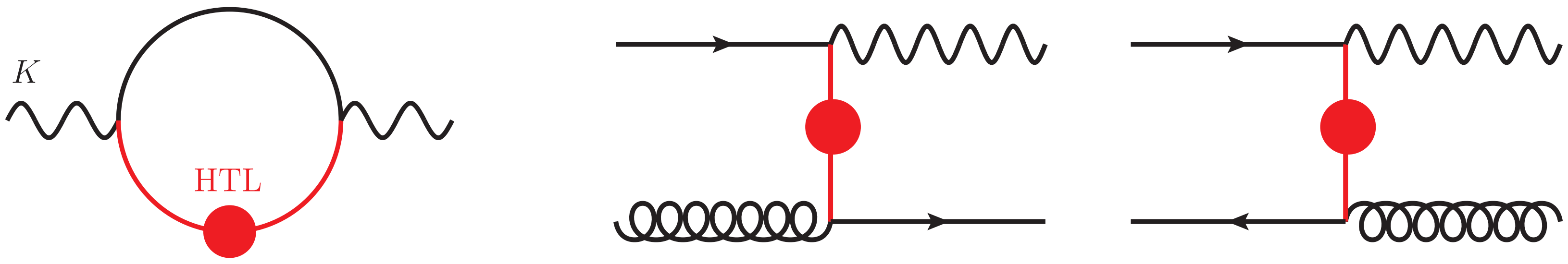
1↔3 processes suppressed by small K^2

- Equivalence with kinetic theory: distributions x matrix elements

- IR divergence (Compton) when t goes to zero

2↔2 processes

- The IR divergence disappears when **Hard Thermal Loop** resummation is performed Braaten Pisarski **NPB337** (1990)



2↔2 processes

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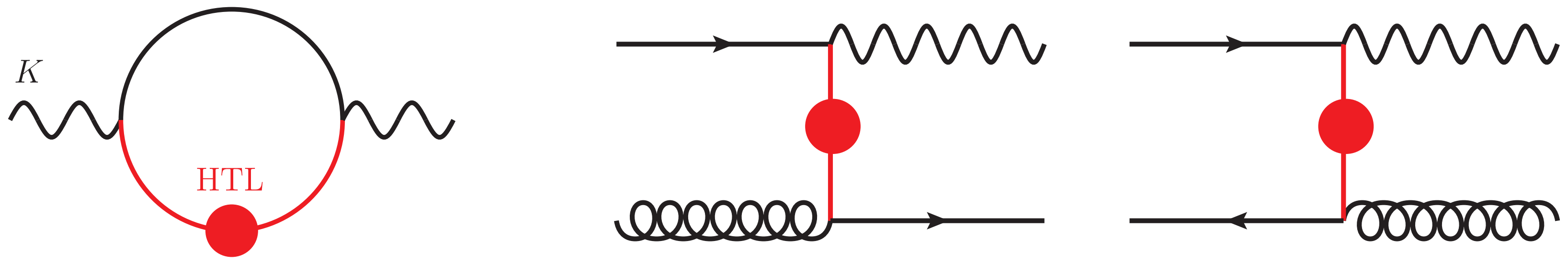
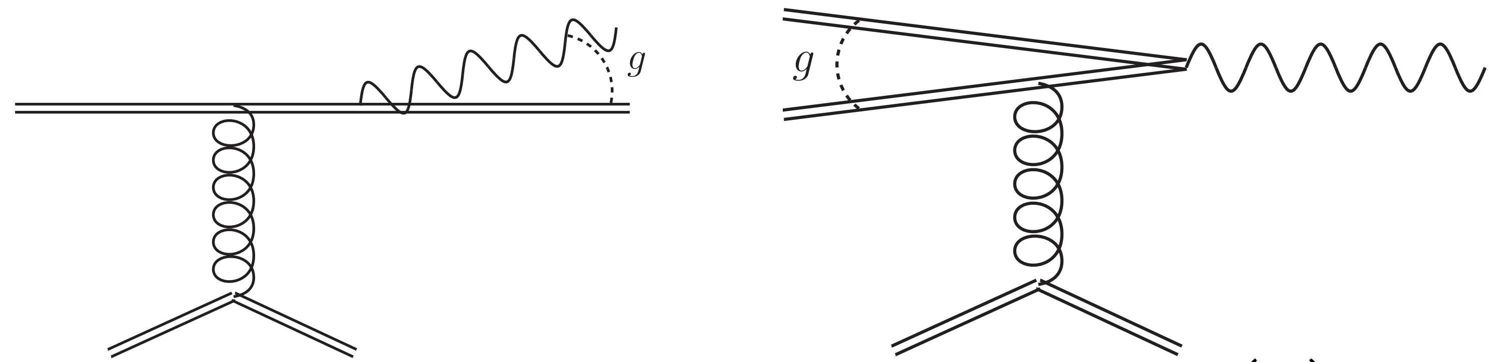


- In the end one obtains the result

$$\left.\frac{d\Gamma_{l^+l^-}(K)}{dk^0 d^3k}\right|_{2\leftrightarrow 2} \propto e^2 g^2 \left[\log\frac{T}{m_\infty} + C_{2\leftrightarrow 2}\left(\frac{k}{T}\right)\right]$$

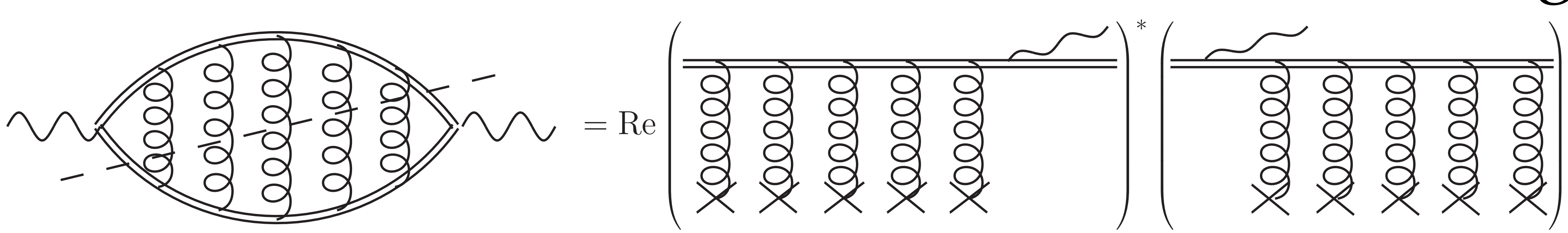
Kapusta Lichard Siebert **PRD44** (1991) Baier Nakkagawa Niegawa Redlich **ZPC53** (1992)

Collinear processes



- These diagrams contribute to LO if small (g) angle radiation/ annihilation Aurenche Gelis Kobes Petitgirard Zaraket 1998-2000
- Virtual photon formation times is then of the same order of the soft scattering rate $\Rightarrow$ interference: *LPM effect*
- Requires resummation of infinite number of ladder diagrams

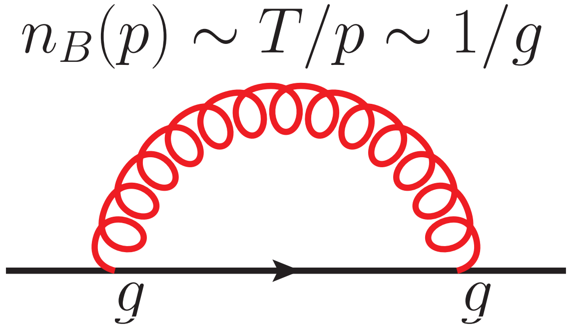
$$\left.\frac{d\Gamma_{l^+l^-}}{dk^0 d^3k}\right|_{\mathrm{coll}} =$$



AMY (2001-02), Aurenche Gelis Moore Zaraket (2002), Aurenche Carrington Gynther (2007)

Beyond leading order

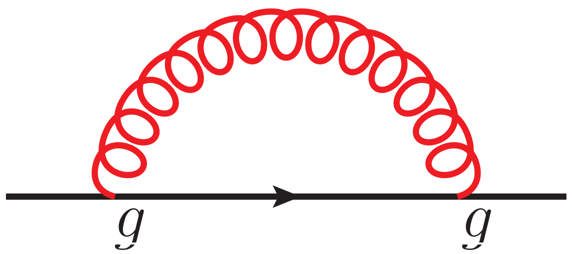
- The soft scale gT introduces $O(g)$ corrections



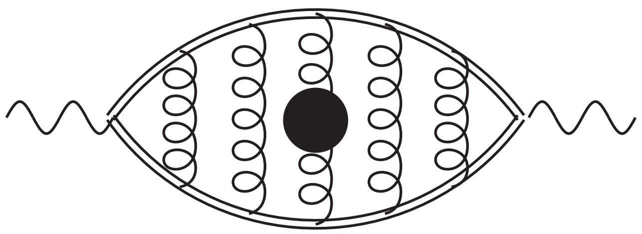
Beyond leading order

- The soft scale gT introduces $O(g)$ corrections

$n_B(p) \sim T/p \sim 1/g$



In the collinear sector: account for 1-loop rungs (related to NLO qhat). Euclidean (EQCD) evaluation



Caron-Huot **PRD79**

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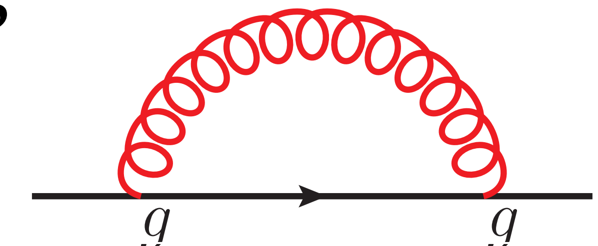

