Anisotropic Hydrodynamics

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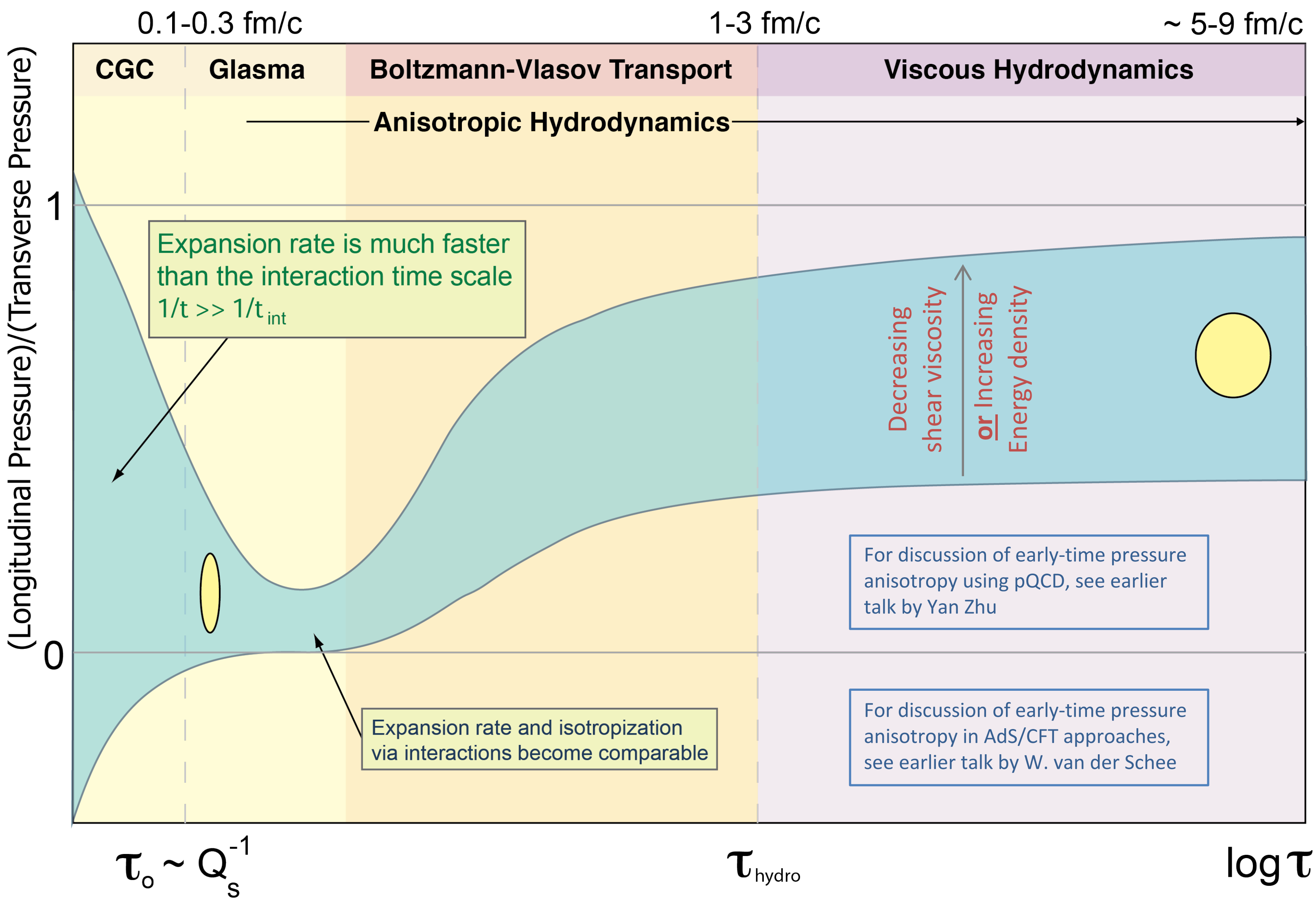
Motivation

- The canonical way to derive viscous hydrodynamics relies on a **linearization around an isotropic equilibrium** state (local rest frame = LRF)
- However, the **QGP is not isotropic in local rest frame (LRF)** → large corrections to ideal hydrodynamics due to strong longitudinal expansion
- Alternative approach**: Anisotropic hydrodynamics** builds in momentum-space anisotropies in the LRF from the beginning
- The goal is to create a quantitatively reliable viscous-hydro-like code that more accurately describes:
    - Early time dynamics
    - Small systems (p+A, p+p)
    - Dynamics near the transverse edges of the overlap region (dilute)
    - Dynamics at forward rapidity (dilute)
    - Temperature-dependent (and potentially large) η/S
- With this, we hope to be able to **more reliably extract the transport coefficients** from data.

