

Lattice calculation of the Polyakov loop and Polyakov loop correlators

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XIIth Quark Confinement and the Hadron Spectrum, Thessaloniki,
08/30/2016

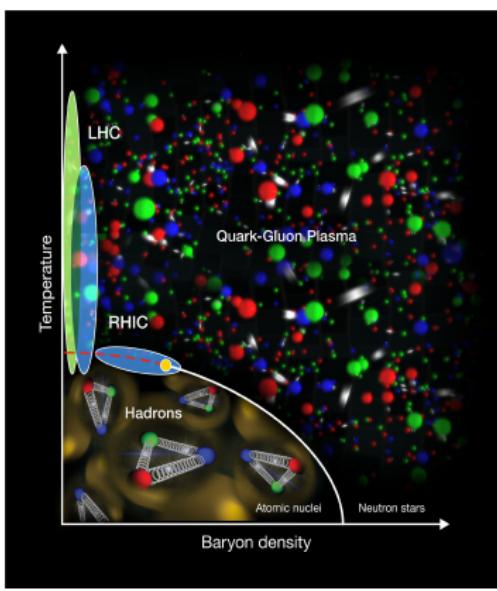
PRD 93 114502 (2016); arXiv:1601:06193; arXiv:1601:08001

Polyakov loop and Polyakov loop correlator in 2+1 flavor QCD

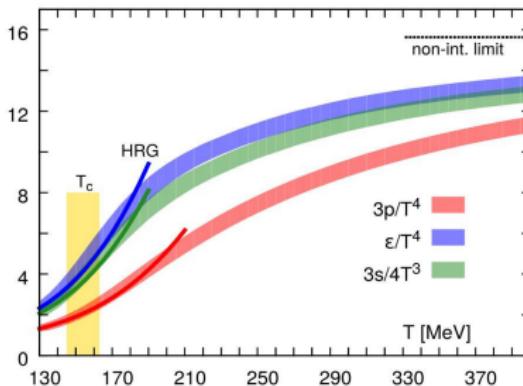
- Overview & introduction
- Main part
 - Deconfinement crossover and critical behavior
 - Entropy of a static quark
 - High temperature
 - Polyakov loop susceptibilities
 - Renormalization schemes
 - Static $Q\bar{Q}$ correlators at finite temperature
 - Weak coupling
- Summary

Quark-Gluon-Plasma - the high-temperature phase of QCD

QCD Phase diagram



QCD Equation of state

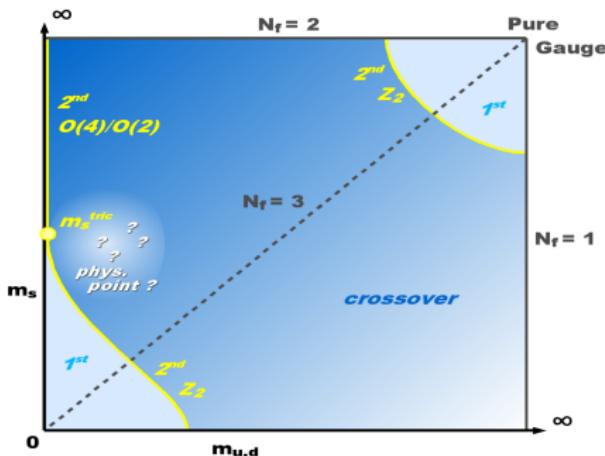


A. Bazavov et al., PRD 90 094503 (2014) [HotQCD]

- Smooth crossover region
- Strongly-coupled QGP**

The QCD crossover transition

Columbia plot

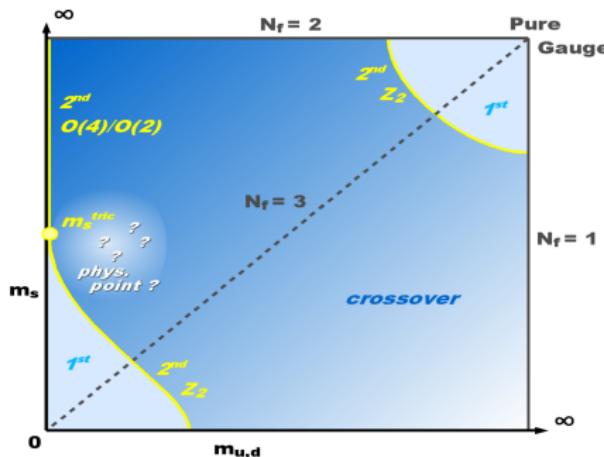


Finite quark masses distort critical behavior.

- crossover at **pseudocritical temperature**
- observables have **different** sensitivities to the **underlying critical behavior**

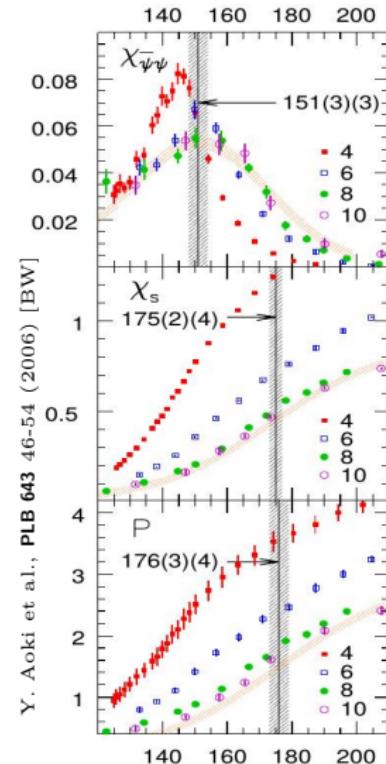
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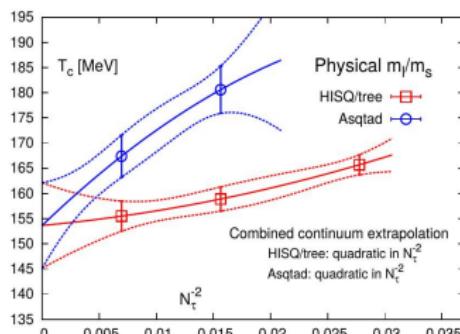


Chiral observables vs deconfinement observables

HotQCD: pseudocritical T_c from **chiral susceptibilities** reproduced

Quark number susc. $\frac{\partial \chi_q}{\partial T}$:

- dominated by **regular part** of free energy
- singular part** not easily accessible



A. Bazavov et al., PRD 85 054503 (2012) [HotQCD]

“Thus, the **Polyakov loop** [...] has **no demonstrated relation to the singular part** of the QCD partition function.”

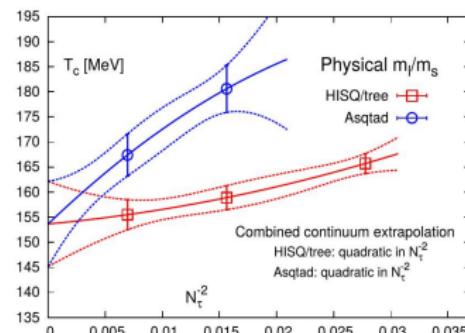
(A. Bazavov et al., ibid.)

