Resummation of large higher order corrections in non-linear QCD evolution

XIIth Quark Confinement
and the Hadron Spectrum
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E. Iancu, J.D. Madrigal, A.H. Mueller, G. Soyez and D.T., Phys. Lett. **B744** (2015) 293, **B750** (2015) 643 and w.i.p.
E. Iancu, A.H. Mueller and D.T., arXiv:1608.05293

Outline

- Perturbation theory in QM and in QFT
- Parton saturation in QCD
- BK equation at NLO and large transverse logarithms
- Unphysical solutions
- Resummation of logarithms to all orders
- Restoration of stability and solutions
- DIS fits, outlook

Perturbation theory in quantum mechanics

Energy levels in quantum mechanics: e.g. add perturbation $-2gx^3 + g^2x^4$ to simple harmonic oscillator

$$E_n = E_n^{(0)} + g^2 E_n^{(1)} + g^4 E_n^{(2)} + \cdots$$

Coefficients $E_n^{(i)}$ are numbers (for given m,ω, …)

Good approximation to keep few terms $n \leq n_0$

(This example needs $n_0 \lesssim 1/g^2$: asymptotic series
Non-perturbative effects and instantons formation)

Perturbation theory in QED, QCD, ...

Generic quantity in field theory with interaction g

$$\sigma = g^{2k}\sigma_0 + g^{2k+2}\sigma_1 + g^{2k+4}\sigma_2 + \cdots$$

E.g. diff. cross section: σ_i functions of particle 4-momenta

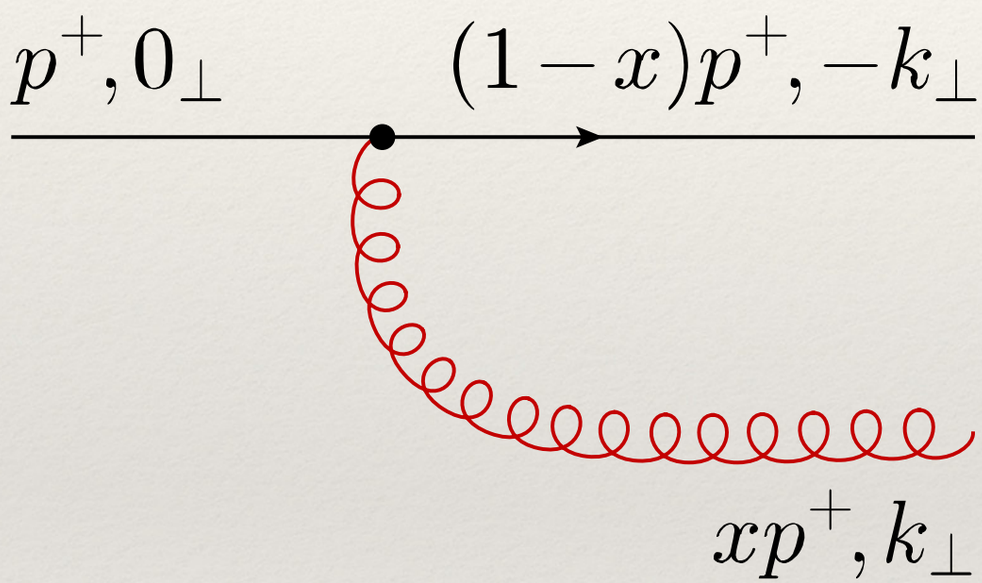
QCD at large distance, series is bad, use NP methods

At short distance $\alpha_s = g_s^2/4\pi \ll 1$, series looks meaningful

But could be $\alpha_s \sigma_{i+1}/\sigma_i \gtrsim 1$ for some momenta : fixed order series not enough, resum in certain kinematic domains

Parton emission in pQCD

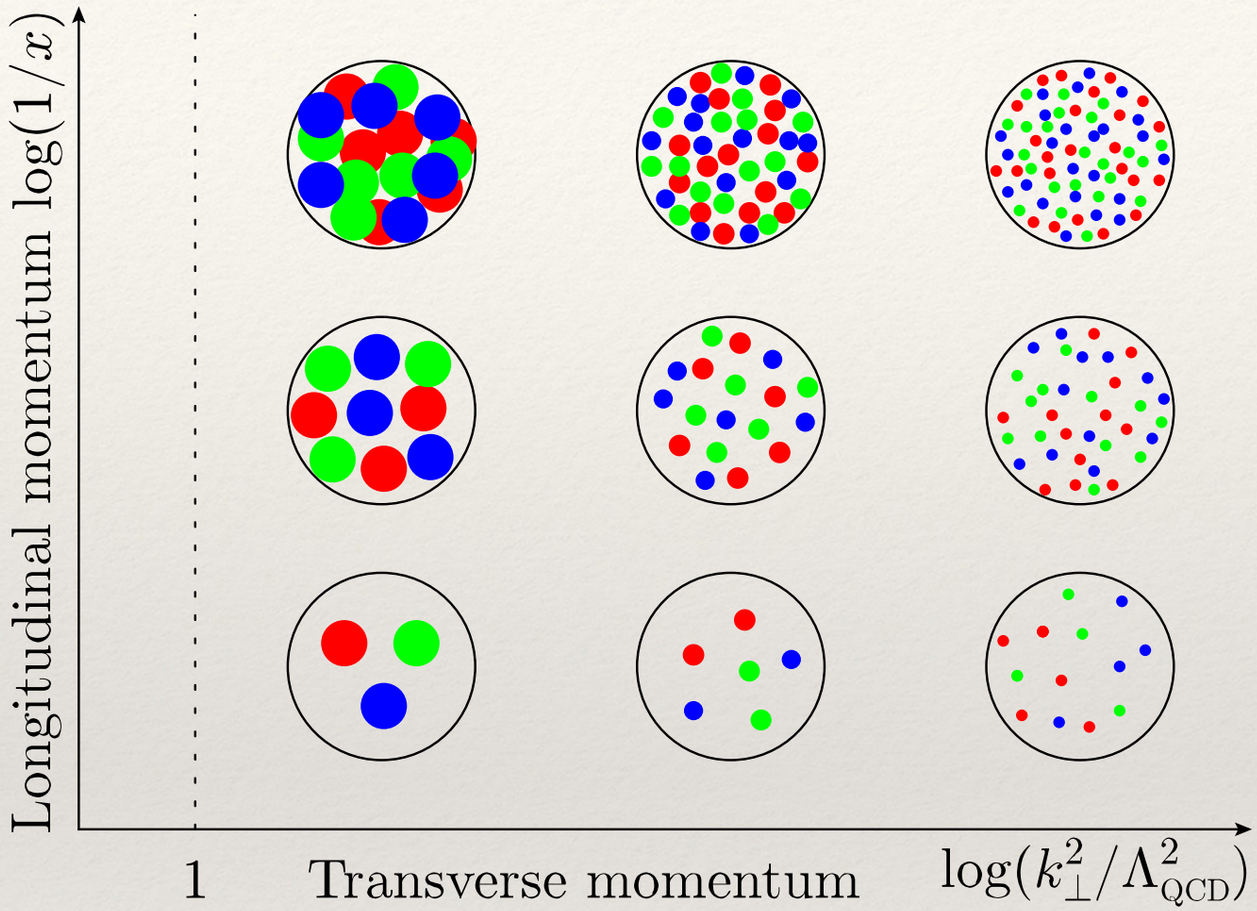
Consider emission of gluon from parent parton



$$\mathrm{d}P = C_\mathrm{R} \frac{\bar{\alpha}_s}{\pi^2} \frac{\mathrm{d}^2 k_\perp}{k_\perp^2} \frac{\mathrm{d}x}{x}$$

Integration over intermediate particles in cascade leads to two types of large logarithms to be resummed: transverse for DGLAP, longitudinal for BFKL

Parton saturation/Color Glass Condensate



Saturation momentum : $g(x, Q_s^2)/Q_s^2 R^2 \sim 1/\alpha_s$

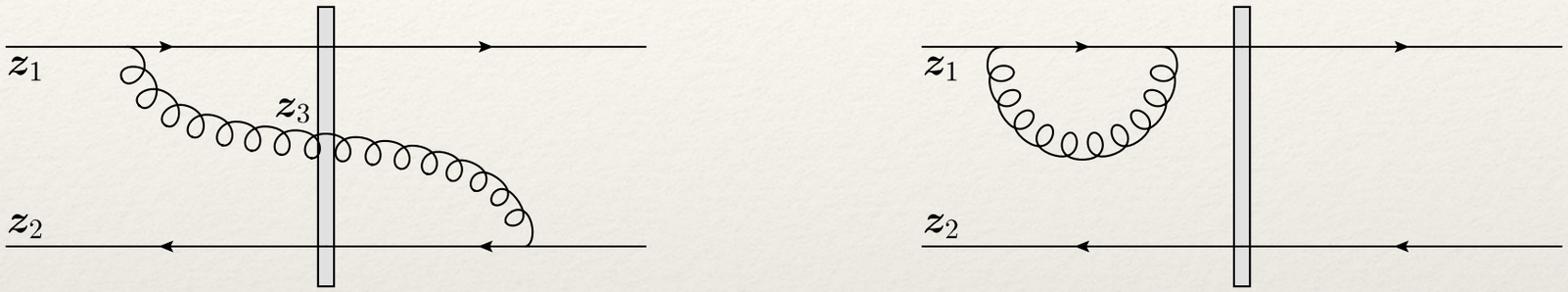
Q_s at small-x much larger than $\Lambda_{\rm QCD} \sim 200{\rm GeV}$