QGP momentum anisotropy cartoon



2^{nd}-order viscous hydrodynamics

For small departures from equilibrium we can linearize

$$\boxed{f(x,p) = f_{\rm eq}\left(\frac{p^\mu u_\mu}{T}\right)(1+\delta f(x,p))}$$

$$\boxed{\begin{aligned} T^{\mu\nu} &= T^{\mu\nu}_{\rm ideal} + \int dP\; p^\mu p^\nu f_{\rm eq}\,\delta f \\ &\equiv T^{\mu\nu}_{\rm ideal} + \Pi^{\mu\nu} \end{aligned}} \longrightarrow \boxed{\Pi^{\mu\nu} = \int dP\; p^\mu p^\nu f_{\rm eq}\,\delta f}$$

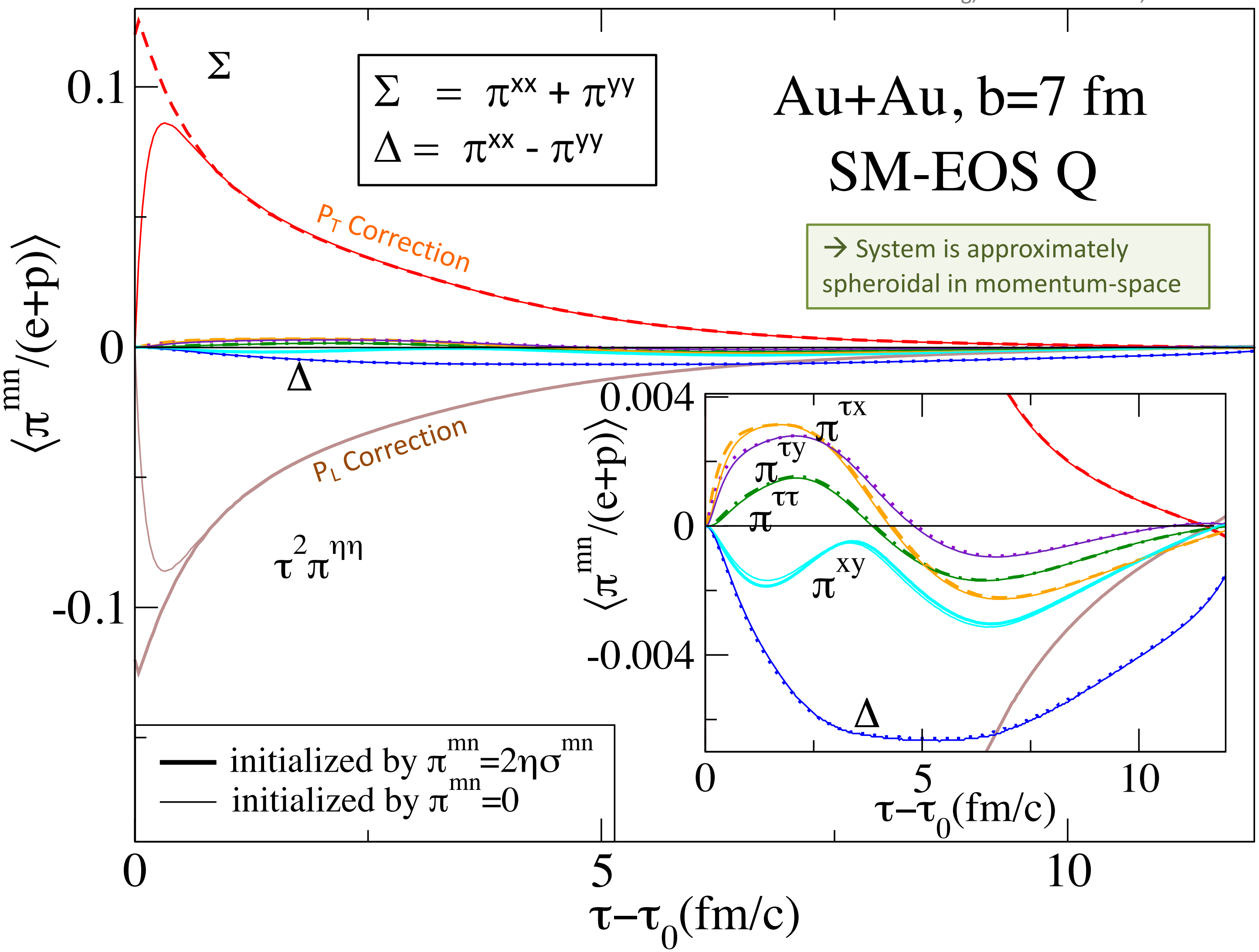
For viscous hydro one expands δf in a gradient expansion: $n^{\rm th}$ order in gradients
→ $n^{\rm th}$-order viscous Hydro

- $1^{\rm st}$ order Hydro : Relativistic Navier-Stokes (parabolic diff eqs → acausal)
  [e.g. Eckart and Landau-Lifshitz]
- $2^{\rm nd}$ order Hydro : Including quadratic gradients fixes causality problem; hyperbolic diff eqs
  [e.g. Israel-Stewart, BRSSS, DNMR (expansion in R^{-1} and Kn), …]
- …

Gradient expansion reliably describes the near-equilibrium limit, but, it is an asymptotic series. [See earlier talk by W. Florkowski]

Indications from Viscous Hydro

H. Song, PhD Dissertation, 0908.3656



Dissipative Anisotropic Hydrodynamics

- There are two ways being actively followed in the literature to address this problem
  A. Linearize around a spheroidal distribution function
     Bazow, Martinez, Molnar, Niemi, Rischke, Heinz, MS
  B. Introduce a generalized anisotropy tensor which replaces the shear stress tensor at LO and linearize around that instead
     Tinti, Ryblewski, Martinez, Nopoush, Alqahtani, Florkowski, MS
- Each of these methods has its own advantages.
- The first can more straightforwardly benefit from the systematic methods introduced in the past to derive the standard 2^{nd} viscous hydrodynamics equations
- The second is more general and can, in principle, even more reliably describe far-from-equilibrium systems; however, the formalism is a bit more complicated.

