Colectivity & Hydrodynamics

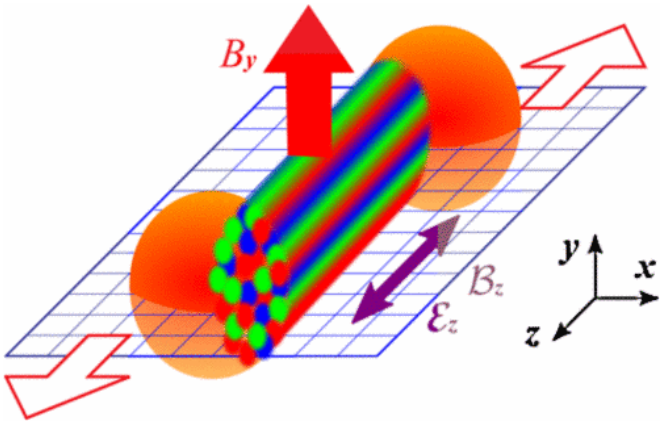
Wojciech Florkowski

Institute of Nuclear Physics, Krakow and Jan Kochanowski University, Kielce, Poland

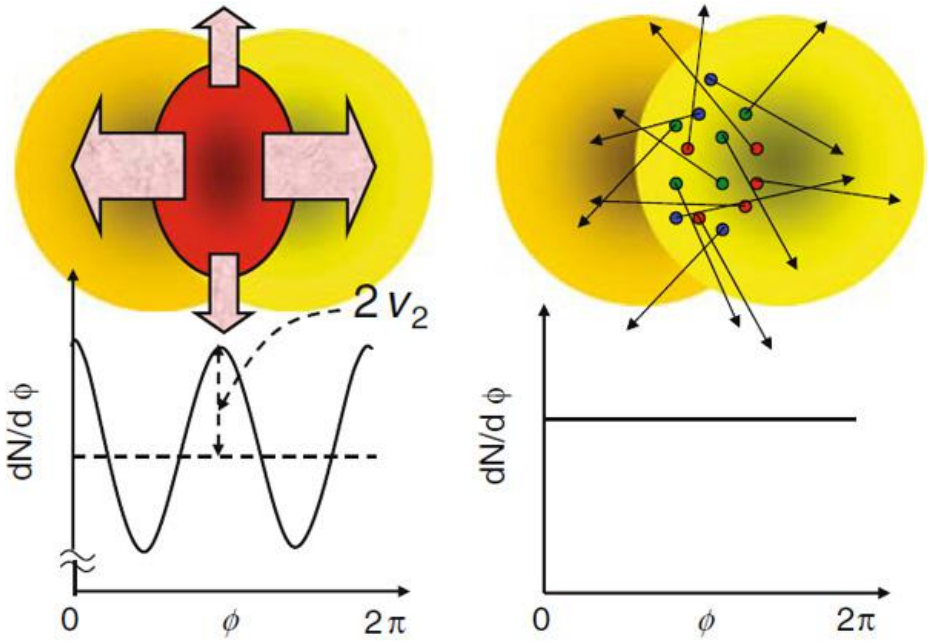
XIIth Quark Confinement and the Hadron Spectrum
August 18, 2016

Elliptic-flow phenomenon

J.-Y. Ollitrault, PRD (1992)



K. Fukushima, D. E. Kharzeev, H. J. Warringa, PRL 104, 212001



T. Hirano, N. van der Kolk, A.Bilandzic, Lect.Notes Phys. 785 (2010) 139

strong elliptic flow (v_2) at RHIC and the LHC
⇒ collective behavior
⇒ correlations between positions and momenta

good description in terms of perfect fluid hydrodynamics
⇒ strongly-interacting system
⇒ cross sections formally set equal to ∞

$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left[1 + 2v_1\cos(\phi) + 2v_2\cos(2\phi) + ...\right]$$

$$v_n(p_T, y) = \frac{\int d\phi \cos(n\phi)\frac{dN}{d\phi}}{\int d\phi \frac{dN}{d\phi}} \equiv \langle\cos(n\phi)\rangle$$

RHIC: early perfect-fluid calculations, U. Heinz et al., E. Shuryak and D. Teaney, ...

Hydrodynamics of perfect fluid

main assumption: system is in local thermal equilibrium

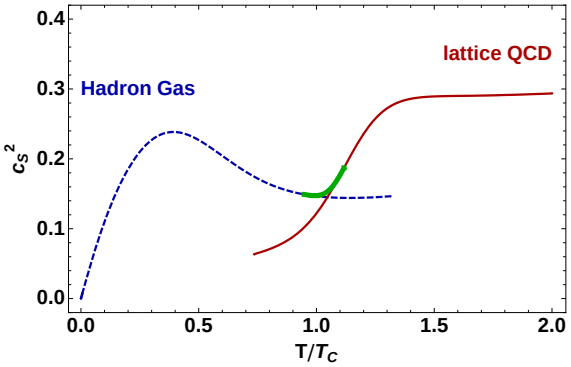
- conservation of the baryon number (and other conserved charges), energy and momentum

$$\partial_\mu N^\mu = 0 \qquad N^\mu \equiv n u^\mu$$

$$\partial_\mu T^{\mu\nu}_{id} = 0 \qquad T^{\mu\nu}_{id} \equiv \mathcal{E} u^\mu u^\nu - \Delta^{\mu\nu}\mathcal{P}$$

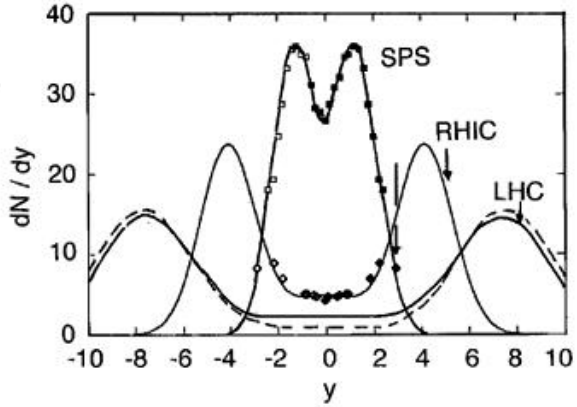
$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu} \qquad \Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu$$