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Are there physical reasons why the **T_c extracted from L is higher** than the T_c extracted from $\chi_{m,l}$, i.e. is this a **manifestation of the broad crossover?** **Does L provide reliable information about T_c in full QCD at all?**

The Polyakov loop L in Lattice Gauge Theory

$$W(\beta, N_\tau, \mathbf{x}) = \prod_{x_0=1}^{N_\tau} U_0(x_0, \mathbf{x})$$

$$L(\beta, N_\tau) = \frac{\text{Tr } W(\beta, N_\tau, \mathbf{x})}{N_c}$$

Trace of a **static quark** propagator,
related to **static quark free energy**

$$\langle L \rangle^r = e^{-N_\tau a C_Q} \langle L \rangle^b = \exp\left[-\frac{F_Q}{T}\right]$$

C_Q introduces **scheme dependence!**

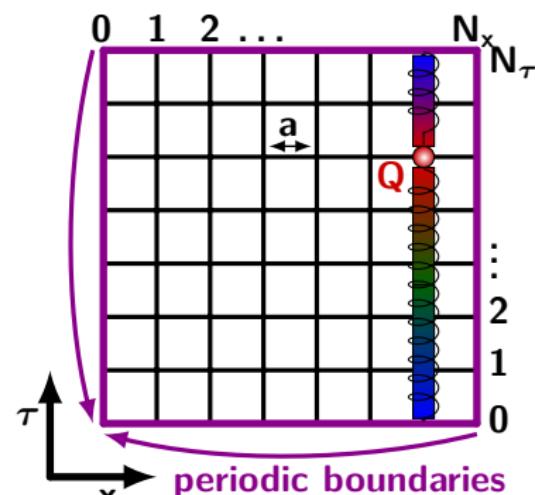
Thermal expectation values

$$\langle \mathcal{O}(T) \rangle = \int \mathcal{D}\phi \mathcal{O}[\phi] e^{-\int \mathcal{L}[\phi; \alpha]}$$

- **inverse temperature** $\frac{1}{T} = a N_\tau$

- Continuum limit for **fixed T** :
vary a and N_τ simultaneously

LGT on a Euclidean space-time grid



$$W(\beta, N_\tau, \mathbf{x}) = \prod_{x_0=1}^{N_\tau} U_0(x_0, \mathbf{x})$$

$$L(\beta, N_\tau) = \frac{\text{Tr } W(\beta, N_\tau, \mathbf{x})}{N_c}$$

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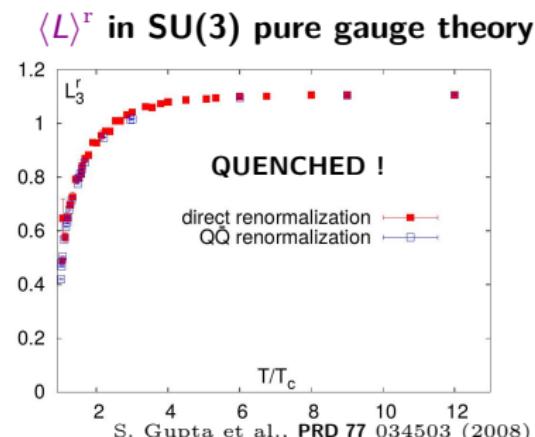
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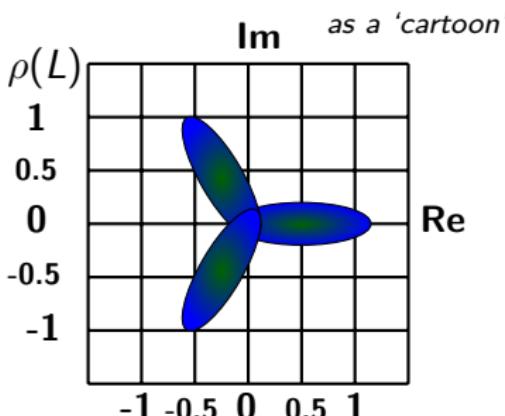
The Polyakov loop L in Lattice Gauge Theory



The **renormalized Polyakov loop** is an **order parameter** of the transition in **pure gauge theory**.

Already $\langle L \rangle^b$ is **discontinuous** at T_c due to the **Z(3) center symmetry**

Center symmetry in $SU(3)$ pure gauge theory

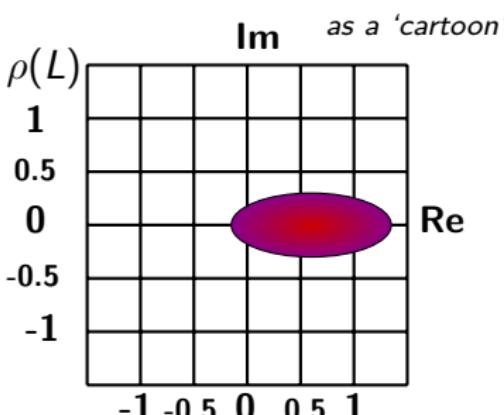


Z(3) center symmetry for $T < T_c$

$$\langle L \rangle = 0 \Leftrightarrow F_Q = \infty$$

Confinement in pure gauge theory

Center symmetry in SU(3) pure gauge theory

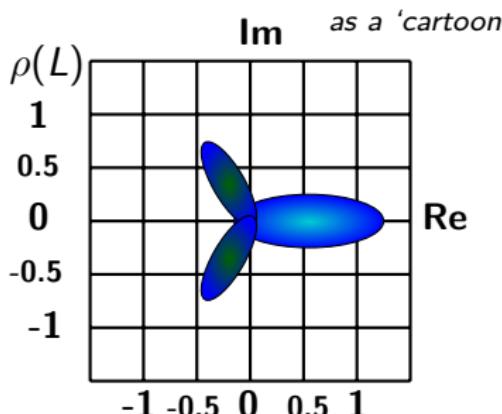


No center symmetry for $T > T_c$

$$\langle L \rangle > 0 \Leftrightarrow F_Q = \text{finite}$$

Deconfinement in pure gauge theory

Center symmetry in SU(3) pure gauge theory



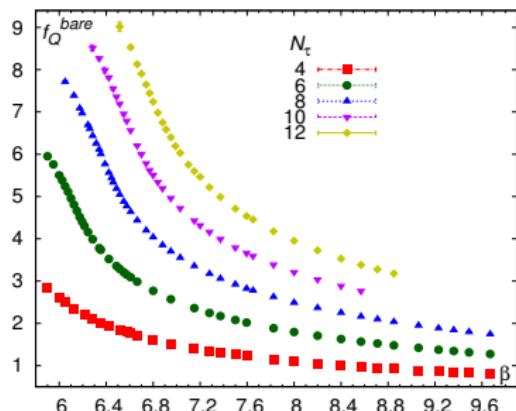
Center symmetry is broken in QCD by sea quarks for $T < T_c$

$\langle L \rangle > 0 \Leftrightarrow F_Q = \text{finite}$ due to **string breaking**

$F_Q \simeq \sum_i E_i$ due to **static hadrons** with energies E_i (cf. HRG models)

Bare Polyakov loop and bare free energy

PRD 93 114502 (2016)



Free energy needs renormalization

$$\langle L \rangle = \langle L^b \rangle e^{-N_\tau a C_Q}$$

$$f_Q = f_Q^b + N_\tau a C_Q$$

$$F_Q = F_Q^b + C_Q$$

$$f_Q^b \equiv F_Q^b / T = -\log \langle L^b \rangle$$

For each N_τ : 31 – 43 temperatures,
HISQ/Tree action, errors: $\mathcal{O}(\alpha_s a^2, a^4)$,
 $\frac{N_\sigma}{N_\tau} = 4$, $m_l = \frac{m_s}{20} \Leftrightarrow m_\pi = 161$ MeV,
mostly HotQCD lattices: A. Bazavov et al.,

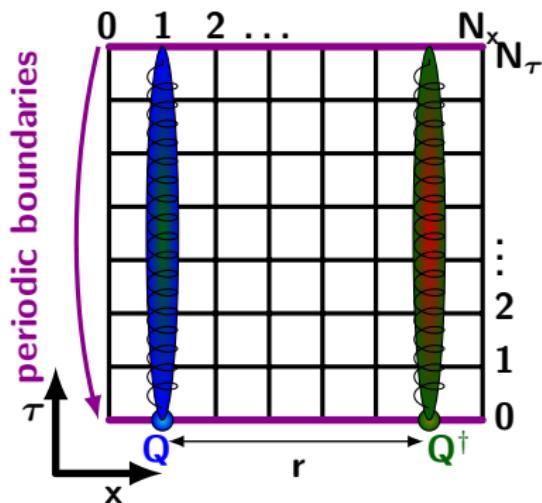
PRD 85 054503 (2012), PRD 90 094503 (2014) [HotQCD]

T range from $0.72 T_c$ up to $30 T_c$

- $C_Q(\beta)$ is **independent of N_τ**
- C_Q has **UV divergence**: $C_Q = \frac{c_Q}{a}$

$$C_Q = \frac{c}{a} + b + \mathcal{O}(a^2)$$
- **Scheme dependence**: $b + \mathcal{O}(a^2)$