Spheroidal expansion method

Viscous Hydrodynamics Expansion

See e.g.
- W. Florkowski and R. Ryblewski, 1007.0130
- M. Martinez and MS, 1007.0889
- D. Bazow, U. Heinz, and MS, 1311.6720
- D. Bazow, U. Heinz, and M. Martinez, 1503.07443
- E. Molnar, H. Niemi, and D. Rischke, 1602.00573; 1606.09019

$$f(\tau, \mathbf{x}, \mathbf{p}) = \underbrace{f_{\rm eq}(\mathbf{p}, T(\tau, \mathbf{x}))}_{\text{Isotropic in momentum space}} + \delta f$$

Anisotropic Hydrodynamics (aHydro) Expansion

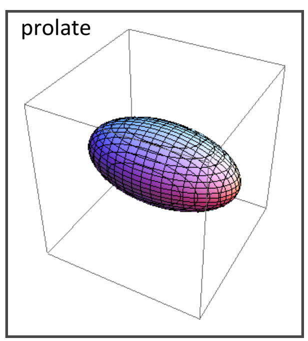
$$f(\tau, \mathbf{x}, \mathbf{p}) = f_{\rm aniso}(\mathbf{p}, \underbrace{\Lambda(\tau, \mathbf{x})}_{T_\perp}, \underbrace{\xi(\tau, \mathbf{x})}_{\text{anisotropy}}\ ) + \delta\tilde{f}$$

Treat this term perturbatively
→ "NLO aHydro"

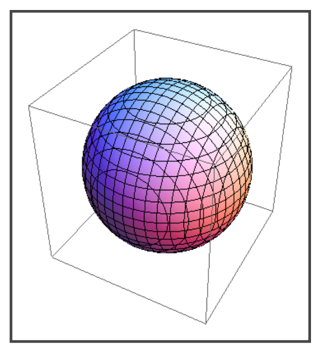
→ "Romatschke-Strickland" form in LRF

$$f^{LRF}_{\rm aniso} = f_{\rm iso}\left(\frac{\sqrt{\mathbf{p}^2 + \xi(\mathbf{x}, \tau) p_z^2}}{\Lambda(\mathbf{x}, \tau)}\right)$$

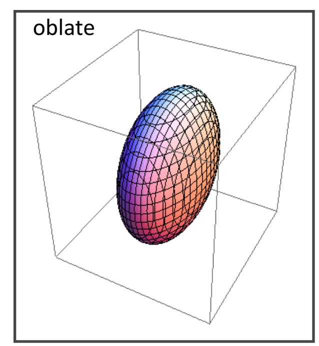
$$\xi = \frac{\langle p_T^2 \rangle}{2\langle p_L^2 \rangle} - 1$$



$-1 < \xi < 0$



$\xi = 0$



$\xi > 0$

Why spheroidal form at LO?

- What is special about this form at leading order?

$$f_{\rm aniso}^{LRF} = f_{\rm iso}\left(\frac{\sqrt{\mathbf{p}^2+\xi(\mathbf{x},\tau)p_z^2}}{\Lambda(\mathbf{x},\tau)}\right)$$

- Gives the ideal hydro limit when $\xi=0$ ($\Lambda \rightarrow T$)
- For longitudinal (0+1d) free streaming, the LRF distribution function is of spheroidal form; limit emerges automatically in 0+1d aHydro

$$\xi_{\rm FS}(\tau) = (1+\xi_0)\left(\frac{\tau}{\tau_0}\right)^2 - 1$$

- Since $f_{iso} \geq 0$, the one-particle distribution function and pressures are ≥ 0 (not guaranteed in standard 2nd-order viscous hydro)
- Reduces to 2nd-order viscous hydrodynamics in limit of small anisotropies

$$\frac{\Pi}{\mathcal{E}_{\rm eq}} = \frac{8}{45}\xi + \mathcal{O}(\xi^2)$$

For 3+1d proof of equivalence to second-order viscous hydrodynamics in the near-equilibrium limit see Tinti 1411.7268.

Generalized aHydro formalism

In generalized aHydro, one assumes that the distribution function is of the form

$$f(x,p) = f_{\rm eq}\left(\frac{\sqrt{p^\mu \Xi_{\mu\nu}(x) p^\nu}}{\lambda(x)}, \frac{\mu(x)}{\lambda(x)}\right) + \delta\tilde{f}(x,p)$$

$$\Xi^{\mu\nu} = \underbrace{u^\mu u^\nu}_{\text{LRF four velocity}} + \underbrace{\xi^{\mu\nu}}_{\text{traceless anisotropy tensor}} - \underbrace{\Delta^{\mu\nu}}_{\text{Transverse projector}} \underbrace{\Phi}_{\text{“Bulk”}}$$

$$u^\mu u_\mu = 1$$
$$\xi^\mu{}_\mu = 0$$
$$\Delta^\mu{}_\mu = 3$$
$$u_\mu \xi^{\mu\nu} = u_\mu \Delta^{\mu\nu} = 0$$

See e.g.

