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XIIth Quark Confinement and the Hadron Spectrum
August 18, 2016
Elliptic-flow phenomenon

J.-Y. Ollitrault, PRD (1992)


Strong elliptic flow ($v_2$) at RHIC and the LHC
⇒ collective behavior
⇒ correlations between positions and momenta

Good description in terms of perfect fluid hydrodynamics
⇒ strongly-interacting system
⇒ cross sections formally set equal to $\infty$


\[
\frac{dN}{d\phi} = \frac{N}{2\pi} \left[ 1 + 2v_1 \cos(\phi) + 2v_2 \cos(2\phi) + \ldots \right]
\]

\[
v_n(p_T, y) = \frac{\int d\phi \cos(n\phi) \frac{dN}{d\phi}}{\int d\phi \frac{dN}{d\phi}} \equiv \langle \cos(n\phi) \rangle
\]

RHIC: early perfect-fluid calculations, U. Heinz et al., E. Shuryak and D. Teaney, ...
Hydrodynamics of perfect fluid

main assumption: system is in local thermal equilibrium

- conservation of the baryon number (and other conserved charges), energy and momentum

\[ \partial_{\mu} N_{\mu} = 0 \quad N_{\mu} \equiv n u_{\mu} \]
\[ \partial_{\mu} T_{\mu\nu}^{id} = 0 \quad T_{\mu\nu}^{id} \equiv \mathcal{E} u_{\mu} u_{\nu} - \Delta_{\mu\nu} P \]

6 independent variables: \((\mathcal{E}, P, n, u_{\mu}(3))\)

6 equations (equation of state \(\mathcal{E}(n, P)\))

in ultra relativistic collisions we may neglect the baryon number conservation
Hydrodynamics of perfect fluid

**main assumption:** system is in local thermal equilibrium

- conservation of energy and momentum

\[
\begin{align*}
\partial_\mu N^\mu &= 0 \\
\partial_\mu T^{\mu\nu}_{id} &= 0 \\
N^\mu &= n u^\mu \\
T^{\mu\nu}_{id} &= E u^\mu u^\nu - \Delta^{\mu\nu} P
\end{align*}
\]

- \( \partial_\mu \equiv \frac{\partial}{\partial x^\mu} \)
- \( \Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu \)

- 5 independent variables \((E, P, n, u^\mu (3))\)
- 5 equations (equation of state \(E(n, P)\))

- in ultra relativistic collisions we may neglect the baryon number conservation


\[ R. \text{ Kuiper and G. Wolschin, Annalen Phys. 16, 67 (2007)} \]
Hydrodynamics of perfect fluid


EOS can be checked experimentally by looking at the HBT correlations that give information about the space-time extensions of the system.

Further evidence from complete hydro simulations:
QGP as almost perfect fluid

- quantum mechanics (Danielewicz, Gyulassy, 1985) / AdS/CFT (Kovtun, Son, Starinets, 2005)
  lower bound on viscosity $\eta/S > 1/4\pi$
  ⇒ one should use relativistic dissipative hydrodynamics
  ⇒ better description (assuming small $\eta/S = \text{const}$)

Viscosity

shear viscosity $\eta$

$\Downarrow$

reaction to a change of shape

\[ \pi_{\text{Navier-Stokes}}^{\mu\nu} \propto \eta (\partial^{\mu} u^{\nu}) \]

bulk viscosity $\zeta$

$\Downarrow$

reaction to a change of volume

\[ \Pi_{\text{Navier-Stokes}} \propto \zeta (\partial_{\mu} u^{\mu}) \]

bulk viscosity and pressure vanish for conformal fluids
The ninth drop touched the eighth drop on 17 April 2014. However, it was still attached to the funnel. On 24 April 2014, Prof. White decided to replace the beaker holding the previous eight drops before the ninth drop fused to them. While the bell jar was being lifted, the wooden base wobbled and the ninth drop snapped away from the funnel.

\[ \eta_{\text{qgp}} > \eta_{\text{pitch}} \]

\[ \eta_{\text{qgp}} \sim 10^{11} \text{ Pa s,} \quad (\eta/s)_{\text{qgp}} < 3/(4\pi)\hbar \quad \text{(from experiment)} \]
Shear and bulk viscosity of the plasma

- $\eta/S$ reaches minimum in the region of the phase transition

- $\zeta/S$ reaches maximum in the region of the phase transition

Dissipative hydrodynamics

Navier-Stokes equations — algebraic equations — $T, u^\mu$ are the only hydrodynamic variables

- energy-momentum conservation

\[ \partial_\mu T_{\text{vis}}^{\mu\nu} = 0 \quad T_{\text{vis}}^{\mu\nu} = \mathcal{E} u^\mu u^\nu - \Delta^{\mu\nu} (P + \Pi) + \pi^{\mu\nu} \]

