Resummation of large higher order corrections in non-linear QCD evolution

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Outline

◆ Perturbation theory in QM and in QFT
◆ Parton saturation in QCD
◆ BK equation at NLO and large transverse logarithms
◆ Unphysical solutions
◆ Resummation of logarithms to all orders
◆ Restoration of stability and solutions
◆ DIS fits, outlook
Perturbation theory in quantum mechanics

Energy levels in quantum mechanics: e.g. add perturbation $-2gx^3 + g^2x^4$ to simple harmonic oscillator

$$E_n = E_n^{(0)} + g^2 E_n^{(1)} + g^4 E_n^{(2)} + \cdots$$

Coefficients $E_n^{(i)}$ are numbers (for given $m, \omega, \ldots$)

Good approximation to keep few terms $n \leq n_0$

(This example needs $n_0 \lesssim 1/g^2$ : asymptotic series Non-perturbative effects and instantons formation)
Perturbation theory in QED, QCD, ...

Generic quantity in field theory with interaction $g$

$$\sigma = g^{2k}\sigma_0 + g^{2k+2}\sigma_1 + g^{2k+4}\sigma_2 + \cdots$$

E.g. diff. cross section: $\sigma_i$ functions of particle 4-momenta

QCD at large distance, series is bad, use NP methods

At short distance $\alpha_s = g_s^2/4\pi \ll 1$, series looks meaningful

But could be $\alpha_s\sigma_{i+1}/\sigma_i \gtrsim 1$ for some momenta: fixed order series not enough, resum in certain kinematic domains
Parton emission in pQCD

Consider emission of gluon from parent parton

\[ p^+, 0_\perp \rightarrow (1 - x)p^+, -k_\perp \]

\[ xp^+, k_\perp \]

Integration over intermediate particles in cascade leads to two types of large logarithms to be resummed: transverse for DGLAP, longitudinal for BFKL
Parton saturation/Color Glass Condensate

Saturation momentum: \( g(x, Q_s^2)/Q_s^2 R^2 \sim 1/\alpha_s \)

\( Q_s \) at small-\( x \) much larger than \( \Lambda_{QCD} \sim 200\text{GeV} \)

High density, weak coupling, non-linear dynamics
Diagrams for dipole evolution

- Probe system with color dipole. Evolution:
  - LO
  - NLO $N_f$
  - NLO $N_C$

- Right mover. Lower $k^+$ longitudinal momentum.
The BK equation at NLO

\[
\frac{dS_{12}}{dY} = \frac{\alpha_s}{2\pi} \int d^2 z_3 \frac{z_{12}^2}{z_{13}^2 z_{23}^2} \left[ 1 + \alpha_s \left( \bar{b} \ln z_{12}^2 - \bar{b} \frac{z_{13}^2 - z_{23}^2}{z_{12}^2} \ln \frac{z_{13}^2}{z_{23}^2} + \frac{67}{36} - \frac{\pi^2}{12} - \frac{5}{18} \frac{N_f}{N_c} \right) \right] (S_{13}S_{32} - S_{12}) \\
+ \frac{\alpha_s^2}{8\pi^2} \int d^2 z_3 \frac{d^2 z_4}{z_{34}^4} \left[ -2 + \frac{z_{13}^2 z_{24}^2 + z_{14}^2 z_{23}^2 - 4z_{12}^2 z_{34}^2}{z_{13}^2 z_{24}^2 - z_{14}^2 z_{23}^2} \ln \frac{z_{13}^2 z_{24}^2}{z_{14}^2 z_{23}^2} \right] \\
\left[ S_{13}S_{34}S_{42} - \frac{1}{2N_c^3} \text{tr} (V_1 V_3 V_4 V_4 V_3 V_2 V_1 V_3) - \frac{1}{2N_c^3} \text{tr} (V_1 V_4 V_3 V_2 V_4 V_1) - S_{13}S_{32} + \frac{1}{N_c^2} S_{12} \right] \\
+ \frac{\alpha_s^2 N_f}{8\pi^2 \frac{N_f}{N_c}} \int d^2 z_3 \frac{d^2 z_4}{z_{34}^4} \left[ 2 - \frac{z_{13}^2 z_{24}^2 + z_{14}^2 z_{23}^2 - 4z_{12}^2 z_{34}^2}{z_{13}^2 z_{24}^2 - z_{14}^2 z_{23}^2} \ln \frac{z_{13}^2 z_{24}^2}{z_{14}^2 z_{23}^2} \right] \\
\left[ S_{14}S_{32} - \frac{1}{N_c^3} \text{tr} (V_1 V_2 V_3 V_4) - \frac{1}{N_c^3} \text{tr} (V_1 V_3 V_1 V_4) + \frac{1}{N_c^2} S_{12}S_{34} - S_{13}S_{32} + \frac{1}{N_c^2} S_{12} \right]
\]