High density, weak coupling, non-linear dynamics

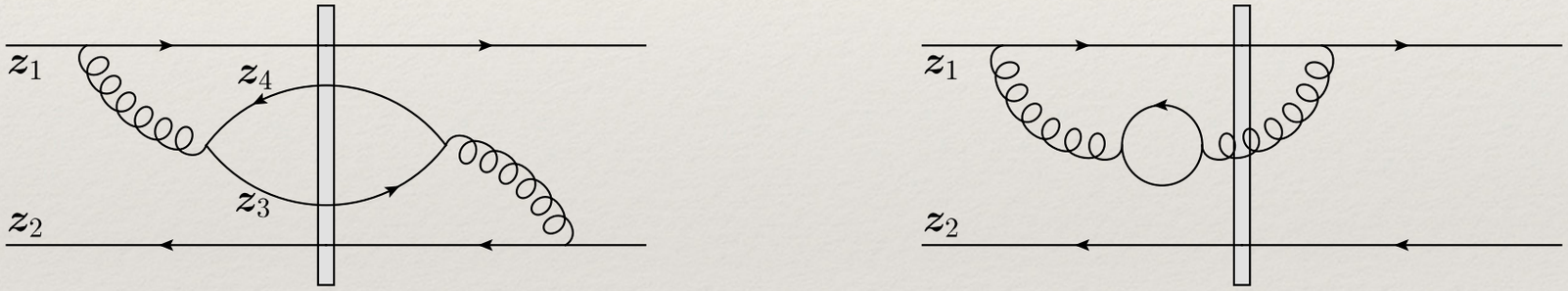
Diagrams for dipole evolution

- Probe system with color dipole. Evolution:

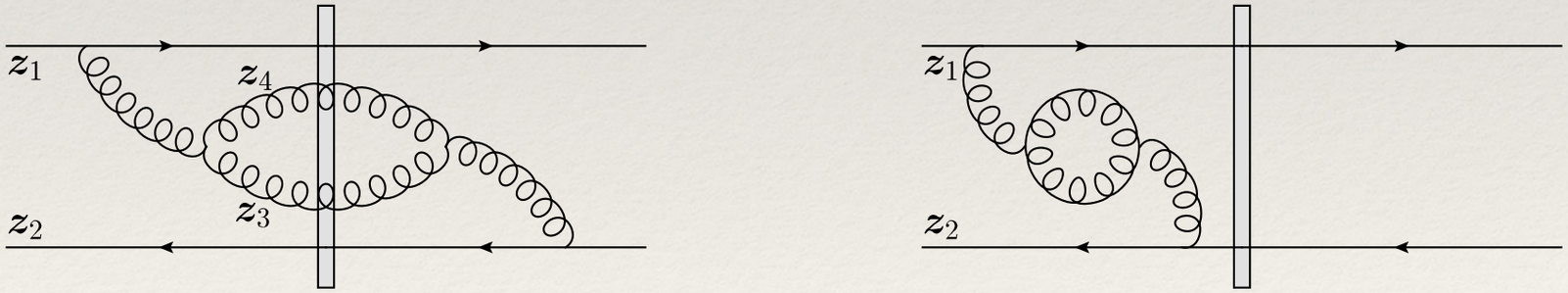
- LO



- NLO N_f



- NLO N_C



- Right mover. Lower k^+ longitudinal momentum.

The BK equation at NLO Balitsky, Chirilli 08

$$\frac{\mathrm{d}S_{12}}{\mathrm{d}Y} = \frac{\bar{\alpha}_s}{2\pi}\int \mathrm{d}^2 z_3\, \frac{z_{12}^2}{z_{13}^2 z_{23}^2}\left[1+\bar{\alpha}_s\left(\bar{b}\,\ln z_{12}^2\mu^2 - \bar{b}\,\frac{z_{13}^2-z_{23}^2}{z_{12}^2}\ln\frac{z_{13}^2}{z_{23}^2} + \frac{67}{36} - \frac{\pi^2}{12} - \frac{5}{18}\frac{N_\mathrm{f}}{N_\mathrm{c}} - \frac{1}{2}\ln\frac{z_{13}^2}{z_{12}^2}\ln\frac{z_{23}^2}{z_{12}^2}\right)\right](S_{13}S_{32}-S_{12})$$

$$+\frac{\bar{\alpha}_s^2}{8\pi^2}\int\frac{\mathrm{d}^2 z_3\,\mathrm{d}^2 z_4}{z_{34}^4}\left[-2+\frac{z_{13}^2 z_{24}^2+z_{14}^2 z_{23}^2-4z_{12}^2 z_{34}^2}{z_{13}^2 z_{24}^2-z_{14}^2 z_{23}^2}\ln\frac{z_{13}^2 z_{24}^2}{z_{14}^2 z_{23}^2} + \frac{z_{12}^2 z_{34}^2}{z_{13}^2 z_{24}^2}\left(1+\frac{z_{12}^2 z_{34}^2}{z_{13}^2 z_{24}^2-z_{14}^2 z_{23}^2}\right)\ln\frac{z_{13}^2 z_{24}^2}{z_{14}^2 z_{23}^2}\right]$$

$$\left[S_{13}S_{34}S_{42} - \frac{1}{2N_\mathrm{c}^3}\mathrm{tr}\left(V_1V_3^\dagger V_4V_2^\dagger V_3V_4^\dagger\right) - \frac{1}{2N_\mathrm{c}^3}\mathrm{tr}\left(V_1V_4^\dagger V_3V_2^\dagger V_4V_3^\dagger\right) - S_{13}S_{32} + \frac{1}{N_\mathrm{c}^2}S_{12}\right]$$

$$+\frac{\bar{\alpha}_s^2}{8\pi^2}\frac{N_\mathrm{f}}{N_\mathrm{c}}\int\frac{\mathrm{d}^2 z_3\,\mathrm{d}^2 z_4}{z_{34}^4}\left[2-\frac{z_{13}^2 z_{24}^2+z_{14}^2 z_{23}^2-z_{12}^2 z_{34}^2}{z_{13}^2 z_{24}^2-z_{14}^2 z_{23}^2}\ln\frac{z_{13}^2 z_{24}^2}{z_{14}^2 z_{23}^2}\right]$$

$$\left[S_{14}S_{32} - \frac{1}{N_\mathrm{c}^3}\mathrm{tr}\left(V_1V_2^\dagger V_3V_4^\dagger\right) - \frac{1}{N_\mathrm{c}^3}\mathrm{tr}\left(V_1V_4^\dagger V_3V_2^\dagger\right) + \frac{1}{N_\mathrm{c}^2}S_{12}S_{34} - S_{13}S_{32} + \frac{1}{N_\mathrm{c}^2}S_{12}\right]$$

$$z_{ij} = z_i - z_j \qquad S_{ij} = \frac{1}{N_\mathrm{c}}\mathrm{tr}\left(V_i^\dagger V_j\right) \qquad V_i^\dagger = \mathrm{P}\exp\left[\mathrm{i}g\int \mathrm{d}z^+ A_a^-(z^+,\boldsymbol{z}_i)t^a\right] \qquad \bar{b} = \frac{11}{12} - \frac{1}{6}\frac{N_\mathrm{f}}{N_\mathrm{c}}$$

See also Kovner, Lublinsky, Mulian 14

Large transverse logs

- Strongly ordered large ``perturbative'' dipoles (DLA)

$$1/Q_s \gg z_{14} \simeq z_{24} \simeq z_{34} \gg z_{13} \simeq z_{23} \gg z_{12}$$

- Large dipoles interact stronger, real terms only (N_f=0)

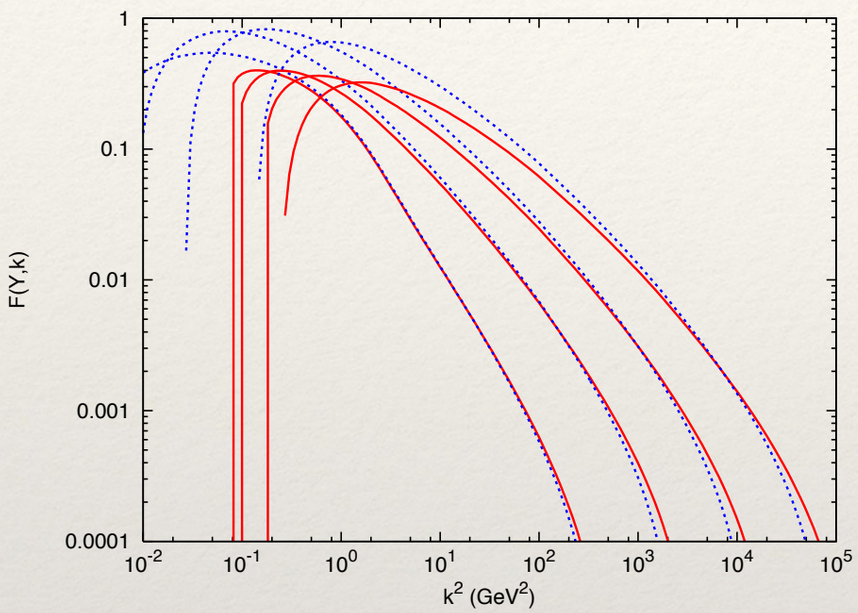
$$\frac{\mathrm{d}T_{12}}{\mathrm{d}Y} = \bar{\alpha}_s \int_{z_{12}^2}^{1/Q_s^2} \mathrm{d}z_{13}^2 \, \frac{z_{12}^2}{z_{13}^4} \left(1 - \bar{\alpha}_s \frac{1}{2} \ln^2 \frac{z_{13}^2}{z_{12}^2} - \bar{\alpha}_s \frac{11}{12} \ln \frac{z_{13}^2}{z_{12}^2} \right) T_{13}$$

