The flow paradigm

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Ultrarelativistic nuclear collisions

• At the LHC: collisions between lead atomic nuclei Pb\(^{82+}\) (Pb-Pb runs in 2010, 2011, 2015)
• proton-Pb collisions in 2012 and 2013
The other facility: RHIC

- The only dedicated heavy-ion collider, at Brookhaven (USA), running since 2000.
- Energy per nucleon-nucleon collision:
  RHIC: up to 200 GeV
  LHC: 5.02 TeV
Nuclear collisions at the LHC

• Lorentz contraction factor \( \sim 2700 \): colliding thin pancakes
• A *deconfined quark-gluon plasma* is created and expands into the vacuum
• The best theoretical description of the expansion is a macroscopic one: a small lump of relativistic fluid (\( v \sim c \)), \( T \sim 200\text{-}300 \) MeV
What we see

Typical Pb-Pb collision “event” at the LHC
The flow paradigm (2010)

- Particles are emitted independently in every event.
- In a hydrodynamic description, the momenta $p$ of outgoing particles are sampled independently from an underlying probability distribution $f(p)$.
- The fluctuations of this single-particle distribution $f(p)$ event to event create the specific correlations seen experimentally.

Alver & Roland arXiv:1003.0194
Consequences of the flow paradigm

• The flow paradigm alone strongly constrains observables, irrespective of any particular hydrodynamic description.

• For instance, the flow paradigm implies that the two-particle correlation is $\langle f(p_1)f(p_2) \rangle - \langle f(p_1) \rangle \langle f(p_2) \rangle$ where $\langle \ldots \rangle =$average over events.

• Viewed as a matrix as a function of $p_1$ and $p_2$, it is a covariance matrix, hence its eigenvalues are all positive.

More consequences

• Experimentally, higher-order correlations are easily measured (8-particle correlations, for instance) because of the large multiplicity & combinatorics.

• The consequences of the flow paradigm for higher-order correlations are not yet fully explored.

• I will present two recent applications of the flow paradigm to higher-order correlations.

Giacalone, Yan, Noronha-Hostler, JYO, 1605.08303
1608.01823
Pseudorapidity, azimuth

- Trajectories of charged particles:
  - polar angle $\theta$
  - (or rapidity $\eta=-\ln\tan(\theta/2)$)
  - azimuthal angle $\phi$
Anisotropic flow

• The single-particle distribution is essentially independent of rapidity $\eta$ but depends on azimuthal angle, $\varphi$.

• Fourier decomposition: $f(\varphi) = \sum_n V_n e^{-i n \varphi}$

• $v_n \equiv |V_n| = \text{anisotropic flow}$

• Flow paradigm: all information contained in $V_n$ and its event-to-event fluctuations.
Anisotropic flow and hydrodynamics

*Initial transverse density profile*

Expansion

*Final distribution*

Elliptic flow $v_2$

Triangular flow $v_3$

In hydrodynamics, anisotropic flow is a response to the anisotropy of the initial density profile.
The centrality dependence of $v_n$

- Root-mean-square values of $v_n$.
- Largest Fourier harmonic is elliptic flow, $v_2$.
- Steep decrease of $v_2$ for central collisions: reflects the elliptic geometry of the overlap area.
- At the qualitative level, the centrality dependence of $v_n$ is naturally explained by hydrodynamics.
1. Symmetric cumulants

Giacalone, Yan, Noronha-Hostler, JYO, 1605.08303
New data from ALICE

• « symmetric cumulants » : specific 4 particle correlations.

• Using the flow paradigm, the symmetric cumulant can be recast as a correlation between the magnitudes of 2 different Fourier harmonics:

\[ SC(4, 2) \equiv \frac{\langle v_4^2 v_2^2 \rangle - \langle v_4^2 \rangle \langle v_2^2 \rangle}{\langle v_4^2 \rangle \langle v_2^2 \rangle} \]

• ALICE has recently measured \( SC(4,2) \) as a function of centrality
New data from ALICE

ALICE Collaboration, arXiv:1604.07663
The *event-plane correlation* measured by ATLAS is in fact a linear (Pearson) correlation between the *complex* flow coefficients $V_4$ and $(V_2)^2$

\[
\cos \Phi_{24} \equiv \frac{\text{Re}\langle V_4 (V_2^*)^2 \rangle}{\sqrt{\langle v_4^2 \rangle \langle v_2^4 \rangle}}
\]

It is also a measure of the correlation between $V_4$ and $V_2$, which involves the *relative angle* and the *magnitudes*.

*Luzum JYO arXiv:1209.2323*
ATLAS « event-plane correlation »

ATLAS Collaboration, arXiv:1403.0489
Can we compare ALICE and ATLAS?

• We have two measures of the correlation between $V_4$ and $V_2$, the *symmetric cumulant (ALICE)* and the *event-plane correlation (ATLAS)*.

• I derive a *quantitative relation* between these two measures, test it on hydro calculations and then on data.
Modeling the correlation

• Decompose $V_4 = V_{4L} + \chi_4 (V_2)^2$, with $\chi_4$ = constant fixed so that linear correlation between the two terms = 0.