- M. Martinez, R. Ryblewski, and MS, 1204.1473
- L. Tinti and W. Florkowski, 1312.6614
- M. Nopoush, R. Ryblewski, and MS, 1405.1355

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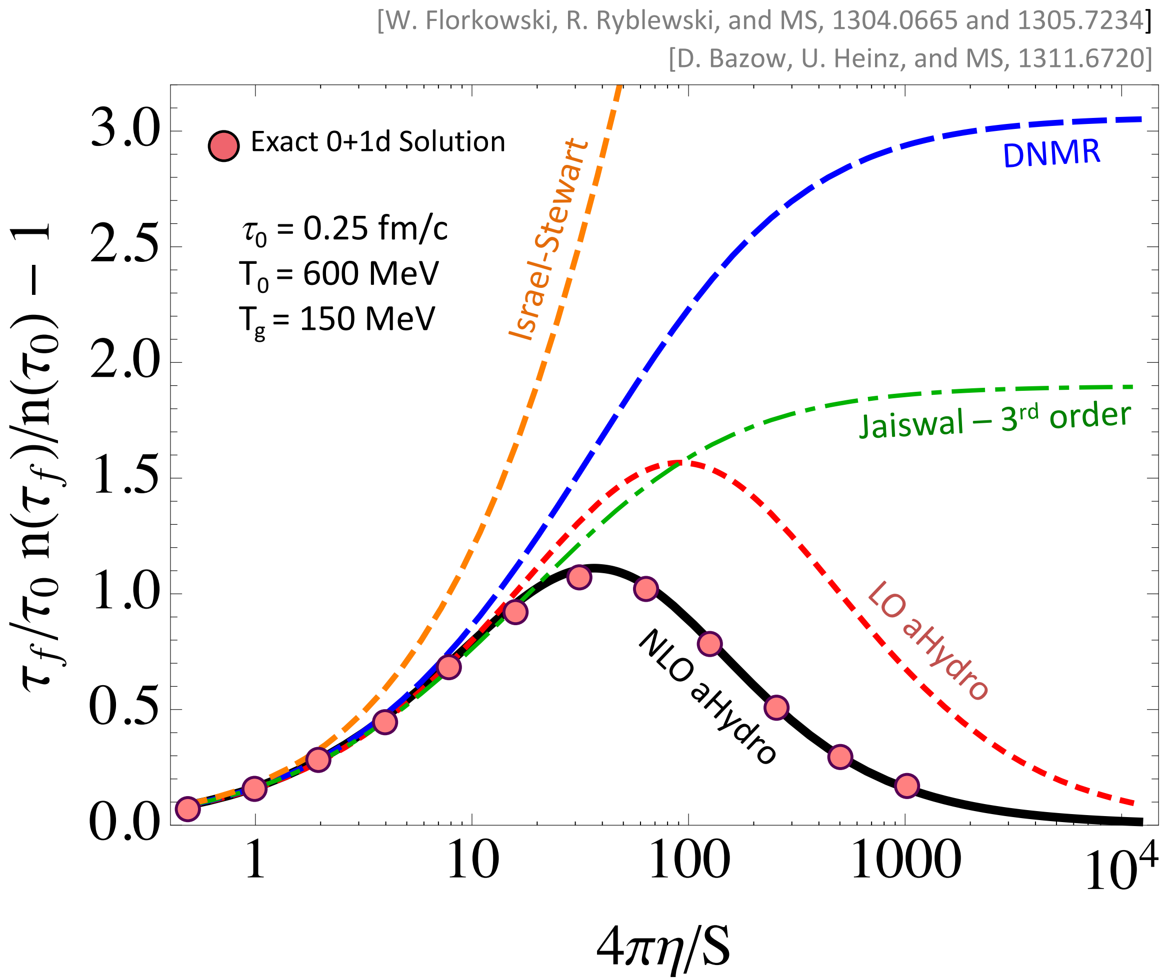
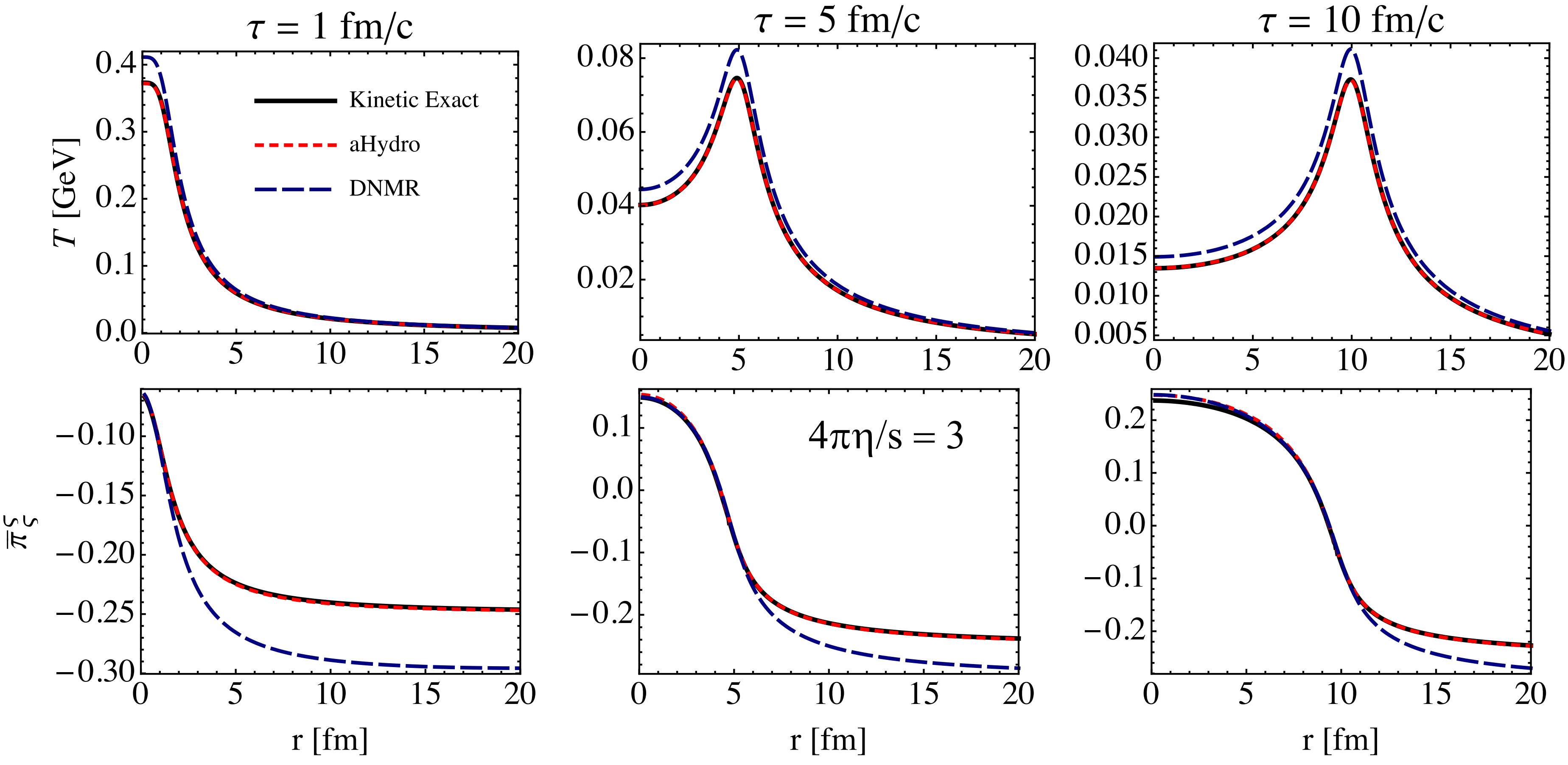
Conformal 0+1d aHydro results