- 6 independent variables: $(\mathcal{E}, \mathcal{P}, n, u^\mu(3))$
- 6 equations (equation of state $\mathcal{E}(n, \mathcal{P})$)



M. Chojnacki, W. Florkowski, Acta Phys.Polon. B38 (2007) 3249-3262

- in ultra relativistic collisions we may neglect the baryon number conservation



R. Kuiper and G. Wolschin, Annalen Phys. 16, 67 (2007)

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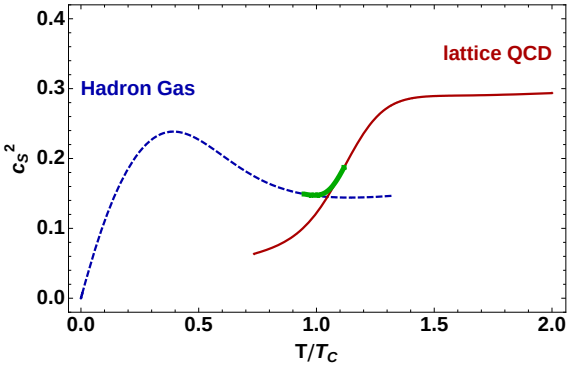
- conservation of energy and momentum

$$\cancel{\partial_\mu N^\mu = 0} \qquad \cancel{N^\mu = n u^\mu}$$

$$\partial_\mu T^{\mu\nu}_{id} = 0 \qquad T^{\mu\nu}_{id} \equiv \mathcal{E} u^\mu u^\nu - \Delta^{\mu\nu}\mathcal{P}$$

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu} \qquad \Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu$$

- 5 independent variables ($\mathcal{E}, \mathcal{P}, \cancel{n}, u^\mu(3)$)
- 5 equations (equation of state $\mathcal{E}(\cancel{n}, \mathcal{P})$)



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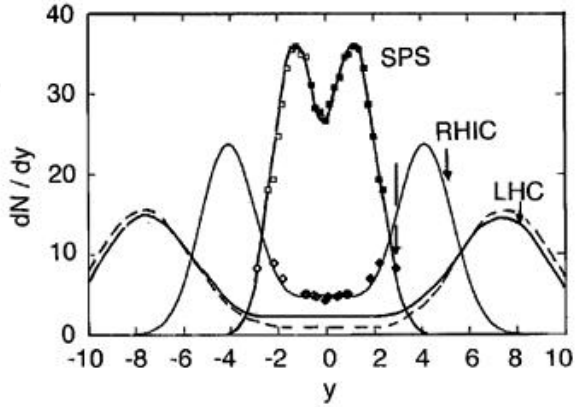
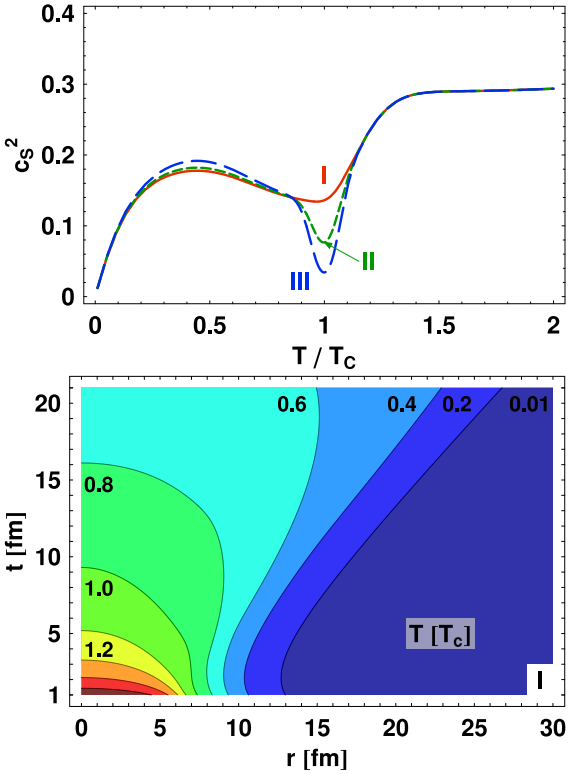


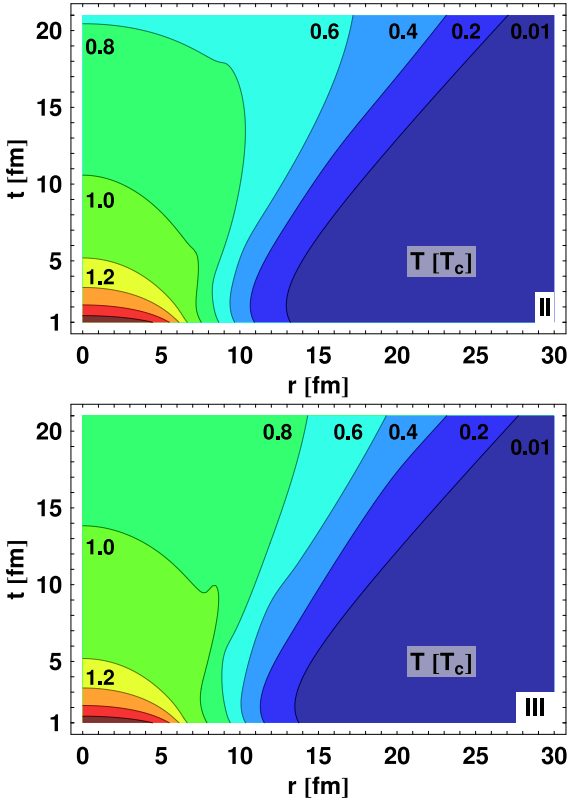
R. Kuiper and G. Wolschin, Annalen Phys. 16, 67 (2007)