\[z_{ij} = z_i - z_j \quad S_{ij} = \frac{1}{N_c} \text{tr} (V_i V_j^\dagger) \quad V_i^\dagger = \text{P exp} \left[ ig \int dz^+ A_a^- (z^+, z_i) t^a \right] \]

\[\bar{b} = \frac{11}{12} - \frac{1}{6} \frac{N_f}{N_c} \]

See also Kovner, Lublinsky, Mulian 14
Large transverse logs

- Strongly ordered large "perturbative" dipoles (DLA)

\[ \frac{1}{Q_s} \gg z_{14} \simeq z_{24} \simeq z_{34} \gg z_{13} \simeq z_{23} \gg z_{12} \]

- Large dipoles interact stronger, real terms only (\(N_f=0\))

\[
\frac{dT_{12}}{dY} = \bar{\alpha}_s \int_{z_{12}^2}^{1/Q_s^2} \frac{dz_{12}^2}{z_{13}^4} \left( 1 - \bar{\alpha}_s \frac{1}{2} \ln^2 \frac{z_{13}^2}{z_{12}^2} - \bar{\alpha}_s \frac{11}{12} \ln \frac{z_{13}^2}{z_{12}^2} \right) T_{13}
\]

- NLO > LO, unstable expansion in coupling.

Simple but general IC: color transparency + saturation

\[
T_{12} = \begin{cases} 
\frac{z_{12}^2 Q_s^2}{\bar{\alpha}_s \Delta Y} & , \ z_{12} Q_s \ll 1 \\
1 & , \ z_{12} Q_s \gg 1
\end{cases} \Rightarrow \frac{\Delta T_{12}}{\bar{\alpha}_s \Delta Y} \simeq z_{12}^2 Q_s^2 \left( \ln \frac{1}{z_{12}^2 Q_s^2} - \bar{\alpha}_s \frac{1}{6} \ln^3 \frac{1}{z_{12}^2 Q_s^2} - \frac{11}{24} \bar{\alpha}_s \ln^2 \frac{1}{z_{12}^2 Q_s^2} \right)
\]
Unstable numerical solutions

Avsar, Stasto, DT, Zaslavsky 11

Lappi, Mantysaari 15

Resummation in non-linear QCD
Two gluons and time ordering (kinematics)

- Hard to soft projectile evolution \( k \ll p \) and \( k^+ \ll p^+ \)
- Energy denominators lead to largest logs when emissions are time-ordered \( \tau_k \approx k^+ z_4^2 \ll \tau_p \approx p^+ z_3^2 \)
- Leads to double log term in NLO BK equation

\[
\Delta T_{12} = \bar{\alpha}_s^2 \int \frac{dp^+}{p^+} \frac{dk^+}{k^+} \Theta\left(p^+ \frac{z_3^2}{z_4^2} - k^+ \right) dz_3^2 dz_4^2 \frac{z_{12}^2}{z_3^2 z_4^2} T(z_4) \rightarrow - \frac{\bar{\alpha}_s^2 \Delta Y}{2} \int_{z_{12}^2}^{1/Q_s^2} \frac{dz_4^2}{z_4^2} \ln^2 \frac{z_4^2}{z_{12}^2} T(z_4)
\]
Resummation of double logs in DLA

- Systematically resum to all orders in non-local equation

\[
\frac{dT(Y, z_{12}^2)}{dY} = \bar{\alpha}_s \int_{z_{12}^2}^{1/Q_s^2} \frac{dz_{13}^2}{z_{13}^2} \frac{z_{12}^2}{z_{13}^2} \Theta \left( Y - \ln \frac{z_{13}^2}{z_{12}^2} \right) T \left( Y - \ln \frac{z_{13}^2}{z_{12}^2}, z_{13}^2 \right)
\]