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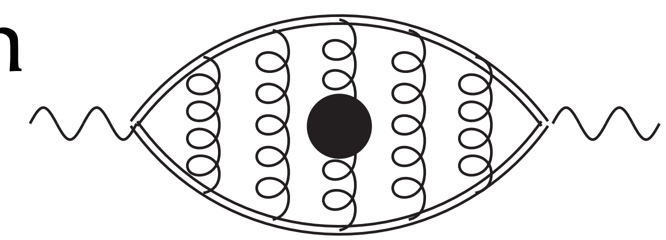
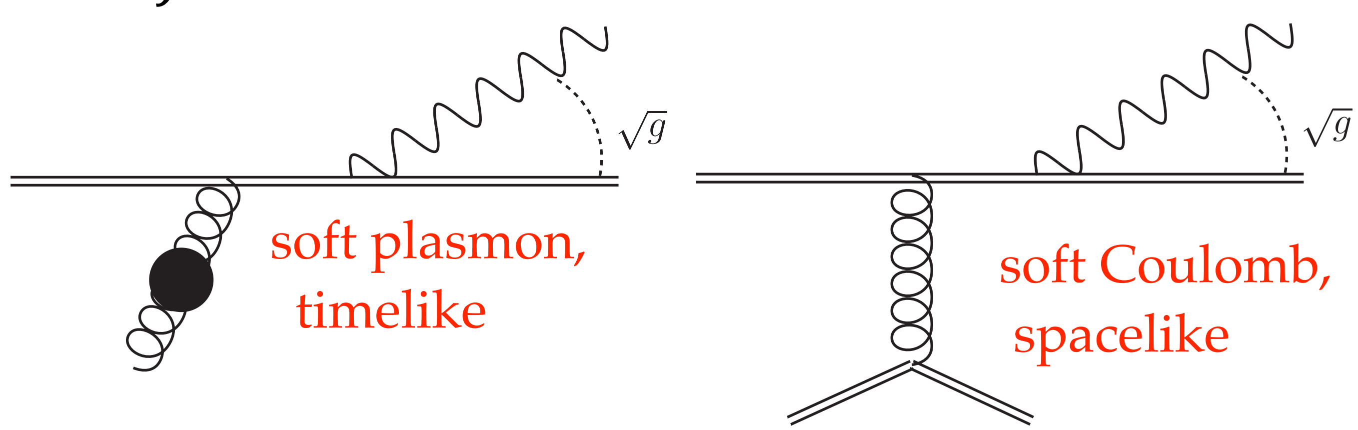
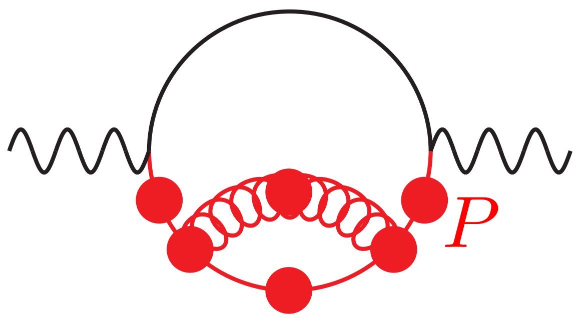
Caron-Huot **PRD79**

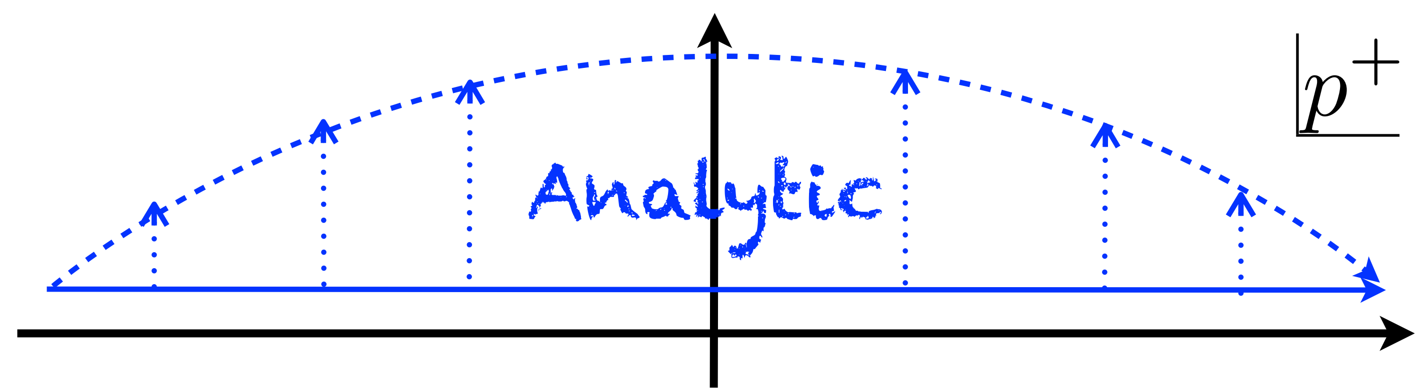


New semi-collinear processes: larger angle radiation, NLO in collinear radiation approx. Requires a *"modified qhat"*, relevance for jets too



Beyond leading order





- Add soft gluons to soft quarks: nasty all-HTL region

Analyticity allows us to take a detour in the complex plane away from the nasty region $\Rightarrow$ compact expression

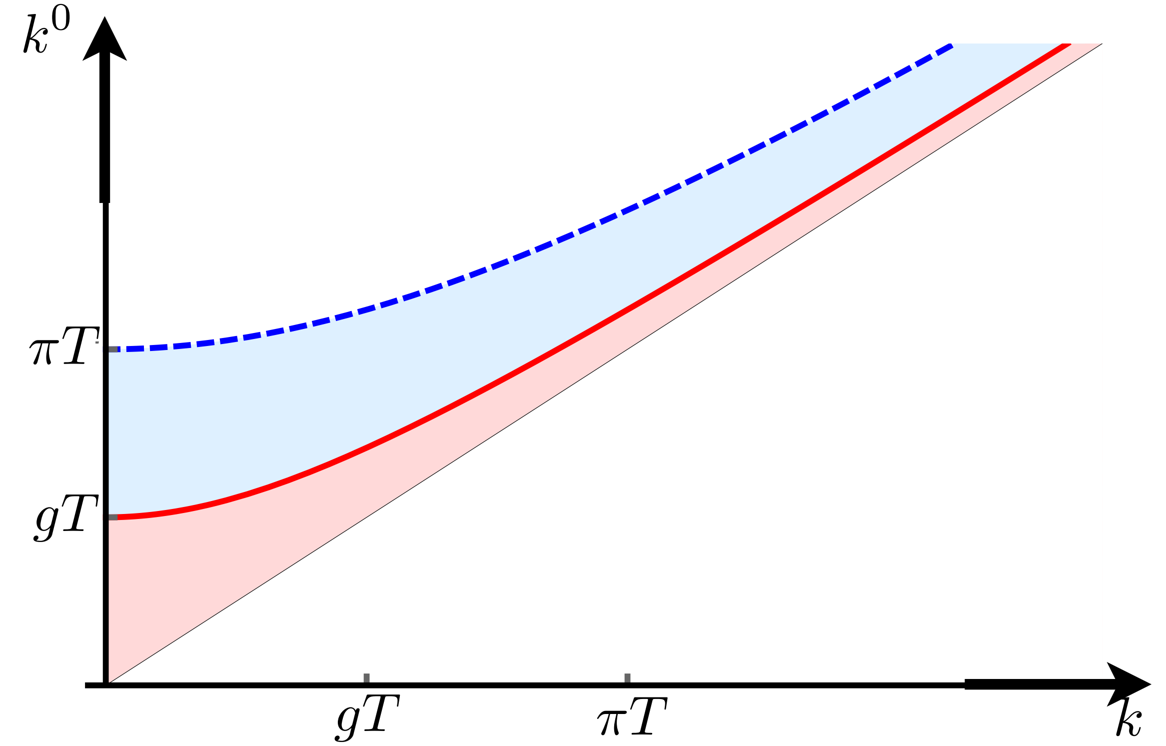
$$\int \frac{d^2p_\perp}{(2\pi)^2}\frac{m_\infty^2}{p_\perp^2+m_\infty^2} \xrightarrow{\text{NLO}} \int \frac{d^2p_\perp}{(2\pi)^2}\frac{m_\infty^2+\delta m_\infty^2}{p_\perp^2+m_\infty^2+\delta m_\infty^2}$$

JG Hong Kurkela Lu Moore Teaney (2013) for photons
JG Moore (2014) for dileptons

NLO at large K^2

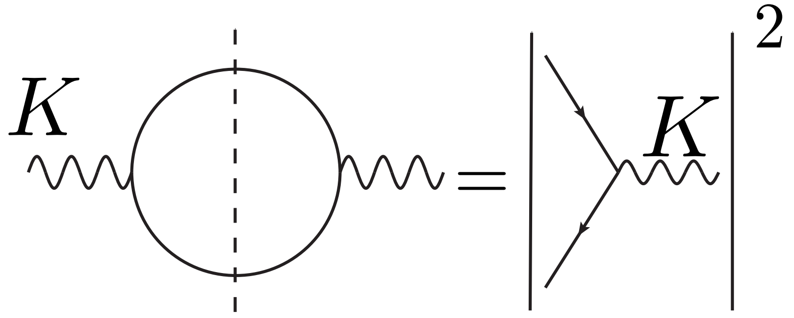
NLO at large K^2

- Before showing any results, let us look at the large-M region



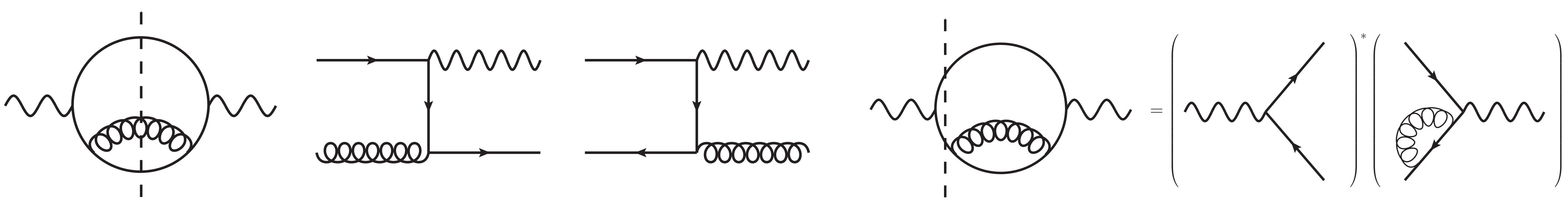
$$k^0 + k \sim T \qquad k^0 - k \sim T$$

- As we have seen, the Born term is proportional to K^2, which is now large ($\sim T^2$), so that the Born term is a well defined LO ($\alpha_{\rm EM} g^0$)



NLO at large K^2

- At NLO, HTL and LPM resummations are no longer necessary
- Very complicated two-loop integrals with intricate kinematics. Interplay of real and virtual corrections with cancellations of IR divergences



Laine **JHEP1305, JHEP1311** (2013)