- NLO > LO, unstable expansion in coupling.
  Simple but general IC: color transparency + saturation

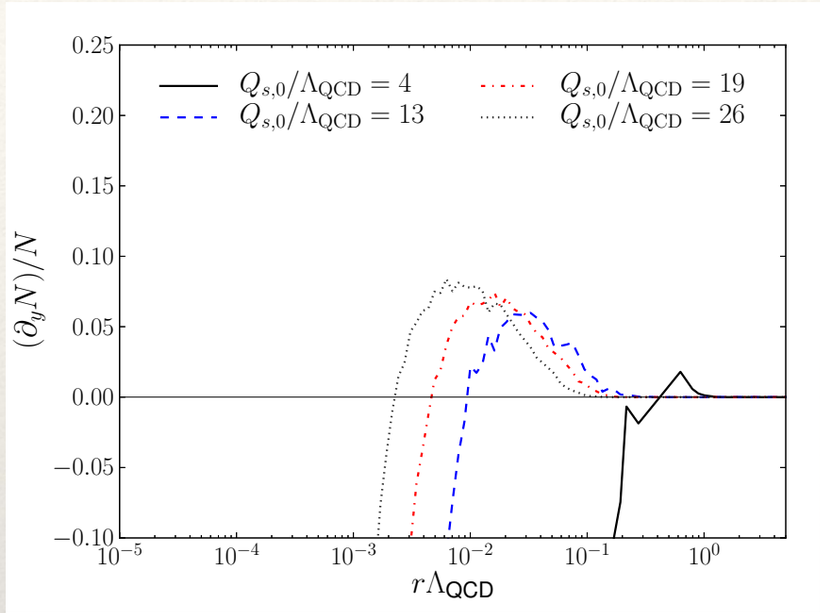
$$T_{12} = \begin{cases} z_{12}^2 Q_{s0}^2 \,, & z_{12} Q_{s0} \ll 1 \\ 1 \,, & z_{12} Q_{s0} \gg 1 \end{cases} \Rightarrow \frac{\Delta T_{12}}{\bar{\alpha}_s \Delta Y} \simeq z_{12}^2 Q_{s0}^2 \left(\ln \frac{1}{z_{12}^2 Q_{s0}^2} - \frac{\bar{\alpha}_s}{6} \ln^3 \frac{1}{z_{12}^2 Q_{s0}^2} - \frac{11 \bar{\alpha}_s}{24} \ln^2 \frac{1}{z_{12}^2 Q_{s0}^2} \right)$$

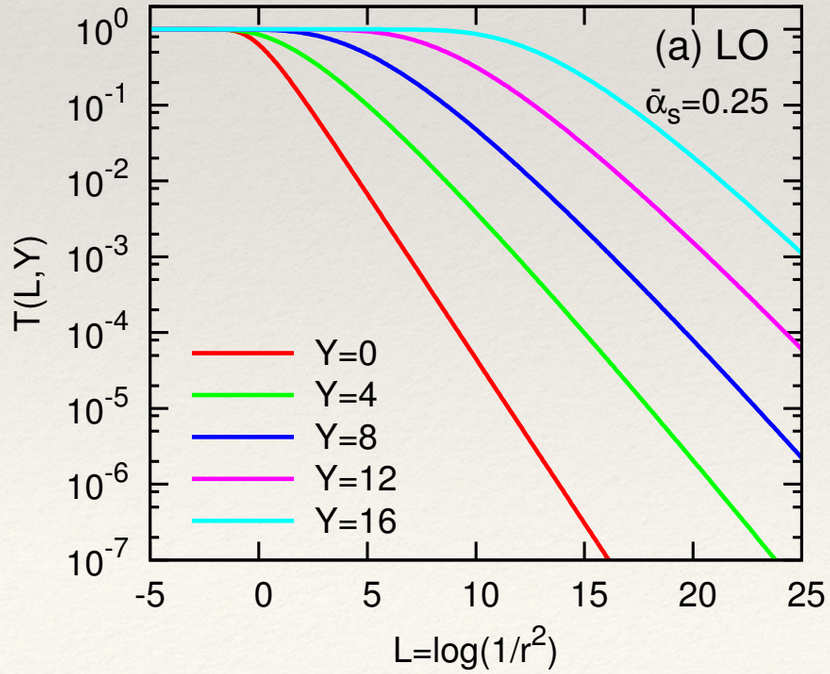
Unstable numerical solutions

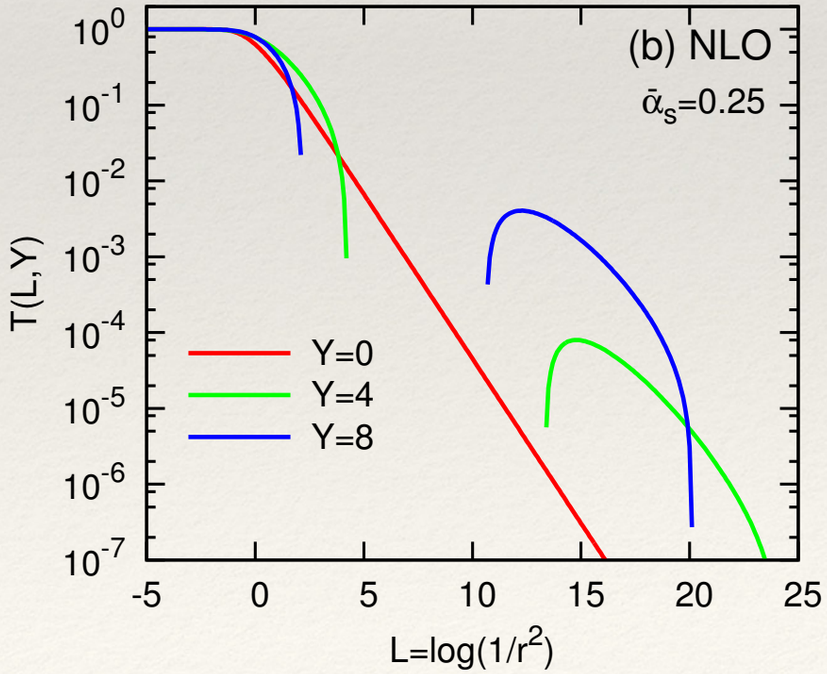
Avsar, Stasto, DT, Zaslavsky 11



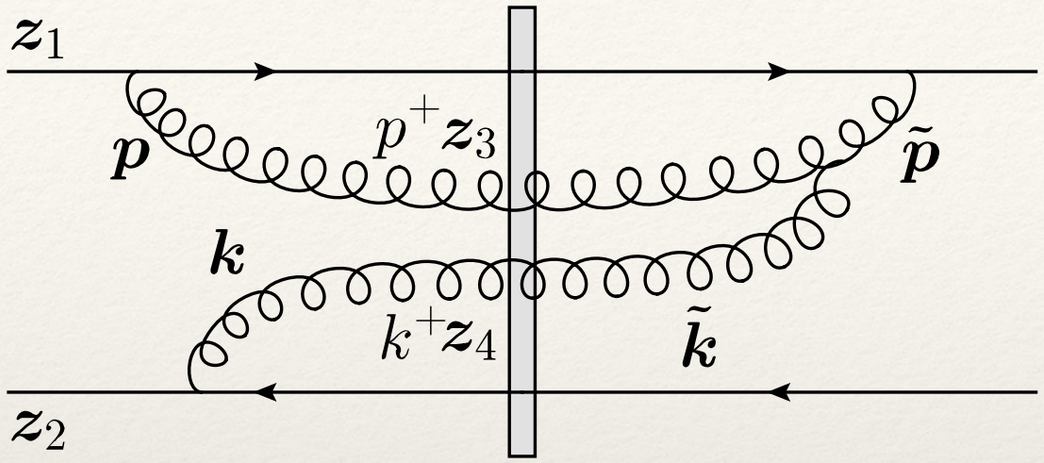
Lappi, Mantysaari 15







Two gluons and time ordering (kinematics)



- Hard to soft projectile evolution $\boldsymbol{k} \ll \boldsymbol{p}$ and $k^+ \ll p^+$
- Energy denominators lead to largest logs when emissions are time-ordered $\tau_k \approx k^+ z_4^2 \ll \tau_p \approx p^+ z_3^2$
- Leads to double log term in NLO BK equation

$$\Delta T_{12} = \bar{\alpha}_s^2 \int \frac{\mathrm{d}p^+}{p^+} \frac{\mathrm{d}k^+}{k^+} \Theta\left(p^+ \frac{z_3^2}{z_4^2} - k^+\right) \mathrm{d}z_3^2 \mathrm{d}z_4^2 \frac{z_{12}^2}{z_3^2 z_4^4} T(z_4) \to -\frac{\bar{\alpha}_s^2 \Delta Y}{2} \int_{z_{12}^2}^{1/Q_s^2} \frac{\mathrm{d}z_4^2}{z_4^4} \ln^2 \frac{z_4^2}{z_{12}^2} T(z_4)$$