Hydrodynamics of perfect fluid

M. Chojnacki, W. Florkowski, Acta Phys.Polon. B38 (2007) 3249, isotherms depending on the form of EOS

EOS can be checked experimentally by looking at the HBT correlations that give information about the space-time extensions of the system





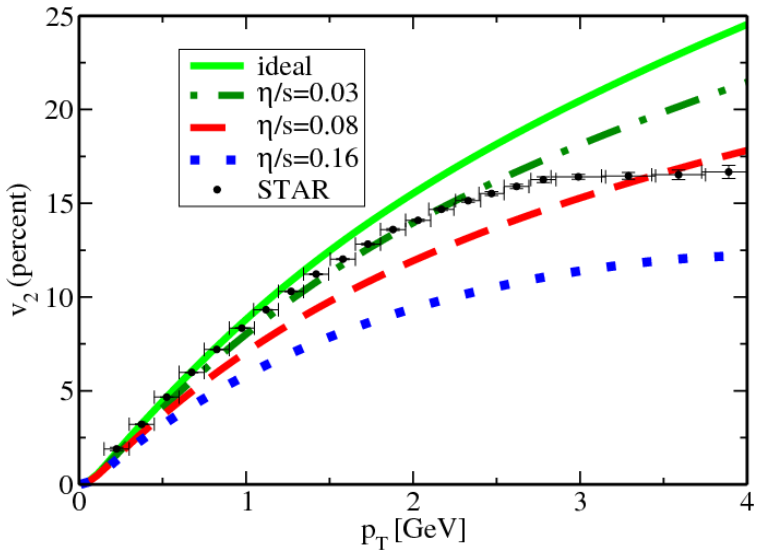
further evidence from complete hydro simulations:
W. Broniowski, M. Chojnacki, WF, A. Kisiel, PRL 101 (2008) 022301
P. Bozek, I. Wyskiel, PRC 79 (2009) 044916; P. Bozek, PRC 83 (2011) 044910

QGP as almost perfect fluid

- quantum mechanics (Danielewicz, Gyulassy, 1985) / AdS/CFT (Kovtun, Son, Starinets, 2005) lower bound on viscosity $\eta/\mathcal{S} > 1/4\pi$

  $\Rightarrow$ one should use relativistic dissipative hydrodynamics

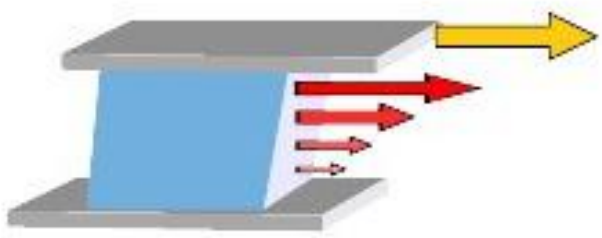
  $\Rightarrow$ better description (assuming small $\eta/\mathcal{S} = \mathrm{const}$)



P. Romatschke and U. Romatschke, Phys. Rev. Lett. 99, 172301 (2007)

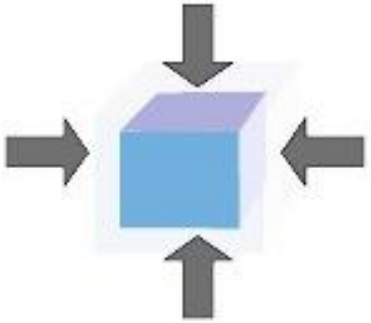
Viscosity

shear viscosity η
$\Downarrow$
reaction to a change of shape



$$\pi^{\mu\nu}_{Navier-Stokes} \propto \eta \left(\partial^{\langle\mu} u^{\nu\rangle}\right)$$

bulk viscosity ζ
$\Downarrow$
reaction to a change of volume



$$\Pi_{Navier-Stokes} \propto \zeta \left(\partial_\mu u^\mu\right)$$

bulk viscosity and pressure vanish for conformal fluids

Pitch drop experiment

John Mainstone (Wikipedia)

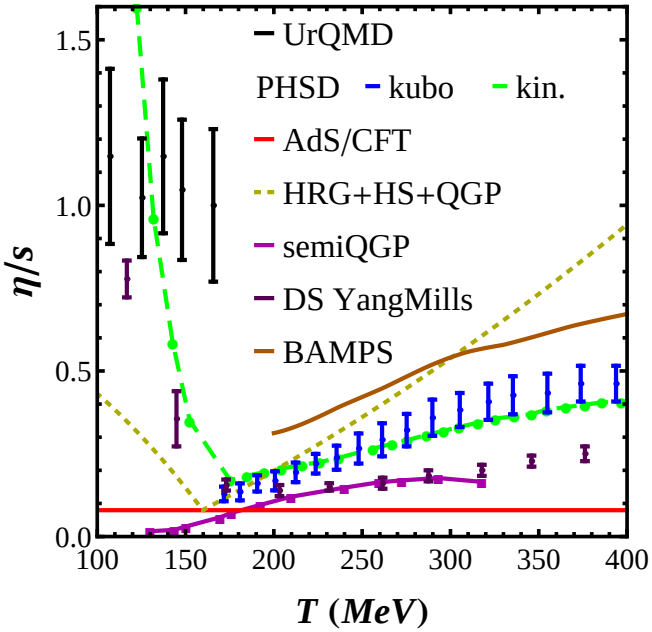
Wikipedia: The ninth drop touched the eighth drop on 17 April 2014. However, it was still attached to the funnel. On 24 April 2014, Prof. White decided to replace the beaker holding the previous eight drops before the ninth drop fused to them. While the bell jar was being lifted, the wooden base wobbled and the ninth drop snapped away from the funnel.