• Just math, no physics input

• Then $\Phi_{24}$ measures the relative magnitude of the 2 terms: 
  $\chi_4^2 \langle v_2^4 \rangle = \langle v_4^2 \rangle \cos^2 \Phi_{24}$
Modeling the correlation

\[ V_4 = V_{4L} + \chi_4 (V_2)^2 \]

• We assume that the two terms are independent (stronger than uncorrelated)

• Then: correlation between \((v_4)^2\) and \((v_2)^2\) is only from the nonlinear part:

\[
\langle v_4^2 v_2^2 \rangle - \langle v_4^2 \rangle \langle v_2^2 \rangle = \chi_4^2 \left( \langle v_2^6 \rangle - \langle v_2^4 \rangle \langle v_2^2 \rangle \right)
\]
Expressing $X_4$ as a function of the event-plane correlation, we obtain:

$$SC(4, 2) = \left( \frac{\langle v_2^6 \rangle}{\langle v_2^4 \rangle \langle v_2^2 \rangle} - 1 \right) \cos^2 \Phi_{24}$$

- symmetric cumulant
- elliptic flow fluctuations
- event-plane correlation
Event-by-event hydrodynamics

We compute both sides of the equation independently in event-by-event viscous hydro with Glauber initial conditions.

The relation is satisfied to a good approximation for all centralities.
ALICE versus ATLAS

Using elliptic flow fluctuations (cumulants) and event plane correlations from ATLAS:

SC(4,2)
Predictions

Same methodology applied to different orders:

\[
SC(4, 3) = \left( \frac{\langle v_2^4 v_3^2 \rangle}{\langle v_2^4 \rangle \langle v_3^2 \rangle} - 1 \right) \cos^2 \Phi_{24}
\]

\[
SC(5, 2) = \left( \frac{\langle v_2^2 v_3^2 \rangle}{\langle v_2^4 \rangle} \frac{\langle v_3^2 \rangle}{\langle v_2^2 \rangle} - 1 \right) \cos^2 \Phi_{235}
\]

\[
SC(5, 3) = \left( \frac{\langle v_2^2 v_3^4 \rangle}{\langle v_2^2 \rangle \langle v_3^2 \rangle} - 1 \right) \cos^2 \Phi_{235}
\]

- Symmetric cumulants
- Flow fluctuations
- Event-plane correlations
Data-driven predictions (no hydro calculation!) using ATLAS results on $v_n$ fluctuations and event-plane correlations.
2. The statistics of $v_2$ fluctuations

Giacalone, Yan, Noronha-Hostler, JYO, 1608.01823
The fluctuations of elliptic flow

• One can measure much more than the rms value of $v_2$. Also higher order moments and cumulants

\[ v_2^{(2)} = \left( \langle v_2^2 \rangle \right)^{1/2} \]
\[ v_2^{(4)} = \left( 2 \langle v_2^2 \rangle^2 - \langle v_2^4 \rangle \right)^{1/4} \]
\[ v_2^{(6)} = \left( \left( \langle v_2^6 \rangle - 9 \langle v_2^4 \rangle \langle v_2^2 \rangle + 12 \langle v_2^2 \rangle^3 / 4 \right) \right)^{1/6} \]

• $v_2^{(4)} < v_2^{(2)}$ if $v_2$ fluctuates

• $v_2^{(4)} = v_2^{(6)}$ if fluctuations are 2-dim. Gaussian.
$v_2$ vs. centrality percentile

- $v_2\{2\}$
- $v_2\{2\}$ (same charge)
- $v_2\{4\}$
- $v_2\{4\}$ (same charge)
- $v_2\{q\text{-dist}\}$
- $v_2\{\text{LYZ}\}$

$ALICE$ 1011.3916
The fluctuations of elliptic flow

Decompose $V_2$ into real (cos) and imaginary (sin) parts

$$V_2 = v_x + i v_y$$

Probability distribution in a hydrodynamic calculation:

Fluctuations of $v_x$ are not symmetric: they have negative skewness

$$s \equiv \langle (v_x - \langle v_x \rangle)^3 \rangle$$
Non-Gaussian fluctuations

We have shown by an expansion in powers of the fluctuations that

\[ v_2\{6\}-v_2\{4\} = \frac{s}{\langle 3v_x^2 \rangle} \]

Negative skewness, \( s < 0 \), naturally explains the small lifting of degeneracy seen by ATLAS:

\[ \frac{v_2\{6\}}{v_2\{4\}} \]
Conclusions

• Bulk (soft particle) correlations of arbitrary order are, at the qualitative level, naturally explained by the flow paradigm and minimal assumptions from hydrodynamics (e.g., eccentricity scaling of $v_2$).

• I have illustrated the predictive power of this approach on two examples: symmetric cumulants and elliptic flow fluctuations.

• The consequences of the flow paradigm for higher-order correlations have not yet been fully explored.
Backup slides
Multiplicity and centrality

- Collisions classified from more central to less central in 5% bins
- More central creates more particles
- A central collision (b=0) typically produces 25000 particles.
Two-particle correlations

proton-proton collision

central Pb+Pb collision
Linear and nonlinear hydro response

• **Hydrodynamics** also predicts the decomposition \( V_4 = V_{4L} + \chi_4(V_2)^2 \), where

• \( V_{4L} = \) response to initial fluctuations in 4th harmonic

• \( \chi_4(V_2)^2 \) = nonlinear response induced by hydrodynamic evolution

Teaney & Yan arXiv:1206.1905

Yan & JYO arXiv:1502.02502
ALICE versus ATLAS

- Agreement not as good as in hydro. why?
- ATLAS event-plane correlations are measured with a large pseudorapidity gap and over a wide interval -4.8 to 4.8
- ALICE SC(4,2) is measured without any gap and over the interval -0.8 to 0.8
- Longitudinal flow fluctuations induce a decoherence which may explain why the ATLAS result is smaller.