Resummation of double logs in DLA

- Systematically resum to all orders in <u>non-local equation</u>

$$\frac{\mathrm{d}T(Y,z_{12}^2)}{\mathrm{d}Y} = \bar{\alpha}_s \int_{z_{12}^2}^{1/Q_s^2} \frac{\mathrm{d}z_{13}^2}{z_{13}^2}\frac{z_{12}^2}{z_{13}^2}\Theta\left(Y - \ln\frac{z_{13}^2}{z_{12}^2}\right) T\left(Y - \ln\frac{z_{13}^2}{z_{12}^2}, z_{13}^2\right)$$

- Mathematically equivalent to <u>local equation</u>

$$\frac{\mathrm{d}T(Y,z_{12}^2)}{\mathrm{d}Y} = \bar{\alpha}_s \int_{z_{12}^2}^{1/Q_s^2} \frac{\mathrm{d}z_{13}^2}{z_{13}^2}\frac{z_{12}^2}{z_{13}^2}\frac{\mathrm{J}_1\left(2\sqrt{\bar{\alpha}_s \ln^2\frac{z_{13}^2}{z_{12}^2}}\right)}{\sqrt{\bar{\alpha}_s \ln^2\frac{z_{13}^2}{z_{12}^2}}} T(Y,z_{13}^2)$$

with modified initial condition (impact factor)

$$T(0,z_{12}^2) \propto \frac{C_\mathrm{F}}{N_\mathrm{c}} z_{12}^2 Q_{s0}^2 \sqrt{\bar{\alpha}_s}\,\mathrm{J}_1\left(2\sqrt{\bar{\alpha}_s \ln^2\frac{1}{z_{12}^2 Q_{s0}^2}}\right)$$

Resummation of double logs in BK

- Promote local equation to include BK physics

$$\frac{\mathrm{d}S_{12}}{\mathrm{d}Y} = \frac{\bar{\alpha}_s}{2\pi}\int \mathrm{d}^2 z_3 \frac{z_{12}^2}{z_{13}^2 z_{23}^2} \frac{\mathrm{J}_1\left(2\sqrt{\bar{\alpha}_s L_{13} L_{23}}\right)}{\sqrt{\bar{\alpha}_s L_{13} L_{23}}} (S_{13}S_{32} - S_{12})$$

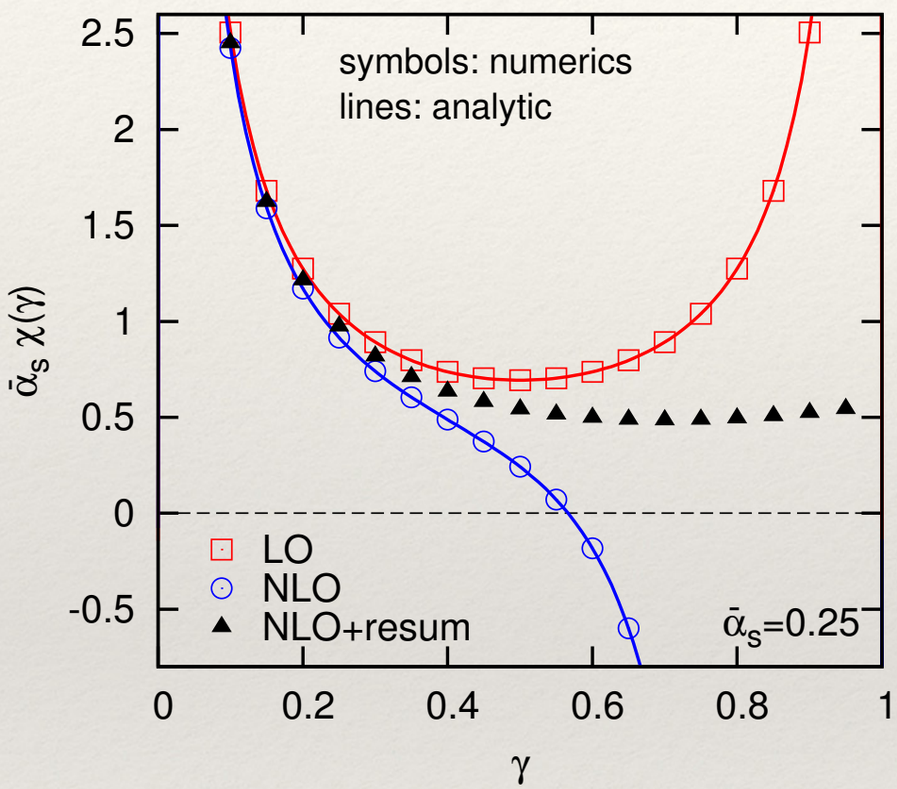
$$\text{with } L_{13}L_{23} = \ln\frac{z_{13}^2}{z_{12}^2}\ln\frac{z_{23}^2}{z_{12}^2}$$

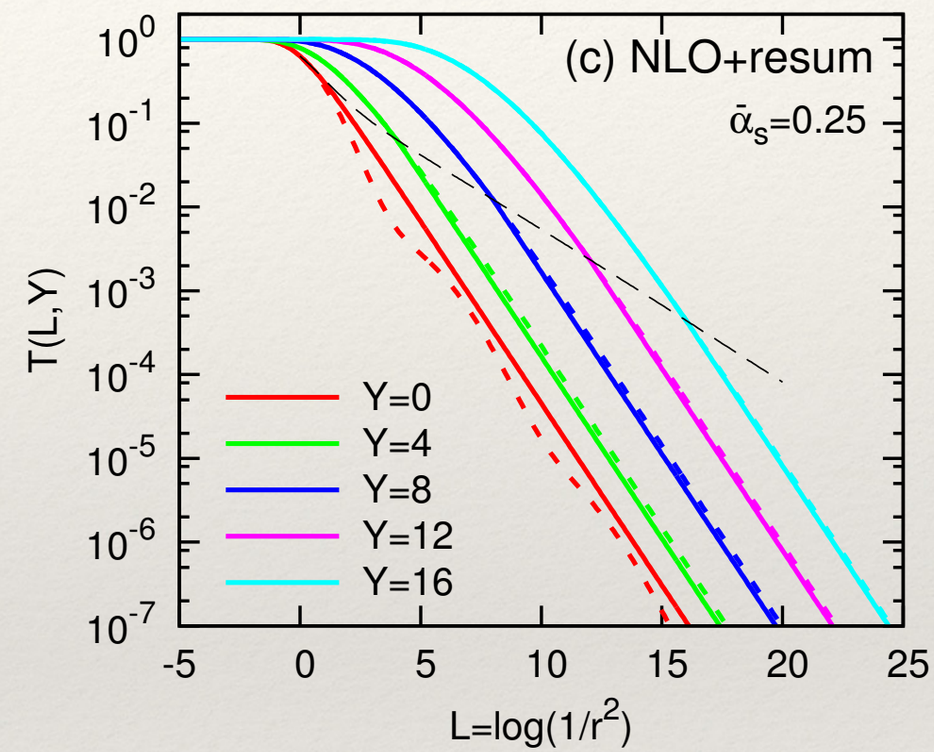
Equivalent non-local equation by Beuf 14

- NLO BK (double log term) when truncated to order $\bar{\alpha}_s^2$
- Exactly resums double log terms to all orders