Matching small and large K^2

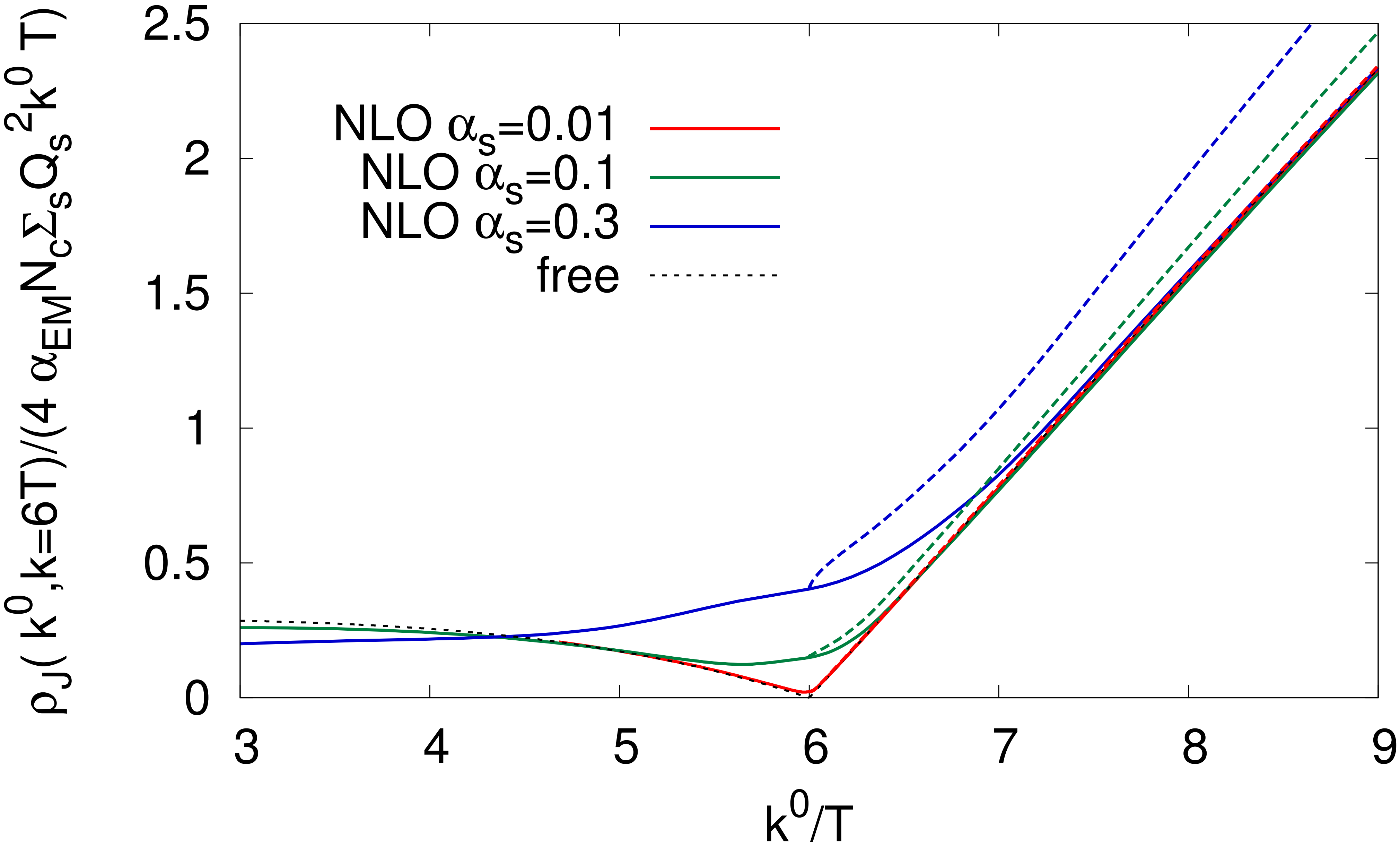
- The large-M calculation diverges logarithmically for $M \to 0$
- The small-M calculation extrapolates for large M to $\rho \propto K^2 + T^2$, in violation of OPE results forbidding a T^2 term Caron-Huot **PRD79** (2009)

A procedure has been devised to combine the two calculations. In a nutshell,

$$\rho_{\rm merge} = \rho_{{\rm large}\,M} + \rho_{\rm LPM} - \rho_{{\rm LPM}\,K^2 \gg T^2}$$

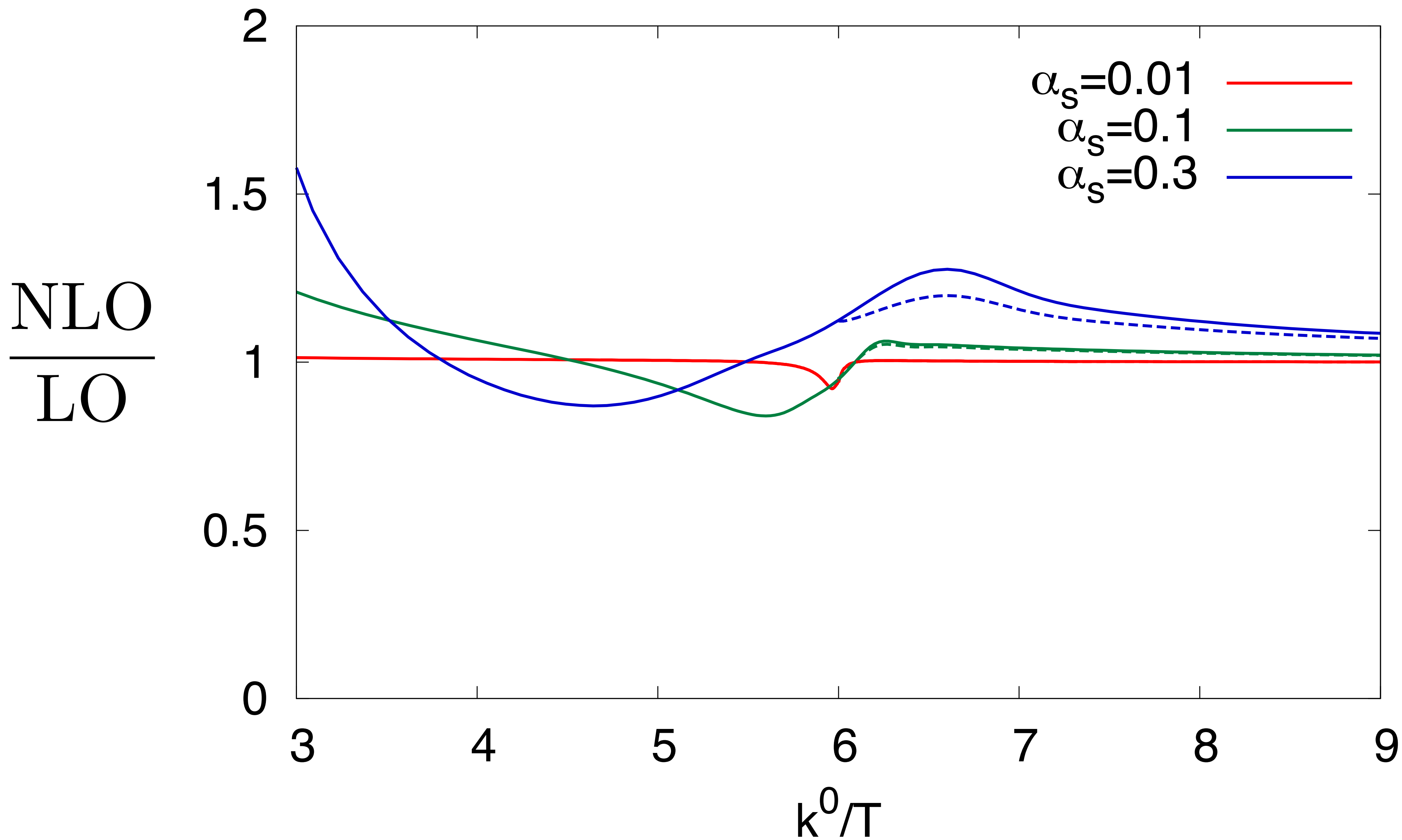
where $\rho_{\rm LPM}$ is the LO collinear part. NLO can be added easily.

Ghisoiu Laine **JHEP1410** (2014), JG Moore **JHEP1412** (2014)



- Full lines: JG Moore, valid at small K^2, does not include Laine (large M)
  Dashed lines: Ghisoiu Laine, valid at large K^2

- At α_s=0.3 the transition at the light cone is smooth
  Ghisoiu Laine **JHEP1410** (2014), JG Moore **JHEP1412** (2014)



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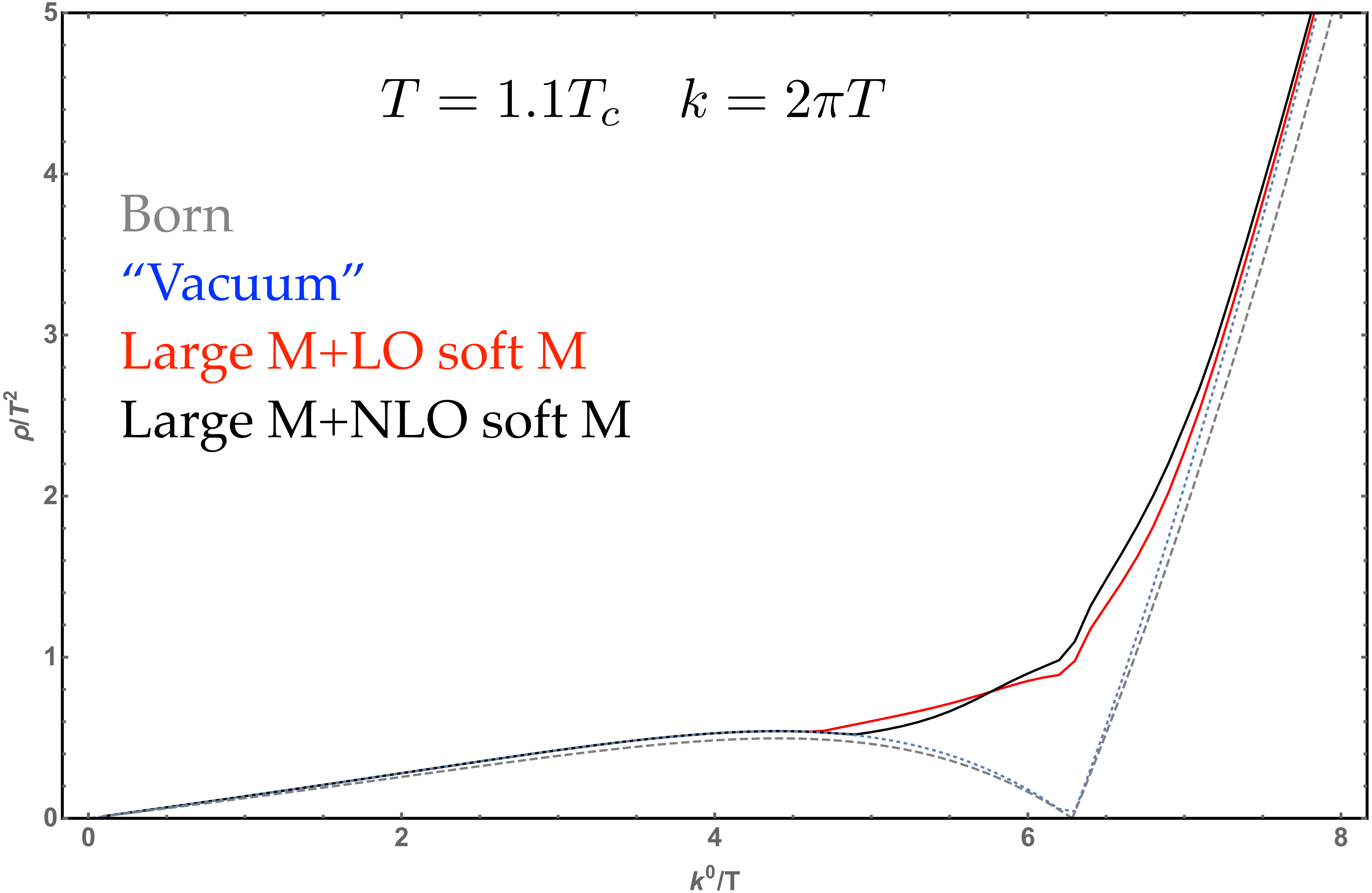
  Ghisoiu Laine **JHEP1410** (2014), JG Moore **JHEP1412** (2014)