Static meson correlators at asymptotically LARGE distances

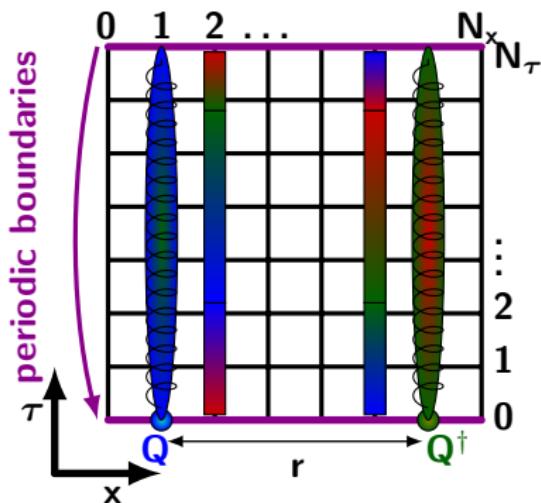


Free energy of static $Q\bar{Q}$ pair:

$$\begin{aligned} f_{Q\bar{Q}}(T, r) &= F_{Q\bar{Q}}(T, r)/T \\ &= - \log \langle L(T, 0)L^\dagger(T, r) \rangle \end{aligned}$$

Polyakov loop correlator $C_P(T, r)$

Static meson correlators at asymptotically LARGE distances



$r \gg 1/T$: static $Q\bar{Q}$ decorrelate

$$\lim_{r \rightarrow \infty} C_P(T, r) = \langle L(T) \rangle^2$$

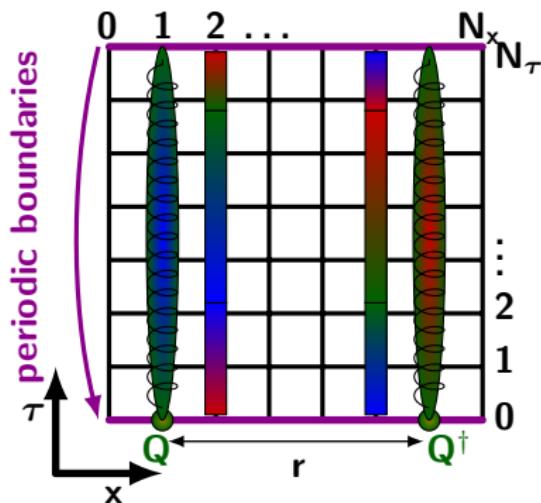
Apparent due to color screening

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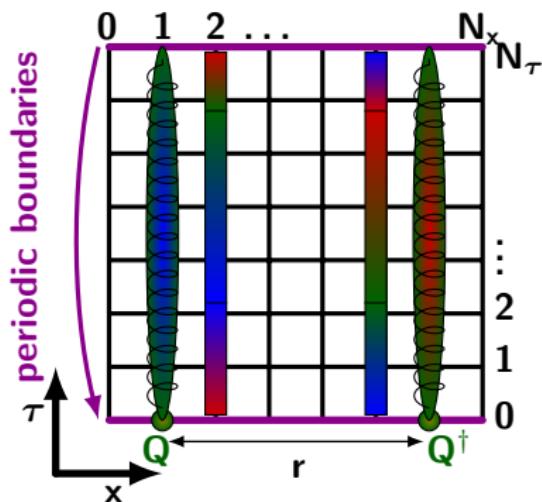
For **any color configuration** of $Q\bar{Q}$

$$\lim_{r \rightarrow \infty} C_S(T, r) = \langle L(T) \rangle^2$$

C_S is defined in **Coulomb gauge** as

$$C_S(T, r) = \frac{1}{3} \sum_{a=1}^3 W_a(T, 0) W_a^\dagger(T, r)$$

Static meson correlators at asymptotically LARGE distances



Free energy of static $Q\bar{Q}$ pair:

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Polyakov loop correlator $C_P(T, r)$

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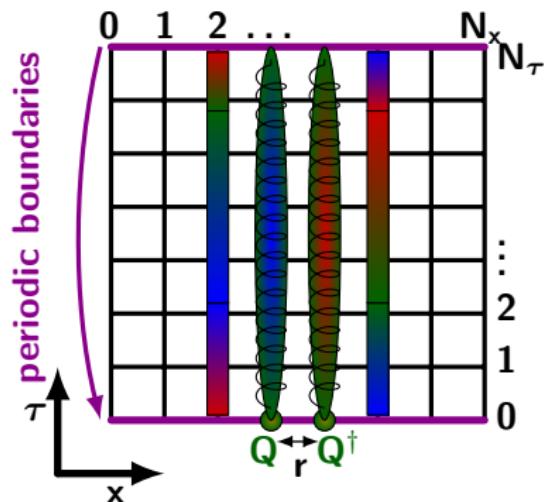
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$$\frac{C_S^r}{C_S^b} = \frac{C_P^r}{C_P^b} = \frac{\langle L^r \rangle^2}{\langle L^b \rangle^2} = \exp [-2N_\tau c_Q]$$

Static meson correlators at asymptotically **SMALL** distances



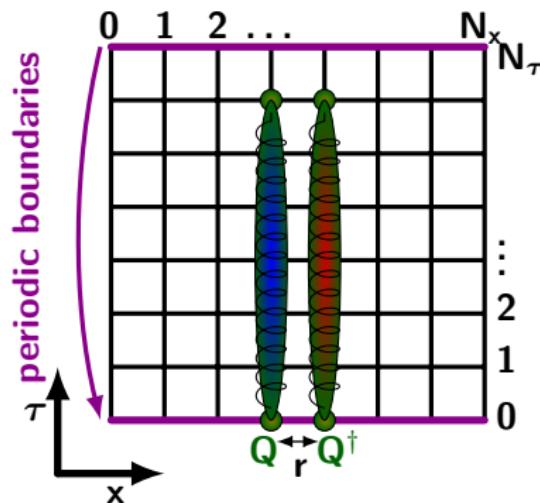
$r \ll \frac{1}{T}$: **small thermal effects** in

$$F_S(T, r) = -T \log \langle C_S(T, r) \rangle$$

For $r \ll \frac{1}{T}$: **vacuum-like** due to
asymptotic freedom



Static meson correlators at asymptotically **SMALL** distances

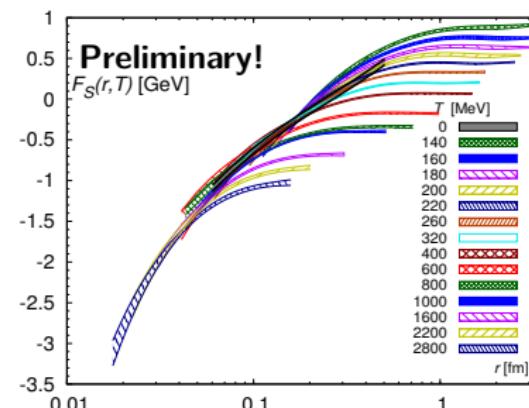


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For $r \ll \frac{1}{T}$: **vacuum-like** due to **asymptotic freedom**

$r \ll \frac{1}{T}$ is a **vacuum-like** regime

$$F_S(T, r) = V_S(r) + \mathcal{O}(rT)$$


$$\frac{F_S^r - F_S^b}{a} = \frac{V_S^r - V_S^b}{a} = -2c_Q$$

Renormalization scheme: $Q\bar{Q}$ procedure

Fix the static energy ($V_S \equiv V$)

$$V^r(\beta, r) = V^b(\beta, r) + 2c_Q(\beta)$$

for each β (β omitted below) to

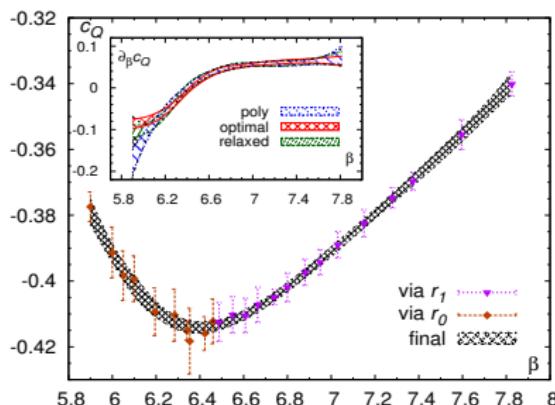
$$V^r(r) = \frac{V_i}{r_i}, \quad r^2 \frac{\partial V(r)}{\partial r} \Big|_{r=r_i} = C_i,$$

with $V_0 = 0.954$, $V_1 = 0.2065$
and $C_0 = 1.65$, $C_1 = 1.0$

- Use HotQCD results for $2c_Q$

A. Bazavov et al., PRD 90 094503 (2014)

- Interpolate in β
- Add $N_\tau c_Q$ to $f_Q^{\text{bare}}(T[\beta, N_\tau])$.