Numerical solution

BFKL on $z_{12}^{2\gamma} \equiv r^{2\gamma}$

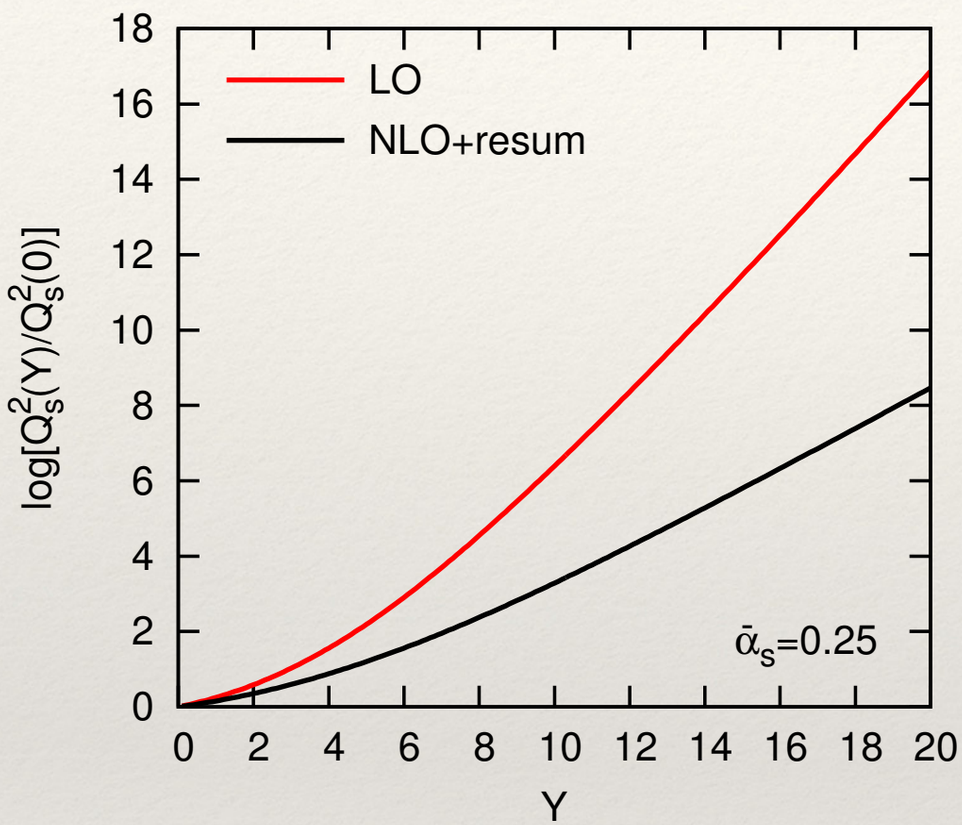


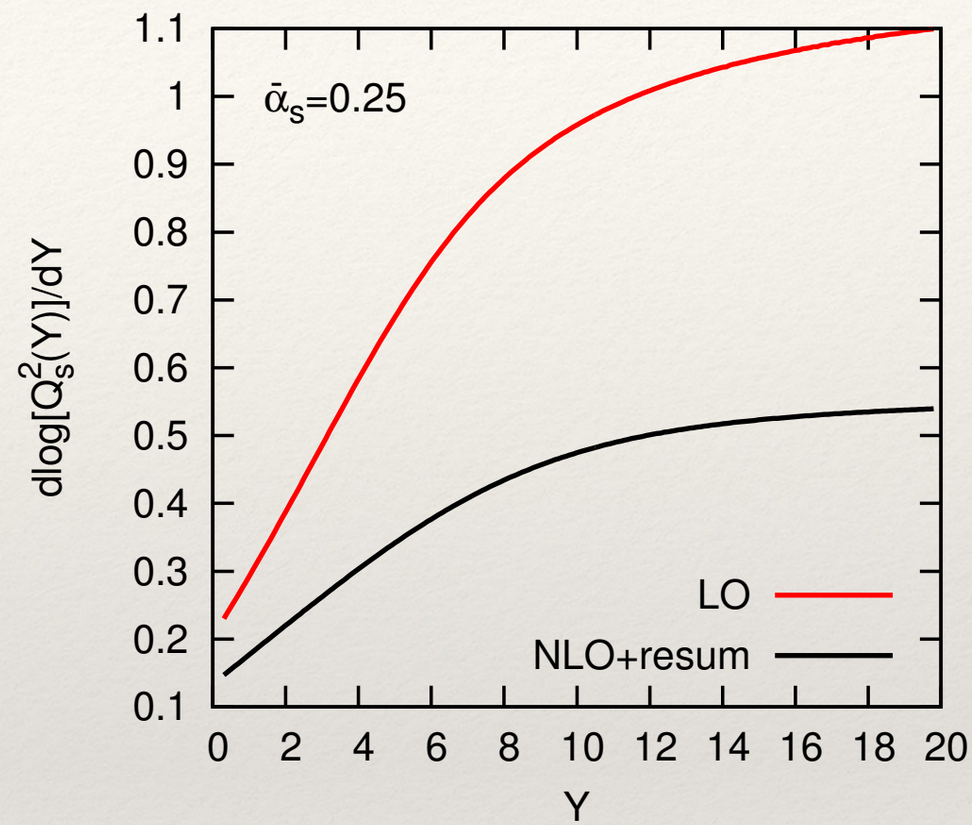


$$\omega_{\rm LO} = \frac{\bar{\alpha}_s}{\gamma} + \frac{\bar{\alpha}_s}{1-\gamma} + \text{finite} \qquad \omega_{\rm NLO} = \frac{\bar{\alpha}_s}{\gamma} + \frac{\bar{\alpha}_s}{1-\gamma} - \frac{\bar{\alpha}_s^2}{(1-\gamma)^3} + \text{finite}$$

$$\omega_{\rm NLO}^{\rm res} = \omega_{\rm NLO} - \frac{\bar{\alpha}_s}{1-\gamma} + \frac{\bar{\alpha}_s^2}{(1-\gamma)^3} + \frac{1}{2}\left[-(1-\gamma) + \sqrt{(1-\gamma)^2 + 4\bar{\alpha}_s}\right] + \text{finite}$$

Numerical solution

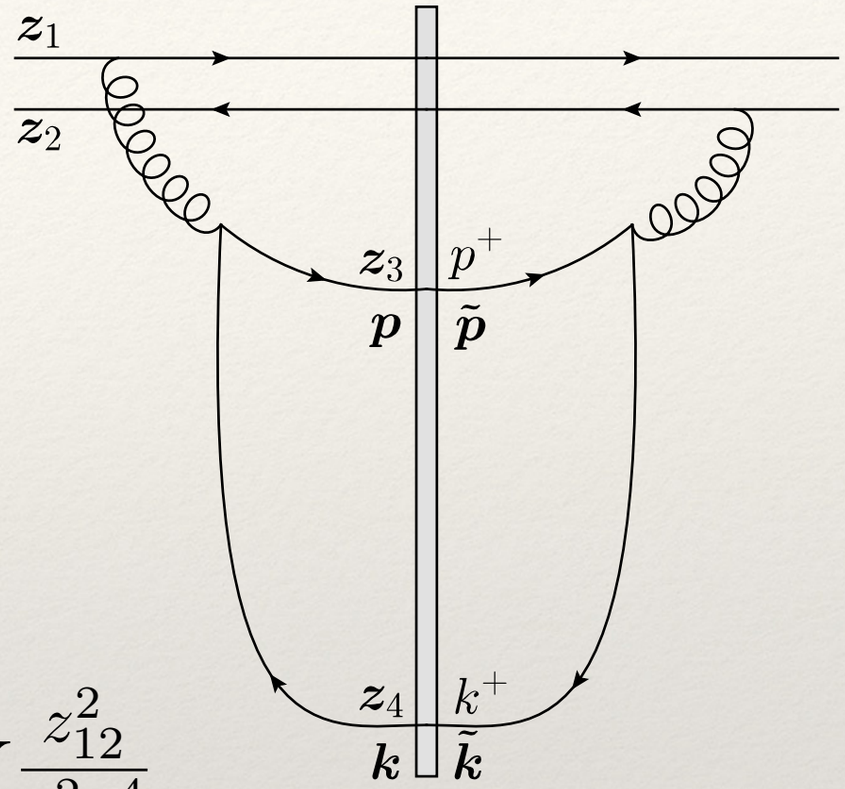




- Considerable speed reduction, roughly factor of 1/2

Single log in quark contribution (dynamics)

- Take $\boldsymbol{k} \ll \boldsymbol{p}$ and $\zeta = k^+/p^+ \ll 1$
  hard to soft projectile evolution



- Quark contribution is easier, no DLs
- Integrate transverse momenta

$$\Sigma\mathcal{A}_{ij} = \frac{\alpha_s^2 N_\mathrm{f}}{2\pi^4}\,\Delta Y \int_0^1 \mathrm{d}\zeta\, \frac{z_{12}^2}{z_4^2}\,\frac{z_3^4+\zeta^2 z_4^4}{(z_3^2+\zeta z_4^2)^4} \simeq \frac{\alpha_s^2 N_\mathrm{f}}{3\pi^4}\,\Delta Y\,\frac{z_{12}^2}{z_3^2 z_4^4}$$

- $\zeta z_4^2 \sim z_3^3$, no time ordering. Integrand P_{qG} split. function
- Insert color structure and scattering

$$\frac{\Delta T_{12}}{\Delta Y} = -\frac{\bar{\alpha}_s^2 N_\mathrm{f}}{6N_\mathrm{c}^3}\, z_{12}^2 \int_{z_{12}^2}^{1/Q_s^2} \frac{\mathrm{d}z_4^2}{z_4^4}\, \ln\frac{z_4^2}{z_{12}^2}\, T(z_4)$$

Relationship to splitting functions

- DGLAP mixes quarks and gluons.
  Largest eigenvalue of moments:

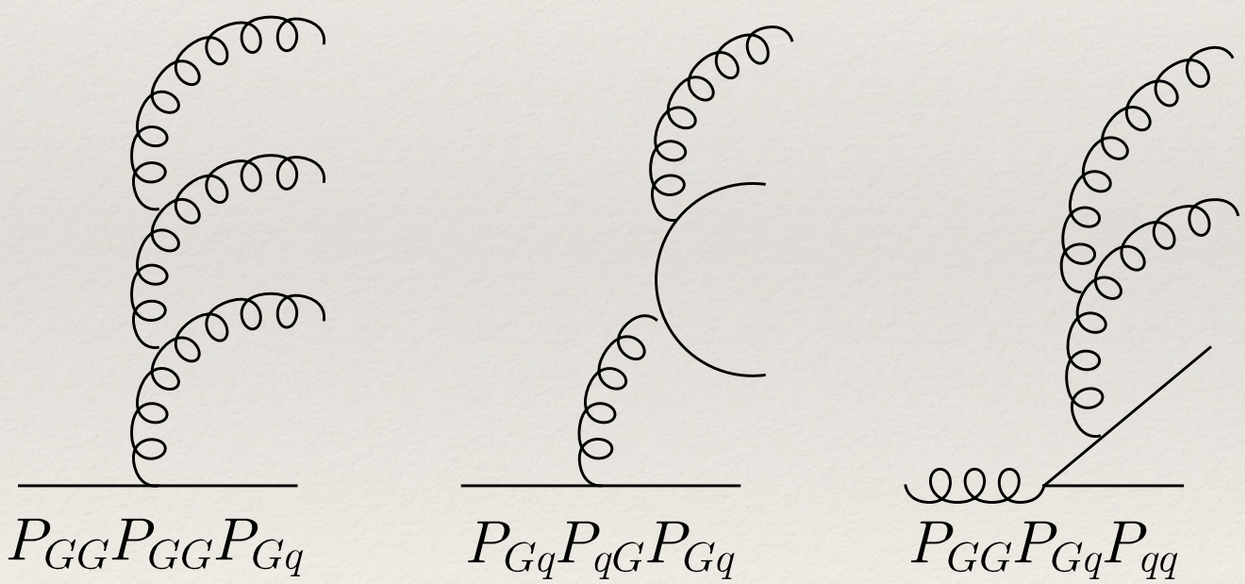
$$\int_0^1 \mathrm{d}z\, z^\omega \left[P_{GG}(z) + \frac{C_\mathrm{F}}{N_\mathrm{c}} P_{qG}(z)\right] = \frac{1}{\omega} \underbrace{-\frac{11}{12} - \frac{N_\mathrm{f}}{6N_\mathrm{c}^3}}_{\equiv A_1} + \mathcal{O}(\omega)$$

- Similar hard to soft gluon diagrams must give -11/12

- All this is DGLAP physics. “Normally” it is soft to hard.