Come visit our website

- http://www.laine.itp.unibe.ch/dilepton-lpm/ Ghisoiu Laine and JG Moore results on a k^0,k mesh, ready for pheno. Used by the McGill group and by Burnier Gastaldi **PRC93** (2015)
- http://www.laine.itp.unibe.ch/dilepton-lattice/ best available pQCD data for the spectral function
  - at finite k: Ghisoiu Laine plus JG Moore plus vacuum corrections to the Born term
  - at zero k: transport peak from Moore Robert (2006), $k^0 > \pi T$, NLO thermal from Aurenche Altherr (1989), vacuum corrections to the Born term. Missing reliable pQCD input in the intermediate region
- JG Laine

Come visit our website



A lattice comparison

EM probes and the lattice

EM probes and the lattice

- What is measured directly is the Euclidean correlator

$$G_E(\tau, k) = \int d^3x J_\mu(\tau, \mathbf{x}) J_\mu(0,0) e^{i\mathbf{k}\cdot\mathbf{x}}$$

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- Analytical continuation $G_E(\tau, k) = G^<(i\tau, k)$

$$G_E(\tau, k) = \int_0^\infty \frac{dk^0}{2\pi} \rho_V(k^0, k) \frac{\cosh\left(k^0(\tau - 1/2T)\right)}{\sinh\left(\frac{k^0}{2T}\right)}$$

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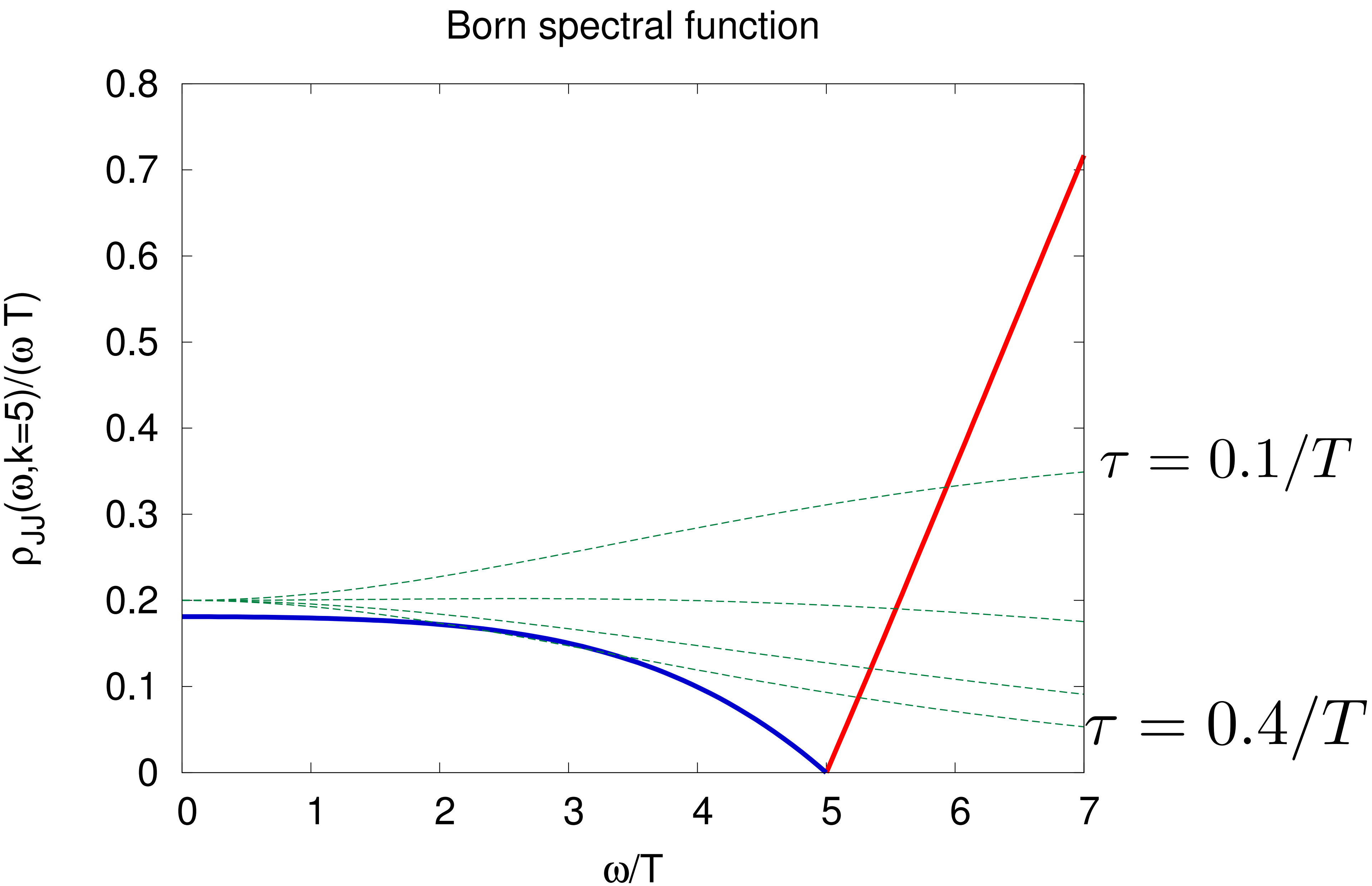
- It contains a lot of info (full spectral function), but hidden in the convolution. Inversion tricky, discrete dataset with errors

At finite momentum

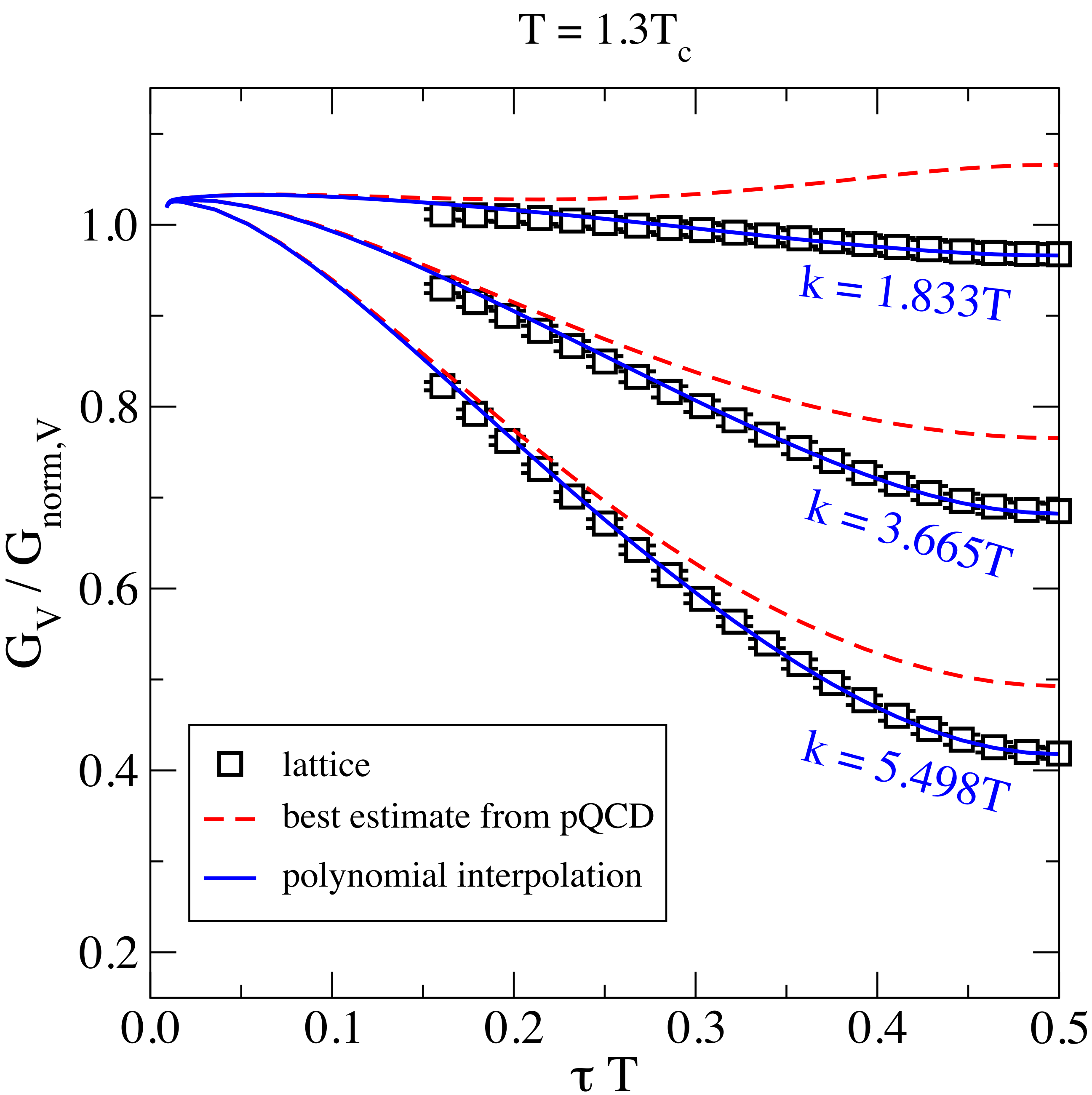
- If k>0 spf describes DIS ($k^0<k$), photons ($k^0=k$) and dileptons ($k^0>k$).