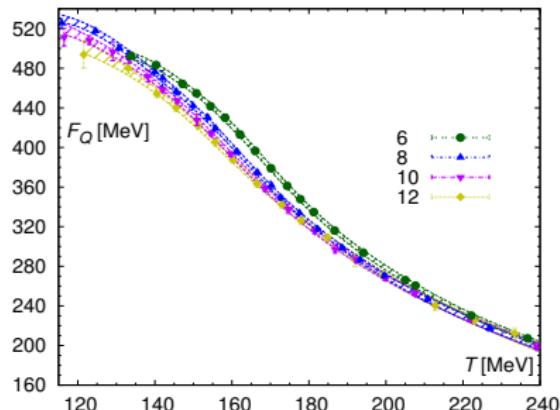
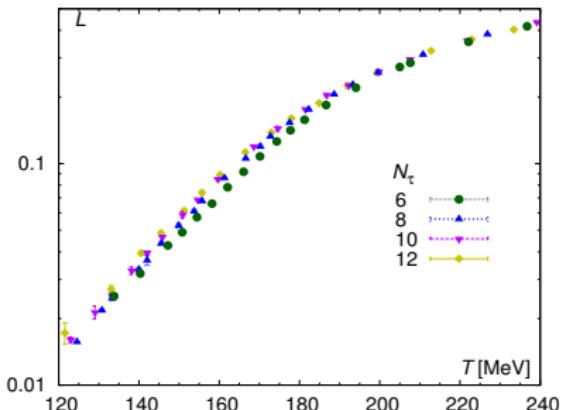
Drawbacks of $Q\bar{Q}$ procedure

- currently limited to $\beta \leq 7.825$

Advantages of $Q\bar{Q}$ procedure

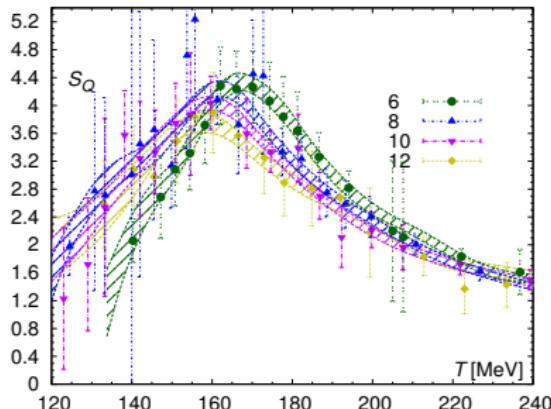
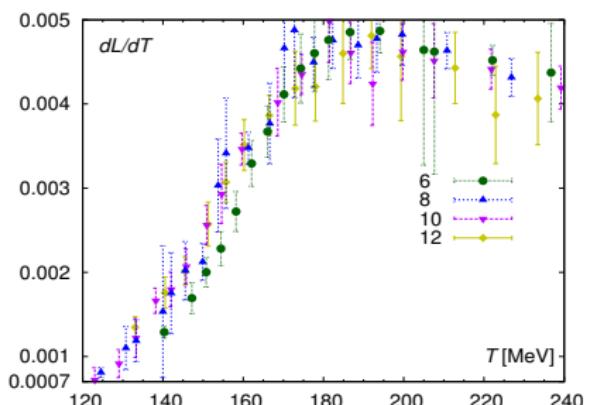
- unambiguous procedure

Renormalized Polyakov loop and renormalized free energy



- Cutoff effects are large in the crossover region for $N_\tau = 6$
- Cutoff effects on par with errors for $T > 200$ MeV
- Errors due to $N_\tau c_Q$ become dominant for high T

Renormalized Polyakov loop and renormalized free energy



'Critical behavior' in the Polyakov loop and in the free energy

Temperature derivative of L

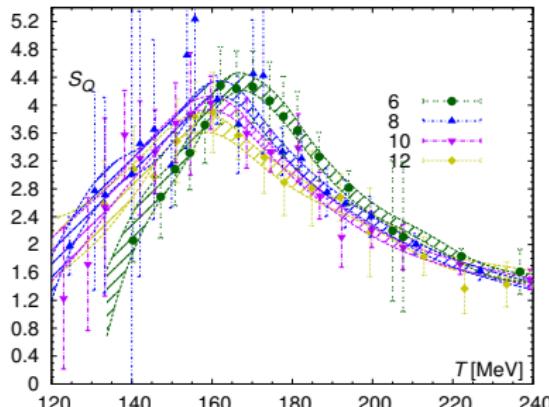
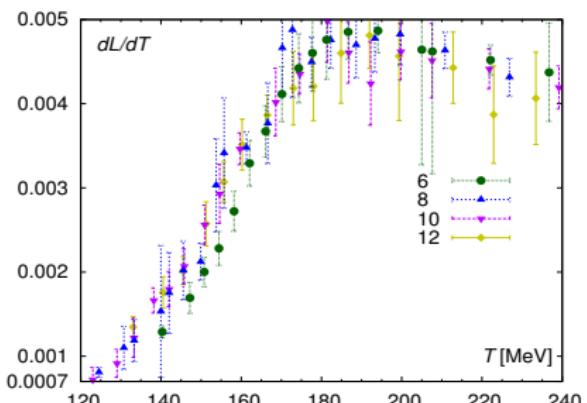
$\frac{dL}{dT}$ peaks at $T \sim 190$ MeV

Temperature derivative of F_Q

$S_Q = -\frac{dF_Q}{dT}$ peaks at $T \sim 160$ MeV

Can we relate the **maxima** to the **deconfinement crossover**?

Renormalized Polyakov loop and renormalized free energy

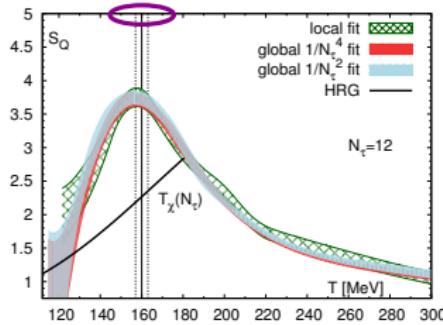
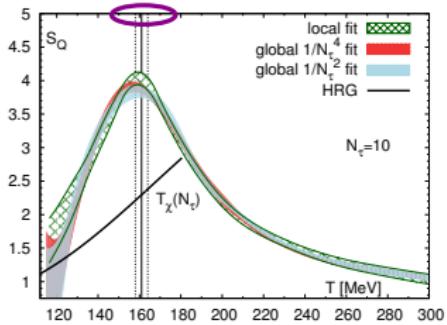
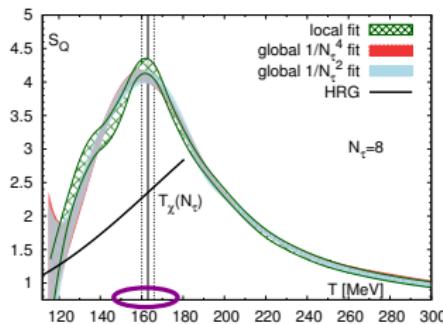
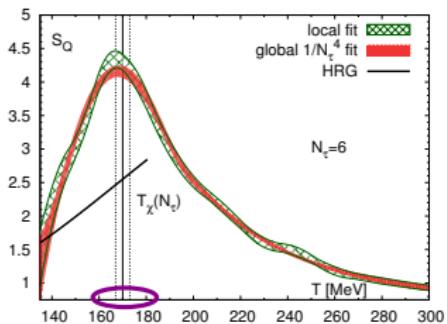


What is the **root cause** of the different inflection points of L and F_Q ?