Soft to hard

- Imagine starting from LM target parton with large q_0^- Evolve up to small projectile by emitting partons with smaller size and smaller minus long. momentum.



$$\eta = \ln \frac{q_0^-}{q^-} = \ln \left(\frac{Q_0^2}{q_0^+} q^+ z_{12}^2 \right) = Y - \ln \frac{1}{z_{12}^2 Q_0^2}$$

DGLAP solution and collinear BK kernel

- General DGLAP solution

$$T_{12}(Y) \approx z_{12}^2 Q_0^2\, xG\left(\eta, \ln\frac{1}{z_{12}^2 Q_0^2}\right) \approx \int \frac{\mathrm{d}\omega}{2\pi \mathrm{i}} \exp\left\{\omega Y + \underbrace{[\bar{\alpha}_s \mathcal{P}(\omega) - \omega - 1]}_{-\gamma} \ln\frac{1}{z_{12}^2 Q_0^2}\right\}$$

- Solve for ω as function of γ. Keep only up to A_1.
  One, two, three, … subleading splittings resummed.
  Exponential combinatorics.

$$\frac{\mathrm{d}S_{12}}{\mathrm{d}Y} = \frac{\bar{\alpha}_s}{2\pi} \int \mathrm{d}^2 z_3 \frac{z_{12}^2}{z_{13}^2 z_{23}^2} \left(\frac{z_{12}^2}{z_>^2}\right)^{\mp \bar{\alpha}_s A_1} \frac{\mathrm{J}_1\left(2\sqrt{\bar{\alpha}_s L_{13} L_{23}}\right)}{\sqrt{\bar{\alpha}_s L_{13} L_{23}}} (S_{13} S_{32} - S_{12})$$

$z_< = \min\{z_{13}, z_{23}\}$. + sign when $z_< < z_{12}$

Running coupling

$$\frac{\mathrm{d}S_{12}}{\mathrm{d}Y} = \frac{\bar{\alpha}_s(\mu)}{2\pi} \int \mathrm{d}^2 z_3 \, \frac{z_{12}^2}{z_{13}^2 z_{23}^2} \left[1 + \bar{\alpha}_s(\mu) \left(\bar{b} \, \ln z_{12}^2 \mu^2 - \bar{b} \, \frac{z_{13}^2 - z_{23}^2}{z_{12}^2} \ln \frac{z_{13}^2}{z_{23}^2} \right) \right] (S_{13} S_{32} - S_{12})$$

- Choose μ to cancel potentially large log in all regions

  Large daughter dipoles : $\mu \approx 1/z_{12}$

  Small daughter dipole : $\mu \approx 1/\min\{z_{13}, z_{23}\}$

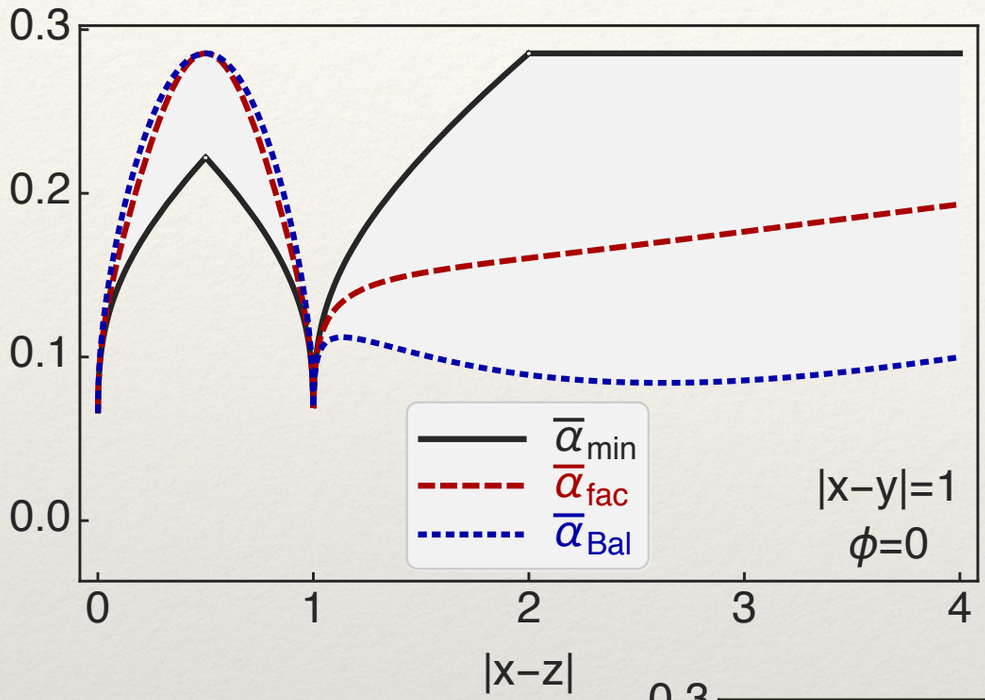
  In general : $\mu \approx 1/\min\{z_{ij}\}$ ✓ Hardest scale

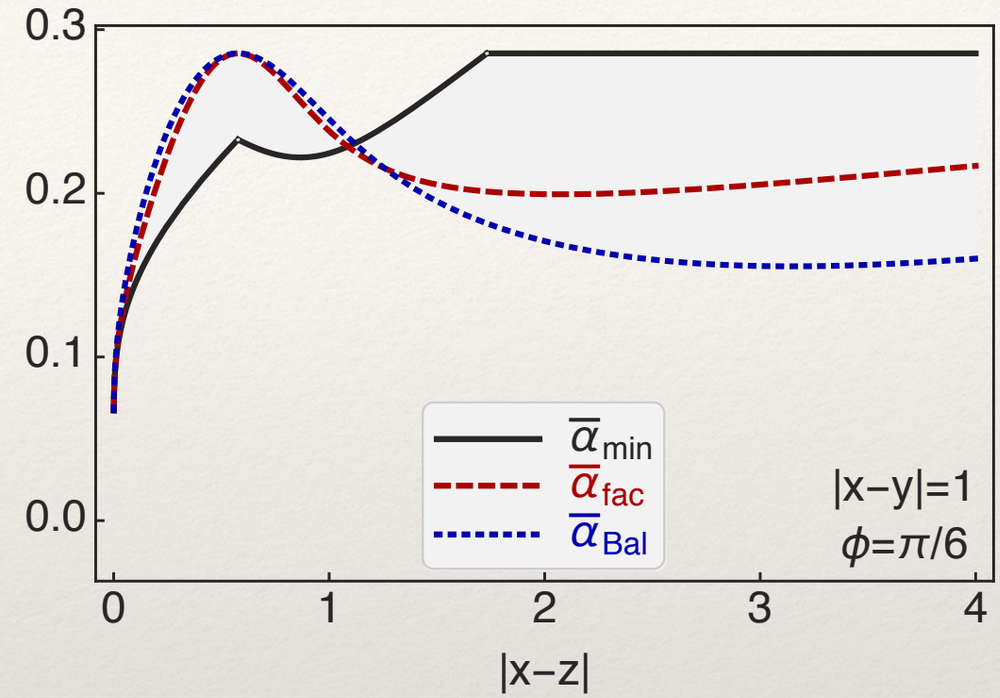
- Balitsky-prescription: ✓, albeit unphysical slow

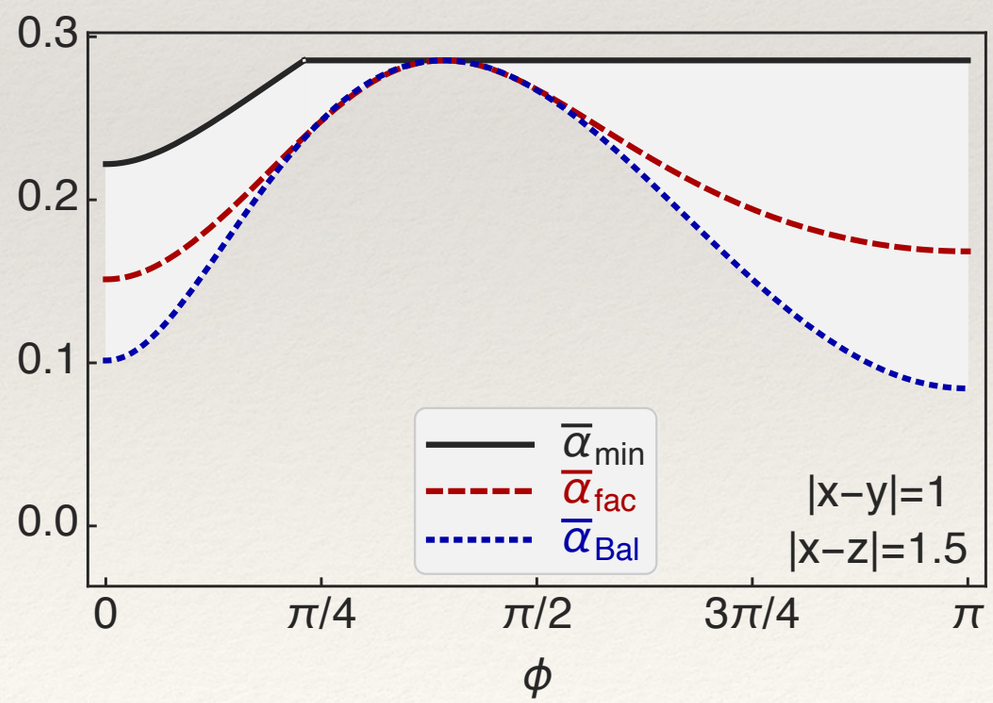
- Choose coefficient of $\bar{b}$ to vanish: ✓

$$\alpha_s = \left[\frac{1}{\alpha_s(z_{12})} + \frac{z_{13}^2 - z_{23}^2}{z_{12}^2} \frac{\alpha_s(z_{13}) - \alpha_s(z_{23})}{\alpha_s(z_{13})\alpha_s(z_{23})} \right]^{-1}$$