- Mathematically equivalent to local equation

\[
\frac{dT(Y, z_{12}^2)}{dY} = \bar{\alpha}_s \int_{z_{12}^2}^{1/Q_s^2} \frac{dz_{13}^2}{z_{13}^2} \frac{z_{12}^2}{z_{13}^2} \frac{J_1 \left( 2\sqrt{\bar{\alpha}_s \ln^2 \frac{z_{13}^2}{z_{12}^2}} \right)}{\sqrt{\bar{\alpha}_s \ln^2 \frac{z_{13}^2}{z_{12}^2}}} T(Y, z_{13}^2)
\]

with modified initial condition (impact factor)

\[
T(0, z_{12}^2) \propto \frac{C_F}{N_c} z_{12}^2 Q_{s0}^2 \sqrt{\bar{\alpha}_s} J_1 \left( 2\sqrt{\bar{\alpha}_s \ln^2 \frac{1}{z_{12}^2 Q_{s0}^2}} \right)
\]
Resummation of double logs in BK

- Promote local equation to include BK physics

\[
\frac{dS_{12}}{dY} = \frac{\bar{\alpha}_s}{2\pi} \int d^2z_3 \frac{z_{12}^2}{z_{13}^2 z_{23}^2} \frac{J_1 \left( 2\sqrt{\bar{\alpha}_s L_{13} L_{23}} \right)}{\sqrt{\bar{\alpha}_s L_{13} L_{23}}} (S_{13}S_{32} - S_{12})
\]

with \( L_{13} L_{23} = \ln \frac{z_{13}^2}{z_{12}^2} \ln \frac{z_{23}^2}{z_{12}^2} \)

Equivalent non-local equation by Beuf 14

- NLO BK (double log term) when truncated to order \( \bar{\alpha}_s^2 \)

- Exactly resums double log terms to all orders
Numerical solution

BFKL on $z_{12}^{2\gamma} \equiv r^{2\gamma}$

\begin{align*}
\omega_{\text{LO}} &= \frac{\tilde{\alpha}_s}{\gamma} + \frac{\tilde{\alpha}_s}{1 - \gamma} + \text{finite} \\
\omega_{\text{NLO}} &= \frac{\tilde{\alpha}_s}{\gamma} + \frac{\tilde{\alpha}_s}{1 - \gamma} - \frac{\tilde{\alpha}_s^2}{(1 - \gamma)^3} + \text{finite} \\
\omega_{\text{NLO}}^{\text{res}} &= \omega_{\text{NLO}} - \frac{\tilde{\alpha}_s}{1 - \gamma} + \frac{\tilde{\alpha}_s^2}{(1 - \gamma)^3} + \frac{1}{2} \left[ -(1 - \gamma) + \sqrt{(1 - \gamma)^2 + 4\tilde{\alpha}_s} \right] + \text{finite}
\end{align*}
Numerical solution

- Considerable speed reduction, roughly factor of 1/2
Single log in quark contribution (dynamics)

- Take $k \ll p$ and $\zeta = k^+/p^+ \ll 1$
- hard to soft projectile evolution
- Quark contribution is easier, no DLs
- Integrate transverse momenta

$$\Sigma A_{i,j} = \frac{\alpha_s^2 N_f}{2\pi^4} \Delta Y \int_0^1 d\zeta \frac{z_1^{22}}{z_3^{24} + \zeta^2 z_4^{24}} \approx \frac{\alpha_s^2 N_f}{3\pi^4} \Delta Y \frac{z_1^{22}}{z_3^{24} z_4^{24}}$$

- $\zeta z_4^2 \sim z_3^3$, no time ordering. Integrand $P_{qG}$ split. function
- Insert color structure and scattering

$$\frac{\Delta T_{12}}{\Delta Y} = -\frac{\bar{\alpha}_s^2 N_f}{6N_c^3} z_{12} \int_{z_{12}^{24}}^{1/Q_s^{24}} \frac{dz_4^{24}}{z_4^{24}} \ln \frac{z_4^{24}}{z_{12}^{24}} T(z_4)$$

Resummation in non-linear QCD
DGLAP mixes quarks and gluons. Largest eigenvalue of moments:

\[
\int_0^1 \! dz \, z^\omega \left[ P_{GG}(z) + \frac{C_F}{N_c} P_{qG}(z) \right] = \frac{1}{\omega} - \frac{11}{12} - \frac{N_f}{6N_c^3} + \mathcal{O}(\omega)
\]

\[\equiv A_1\]

Similar hard to soft gluon diagrams must give -11/12

All this is DGLAP physics. “Normally” it is soft to hard.
Imagine starting from LM target parton with large \( q_0^- \). Evolve up to small projectile by emitting partons with smaller size and smaller minus long. momentum.