$$\eta_{\rm qgp} > \eta_{\rm pitch}$$

$$\eta_{\rm qgp} \sim 10^{11}\ \text{Pa s}, \qquad (\eta/s)_{\rm qgp} < 3/(4\pi)\hbar \quad \text{(from experiment)}$$

Shear and bulk viscosity of the plasma

- $\eta/\mathcal{S}$ reaches minimum in the region of the phase transition



S. I. Finazzo et al., arXiv:1412.2968

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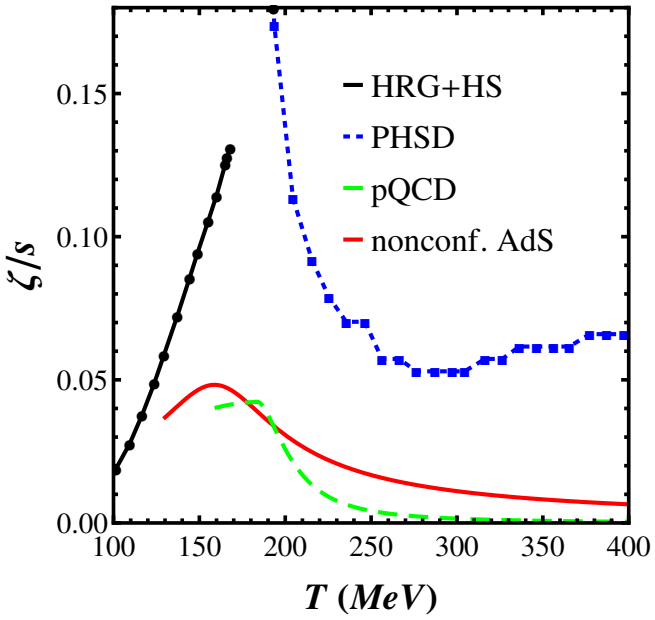

J. Noronha-Hostler

see also: WF, K. Tywoniuk, N. Su, R. Ryblewski, APPB 47 (2016) 1833, arXiv:1509.01242, calculations for the Gribov-Zwanziger plasma

Navier-Stokes equations — algebraic equations — T, u^μ are the only hydrodynamic variables

- energy-momentum conservation

$$\partial_\mu T^{\mu\nu}_{vis} = 0 \qquad T^{\mu\nu}_{vis} = \mathcal{E} u^\mu u^\nu - \Delta^{\mu\nu}(\mathcal{P} + \Pi) + \pi^{\mu\nu}$$

- number of equations: 5 + 6 ($\mathcal{E}, \mathcal{P}, u^\mu(3), \Pi, \pi^{\mu\nu}(5)$)
- number of equations: 4 + 1 (equation of state $\mathcal{E}(\mathcal{P})$)
- we need 6 extra equations - different methods possible

$$\dot{\Pi} + \frac{\Pi}{\tau_\Pi} = -\beta_\Pi \theta, \qquad \theta = \partial_\mu u^\mu$$

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi \sigma^{\mu\nu}, \qquad \sigma^{\mu\nu}\text{- shear flow tensor constructed from the velocity field } u^\mu$$

kinetic coefficients: $\tau_\Pi \beta_\Pi = \zeta \rightarrow$ bulk viscosity, $\tau_\pi \beta_\pi = \eta \rightarrow$ shear viscosity

Israel-Stewart equations — $\Pi, \pi^{\mu\nu}$ promoted to dynamic variables — non-hydrodynamic modes are introduced with the appropriate relaxation times τ_Π, τ_π

- energy-momentum conservation

$$\partial_\mu T^{\mu\nu}_{vis} = 0 \qquad T^{\mu\nu}_{vis} = \mathcal{E} u^\mu u^\nu - \Delta^{\mu\nu}(\mathcal{P} + \Pi) + \pi^{\mu\nu}$$

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$$\dot{\Pi} + \frac{\Pi}{\tau_\Pi} = -\beta_\Pi \theta + \lambda_{\Pi\pi} \pi^{\mu\nu} \sigma_{\mu\nu}$$

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi \sigma^{\mu\nu} - \tau_{\pi\pi} \pi^{\langle\mu}_{\gamma} \sigma^{\nu\rangle\gamma} + \lambda_{\pi\Pi} \Pi \sigma^{\mu\nu}$$

$\tau_\Pi \beta_\Pi = \zeta \rightarrow$ bulk viscosity, $\tau_\pi \beta_\pi = \eta \rightarrow$ shear viscosity

Müller-Israel-Stewart (MIS)

- energy-momentum conservation

$$\partial_\mu T^{\mu\nu}_{vis} = 0 \qquad T^{\mu\nu}_{vis} = \mathcal{E} u^\mu u^\nu - \Delta^{\mu\nu}(\mathcal{P} + \Pi) + \pi^{\mu\nu}$$

- number of equations: 5 + 6 ($\mathcal{E}, \mathcal{P}, u^\mu(3), \Pi, \pi^{\mu\nu}(5)$)
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- we need 6 extra equations - different methods possible

$$\dot{\Pi} + \frac{\Pi}{\tau_\Pi} = -\beta_\Pi \theta - \frac{\zeta T}{2\tau_\Pi} \Pi \, \partial_\lambda \left(\frac{\tau_\Pi}{\zeta T} u^\lambda \right)$$

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi \sigma^{\mu\nu} - \frac{\eta T}{2\tau_\pi} \pi^{\mu\nu} \, \partial_\lambda \left(\frac{\tau_\pi}{\eta T} u^\lambda \right)$$

New approaches add new terms: Baier, Romatschke, Son, Starinets, Stephanov (BRSSS) symmetry arguments due to conformal symmetry, ...