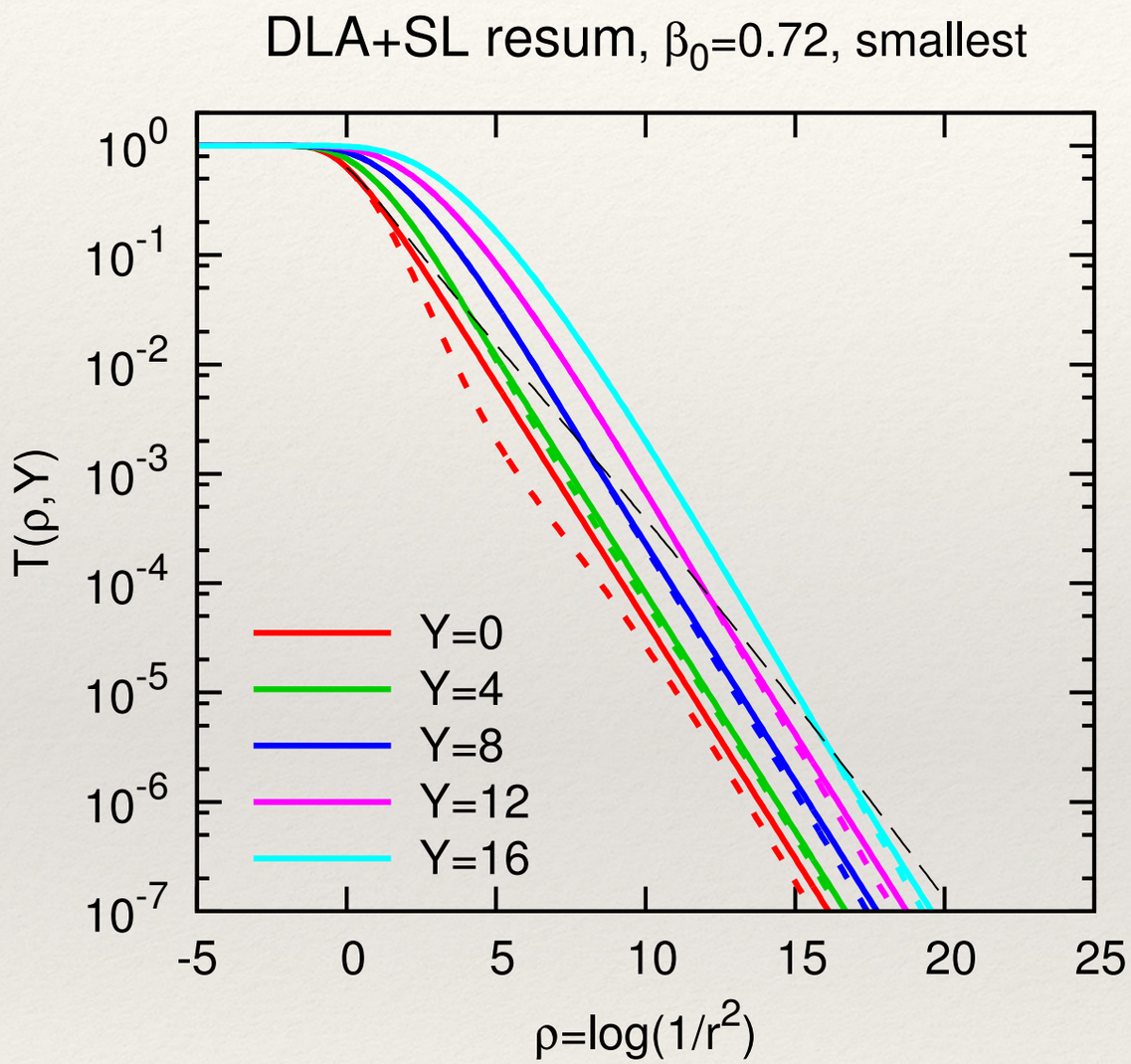
Couplings comparison

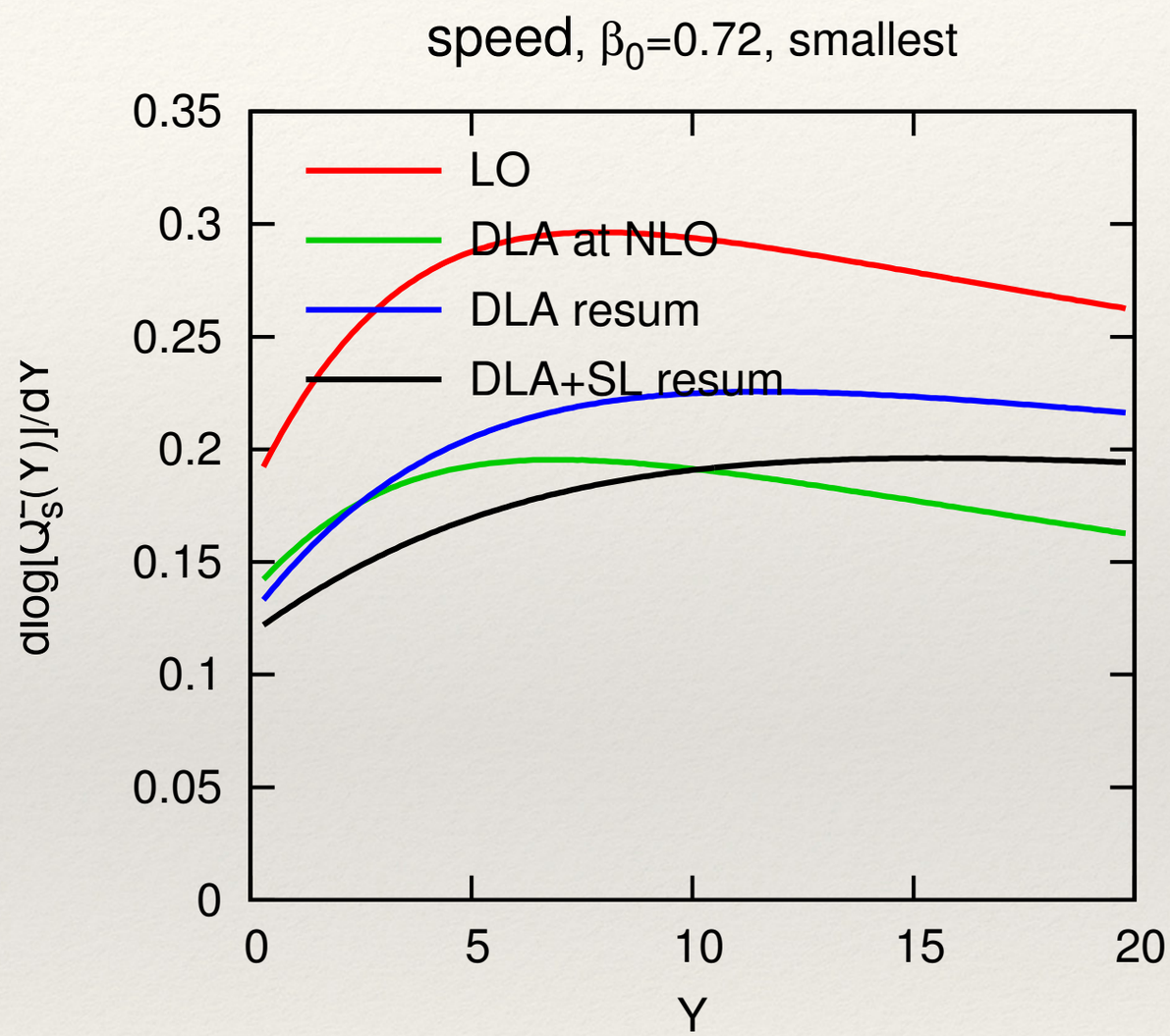






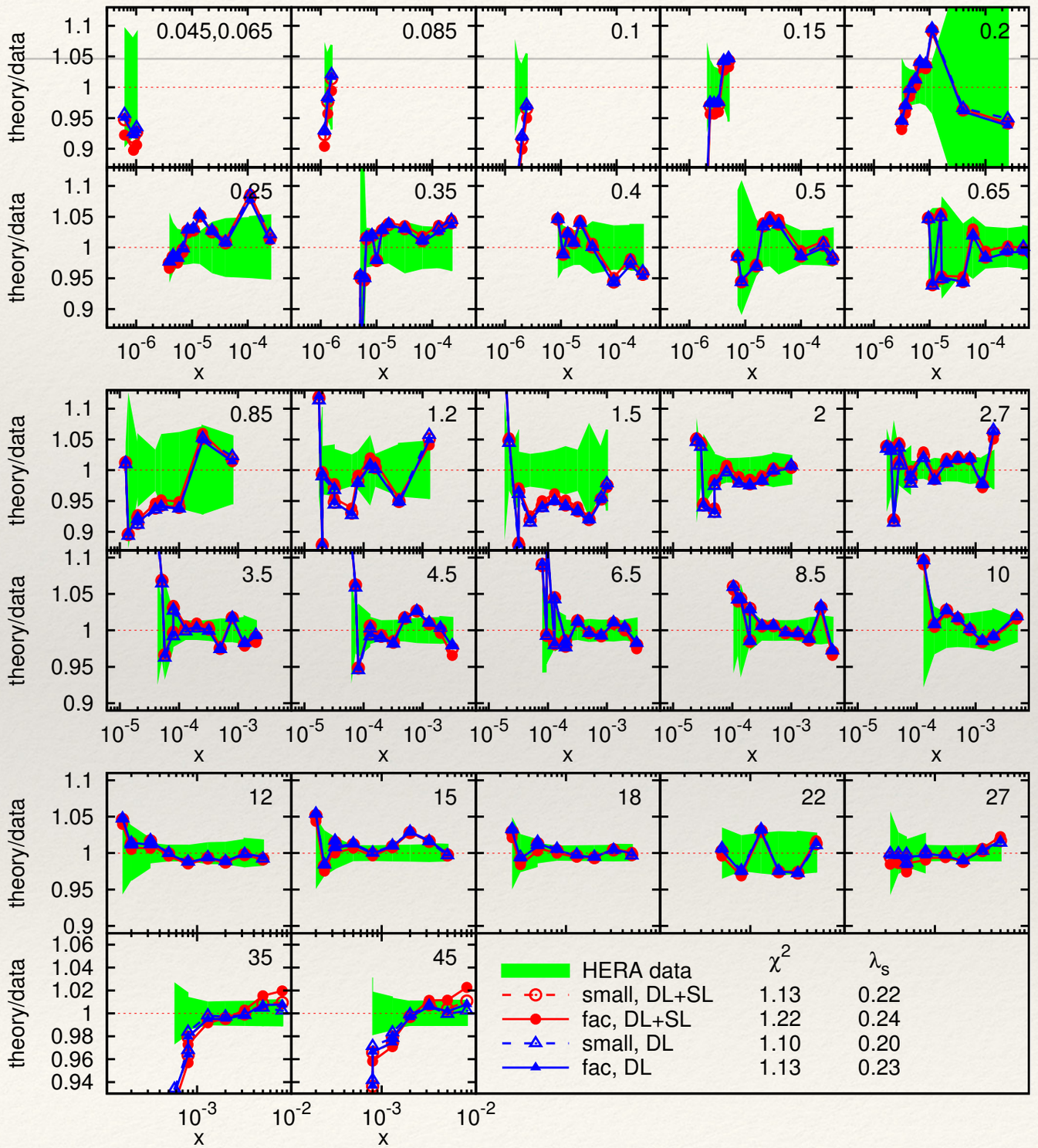
Numerical solution (prescirption:small)

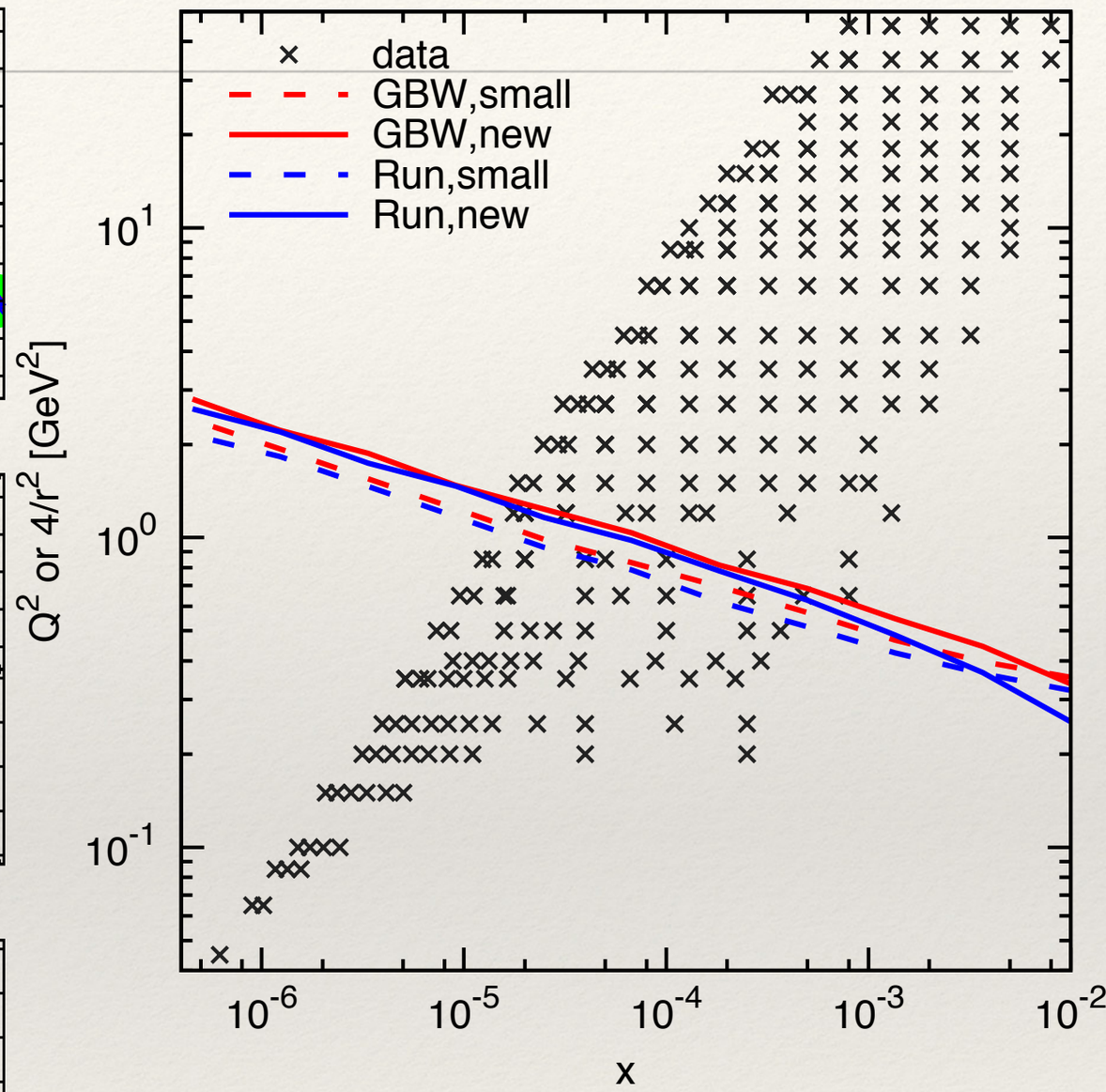




See also Lappi Mantysaari 16

Fit





See also Albacete 15 (Beuf eqn)

Fit

init cdt.	RC schm	sing. logs	χ^2 per data point			parameters				
			$\sigma_{\rm red}$	$\sigma_{\rm red}^{c\bar{c}}$	F_L	R_p[fm]	Q_0[GeV]	C_α	p	$C_{\rm MV}$
GBW	small	yes	1.135	0.552	0.596	0.699	0.428	2.358	2.802	-
GBW	fac	yes	1.262	0.626	0.602	0.671	0.460	0.479	1.148	-
rcMV	small	yes	1.126	0.578	0.592	0.711	0.530	2.714	0.456	0.896
rcMV	fac	yes	1.222	0.658	0.595	0.681	0.566	0.517	0.535	1.550
GBW	small	no	1.121	0.597	0.597	0.716	0.414	6.428	4.000	-
GBW	fac	no	1.164	0.609	0.594	0.697	0.429	1.195	4.000	-
rcMV	small	no	1.097	0.557	0.593	0.723	0.497	7.393	0.477	0.816
rcMV	fac	no	1.128	0.573	0.591	0.703	0.526	1.386	0.502	1.015

init cdt.	RC schm	sing. logs	χ^2/npts for $Q^2_{\rm max}$			
			50	100	200	400
GBW	small	yes	1.135	1.172	1.355	1.537
GBW	fac	yes	1.262	1.360	1.654	1.899
rcMV	small	yes	1.126	1.172	1.167	1.158
rcMV	fac	yes	1.222	1.299	1.321	1.317
GBW	small	no	1.121	1.131	1.317	1.487
GBW	fac	no	1.164	1.203	1.421	1.622
rcMV	small	no	1.097	1.128	1.095	1.078
rcMV	fac	no	1.128	1.177	1.150	1.131

- No anomalous dimension in initial condition
- Including single logs: more physical parameters
- MV model: can be extrapolated to higher Q^2
- Smallest dipole prescription: best fit
  B-prescription: not very good

Conclusion - Outlook

- Stable, slow, evolution with resummed dominant logs
- Insert formalism into more exclusive observables
  e.g. particle production at forward rapidity and calculate
- Understand better hard to soft DGLAP evolution
- Structurally different evolution than the SUSY one