\[
\eta = \ln \frac{q_0^-}{q^-} = \ln \left( \frac{Q_0^2}{q_0^+} q^+ z_{12}^2 \right) = Y - \ln \frac{1}{z_{12}^2 Q_0^2}
\]
General DGLAP solution

\[ T_{12}(Y) \approx z_{12}^2 Q_0^2 x G \left( \eta, \ln \frac{1}{z_{12}^2 Q_0^2} \right) \approx \int \frac{d\omega}{2\pi i} \exp \left\{ \omega Y + \left[ \bar{\alpha}_s \mathcal{P}(\omega) - \omega - 1 \right] \ln \frac{1}{z_{12}^2 Q_0^2} \right\} \]

Solve for \( \omega \) as function of \( \gamma \). Keep only up to \( A_1 \). One, two, three, … subleading splittings resummed. Exponential combinatorics.

\[ \frac{dS_{12}}{dY} = \frac{\bar{\alpha}_s}{2\pi} \int d^2 z_3 \frac{z_{12}^2}{z_{13}^2 z_{23}^2} \left( \frac{z_{12}^2}{z_{<}} \right)^{\bar{\alpha}_s A_1} J_1 \left( 2\sqrt{\bar{\alpha}_s L_{13} L_{23}} \right) \frac{\sqrt{\bar{\alpha}_s L_{13} L_{23}}}{(S_{13}S_{32} - S_{12})} \]

\[ z_{<} = \min\{z_{13}, z_{23}\} \text{. + sign when } z_{<} < z_{12} \]
Running coupling

\[ \frac{dS_{12}}{dY} = \frac{\bar{\alpha}_s(\mu)}{2\pi} \int d^2 z_3 \frac{z_{12}^2}{z_{13}^2 z_{23}^2} \left[ 1 + \bar{\alpha}_s(\mu) \left( \bar{b} \ln z_{12}^2 \mu^2 - \bar{b} \frac{z_{13}^2 - z_{23}^2}{z_{12}^2} \ln \frac{z_{13}^2}{z_{23}^2} \right) \right] (S_{13}S_{32} - S_{12}) \]

- Choose \( \mu \) to cancel potentially large log in all regions
  - Large daughter dipoles: \( \mu \approx 1/z_{12} \)
  - Small daughter dipole: \( \mu \approx 1/\min\{z_{13}, z_{23}\} \)
  - In general: \( \mu \approx 1/\min\{z_{i,j}\} \quad \checkmark \quad \text{Hardest scale} \)

- Balitsky-prescription: \( \checkmark \), albeit unphysical slow

- Choose coefficient of \( \bar{b} \) to vanish: \( \checkmark \)

\[ \alpha_s = \left[ \frac{1}{\alpha_s(z_{12})} + \frac{z_{13}^2 - z_{23}^2}{z_{12}^2} \frac{\alpha_s(z_{13}) - \alpha_s(z_{23})}{\alpha_s(z_{13}) \alpha_s(z_{23})} \right]^{-1} \]
Couplings comparison

\[ \alpha_{\min} \quad \alpha_{\text{fac}} \quad \alpha_{\text{Bal}} \]

\[ |x-y|=1 \quad \phi=0 \]

\[ |x-z| \]

\[ \phi = \pi/6 \]

\[ |x-y|=1 \quad \phi=\pi/6 \]

\[ \phi \]

\[ \alpha_{\min} \quad \alpha_{\text{fac}} \quad \alpha_{\text{Bal}} \]

\[ |x-z|=1.5 \]

\[ \phi \]

Resummation in non-linear QCD
Numerical solution (prescription: small)

DLA+SL resum, $\beta_0=0.72$, smallest

$T(\rho, Y)$

speed, $\beta_0=0.72$, smallest

See also Lappi Mantysaari 16
Resummation in non-linear QCD

See also Albacete 15 (Beuf eqn)
Fit

- No anomalous dimension in initial condition
- Including single logs: more physical parameters
- MV model: can be extrapolated to higher $Q^2$
- Smallest dipole prescription: best fit
- B-prescription: not very good

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Conclusion - Outlook

- Stable, slow, evolution with resummed dominant logs

- Insert formalism into more exclusive observables e.g. particle production at forward rapidity and calculate

- Understand better hard to soft DGLAP evolution

- Structurally different evolution than the SUSY one