Fitting to the lattice

- Getting the Euclidean correlator from the pQCD results is straightforward. It overshoots the lattice data (hold on). Too much spf in the ill-constrained spacelike region?
- Try a fitting Ansatz: perturbative, thermal spf above $M \sim \pi T$. Fifth-degree polynomial in k^0, with odd powers only, below $M \sim \pi T$ (*three coefficients*).
- Constrain two coefficients by requiring smoothness in spf and first derivative at the matching point. Fit the remaining coefficient to lattice data. Higher order odd polynomials also examined
  JG Kaczmarek Laine Meyer **PRD94** (2016)

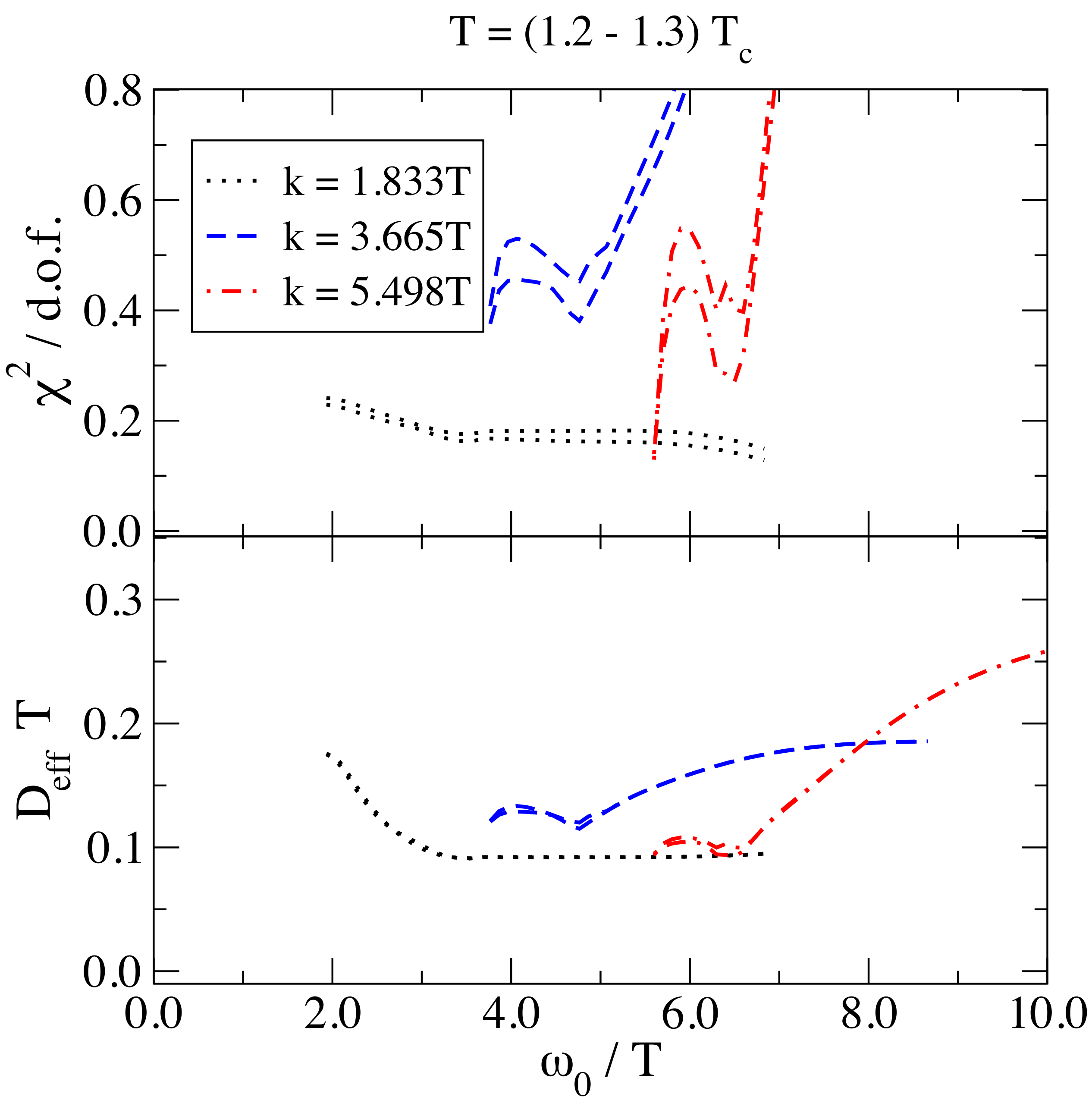
Fitting to the lattice



- Continuum-extrapolated quenched lattice data
- Results qualitatively similar at T=1.1T_c
- Lattice continuum extrapolation reliable only from τT>0.22
- Matching point at k^0=k+1.5T

JG Kaczmarek Laine Meyer **PRD94** (2016)

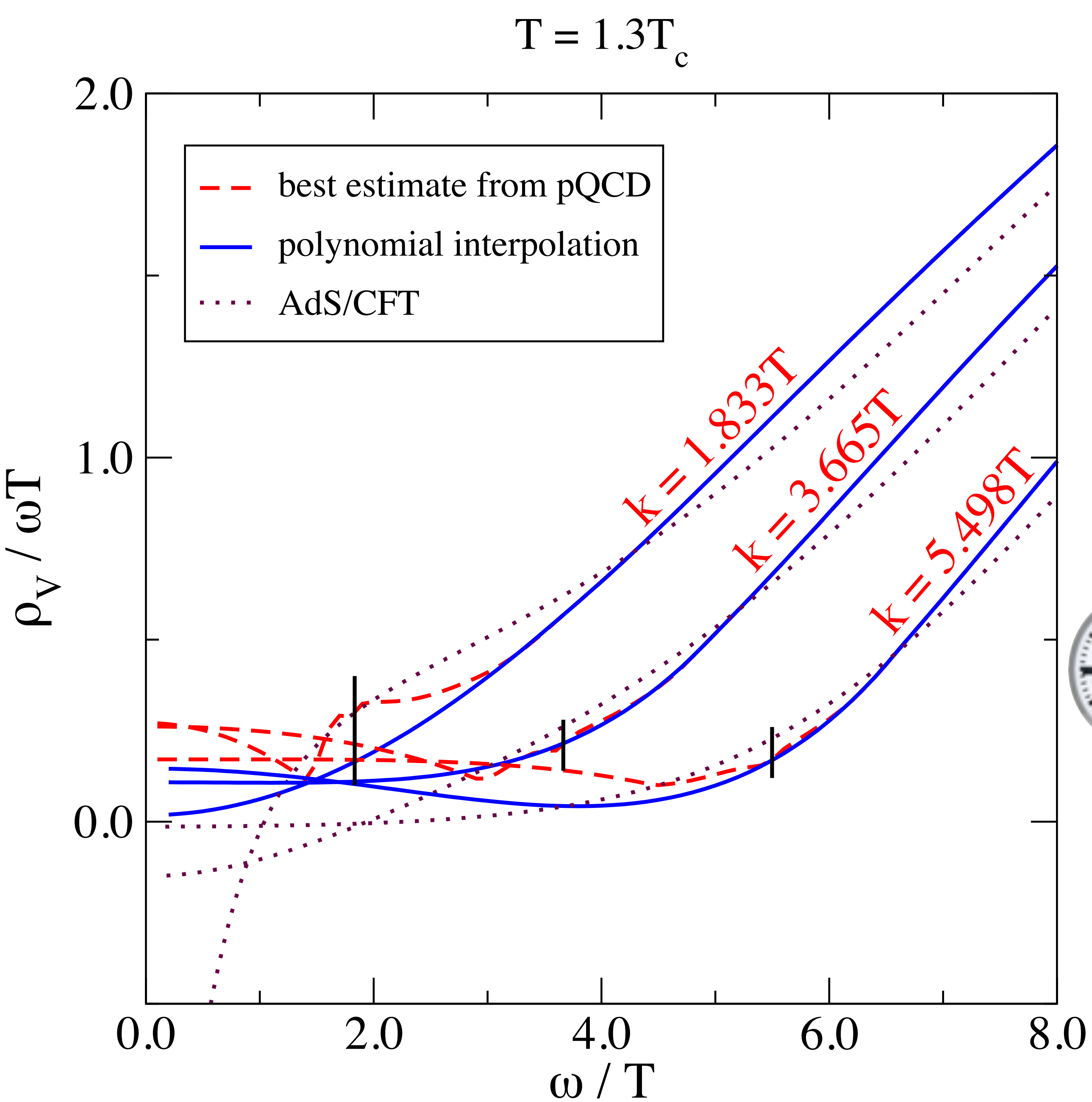
Fitting to the lattice



- Results qualitatively similar at T=1.1T_c
- The fit has a good χ^2, which also has a local minimum for $M \sim \pi T$ and the spf at the photon point is stable against varying the matching point
- D_{eff} proportional to spf at photon point (hold on), quite stable too

JG Kaczmarek Laine Meyer **PRD94** (2016)