- Number (entropy) production vanishes in two limits: ideal hydrodynamic and free streaming limits
- In the conformal model which we are testing with, number density is proportional to entropy density

1+1d aHydro solution for Gubser Flow

M. Nopoush, R. Ryblewski, and MS, 1410.6790
Exact kinetic solution: G. Denicol, U.W. Heinz, M. Martinez, J. Noronha, and MS, 1408.5646 and 1408.7048



3+1d aHydro Equations of Motion

- Assuming an ellipsoidal form for the anisotropy tensor (ignoring off-diagonal components for now), one has seven degrees of freedom: ξ_x, ξ_y, ξ_z, u_x, u_y, u_z, and λ.
- For the EoS we use a lattice-based EoS with the effective temperature T determined via Landau matching.

$$
\begin{aligned}
D_u\mathcal{E} + \mathcal{E}\theta_u + \mathcal{P}_x u_\mu D_x X^\mu + \mathcal{P}_y u_\mu D_y Y^\mu + \mathcal{P}_z u_\mu D_z Z^\mu &= 0\,,\\
D_x\mathcal{P}_x + \mathcal{P}_x\theta_x - \mathcal{E} X_\mu D_u u^\mu - \mathcal{P}_y X_\mu D_y Y^\mu - \mathcal{P}_z X_\mu D_z Z^\mu &= 0\,,\\
D_y\mathcal{P}_y + \mathcal{P}_y\theta_y - \mathcal{E} Y_\mu D_u u^\mu - \mathcal{P}_x Y_\mu D_x X^\mu - \mathcal{P}_z Y_\mu D_z Z^\mu &= 0\,,\\
D_z\mathcal{P}_z + \mathcal{P}_z\theta_z - \mathcal{E} Z_\mu D_u u^\mu - \mathcal{P}_x Z_\mu D_x X^\mu - \mathcal{P}_y Z_\mu D_y Y^\mu &= 0\,.
\end{aligned}
$$

First Moment

$$
\begin{aligned}
D_u\mathcal{I}_x + \mathcal{I}_x(\theta_u + 2u_\mu D_x X^\mu) &= \frac{1}{\tau_{\rm eq}}(\mathcal{I}_{\rm eq} - \mathcal{I}_x)\,,\\
D_u\mathcal{I}_y + \mathcal{I}_y(\theta_u + 2u_\mu D_y Y^\mu) &= \frac{1}{\tau_{\rm eq}}(\mathcal{I}_{\rm eq} - \mathcal{I}_y)\,,\\
D_u\mathcal{I}_z + \mathcal{I}_z(\theta_u + 2u_\mu D_z Z^\mu) &= \frac{1}{\tau_{\rm eq}}(\mathcal{I}_{\rm eq} - \mathcal{I}_z)\,.
\end{aligned}
$$

Second Moment

Florkowski, Haque, Nopoush, Ryblewski, MS, forthcoming

Implementing the equation of state

R Ryblewski and F. Florkowski, 1204.2624
M. Alqahtani, M. Nopoush, and MS, 1509.02913; 1605.02101