- number of equations: $5 + 6 (\mathcal{E}, P, u^\mu (3), \Pi, \pi^{\mu\nu} (5))$
- number of equations: $4 + 1$ (equation of state $\mathcal{E}(P)$)
- we need 6 extra equations - different methods possible

\[
\begin{align*}
\dot{\Pi} + \frac{\Pi}{\tau_\Pi} &= -\beta_\Pi \theta, \quad \theta = \partial_\mu u^\mu \\
\dot{\pi}^{\mu\nu} + \frac{\pi^{\mu\nu}}{\tau_\pi} &= 2\beta_\pi \sigma^{\mu\nu}, \quad \sigma^{\mu\nu} - \text{shear flow tensor constructed from the velocity field } u^\mu
\end{align*}
\]

kinetic coefficients: $\tau_\Pi \beta_\Pi = \zeta \rightarrow \text{bulk viscosity}, \tau_\pi \beta_\pi = \eta \rightarrow \text{shear viscosity}$
Dissipative hydrodynamics

Israel-Stewart equations — $\Pi, \pi^{\mu\nu}$ promoted to dynamic variables — non-hydrodynamic modes are introduced with the appropriate relaxation times $\tau_\Pi, \tau_\pi$

- Energy-momentum conservation

$$\partial_\mu T^\mu_{\text{vis}} = 0 \quad T^\mu_{\text{vis}} = \mathcal{E} u^\mu u^\nu - \Delta^\mu\nu (P + \Pi) + \pi^{\mu\nu}$$

- Number of equations: $5 + 6$ ($\mathcal{E}, P, u^\mu (3), \Pi, \pi^{\mu\nu} (5)$)

- Number of equations: $4 + 1$ (equation of state $\mathcal{E}(P)$)

- We need 6 extra equations - different methods possible

$$\dot{\Pi} + \frac{\Pi}{\tau_\Pi} = -\beta_\Pi \theta + \lambda_\Pi \pi^{\mu\nu} \sigma_{\mu\nu}$$

$$\dot{\pi}^{\mu\nu} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi \sigma^{\mu\nu} - \tau_\pi \pi^{\mu\nu} \sigma^{\mu\nu} + \lambda_\pi \Pi \rho^{\mu\nu}$$

$\tau_\Pi \beta_\Pi = \zeta \rightarrow$ bulk viscosity, $\tau_\pi \beta_\pi = \eta \rightarrow$ shear viscosity
Müller-Israel-Stewart (MIS)

- energy-momentum conservation

\[ \partial_\mu T_{\text{vis}}^{\mu\nu} = 0 \quad T_{\text{vis}}^{\mu\nu} = \varepsilon u^\mu u^\nu - \Delta^{\mu\nu}(\mathcal{P} + \Pi) + \pi^{\mu\nu} \]

- number of equations: 5 + 6 (\varepsilon, \mathcal{P}, u^\mu (3), \Pi, \pi^{\mu\nu} (5))
- number of equations: 4 + 1 (equations of state \varepsilon(\mathcal{P}))
- we need 6 extra equations - different methods possible

\[ \dot{\Pi} + \frac{\Pi}{\tau_{\Pi}} = -\beta_\Pi \theta - \frac{\zeta T}{2\tau_{\Pi}} \Pi \partial_\lambda \left( \frac{\tau_{\Pi}}{\zeta T} u^\lambda \right) \]

\[ \dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_{\pi}} = 2\beta_\pi \sigma^{\mu\nu} - \frac{\eta T}{2\tau_{\pi}} \pi^{\mu\nu} \partial_\lambda \left( \frac{\tau_{\pi}}{\eta T} u^\lambda \right) \]
Dissipative hydrodynamics

New approaches add new terms: Baier, Romatschke, Son, Starinets, Stephanov (BRSSS)
symmetry arguments due to conformal symmetry, ...

- energy-momentum conservation

\[
\partial_\mu T^{\mu\nu}_{\text{vis}} = 0 \\
T^{\mu\nu}_{\text{vis}} = \mathcal{E} u^\mu u^\nu - \Delta^{\mu\nu} (P + \Pi) + \pi^{\mu\nu}
\]

- number of equations: 5 + 6 (\(\mathcal{E}, P, u^\mu (3), \Pi, \pi^{\mu\nu} (5)\))
- number of equations: 4 + 1 (equations of state \(\mathcal{E}(P)\))
- we need 6 extra equations - different methods possible

\[
\Pi + \frac{\pi^{\mu\nu}}{\tau_\pi} = 0 \\
\dot{\pi}^{(\mu\nu)} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi \sigma^{\mu\nu} - \frac{4}{3} \pi^{\mu\nu} \theta + \frac{\lambda_1}{\tau_\pi \eta^2} \pi^{(\mu}_{\lambda} \pi^{\nu)\lambda} + \text{terms including vorticity and curvature}
\]
Dissipative hydrodynamics