- In principle the **entropy** $S_Q(T) = -\frac{dF_Q(T)}{dT}$ is a **measurable quantity**.
- The **inflection point** of F_Q is **renormalization scheme independent**.
- The **inflection point** of L is **scheme dependent!**

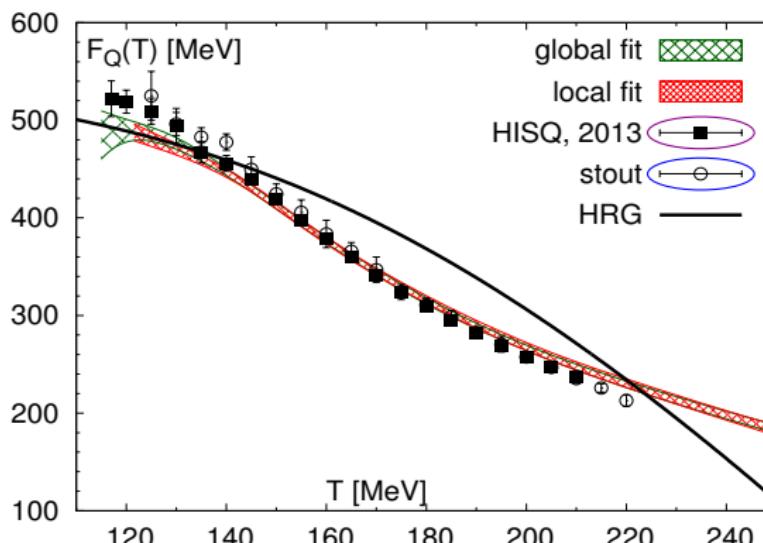
(arXiv:1601:06193)

T_c from chiral observables vs the peak of the entropy



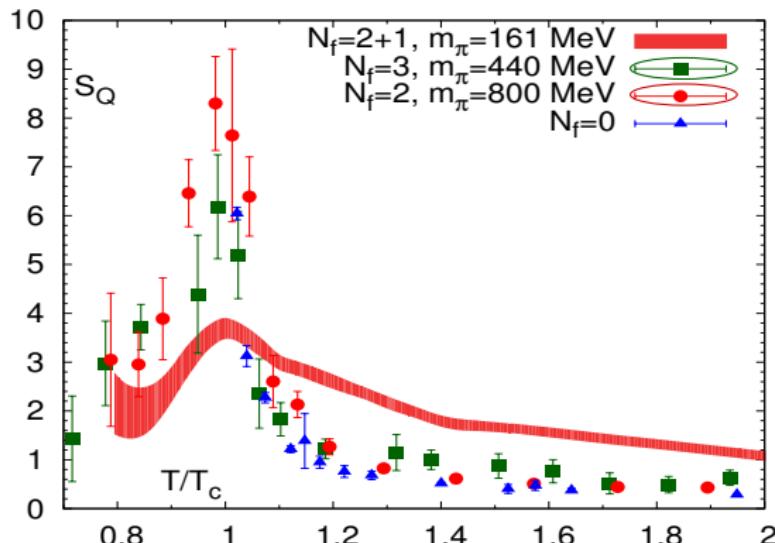
T_χ defined via $O(2)$ scaling fits to $\chi_{m,I}$ A. Bazavov et al., PRD 85 054503 (2012) [HotQCD]

Free energy and hadron resonance gas

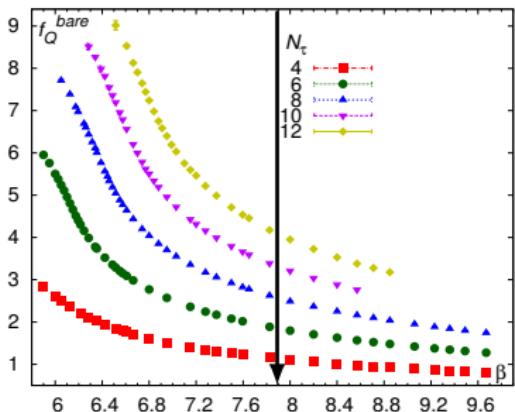


- F_Q is for low T below and for high T above the older HISQ result, due to **better continuum limit and renormalization constant**.
- Hadron resonance gas agrees with our data up to $T \lesssim 135$ MeV.

Critical behavior of the entropy



- The **peak decreases for lower quark masses** and for finer lattices.
 → interpret critical behavior as **melting of the static-light mesons**.
- The entropy peaks at $T_S = 153^{+6.5}_{-5} \text{ MeV}$ in the continuum limit.



$$T(\beta, N_\tau) = T(\beta^{\text{ref}}, N_\tau^{\text{ref}}) \text{ implies}$$

$$\begin{aligned} c_Q(\beta) &= \frac{1}{N_\tau} \left\{ N_\tau^{\text{ref}} c_Q(\beta^{\text{ref}}) \right. \\ &\quad \left. + f_Q^b(\beta^{\text{ref}}, N_\tau^{\text{ref}}) - f_Q^b(\beta, N_\tau) \right\} \\ &\text{infer } c_Q(\beta) \text{ from } c_Q(\beta^{\text{ref}}) \end{aligned}$$

Essential caveat:

The approach is invalid if **cutoff effects persist after renormalization.**

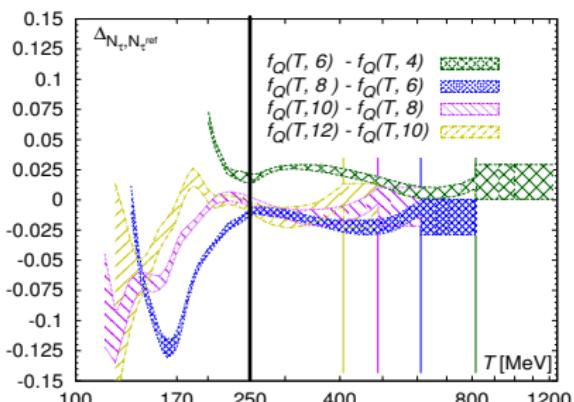
How to renormalize for $\beta > 7.825$?

Direct renormalization scheme

S. Gupta et al., PRD 77 034503 (2008)

$$f_Q(T(\beta, N_\tau), N_\tau) = f_Q^b(\beta, N_\tau) + N_\tau c_Q(\beta)$$

Renormalization scheme: direct renormalization



$$\Delta_{N_\tau, N_\tau^{\text{ref}}} = f_Q^r(\beta, N_\tau) - f_Q^r(\beta^{\text{ref}}, N_\tau^{\text{ref}})$$

$T < 250$ MeV: *large, fluctuating*

$T > 250$ MeV: *small, rather flat*

We estimate $\Delta_{N_\tau, N_\tau^{\text{ref}}}$ for $\beta > 7.825$ as constant with conservative error.

$T(\beta, N_\tau) = T(\beta^{\text{ref}}, N_\tau^{\text{ref}})$ implies

$$c_Q(\beta) = \frac{1}{N_\tau} \left\{ N_\tau^{\text{ref}} c_Q(\beta^{\text{ref}}) + \Delta_{N_\tau, N_\tau^{\text{ref}}} \right. \\ \left. + f_Q^b(\beta^{\text{ref}}, N_\tau^{\text{ref}}) - f_Q^b(\beta, N_\tau) \right\}$$

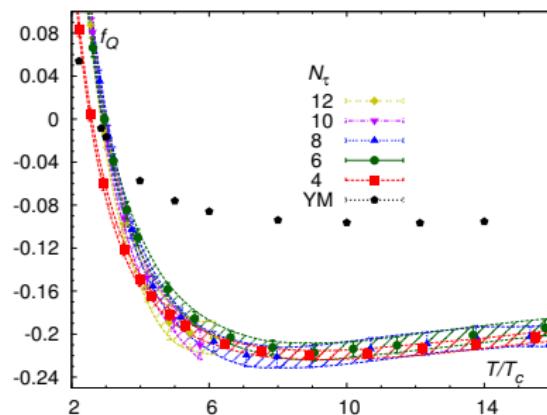
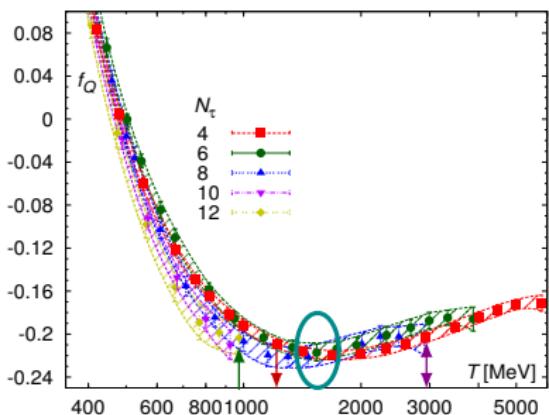
infer $c_Q(\beta)$ from $c_Q(\beta^{\text{ref}})$