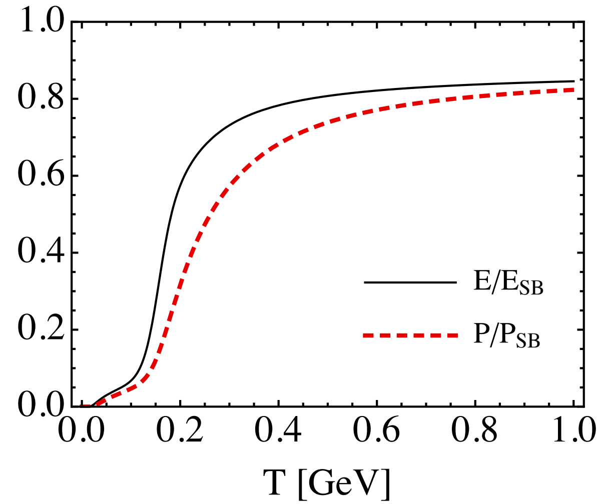
Standard Method

$$n(\Lambda,\xi) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} f_{\rm aniso} = \frac{n_{\rm iso}(\Lambda)}{\sqrt{1+\xi}}$$

$$\mathcal{E}(\Lambda,\xi) = T^{\tau\tau} = \mathcal{R}(\xi)\,\mathcal{E}_{\rm iso}(\Lambda)$$

$$\mathcal{P}_\perp(\Lambda,\xi) = \frac{1}{2}\left(T^{xx}+T^{yy}\right) = \mathcal{R}_\perp(\xi)\mathcal{P}_{\rm iso}(\Lambda)$$

$$\mathcal{P}_L(\Lambda,\xi) = -T^\varsigma_\varsigma = \mathcal{R}_{\rm L}(\xi)\mathcal{P}_{\rm iso}(\Lambda)$$

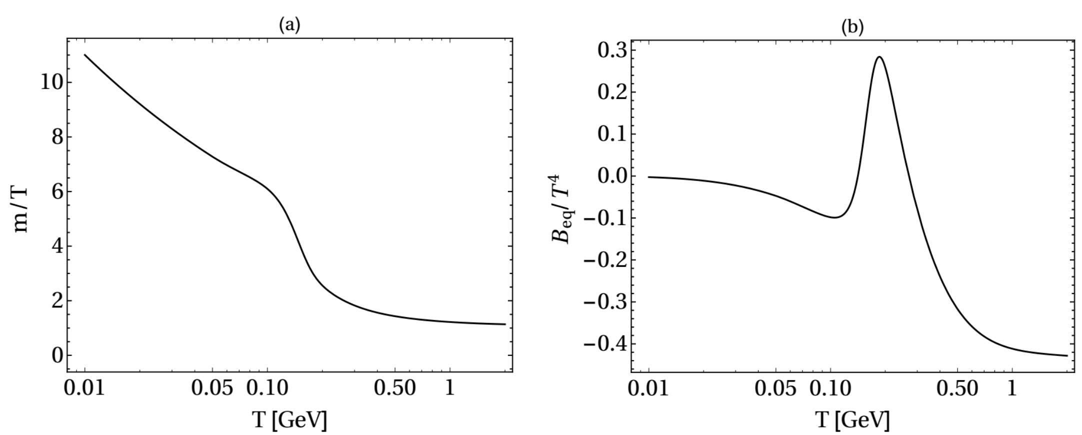


Quasiparticle Method

$$T^{\mu\nu}_{\rm eq} = T^{\mu\nu}_{\rm kinetic,eq} + g^{\mu\nu} B_{\rm eq}$$

$$p^\mu \partial_\mu f + \frac{1}{2}\partial_i m^2 \partial^i_{(p)} f = -\mathcal{C}[f]$$

$$\partial_\mu B = -\frac{1}{2}\partial_\mu m^2 \int dP\, f(x,p)$$



Anisotropic "Cooper-Frye" Freezeout

Bazow, Heinz, Martinez, Nopoush, Ryblewski, MS, 1506.05278
Florkowski, Haque, Nopoush, Ryblewski, MS, forthcoming

- Use same ellipsoidal form for "anisotropic freeze-out" at LO.
- From includes both shear and bulk corrections to to the distribution function.
- Use energy density (scalar) to determine the freeze-out hyper-surface Σ → e.g. $T_{\text{eff,FO}}$ = 150 MeV

$$f(x,p) = f_{\text{iso}}\left(\frac{1}{\lambda}\sqrt{p_\mu \Xi^{\mu\nu} p_\nu}\right)$$

$$\Xi^{\mu\nu} = \underbrace{u^\mu u^\nu}_{\text{isotropic}} + \underbrace{\xi^{\mu\nu}}_{\text{anisotropy tensor}} - \underbrace{\Phi \Delta^{\mu\nu}}_{\text{bulk correction}}$$

$$\xi^{\mu\nu}_{\text{LRF}} \equiv \text{diag}(0, \xi_x, \xi_y, \xi_z)$$

$$\xi^\mu{}_\mu = 0 \qquad u_\mu \xi^\mu{}_\nu = 0$$

$$\left(p^0 \frac{dN}{dp^3}\right)_i = \frac{\mathcal{N}_i}{(2\pi)^3} \int f_i(x,p)\, p^\mu d\Sigma_\mu \,,$$