New approaches add new terms: Denicol, Niemi, Molnar, Rischke (DNMR) simultaneous expansion in the Knudsen number and inverse Reynolds number

- energy-momentum conservation

\[ \partial_{\mu} T_{\text{vis}}^{\mu\nu} = 0 \]
\[ T_{\text{vis}}^{\mu\nu} = \varepsilon u^{\mu} u^{\nu} - \Delta^{\mu\nu} (\mathcal{P} + \Pi) + \pi^{\mu\nu} \]

- number of equations: 5 + 6 (\( \varepsilon, \mathcal{P}, u^{\mu}(3), \Pi, \pi^{\mu\nu} (5) \))
- number of equations: 4 + 1 (equations of state \( \varepsilon(\mathcal{P}) \))
- we need 6 extra equations - different methods possible

\[ \dot{\Pi} + \frac{\Pi}{\tau_{\Pi}} = -\beta_{\Pi} \theta - \delta_{\Pi\Pi} \Pi \theta + \lambda_{\Pi\pi} \pi^{\mu\nu} \sigma_{\mu\nu} \]
\[ \dot{\pi}^{(\mu\nu)} + \frac{\pi^{(\mu\nu)}}{\tau_{\pi}} = 2\beta_{\pi} \sigma^{\mu\nu} + 2\pi^{(\mu}_{\gamma} \omega^{\nu)}_{\gamma} - \delta_{\pi\pi} \pi^{\mu\nu} \theta - \tau_{\pi\pi} \pi^{(\mu}_{\gamma} \sigma^{\nu)}_{\gamma} + \lambda_{\pi\Pi} \Pi \sigma^{\mu\nu} \]

RTA version of the Boltzmann kinetic equation, neglected vorticity for standard form of the collision term additional terms (with new kinetic coefficients) appear shear-bulk coupling \( \eta - \zeta \)
"Standard model" of evolution of matter produced in heavy-ion collisions

- initial conditions including fluctuations ($0 < \tau_0 \lesssim 1 \text{ fm}$)
- non-equilibrium phase and THERMALIZATION ($\rightarrow$HYDRODYNAMIZATION) of matter ($\tau_0 < \tau \lesssim 1 \text{ fm}$) ⇒ emission of hard probes: heavy quarks, photons, jets
- hydrodynamic expansion (expansion and cooling) ($1 \text{ fm} \lesssim \tau \lesssim 10 \text{ fm}$)

Microscopic description of such a many body system is very complicated ↓ effective description in terms of fluid mechanics

- phase transition from QGP to hadron gas (encoded in the equation of state)
- freeze-out and free streaming of hadrons ($10 \text{ fm} \lesssim \tau$)
Hydrodynamics vs. kinetic theory

In the kinetic theory the basic quantity is the one-particle phase-space distribution function:

- **Perfect-fluid hydrodynamics**: local thermal equilibrium
  \[
  f(x, p) = f_{iso} \left( \frac{p \mu u_{\mu}}{T(x)} \right) \Rightarrow T_{id}^{\mu\nu} = \int dP p^\mu p^\nu f(x, p)
  \]

- **Dissipative hydrodynamics**: linear deviations from local equilibrium
  \[
  f(x, p) = f_{iso} \left( \frac{p \mu u_{\mu}}{T(x)} \right) + \delta f(x, p) \Rightarrow T_{vis}^{\mu\nu} = T_{id}^{\mu\nu} + \delta T_{id}^{\mu\nu}
  \]

**URHIC → Extreme spacetime scales**

- Very small systems
- Very large gradients
- Very fast expansion

Dissipative corrections are substantial

Standard dissipative hydrodynamics assumes that the system is always close to local equilibrium, this is in contrast with microscopic calculations showing that plasma is highly anisotropic in the momentum space (at the early stages of the evolution).
Relaxation-time-approximation (RTA) kinetic equation

\[ p^\mu \partial_\mu f(x, p) = p \cdot u \frac{f_{eq}(x, p) - f(x, p)}{\tau_{eq}} \]

solutions known for one-dimensional, boost-invariant system, G. Baym, PLB 138 (1984)
used for comparisons between the underlying kinetic theory and hydrodynamic approaches

One dimensional expansion

1. conformal case
2. non-conformal case
3. non-conformal case with quantum statistics
4. mixtures

(1+1)D flow with Gubser symmetry

Denicol, Heinz, Martinez, Noronha, Strickland
\[ \tau_n \dot{\Pi} + \Pi = -\frac{\zeta}{\tau} - \frac{1}{2} \tau_n \Pi \left[ \frac{1}{\tau} - \left( \frac{\dot{\zeta}}{\zeta} + \frac{\dot{\tau}}{\tau} \right) \right] \]  