- energy-momentum conservation

$$\partial_\mu T^{\mu\nu}_{vis} = 0 \qquad T^{\mu\nu}_{vis} = \mathcal{E} u^\mu u^\nu - \Delta^{\mu\nu}(\mathcal{P} + \Pi) + \pi^{\mu\nu}$$

- number of equations: 5 + 6 ($\mathcal{E}, \mathcal{P}, u^\mu(3), \Pi, \pi^{\mu\nu}(5)$)
- number of equations: 4 + 1 (equations of state $\mathcal{E}(\mathcal{P})$)
- we need 6 extra equations - different methods possible

$$\begin{aligned} \Pi &= 0 \\ \dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} &= 2\beta_\pi \sigma^{\mu\nu} - \frac{4}{3}\pi^{\mu\nu}\theta + \frac{\lambda_1}{\tau_\pi \eta^2}\pi^{\langle\mu}_{\ \lambda}\pi^{\nu\rangle\lambda} \quad \text{(+ terms including vorticity and curvature)} \end{aligned}$$

Dissipative hydrodynamics

New approaches add new terms: Denicol, Niemi, Molnar, Rischke (DNMR)
simultaneous expansion in the Knudsen number and inverse Reynolds number

- energy-momentum conservation

$$\partial_\mu T^{\mu\nu}_{vis} = 0 \qquad T^{\mu\nu}_{vis} = \mathcal{E} u^\mu u^\nu - \Delta^{\mu\nu}(\mathcal{P} + \Pi) + \pi^{\mu\nu}$$

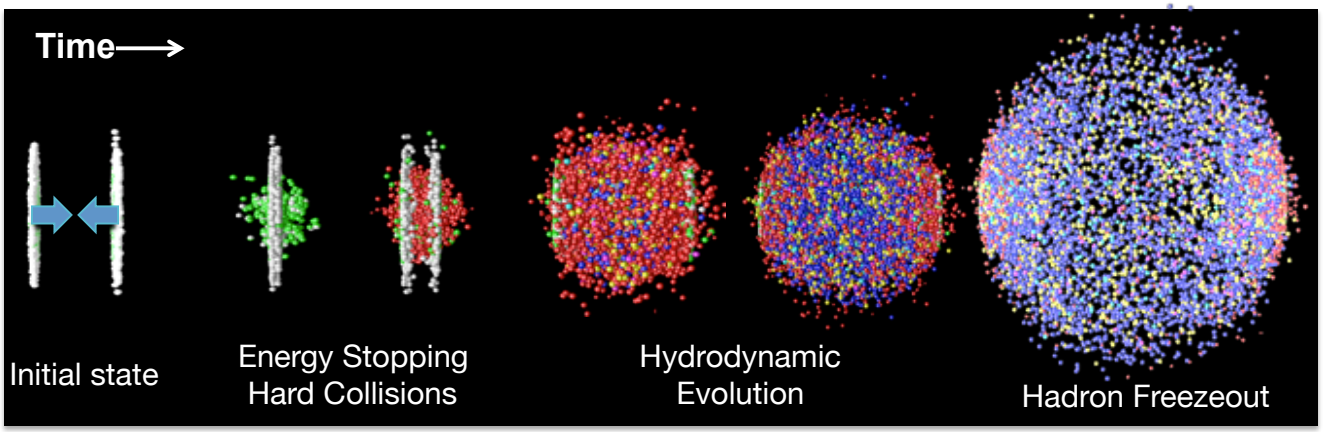
- number of equations: 5 + 6 ($\mathcal{E}, \mathcal{P}, u^\mu(3), \Pi, \pi^{\mu\nu}(5)$)
- number of equations: 4 + 1 (equations of state $\mathcal{E}(\mathcal{P})$)
- we need 6 extra equations - different methods possible

$$\dot{\Pi} + \frac{\Pi}{\tau_\Pi} = -\beta_\Pi \theta - \delta_{\Pi\Pi}\Pi\theta \underline{+\lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}}$$

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi\sigma^{\mu\nu} + 2\pi_\gamma^{\langle\mu}\omega^{\nu\rangle\gamma} - \delta_{\pi\pi}\pi^{\mu\nu}\theta \underline{-\tau_{\pi\pi}\pi_\gamma^{\langle\mu}\sigma^{\nu\rangle\gamma} + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu}}$$

RTA version of the Boltzmann kinetic equation, neglected vorticity
for standard form of the collision term additional terms (with new kinetic coefficients) appear
shear-bulk coupling $\eta - \zeta$

"Standard model" of evolution of matter produced in heavy-ion collisions



T. K. Nayak, Lepton-Photon 2011 Conference

- initial conditions including fluctuations ($0 < \tau_0 \lesssim 1\,\mathrm{fm}$)
- non-equilibrium phase and THERMALIZATION (→HYDRODYNAMIZATION) of matter ($\tau_0 < \tau \lesssim 1\,\mathrm{fm}$)
  $\Rightarrow$ emission of hard probes: heavy quarks, photons, jets
- hydrodynamic expansion (expansion and cooling) ($1\,\mathrm{fm} \lesssim \tau \lesssim 10\,\mathrm{fm}$)

microscopic description of such a many body system is very complicated
$\Downarrow$
effective description in terms of fluid mechanics

- phase transition from QGP to hadron gas (encoded in the equation of state)
- freeze-out and free streaming of hadrons ($10\,\mathrm{fm} \lesssim \tau$)

Hydrodynamics vs. kinetic theory

in the kinetic theory the basic quantity is the one-particle phase-space distribution function

- perfect-fluid hydrodynamics: local thermal equilibrium

$$f(x,p) = f_{\rm iso}\left(\frac{p^\mu u_\mu}{T(x)}\right) \qquad \Rightarrow \qquad T^{\mu\nu}_{id} = \int dP\, p^\mu p^\nu f(x,p)$$

- dissipative hydrodynamics: linear deviations from local equilibrium

$$f(x,p) = \underbrace{f_{\rm iso}\left(\frac{p^\mu u_\mu}{T(x)}\right)}_{\rm LO} + \underbrace{\delta f(x,p)}_{\rm NLO} \qquad \Rightarrow \qquad T^{\mu\nu}_{vis} = T^{\mu\nu}_{id} + \delta T^{\mu\nu}_{id}$$