Essential caveat:

The approach is invalid if **cutoff effects persist after renormalization.**

- Compute cutoff effects for low β and include in relation.
- Estimate cutoff effects for high β and include as well.
- Finally check consistency! ✓

Free energy at high temperatures and quenching effects



Direct renormalization in **two steps**

$$\begin{array}{lll} (\beta^{\text{ref}}, N_\tau^{\text{ref}}) & \max(T) & \rightarrow (\beta, N_\tau) \\ (7.825, 4) & 1222 \text{ MeV} & \rightarrow (8.850, 12) \\ (8.850, 4) & 2920 \text{ MeV} & \rightarrow (9.670, 8) \end{array}$$

$$\max(T) = 5814 \text{ MeV for } N_\tau = 4$$

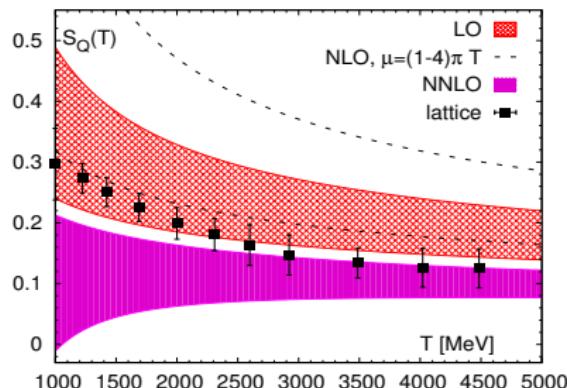
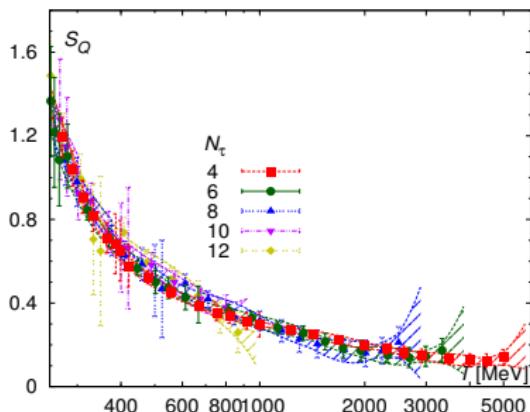
Cutoff effects flip at $T \sim 1.6$ GeV.

- Minimum of f_Q at $T \sim 10T_c$ ($N_\tau = 4$) in pure gauge theory

S. Gupta et al., PRD 77 034503 (2008)

- **Quark contribution** to f_Q for $T \gtrsim 4T_c$ is apparently $\sim 60\%$.
- Large **charm** contribution...?

Entropy at high temperatures and weak-coupling results



Static energies from **lattice and weak coupling approaches** differ by **unphysical additive divergences**.

Avoided when studying **derivatives**, i.e. **static $Q\bar{Q}$ force** or **entropy**

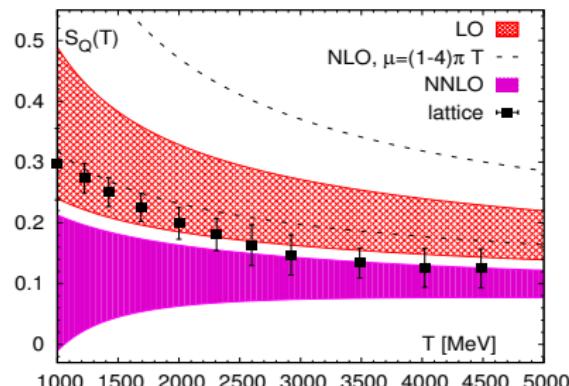
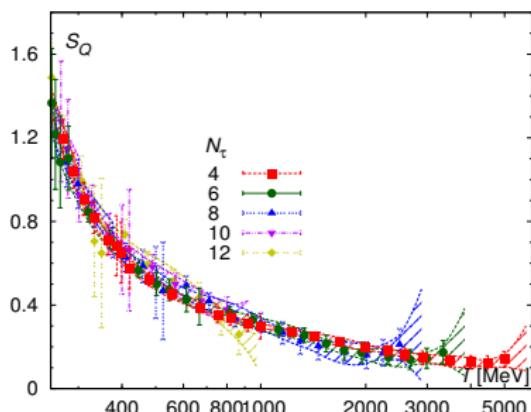
Cutoff effects in $S_Q(T)$ are **small**.

We compare $S_Q(T, 4)$ with weak coupling calculation for 3 flavors.

M. Berwein et al., PRD 93 034010 (2016)

For $T \gtrsim 3$ GeV, $S_Q(T, 4)$ agrees with NNLO. The continuum limit should agree for lower T already.

Entropy at high temperatures and weak-coupling results



Static energies from **lattice and weak coupling approaches** differ by **unphysical additive divergences**.

Avoided when studying **derivatives**, i.e. **static $Q\bar{Q}$ force** or **entropy**

Cutoff effects in $S_Q(T)$ are **small**.

We compare $S_Q(T, 4)$ with weak coupling calculation for 3 flavors.

⇒ M. Berwein, Thu 09/01/16, 17:00, Sec. D

For $T \gtrsim 3$ GeV, $S_Q(T, 4)$ agrees with NNLO. The continuum limit should agree for lower T already.

Renormalization scheme: gradient flow

Gradient flow approach

M. Lüscher, JHEP 1008 071 (2010)

Diffusion-type field evolution in an artificial **fifth dimension t**

$$\dot{V}_\mu = -g_0^2 \{\partial_\mu S[V]\} V_\mu$$

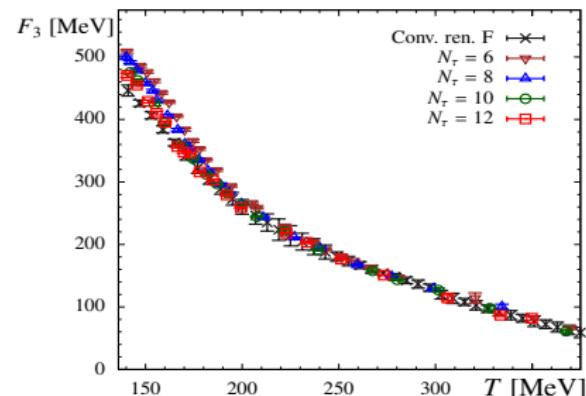
Fields V_μ at finite flow time

$$V_\mu \equiv V_\mu(x, t), \quad V_\mu(x, 0) = U_\mu(x)$$

are smeared out over length scale $f_t = \sqrt{8t}$, have no short distance singularities, **no UV divergences**

fixed flow time t defines a specific renormalization scheme, if

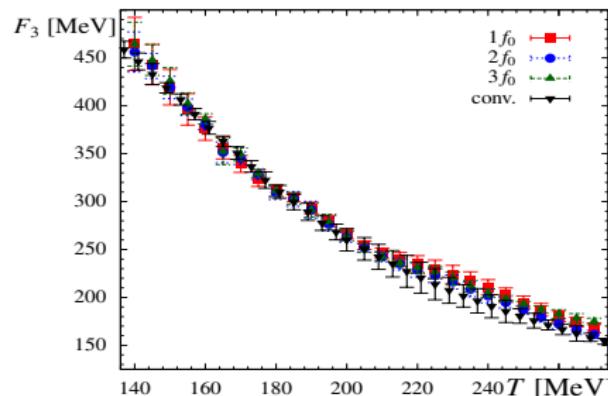
$$a \ll f_t = \sqrt{8t} \ll 1/T = aN_\tau$$



P. Petreczky, H.-P. Schadler, PRD 92 094517 (2015)

- $T \lesssim 400$ MeV: f_t dependence mild, constant differences.
- Cross-check of $Q\bar{Q}$ procedure with result at flow time f_t .