(A)


\[ \tau_n \dot{\Pi} + \Pi = -\frac{\zeta}{\tau} - \frac{4}{3} \tau_n \Pi \frac{1}{\tau} \]  

(B)


\[ \tau_n \dot{\Pi} + \Pi = -\frac{\zeta}{\tau} \]  

(C)

exact solution and all 2nd order viscous hydrodynamics variations tend toward the 1st order solution at late times

none of the 2nd order viscous hydrodynamics variations seems to qualitatively describe the early-time evolution of the bulk viscous pressure in all cases

there is something incomplete in the manner in which 2nd order viscous hydrodynamics treats the bulk pressure (neglected shear-bulk coupling)
Bulk viscous pressure evolution within full viscous anisotropic hydrodynamics with SHEAR-BULK COUPLING

the shear-bulk couplings are extremely important for correct description of the bulk viscous correction
Hydrodynamic gradient expansion

M. P. Heller, R. A. Janik, M. Spalinski, P. Witaszczyk

Formal expansion of $T^{\mu\nu}$ in gradients of hydrodynamic variables $T$ and $u^\mu$

$$T^{\mu\nu} = T^{\mu\nu}_{eq} + \text{powers of gradients of } T \text{ and } u^\mu$$

Simple structures for boost-invariant flow with the relaxation time $\tau_\pi = \frac{c}{T}$, for example, $T$ is expanded around the Bjorken flow

$$T = T_0 \left( \frac{\tau_0}{\tau} \right)^{1/3} \left( 1 + \sum_{n=1}^{\infty} \left( \frac{C}{T_0 \tau_0} \right)^n t_n \left( \frac{\tau_0}{\tau} \right)^{2n/3} \right)$$

Change to $g(w)$

$$g = \frac{1}{T} \frac{dw}{d\tau}, \quad w = \tau T, \quad \Delta = \frac{\Delta P}{P} = 3 \frac{P_\parallel - P_\perp}{\varepsilon} = 12 \left( g - \frac{2}{3} \right)$$

The gradient expansion for boost-invariant flow takes the form of an expansion

$$g(w) = \sum_{n=0}^{\infty} g_n w^{-n}, \quad g_0 = \frac{2}{3}$$
Hydrodynamic gradient expansion

RTA - gradient expansion for the RTA kinetic-theory model – Heller, Kurkela, Spalinski, to be published
WF, R. Ryblewski, M. Spalinski, arXiv:1608.07558

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compared to two versions of anisotropic hydrodynamics:
AH1: L. Tinti, WF, PRC89 (2014) 034907
AH2: L. Tinti, PRC in print

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the series $g_n$, has vanishing radius of convergence, the Borel transform of $g$ is introduced, analytic continuation using diagonal Padé approximants of order 70 is done

$$g_B(\xi) = \sum_{n=0}^{\infty} \frac{g_n}{n!} \xi^n,$$

the cut along the real axis indicates the presence of a single nonhydrodynamic mode, which is purely decaying, as in MIS theory.
I do not know, see the Round Table discussion on Friday

one may speculate: 10 equations for the energy-momentum tensor, 4 follow from the energy-momentum tensor, 6 describe dissipation, characteristic damping rate of about 1 fm/c, hydro should work for systems with a size of about few fm, ...

Shuryak: consequence of the QCD (approximate) scale invariance

Romatschke, Spalinski: hydrodynamic description is valid if hydro is insensitive to non-hydrodynamic modes – criterion satisfied also for small systems, hydro description is connected with hydrodynamic modes that are universal (the same viscosity coefficients for different systems, ...)

...............
SUMMARY

QCD EOS

known from first principles at $\mu_B = 0$, restricted by the data analysis of the HBT correlations, semi hard (large values of $c_s$), faster evolution of the system, shorter time for equilibration of matter produced in heavy-ion collisions, smooth background in the Early Universe (does not lead to structures typical for the first order phase transition)

THERMALIZATION (LOCAL EQUILIBRATION) $\rightarrow$ HYDRODYNAMIZATION

small shear viscosity to entropy ratio + large gradients $(1/t)$ = large pressure corrections

AdS/CFT $\rightarrow$ large pressure corrections

BULK VISCOSITY

correlation $\rightarrow$ large pressure corrections

large at the QCD phase transition, strongly coupled to the shear sector (shear viscosity), to be included (more systematically) in cosmological models or astrophysical calculations (??), to be extracted from heavy-ion collisions

RELATIVISTIC VISCOSOUS HYDRODYNAMICS

one should use the complete versions of relativistic hydrodynamics or possible extensions such as ANISOTROPIC HYDRODYNAMICS $\rightarrow$ see the next talk by M. Strickland

(HYDRODYNAMIC) GRADIENT EXPANSION

useful for comparisons of different theories, gives information about the close-to-equilibrium behavior and non-hydrodynamical modes