URHIC → EXTREME SPACETIME SCALES
very small systems
very large gradients
very fast expansion
⇓
dissipative corrections are substantial

standard dissipative hydrodynamics assumes that the system is always close to local equilibrium, this is in contrast with microscopic calculations showing that plasma is highly anisotropic in the momentum space (at the early stages of the evolution)

Exact solutions of the Boltzmann equation

Relaxation-time-approximation (RTA) kinetic equation

$$p^\mu \partial_\mu f(x,p) = p \cdot u \, \frac{f_{\rm eq}(x,p) - f(x,p)}{\tau_{\rm eq}}$$

solutions known for one-dimensional, boost-invariant system, G. Baym, PLB 138 (1984)
used for comparisons between the underlying kinetic theory and hydrodynamic approaches

One dimensional expansion

Denicol, Maksymiuk, Ryblewski, Strickland, WF

1. conformal case
2. non-conformal case
3. non-conformal case with quantum statistics
4. mixtures

(1+1)D flow with Gubser symmetry

Denicol, Heinz, Martinez, Noronha, Strickland

BULK PRESSURE EVOLUTION

$$\tau_\Pi \dot{\Pi} + \Pi = -\frac{\zeta}{\tau} - \frac{1}{2}\tau_\Pi \Pi \left[\frac{1}{\tau} - \left(\frac{\dot{\zeta}}{\zeta} + \frac{\dot{T}}{T}\right)\right] \qquad (A)$$

Muronga, Phys. Rev. C69 (2004) 034903
Heinz, Song, Chaudhuri, Phys. Rev. C73 (2006) 034904

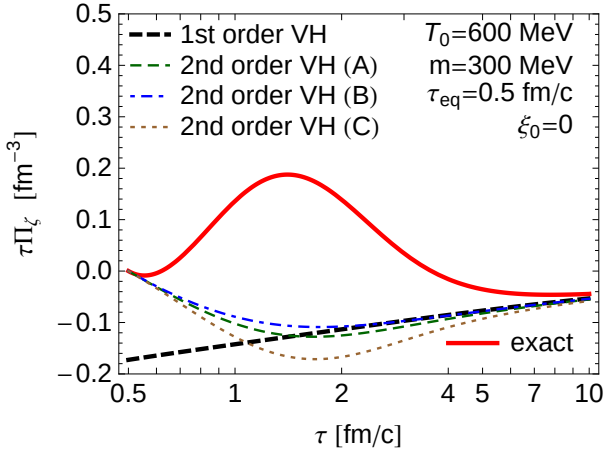
$$\tau_\Pi \dot{\Pi} + \Pi = -\frac{\zeta}{\tau} - \frac{4}{3}\tau_\Pi \Pi \frac{1}{\tau} \qquad (B)$$

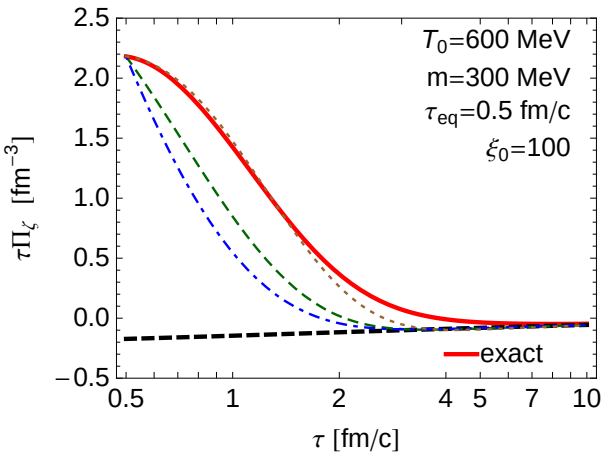
Jaiswal, Bhalerao, Pal, Phys. Rev. C87 (2013) 021901

$$\tau_\Pi \dot{\Pi} + \Pi = -\frac{\zeta}{\tau} \qquad (C)$$

Heinz, Song, Chaudhuri, Phys. Rev. C73 (2006) 034904

WF, E. Maksymiuk, R. Ryblewski, M.Strickland, Phys.Rev. C89 (2014) 054908





exact solution and all 2nd order viscous hydrodynamics variations tend toward the 1st order solution at late times

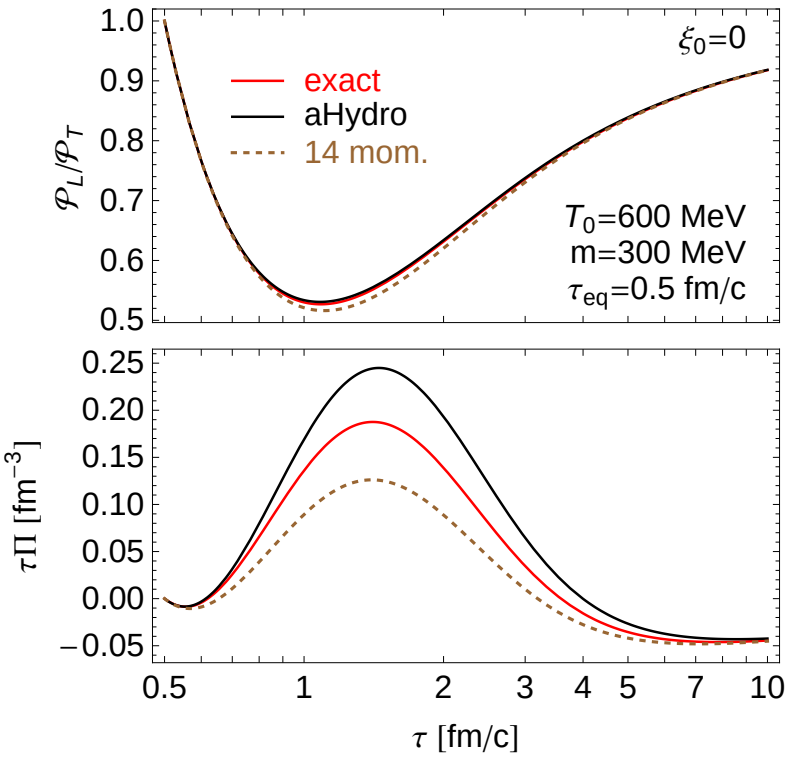
none of the 2nd order viscous hydrodynamics variations seems to qualitatively describe the early-time evolution of the bulk viscous pressure in all cases