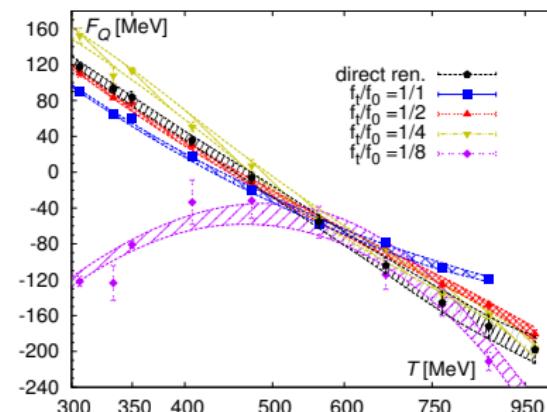
Gradient flow renormalization at high temperatures



P. Petreczky, H.-P. Schadler, PRD 92 094517 (2015)

The continuum limit at low T is within errors independent of f_t .

Higher T (smaller a): $0 < f_t \leq 1$



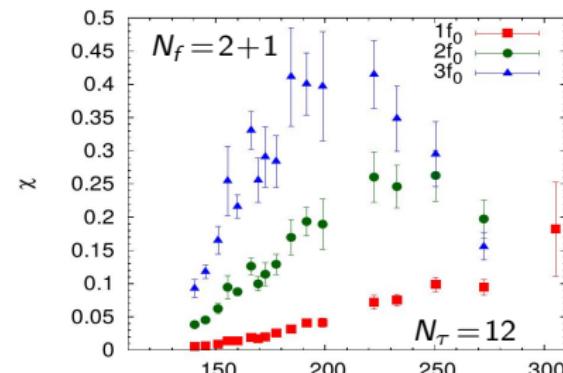
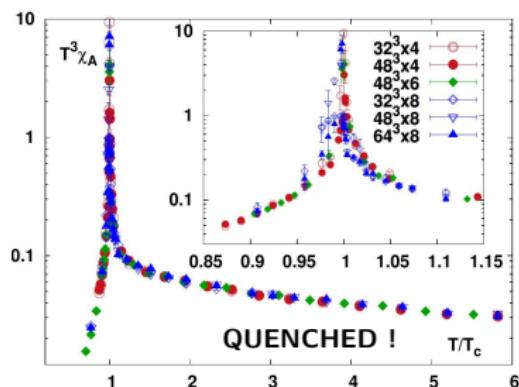
$F_Q(T, 12)$ via direct renormalization & gradient flow for different f_t .

Strong f_t dependent cutoff effects

At LO: smaller S_Q for larger f_t .

Larger N_τ needed to afford smaller flow times at higher temperatures

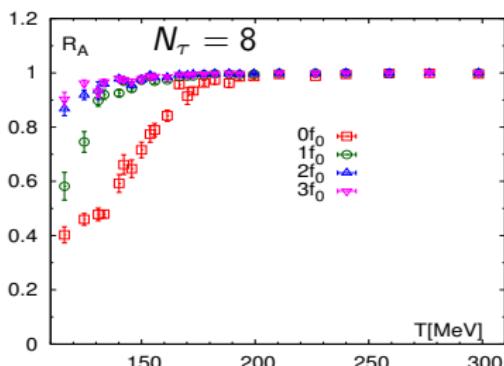
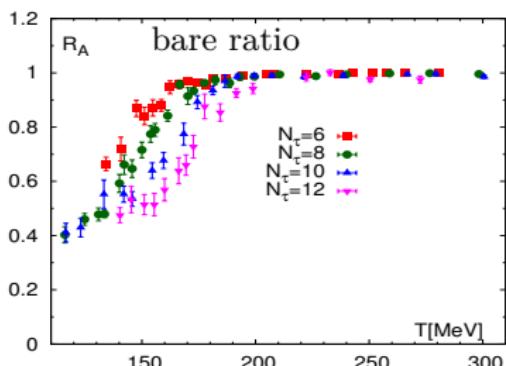
Polyakov loop susceptibilities



P.M. Lo et al., PRD 88 074502 (2013)

- **Polyakov loop susceptibility:** $\chi_A = (VT^3) (\langle |L|^2 \rangle - \langle |L| \rangle^2)$
- Mixes different representations: $9 \langle |L_3|^2 \rangle = 8 \langle L_8 \rangle - 1$
- **Casimir scaling violations** (P. Petreczky, H.-P. Schadler, PRD 92 094517 (2015))
 - no $Q\bar{Q}$ scheme, renormalize 2+1 flavor HISQ data via gradient flow
- χ_A strongly f_t dependent, no indication for critical behavior

Ratios of Polyakov loop susceptibilities

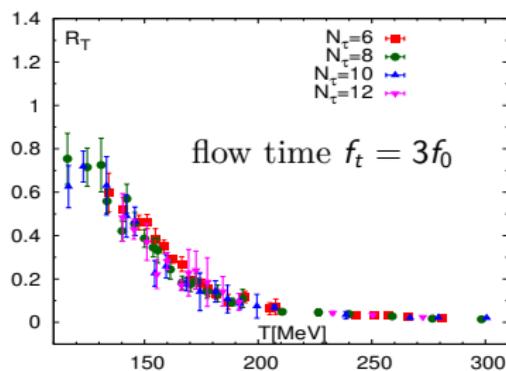
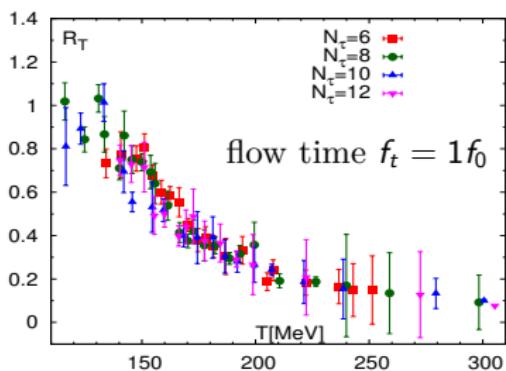


- **Longitudinal** and **transverse** Polyakov loop susceptibilities:

$$\chi_L = (VT^3) (\langle \text{Re } L^2 \rangle - \langle \text{Re } L \rangle^2), \quad \chi_T = (VT^3) (\langle \text{Im } L^2 \rangle)$$

- $R_A = \chi_A / \chi_L$: step function behavior **cannot be related to crossover**.

Ratios of Polyakov loop susceptibilities



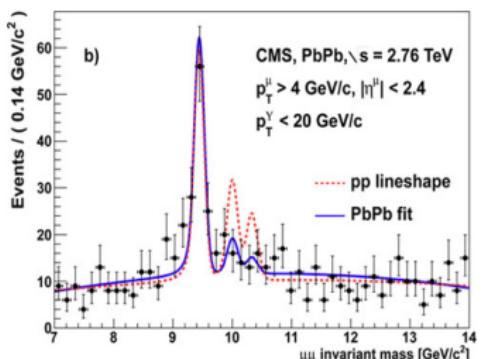
- **Longitudinal** and **transverse** Polyakov loop susceptibilities:

$$\chi_L = (VT^3) (\langle \text{Re } L^2 \rangle - \langle \text{Re } L \rangle^2), \quad \chi_T = (VT^3) (\langle \text{Im } L^2 \rangle)$$

- $R_T = \chi_T / \chi_L$: **crossover pattern** for $f_t \geq f_0$, exposes **critical behavior**.