Fitting to the lattice

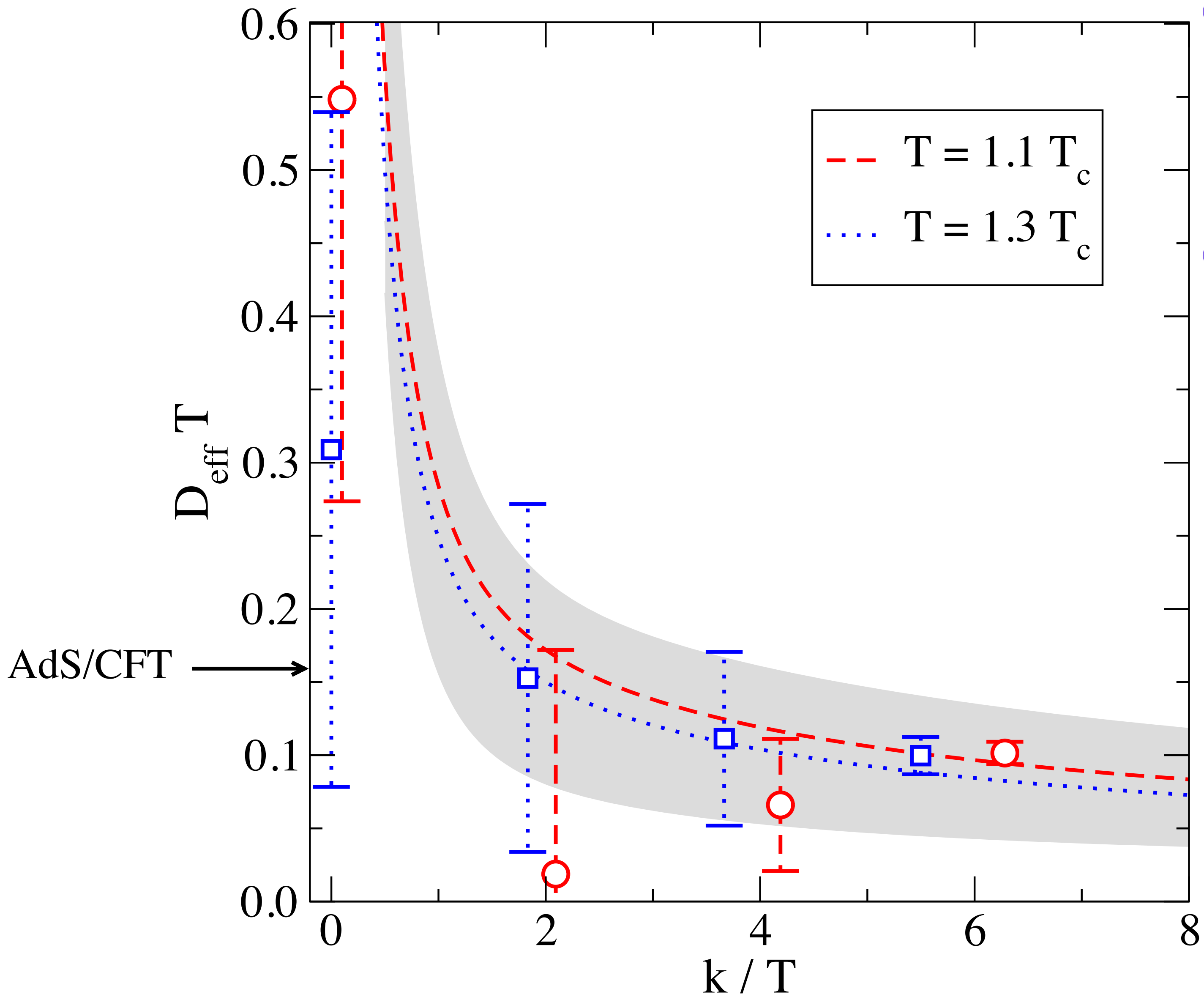


- At the photon point modest changes from pQCD expectations (below 20% except perhaps at the smallest *k*s, also at 1.1 T_c). Good for pheno!

AdS/CFT curve adjusted to asymptote to the bare QCD result (extra symmetries make T=0 curve coupling-independent)

JG Kaczmarek Laine Meyer **PRD94** (2016). AdS/CFT: Caron-Huot Kovtun Moore Starinets Yaffe **JHEP0612** (2006)

- Define $D_{\rm eff}(k) \equiv \begin{cases} \dfrac{\rho_{\rm V}(k,\mathbf{k})}{2\chi_{\rm q} k} & , \quad k>0 \\ \lim\limits_{\omega\to 0^+} \dfrac{\rho^{ii}(\omega,\mathbf{0})}{3\chi_{\rm q}\omega} & , \quad k=0 \end{cases}$



- In the hydro limit $k \ll T$ $D_{\rm eff} \to D$

$$\sigma = e^2 \sum_{f=1}^{N_{\rm f}} Q_f^2 \chi_{\rm q} D$$

- Lattice errors from bootstrap samples
- At large momentum excellent agreement with NLO pQCD from before. At finite $k>0$ this method could be a more controlled approach to the extraction of σ, w/o the large uncertainties associated with the transport peak at k=0.
- Try this for shear?

JG Kaczmarek Laine Meyer **PRD94** (2016). NLO pQCD: JG Hong Kurkela Lu Moore Teaney **JHEP05** (2013)

Conclusions

- NLO calculations for dileptons are now available over a wide range of invariant masses (at finite k)
- In both cases convergence seems reasonable. At small K^2 transition to photon is smooth
- A collection of the best available data has been prepared and is ready for use by pheno/lattice practitioners
- Comparison with lattice with a simple, motivated Ansatz gives a good fit and seems to further suggest stability (at the tens of % level) of the pQCD rates

Backup

LPM resummation

- Quark statistical functions × DGLAP splitting × transverse evolution

$$\frac{d\Gamma}{d^3k} = \frac{\alpha}{\pi^2 k} \int \frac{dp^+}{2\pi} n_{\mathrm{F}}(k+p^+)[1-n_{\mathrm{F}}(p^+)] \frac{(p^+)^2+(p^++k)^2}{2(p^+(p^++k))^2} \lim_{\mathbf{x}_\perp \to 0} 2\mathrm{Re}\, \boldsymbol{\nabla}_{\mathbf{x}_\perp} \mathbf{f}(x_\perp)$$

$$\begin{array}{ccccc} x^+ & \gg & x_\perp & \gg & x^- \\ 1/g^2T & \gg & 1/gT & \gg & 1/T \end{array}$$

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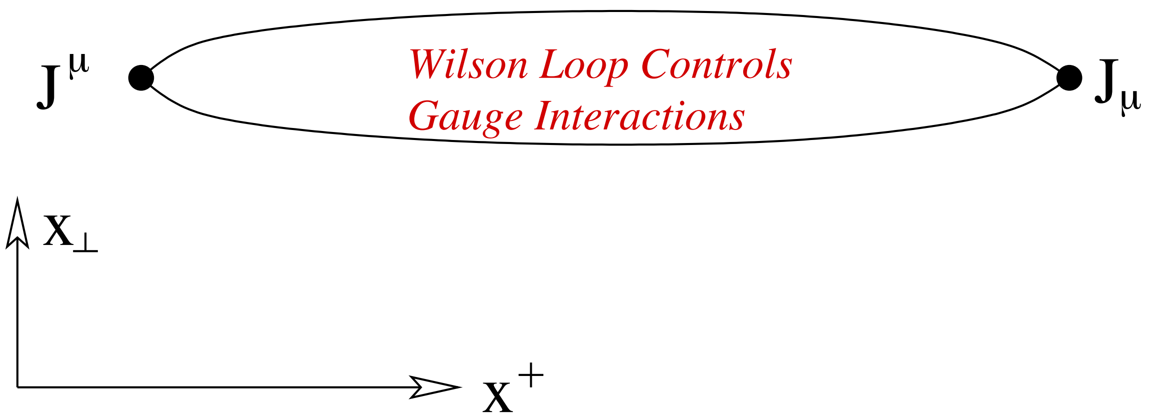
- Transverse diffusion and Wilson-loop correlators evolve the transverse density **f** *along the spacetime light-cone*

$$-2i\nabla\delta^2(\mathbf{x}_\perp) = \left[\frac{ik}{2p^+(k+p^+)}\left(m_\infty^2 - \nabla^2_{\mathbf{x}_\perp}\right) + \mathcal{C}(x_\perp)\right]\mathbf{f}(\mathbf{x}_\perp)$$

$$\begin{array}{ccccc} x^+ & \gg & x_\perp & \gg & x^- \\ 1/g^2T & \gg & 1/gT & \gg & 1/T \end{array}$$

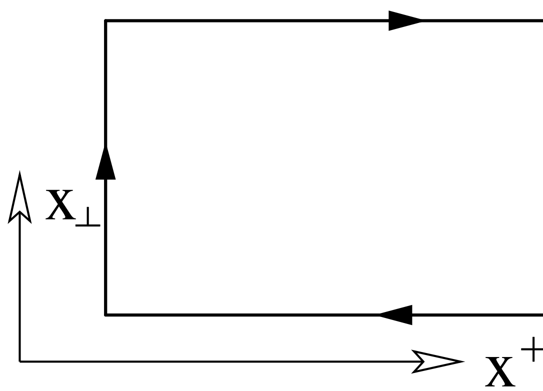


Zakharov 1996-98 AMY 2001-02

LPM resummation: two inputs

- Asymptotic mass $m_\infty^2 = 2g^2 C_R \left(\int \frac{d^3p}{(2\pi)^3} \frac{n_B(p)}{p} + \int \frac{d^3p}{(2\pi)^3} \frac{n_F(p)}{p} \right)$
- Light-cone Wilson loop, related to $\hat{q}$

$$\hat{q} \equiv \int_0^{q_{\max}} \frac{d^2 q_\perp}{(2\pi)^2} q_\perp^2 C(q_\perp)$$



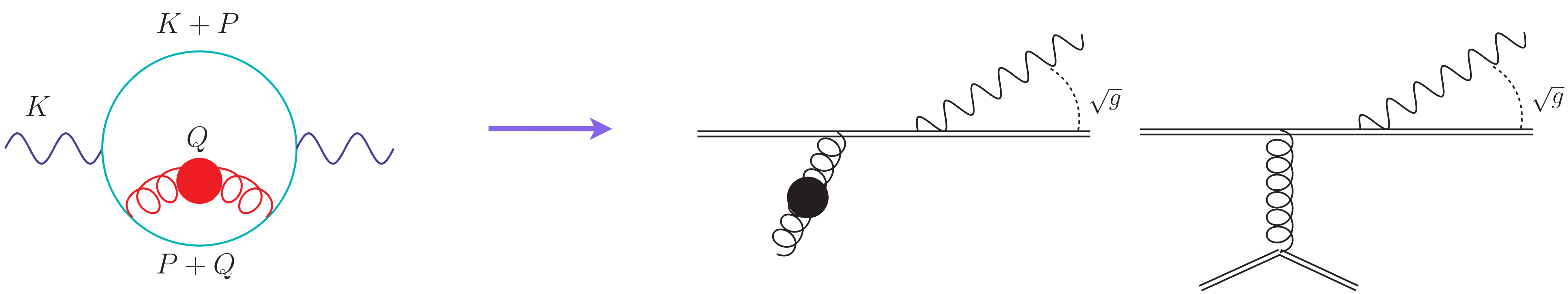
$\propto e^{\mathcal{C}(x_\perp)L}$

BDMPS-Z, Wiedemann, Casalderrey-Solana Salgado, D'Eramo Liu Rajagopal, Benzke Brambilla Escobedo Vairo

- Soft contribution becomes Euclidean! Caron-Huot **PRD79** (2008), can be "easily" computed in perturbation theory Possible lattice measurements Laine Rothkopf **JHEP1307** (2013) Panero Rummukainen Schäfer **1307.5850** talk by Panero