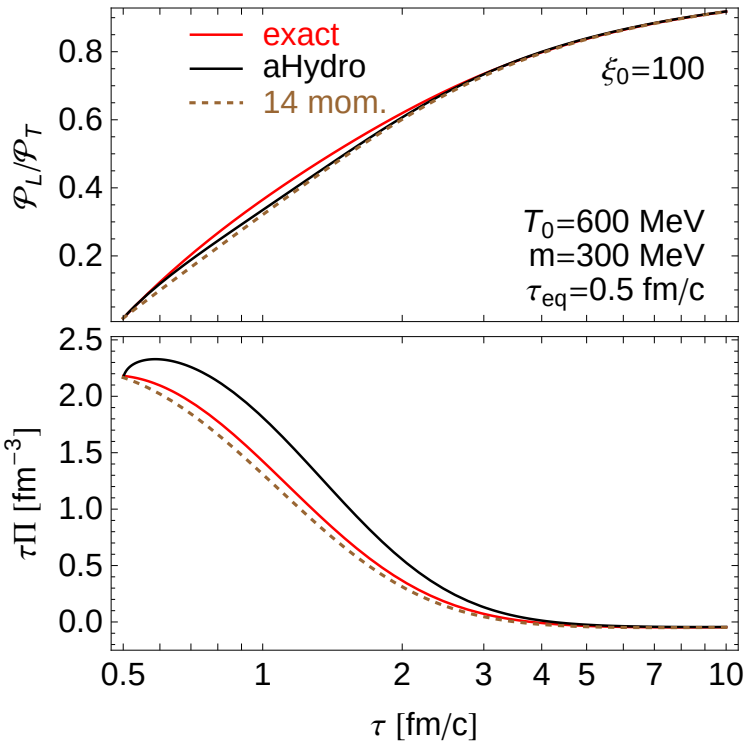
there is something incomplete in the manner in which 2nd order viscous hydrodynamics treats the bulk pressure (neglected shear–bulk coupling)

Bulk viscous pressure evolution within full viscous anisotropic hydrodynamics with SHEAR-BULK COUPLING

G. Denicol, S. Jeon, C. Gale, Phys.Rev. C90 (2014) 024912

G. Denicol, R. Ryblewski, WF, M.Strickland, Phys.Rev. C90 (2014) 044905





the shear-bulk couplings are extremly important for correct description of the bulk viscous correction

Hydrodynamic gradient expansion

M. P. Heller, R. A. Janik, M. Spalinski, P. Witaszczyk
Formal expansion of $T^{\mu\nu}$ in gradients of hydrodynamic variables T and u^μ

$$T^{\mu\nu} \quad = \quad T^{\mu\nu}_{\rm eq} + \text{powers of gradients of } T \text{ and } u^\mu$$

Simple structures for boost-invariant flow with the relaxation time $\tau_\pi = \frac{c}{T}$,
for example, T is expanded around the Bjorken flow

$$T = T_0 \left(\frac{\tau_0}{\tau}\right)^{1/3} \left(1 + \sum_{n=1}^{\infty} \left(\frac{c}{T_0 \tau_0}\right)^n t_n \left(\frac{\tau_0}{\tau}\right)^{2n/3}\right)$$

Change to $g(w)$

$$g = \frac{1}{T}\frac{dw}{d\tau}, \qquad w = \tau T, \quad \Delta = \frac{\Delta P}{P} = 3\frac{P_\parallel - P_\perp}{\varepsilon} = 12\left(g - \frac{2}{3}\right)$$

The gradient expansion for boost-invariant flow takes the form of an expansion

$$g(w) = \sum_{n=0}^{\infty} g_n w^{-n}, \quad g_0 = \frac{2}{3}$$

Hydrodynamic gradient expansion

RTA - gradient expansion for the RTA kinetic-theory model – Heller, Kurkela, Spalinski, to be published
WF, R. Ryblewski, M. Spalinski, arXiv:1608.07558

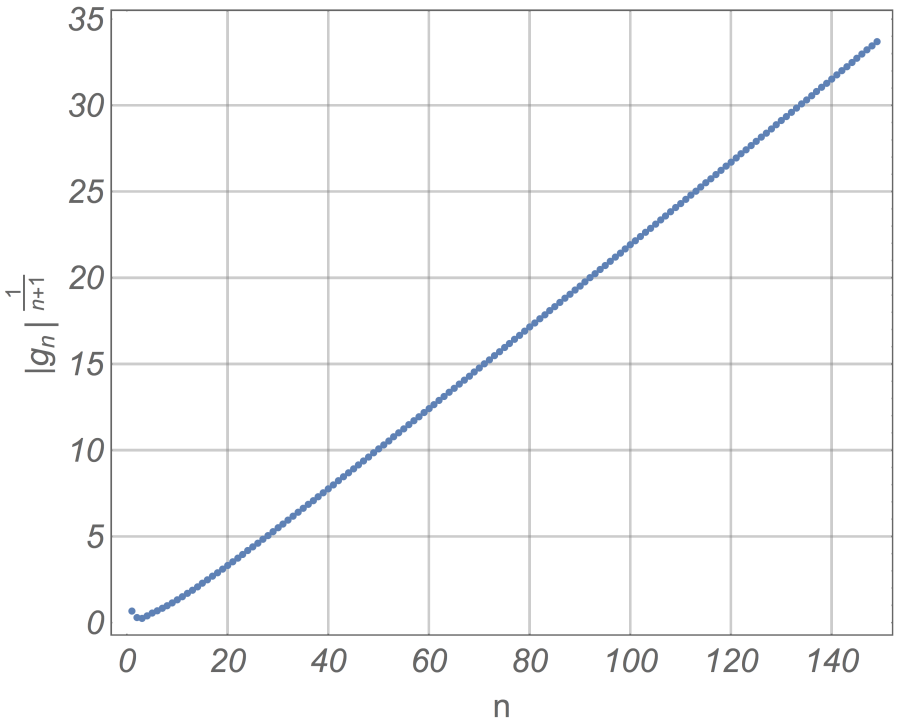
n	RTA	BRSS	DNMR	MIS
0	2/3	2/3	2/3	2/3
1	4/45	4/45	4/45	4/45
2	16/945	16/945	16/945	8/135
3	−208/4725	−1712/99225	-304/33075	112/2025