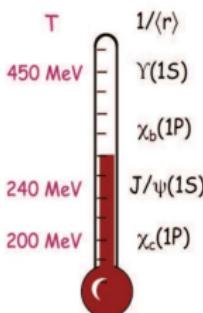
Heavy-ion collisions: quarkonium suppression in QGP

Υ suppression in HIC



Measured **rates of higher states**
vs ground state are reduced

Sequential melting



T. Matsui, H. Satz, PLB 178 (1986)

Sequential melting of quarkonia as
indicator of **temperature** of QGP

Heavy quarks are sensitive probes for properties of QGP

Effective field theories for non-relativistic heavy quarks

- **Hierarchy of scales:**

$$M \gg Mv \gg Mv^2$$

- Integrate out $M \Rightarrow \text{NRQCD}$
- Integrate out $Mv \Rightarrow \text{pNRQCD}$
- Color-singlet and -octet states
- Treat relative coordinate $r \sim 1/(Mv)$ as parameter
- Matching coefficients: **potentials**

N. Brambilla et al., RMP77 1423 (2005)

- **Thermal scales for $T > 0$:**

$$T, gT, g^2 T$$

- $\frac{1}{r}$ vs thermal scales
- Debye screening: $\mathbf{m}_D \sim gT$
- Landau damping
- Singlet-to-octet transitions
- Potentials become **complex**

N. Brambilla et al., PRD78 014017 (2008)

$$V_S(r, T) = -C_F \alpha_s \left(\frac{e^{-rm_D}}{r} - \frac{2iT}{rm_D} \int_0^\infty dx \frac{\sin(rm_D x)}{(x^2 + 1)^2} + (\mathbf{m}_D + iT) \right) + \mathcal{O}(g^4)$$

M. Laine et al., JHEP 0703 054 (2007)

Free energies of heavy quark states

$$V_S(r, T) = -C_F \alpha_s \left(\frac{e^{-r m_D}}{r} - \frac{2iT}{rm_D} \int_0^\infty dx \frac{\sin(rm_D x)}{(x^2 + 1)^2} + (m_D + iT) \right) + \mathcal{O}(g^4)$$

- Calculation of complex potential at $T > 0$ is difficult
- Potentials are related to the **Polyakov loop correlator**

M. Laine et al., ibid.

$$C_P(r, T) = e^{-F_{Q\bar{Q}}(r, T)} = \frac{1}{9} e^{-V_S/T} + \frac{8}{9} e^{-V_O/T} + \mathcal{O}(g^6) \text{ for } rT \ll 1$$

- Singlet and octet free energies

P. Petreczky, EPJ C43 (2005)

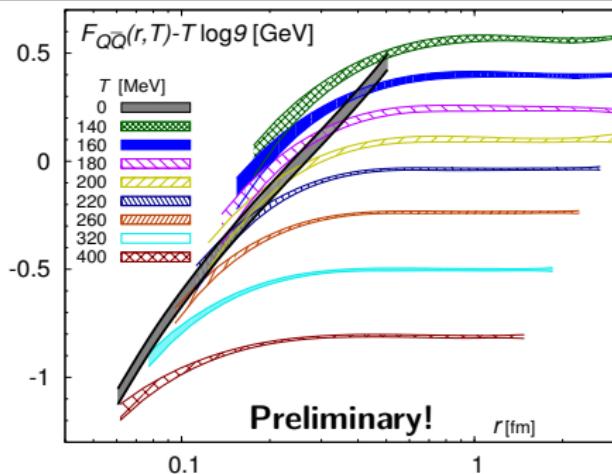
$$C_P(r, T) = e^{-F_{Q\bar{Q}}(r, T)} = \frac{1}{9} e^{-F_S/T} + \frac{8}{9} e^{-F_O/T} = \frac{1}{9} C_S(r, T) + \frac{8}{9} C_O(r, T)$$

- Singlet free energy and real part of singlet potential can be related

$$F_S(r, T) = \operatorname{Re} V_S(r, T) + \mathcal{O}(g^4) = -C_F \alpha_s \left(\frac{e^{-r m_D}}{r} + m_D \right) + \mathcal{O}(g^4)$$

- C_S and C_O are **gauge dependent**, meaning is not completely obvious

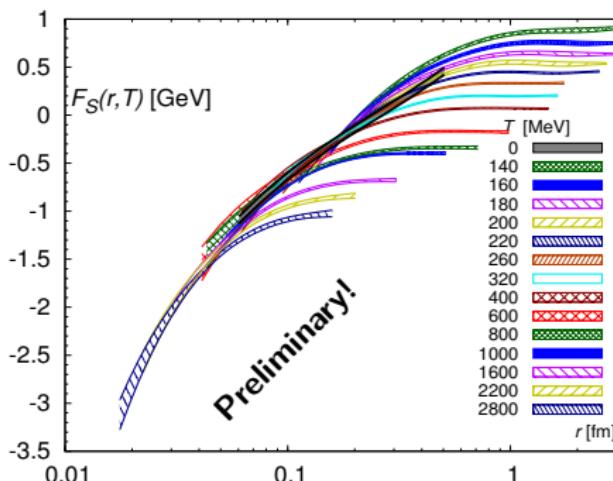
Polyakov loop correlator and $Q\bar{Q}$ free energy



$$e^{-\frac{F_{Q\bar{Q}}(r, T)}{T}} = C_P(r, T) = \langle L(T, 0) L^\dagger(T, r) \rangle = \frac{1}{9} e^{-\frac{F_S(r, T)}{T}} + \frac{8}{9} e^{-\frac{F_O(r, T)}{T}}$$

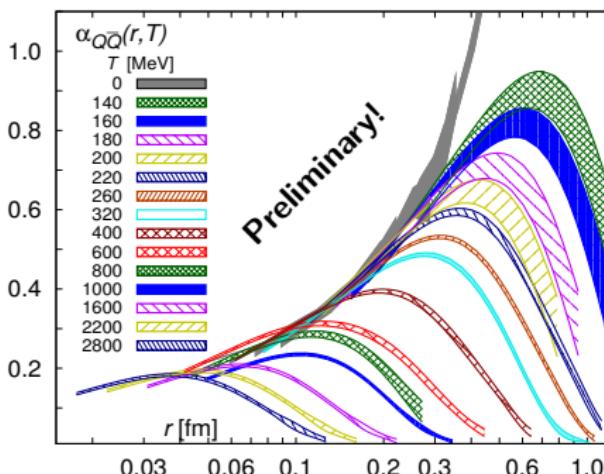
- Rather close to $T=0$ static energy $V_S(r)$ up to $r \sim \frac{0.15}{T}$
- Quantitative agreement with S. Borsanyi et al., JHEP 1504 (2015) [BW]

Singlet free energy in Coulomb gauge



- **Singlet free energy:** $e^{-F_S(r, T)/T} = C_S(r, T) = \frac{1}{3} \langle \sum_{a=1}^3 W_a(T, 0) W_a^\dagger(T, r) \rangle$
- Wilson line correlator requires explicit **gauge fixing** (Coulomb gauge)
- Consistent with $T=0$ static energy $V_S(r)$ up to $r \sim \frac{0.45}{T}$

Effective coupling: confining and screening regimes



- Effective coupling $\alpha_{Q\bar{Q}}(r, T)$ is a proxy for the **force**

$$\alpha_{Q\bar{Q}}(r, T) = \frac{r^2}{C_F} \frac{\partial E(r, T)}{\partial r}, \quad E = \{F_S(r, T), V_S(r)\}$$

- $\alpha_{Q\bar{Q}}(r, T) \gtrsim 0.5$ for $T \lesssim 2T_c$: QGP in HIC is **strongly coupled**

- We extract different deconfinement observables from the renormalized Polyakov loop. Our analysis is firmly based on the $Q\bar{Q}$ procedure.
- Renormalization scheme dependence leads to an inflection point of the Polyakov loop at $T \sim 180\text{-}200\text{ MeV}$.
- We see in the entropy $S_Q(T) = -\frac{dF_Q(T)}{dT}$ and in the ratio of Polyakov susceptibilities $R_T(T) = \frac{\chi_T(T)}{\chi_L(T)}$ crossover behavior at $T \sim T_c$.
- We extract $T_S = 153_{-5}^{+6.5}\text{ MeV}$ from the entropy, in agreement with $T_\chi = 160(6)\text{ MeV}$ (chiral susceptibilities, O(2) scaling fits, $\frac{m_l}{m_s} = 1/20$).

N_τ	∞	12	10	8	6
T_S	$153_{-5}^{+6.5}$	157.5(6)	159(4.5)	162(4.5)	167.5(4.5)
T_χ	160(6)	161(2)	[162(2)]*	164(2)	171(2)

- **Weak-coupling behavior** of the Polyakov loop sets in for $T \sim 3\text{ GeV}$.

- Continuum limit of static quark correlators in $N_f = 2+1$ QCD up to $T \sim 2.9$ GeV and down to $r \sim 0.018$ fm.
- Static $Q\bar{Q}$ correlators show **remnants of confinement**, up to $T \sim 300$ MeV QGP is strongly coupled.
- Onset of thermal effects strongly depends on individual observables, is much faster if color octet states contribute.
- Further analysis and precision test of perturbation theory for static correlators at finite T are in preparation.