The semi-collinear region

- Seemingly different processes boiling down to wider-angle radiation



P semi-collinear
Q soft

Q soft plasmon, timelike

Q soft cut, spacelike

- Evaluation: introduce "*modified* $\hat{q}$" that keep tracks of the changes in the small light-cone component p^- of the quarks

"*standard*" $$\frac{\hat{q}}{g^2C_R} \equiv \frac{1}{g^2C_R}\int\frac{d^2q_\perp}{(2\pi)^2}q_\perp^2\mathcal{C}(q_\perp) \propto \int d^4Q\langle F^{+\mu}(Q)F^{+}{}_{\mu}(-Q)\rangle_{q^-=0}$$

"*modified*" $$\frac{\hat{q}(\delta E)}{g^2C_R} \propto \int d^4Q\langle F^{+\mu}(Q)F^{+}{}_{\mu}(-Q)\rangle_{q^-=\delta E}$$

- The "*modified* $\hat{q}$" can also be evaluated in EQCD

Euclideanization of light-cone soft physics

For v=x_z/t=∞ correlators (such as propagators) are the equal time Euclidean correlators.

$$G^{>}(t=0,\mathbf{x}) = \sum\!\!\!\!\!\!\!\int_p G_E(\omega_n, p)e^{i\mathbf{p}\cdot\mathbf{x}}$$

- Causality: retarded functions analytic for positive imaginary parts of all *timelike* and *lightlike* variables: the above result can be extended to the lightcone

$$G^{>}(t=x_z,\mathbf{x}_\perp) = \sum\!\!\!\!\!\!\!\int_p G_E(\omega_n, p_\perp, p_z + i\omega_n)e^{i(\mathbf{p}_\perp\cdot\mathbf{x}_\perp + p_z x_z)}$$

- The sums are dominated by the zero mode for soft physics=>EQCD!
- Equivalent to sum rules   Caron-Huot **PRD79** (2009)

Summary

- LO rate

$$(2\pi)^3 \frac{d\Gamma}{d^3k}\bigg|_{\rm LO} = \mathcal{A}(k)\overbrace{\left[\log\frac{T}{m_\infty} + C_{2\to 2}(k) + C_{\rm coll}(k)\right]}^{C_{\rm LO}(k)}$$

$$\mathcal{A}(k) = \alpha_{\rm EM}\, g^2\; C_F T^2 \frac{n_{\rm F}(k)}{2k}\sum_f Q_f^2 d_f$$

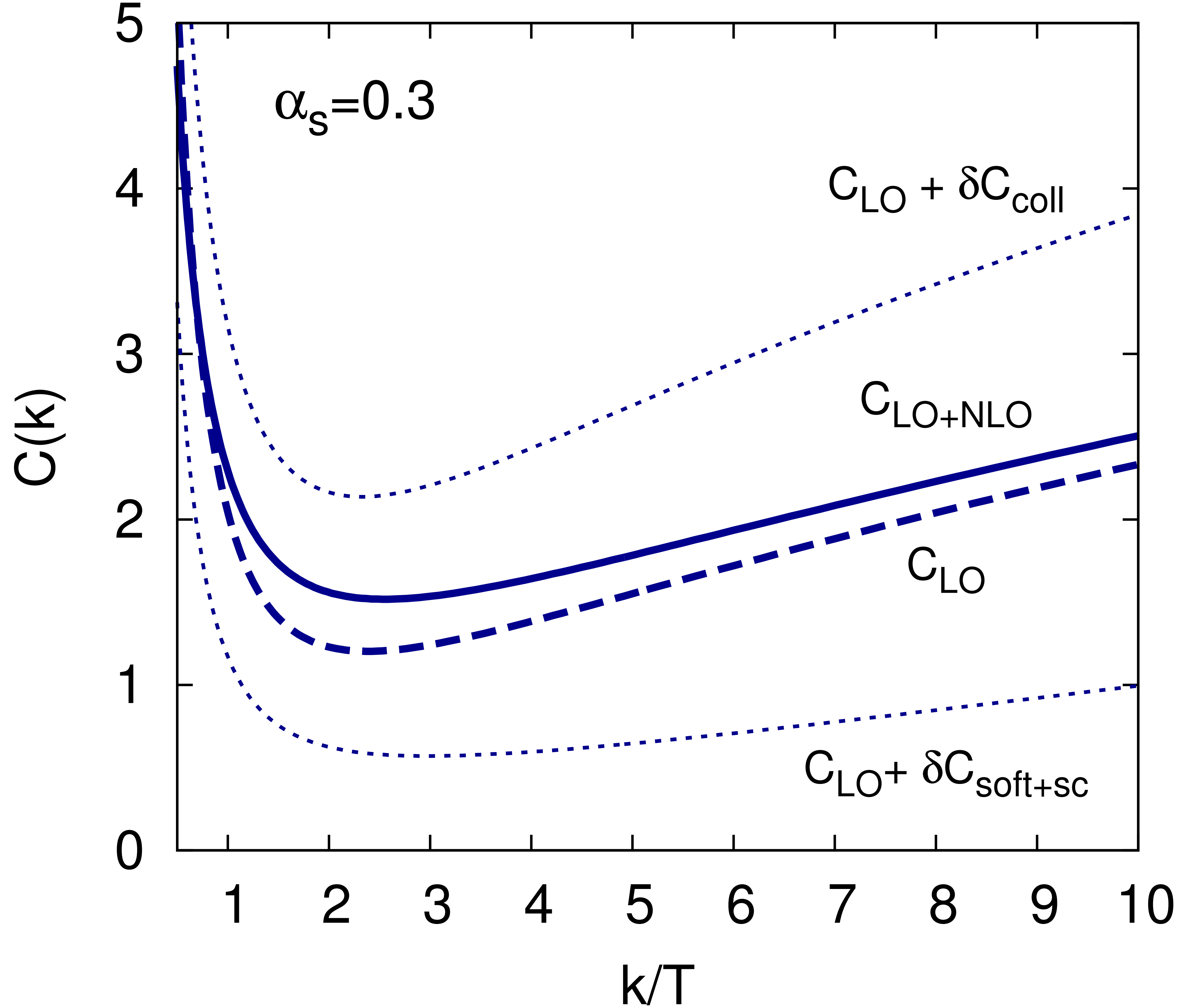
- NLO correction

$$(2\pi)^3 \frac{d\delta\Gamma}{d^3k}\bigg|_{\rm NLO} = \mathcal{A}(k)\overbrace{\left[\underbrace{\frac{\delta m_\infty^2}{m_\infty^2}\log\frac{\sqrt{2Tm_D}}{m_\infty} + \frac{\delta m_\infty^2}{m_\infty^2} C_{\rm soft+sc}(k)}_{\delta C_{\rm soft+sc}(k)} + \underbrace{\frac{\delta m_\infty^2}{m_\infty^2} C^{\delta m}_{\rm coll}(k) + \frac{g^2 C_A T}{m_D} C^{\delta \mathcal{C}}_{\rm coll}(k)}_{\delta C_{\rm coll}(k)}\right]}^{\delta C_{\rm NLO}(k)}$$