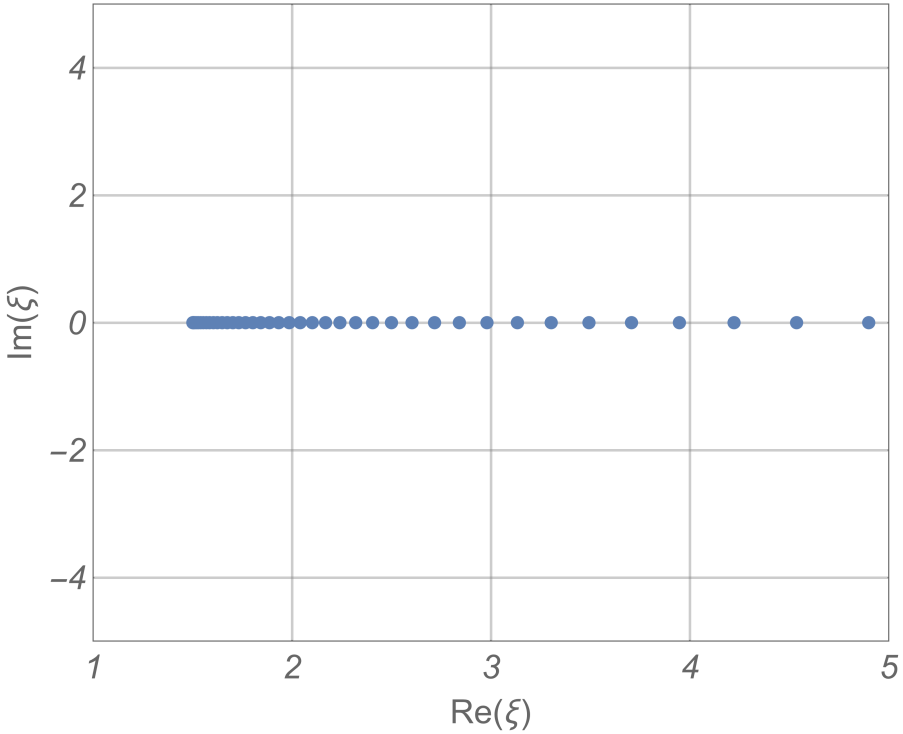
compared to two versions of anisotropic hydrodynamics:
AH1: L. Tinti, WF, PRC89 (2014) 034907
AH2: L. Tinti, PRC in print

n	RTA	BRSS	AH I	AH II
0	2/3	2/3	2/3	2/3
1	4/45	4/45	4/45	4/45
2	16/945	16/945	8/945	16/945
3	−208/4725	−1712/99225	−184/4725	−176/6615

the series g_n, has vanishing radius of convergence,the Borel transform of g is introduced, analytic continuation using diagonal Padé approximants of order 70 is done

$$g_B(\xi) = \sum_{n=0}^{\infty} \frac{g_n}{n!} \xi^n, \tag{1}$$





the cut along the real axis indicates the presence of a single nonhydrodynamic mode, which is is purely decaying, as in MIS theory.

WHY HYDRO WORKS?

- I do not know, see the Round Table discussion on Friday
- one may speculate: 10 equations for the energy-momentum tensor, 4 follow from the energy-momentum tensor, 6 describe dissipation, characteristic damping rate of about 1 fm/c, hydro should work for systems with a size of about few fm, ...
- Shuryak: consequence of the QCD (approximate) scale invariance
- Romatschke, Spalinski: hydrodynamic description is valid if hydro is insensitive to non-hydrodynamic modes – criterion satisfied also for small systems, hydro description is connected with hydrodynamic modes that are universal (the same viscosity coefficients for different systems, ...)
- .....

SUMMARY

- QCD EOS

  known from first principles at $\mu_B = 0$, restricted by the data analysis of the HBT correlations, semi hard (large values of c_s), faster evolution of the system, shorter time for equilibration of matter produced in heavy-ion collisions, smooth background in the Early Universe (does not lead to structures typical for the first order phase transition)

- THERMALIZATION (LOCAL EQUILIBRATION) → HYDRODYNAMIZATION

  small shear viscosity to entropy ratio + large gradients (1/t) = large pressure corrections

  AdS/CFT → large pressure corrections

- BULK VISCOSITY

  large at the QCD phase transition, strongly coupled to the shear sector (shear viscosity), to be included (more systematically) in cosmological models or astrophysical calculations (?), to be extracted from heavy-ion collisions

- RELATIVISTIC VISCOUS HYDRODYNAMICS

  one should use the complete versions of relativistic hydrodynamics or possible extensions such as ANISOTROPIC HYDRODYNAMICS→ see the next talk by M. Strickland

- (HYDRODYNAMIC) GRADIENT EXPANSION

  useful for comparisons of different theories, gives information about the close-to-equilibrium betavior and non-hydrodynamical modes