NOTE: Usual 2nd-order viscous hydro form

$$f(p,x) = f_{\text{eq}}\left[1 + (1 - a f_{\text{eq}}) \frac{p_\mu p_\nu \Pi^{\mu\nu}}{2(\epsilon + P)T^2}\right]$$

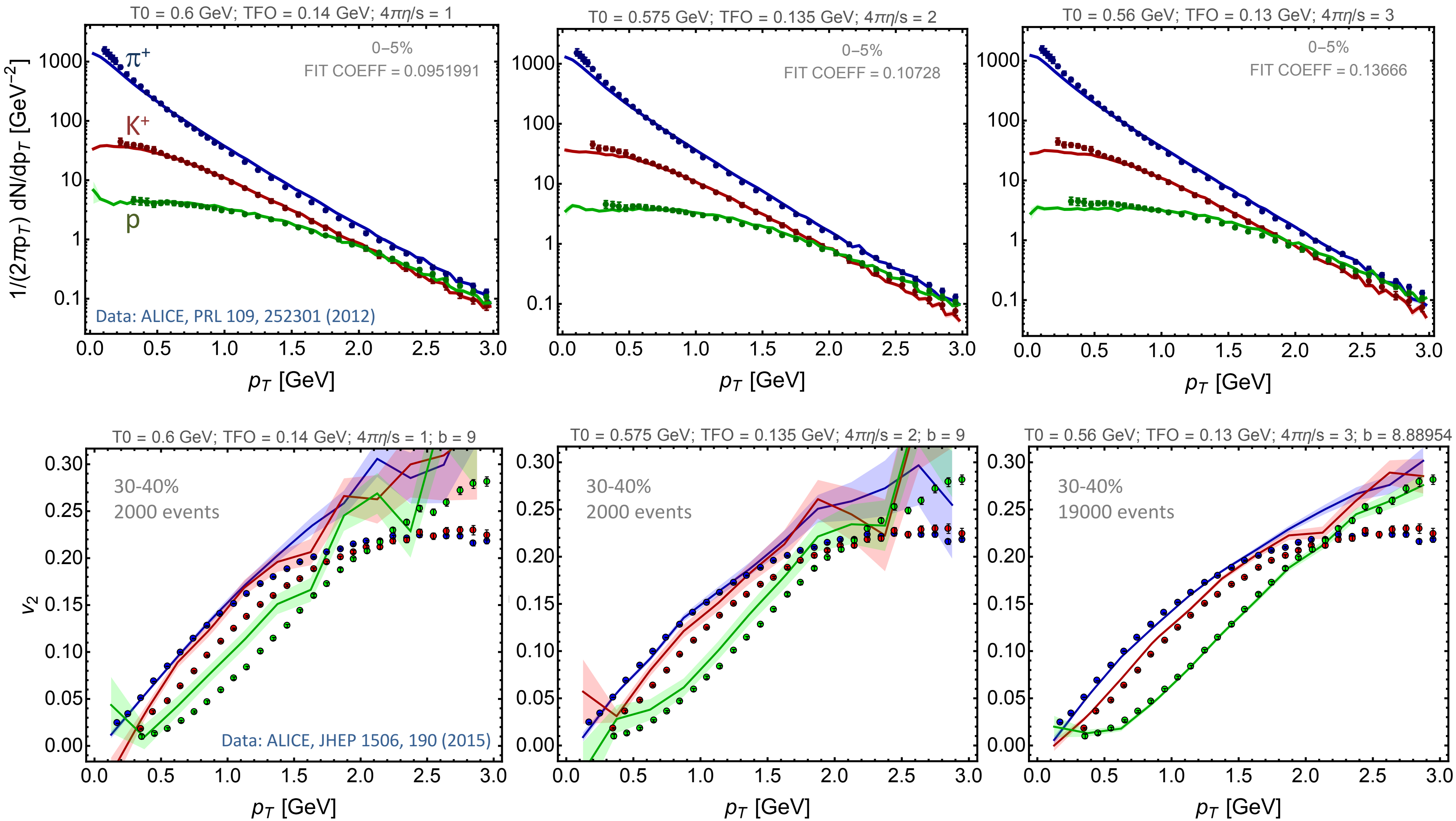
$$f_{\text{eq}} = 1/[\exp(p \cdot u/T) + a] \qquad a = \text{-}1,\ +1,\ \text{or}\ 0$$

- This form suffers from the problem that the distribution function can be negative in some regions of phase space → unphysical
- Problem becomes worse when including bulk viscous correction (see forthcoming slides).

Preliminary results – Glauber ICs

Florkowski, Haque, Nopoush, Ryblewski, MS, forthcoming

LHC 2.76 TeV Pb+Pb collisions; top row shows spectra, bottom row shows differential v_2



Discussion I

- 3+1d aHydro calculation includes effects of both bulk and shear viscosity.
- In the RTA model used, the bulk viscosity is related to the shear viscosity via

$$\frac{\zeta}{\tau_\pi} = \left(\frac{1}{3} - c_s^2\right)(\mathcal{E} + \mathcal{P}) - \frac{2}{9}(\mathcal{E} - 3\mathcal{P})$$
$$\frac{\eta}{\tau_\pi} = \frac{4}{5}\mathcal{P} + \frac{1}{15}(\mathcal{E} - 3\mathcal{P})$$