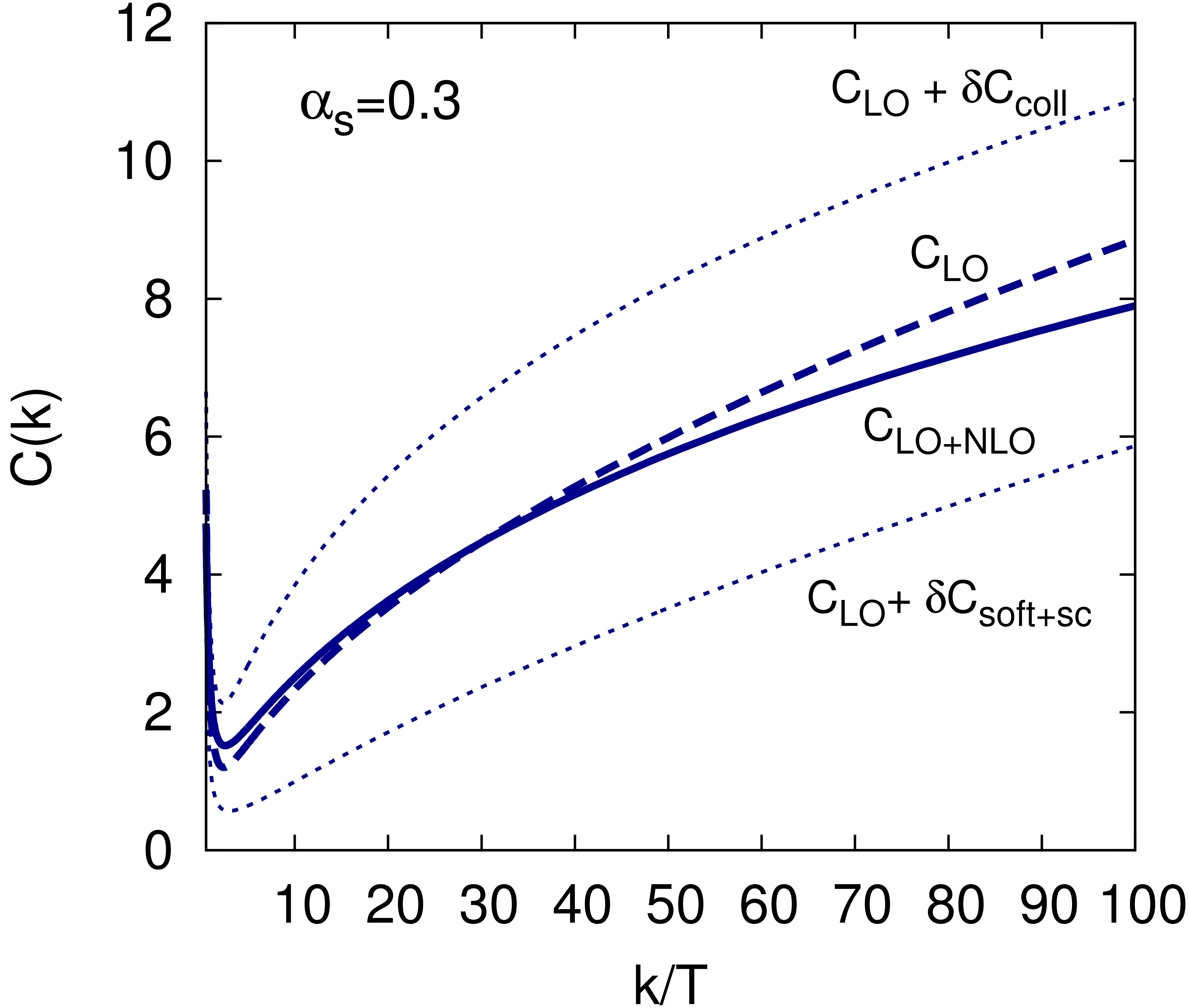
- Fits available in the paper

JG Hong Kurkela Lu Moore Teaney **JHEP0513 (2013)**

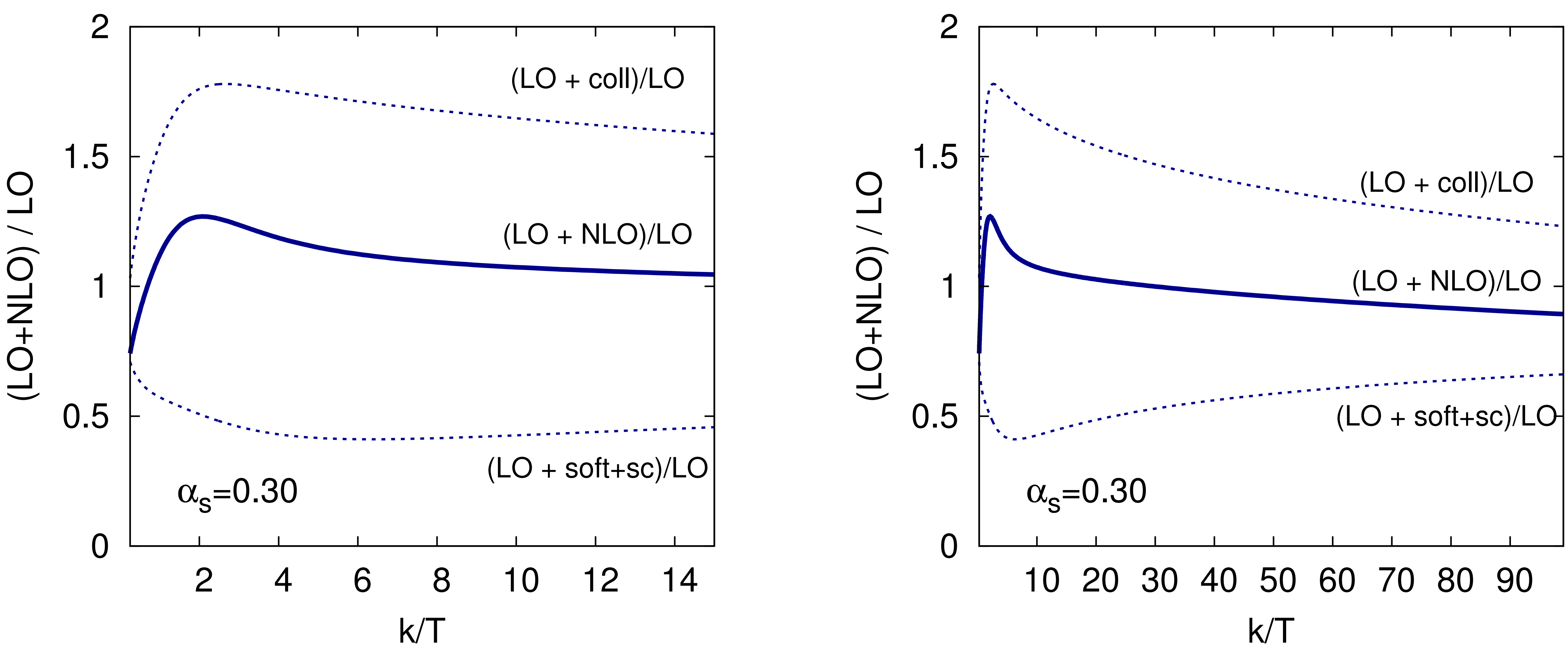
$$
(2\pi)^3 \frac{d\delta\Gamma}{d^3k}\bigg|_{\mathrm{NLO}} = \mathcal{A}(k) \overbrace{\left[\underbrace{\frac{\delta m_\infty^2}{m_\infty^2}\log\frac{\sqrt{2Tm_D}}{m_\infty} + \frac{\delta m_\infty^2}{m_\infty^2} C_{\mathrm{soft+sc}}(k)}_{\delta C_{\mathrm{soft+sc}}(k)} + \underbrace{\frac{\delta m^2_\infty}{m^2_\infty} C^{\delta m}_{\mathrm{coll}}(k) + \frac{g^2 C_A T}{m_D} C^{\delta \mathcal{C}}_{\mathrm{coll}}(k)}_{\delta C_{\mathrm{coll}}(k)} \right]}^{\delta C_{\mathrm{NLO}}(k)}
$$

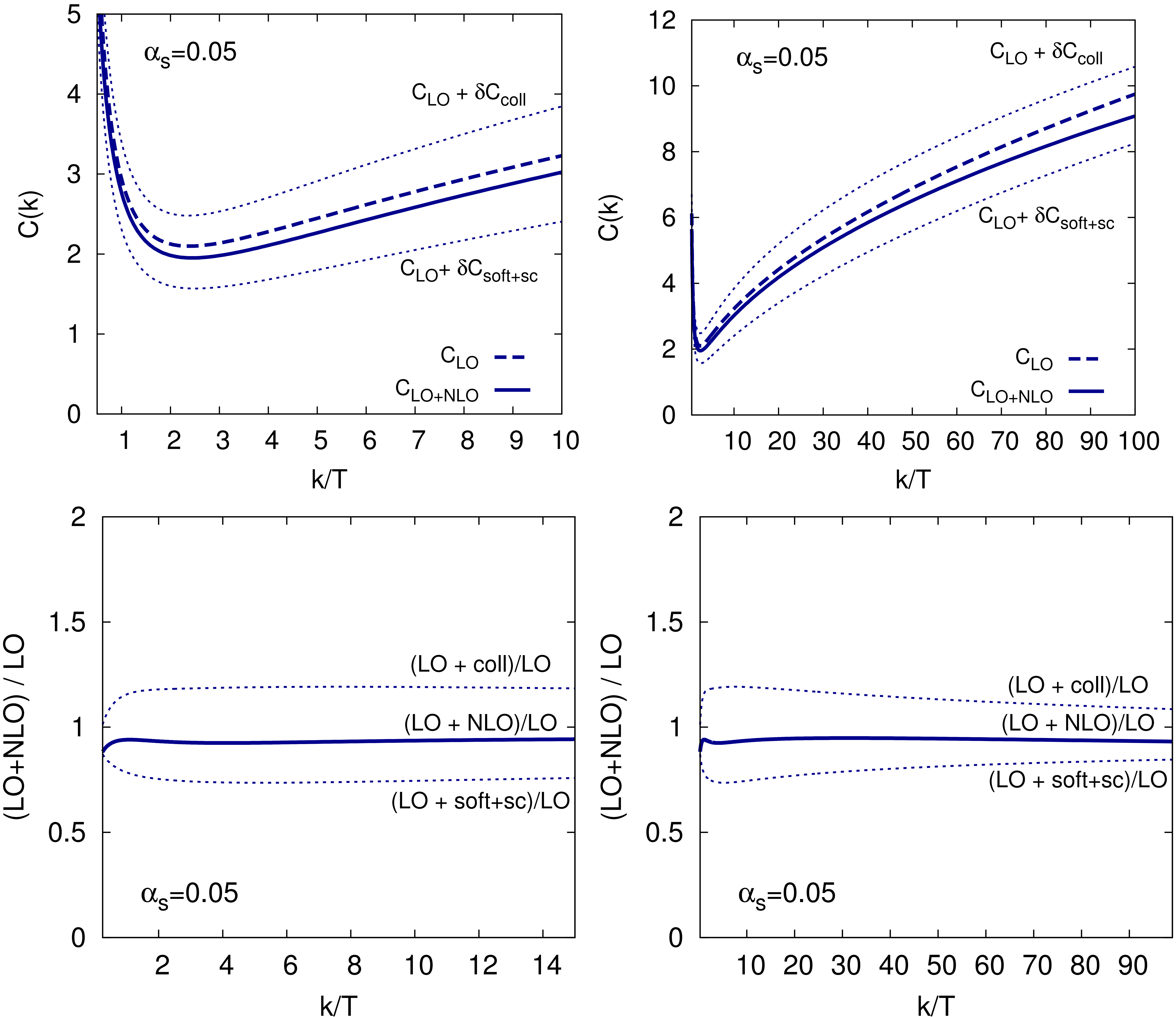


$$(2\pi)^3 \frac{d\delta\Gamma}{d^3k}\bigg|_{\rm NLO} = \mathcal{A}(k)\left[\overbrace{\underbrace{\frac{\delta m_\infty^2}{m_\infty^2}\log\frac{\sqrt{2Tm_D}}{m_\infty} + \frac{\delta m_\infty^2}{m_\infty^2}C_{\rm soft+sc}(k)}_{\delta C_{\rm soft+sc}(k)} + \underbrace{\frac{\delta m_\infty^2}{m_\infty^2}C^{\delta m}_{\rm coll}(k) + \frac{g^2C_AT}{m_D}C^{\delta\mathcal{C}}_{\rm coll}(k)}_{\delta C_{\rm coll}(k)}}^{\delta C_{\rm NLO}(k)}\right]$$



$$(2\pi)^3 \frac{d\delta\Gamma}{d^3k}\bigg|_{\mathrm{NLO}} = \mathcal{A}(k) \overbrace{\left[\underbrace{\frac{\delta m_\infty^2}{m_\infty^2} \log \frac{\sqrt{2Tm_D}}{m_\infty} + \frac{\delta m_\infty^2}{m_\infty^2} C_{\mathrm{soft+sc}}(k)}_{\delta C_{\mathrm{soft+sc}}(k)} + \underbrace{\frac{\delta m_\infty^2}{m_\infty^2} C_{\mathrm{coll}}^{\delta m}(k) + \frac{g^2 C_A T}{m_D} C_{\mathrm{coll}}^{\delta\mathcal{C}}(k)}_{\delta C_{\mathrm{coll}}(k)} \right]}^{\delta C_{\mathrm{NLO}}(k)}$$





α_s=0.05
C_{LO} + δC_{coll}
C_{LO}+ $\delta C_{soft+sc}$
C_{LO}
C_{LO+NLO}
C(k)
k/T
(LO+NLO) / LO
(LO + coll)/LO
(LO + NLO)/LO
(LO + soft+sc)/LO