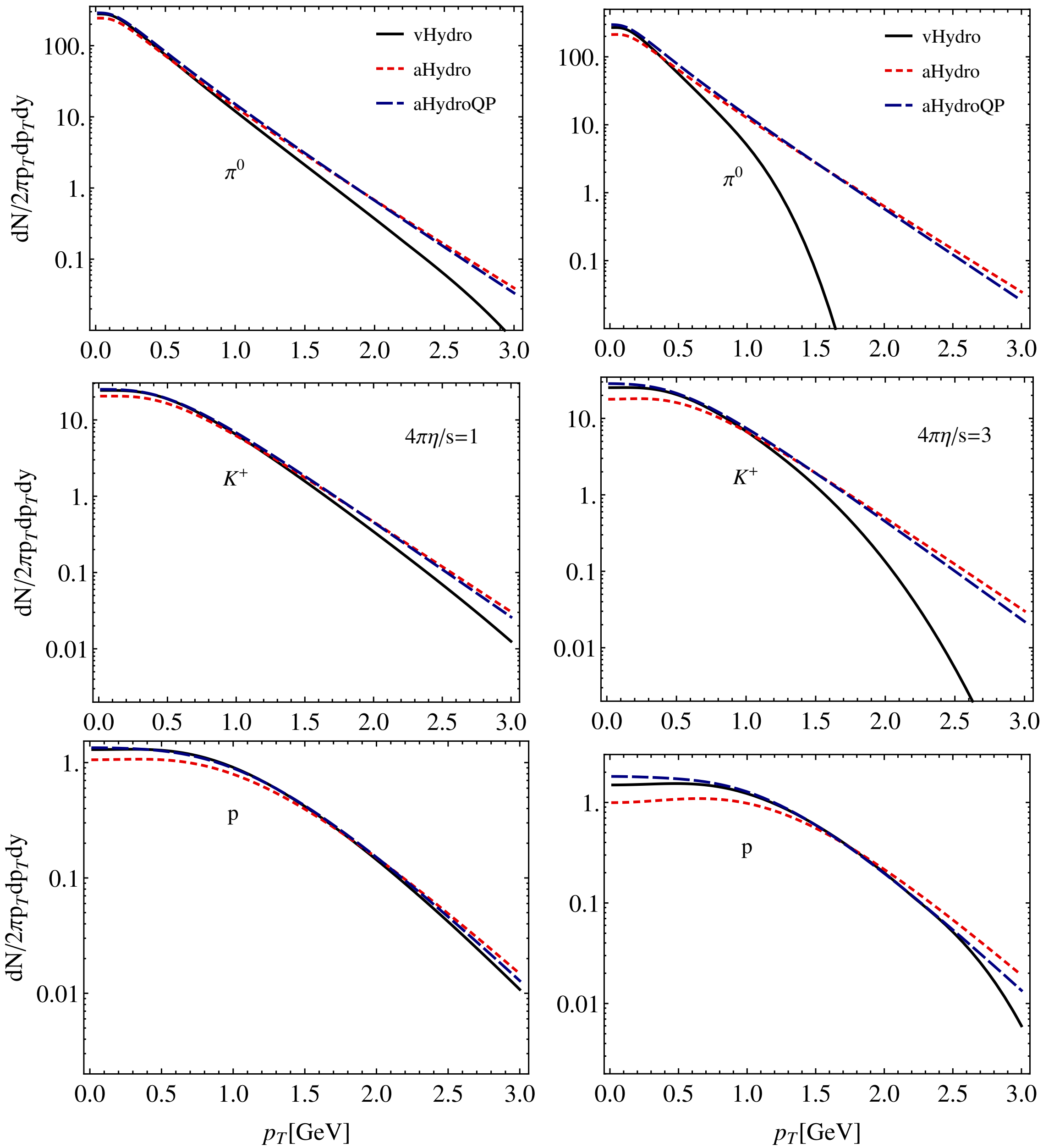
  so there is only one independent transport coefficient (this also holds true for all other higher-order transport coefficients when using RTA).

- Our preliminary findings suggest that $\eta/s \cong 0.23$.
- This is different than the recent vHydro results of Ryu, et al 1502.01675, who found that including bulk viscosity gives $\eta/s \cong 0.095$.

Discussion II

M. Alqahtani, M. Nopoush, and MS, 1605.02101

- The difference can be due to many factors (slightly different EoS, different initial conditions, etc.).
- However, one very troubling feature of the ``standard'' **vHydro bulk corrrection** is that it can **cause the primordial particle spectra to become negative** at (not so) high momentum**.**
- This unphysical effect can cause one to overestimate the effect of increasing the shear viscosity to entropy density ratio.
- aHydro does not suffer from this problem.



Conclusions and Outlook

- Anisotropic hydrodynamics builds upon prior advances in relativistic hydrodynamics in an attempt to create a more quantitatively reliable model of QGP evolution.
- It incorporates some “facts of life” specific to the conditions generated in relativistic heavy ion collisions and, in doing so, optimizes the dissipative hydrodynamics approach.
- We now have a 3+1d aHydro code with realistic EoS, anisotroic freeze-out, etc which we are using to extract QGP transport coefficients.
- Our preliminary fits to experimental data look quite good; we can fit spectra and v_2 including the mass splitting between different hadronic species. Need more statistics and tuning…
- Our preliminary findings suggest $\eta/s \cong 0.23$ for LHC 2.76 TeV Pb+Pb collisions.