The flow paradigm

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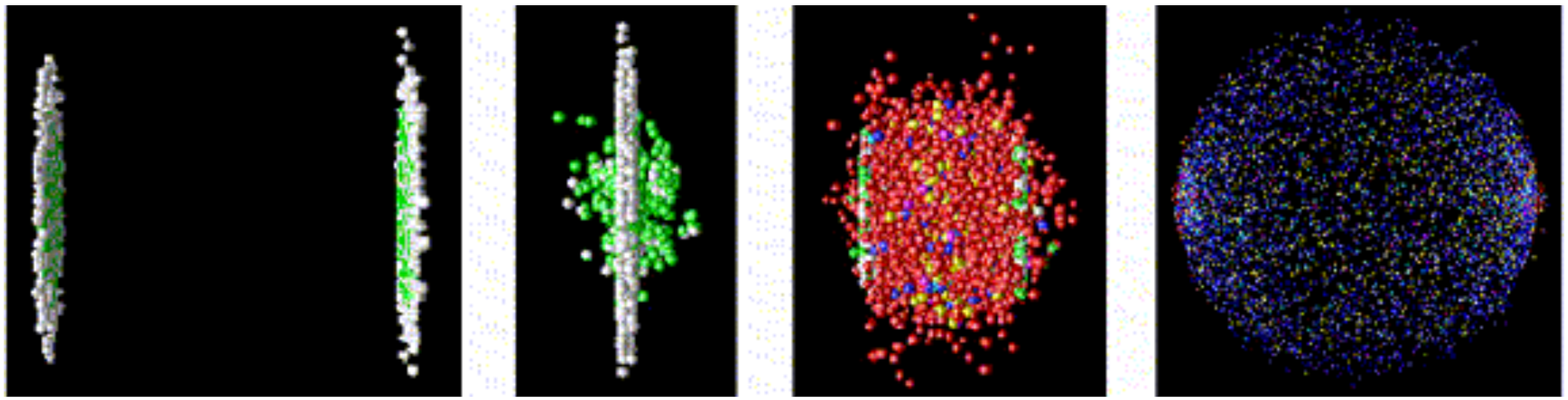
Ultrarelativistic nuclear collisions

- At the LHC: collisions between lead atomic nuclei Pb^{82+} (Pb-Pb runs in 2010, 2011, 2015)
- proton-Pb collisions in 2012 and 2013

The other facility : RHIC

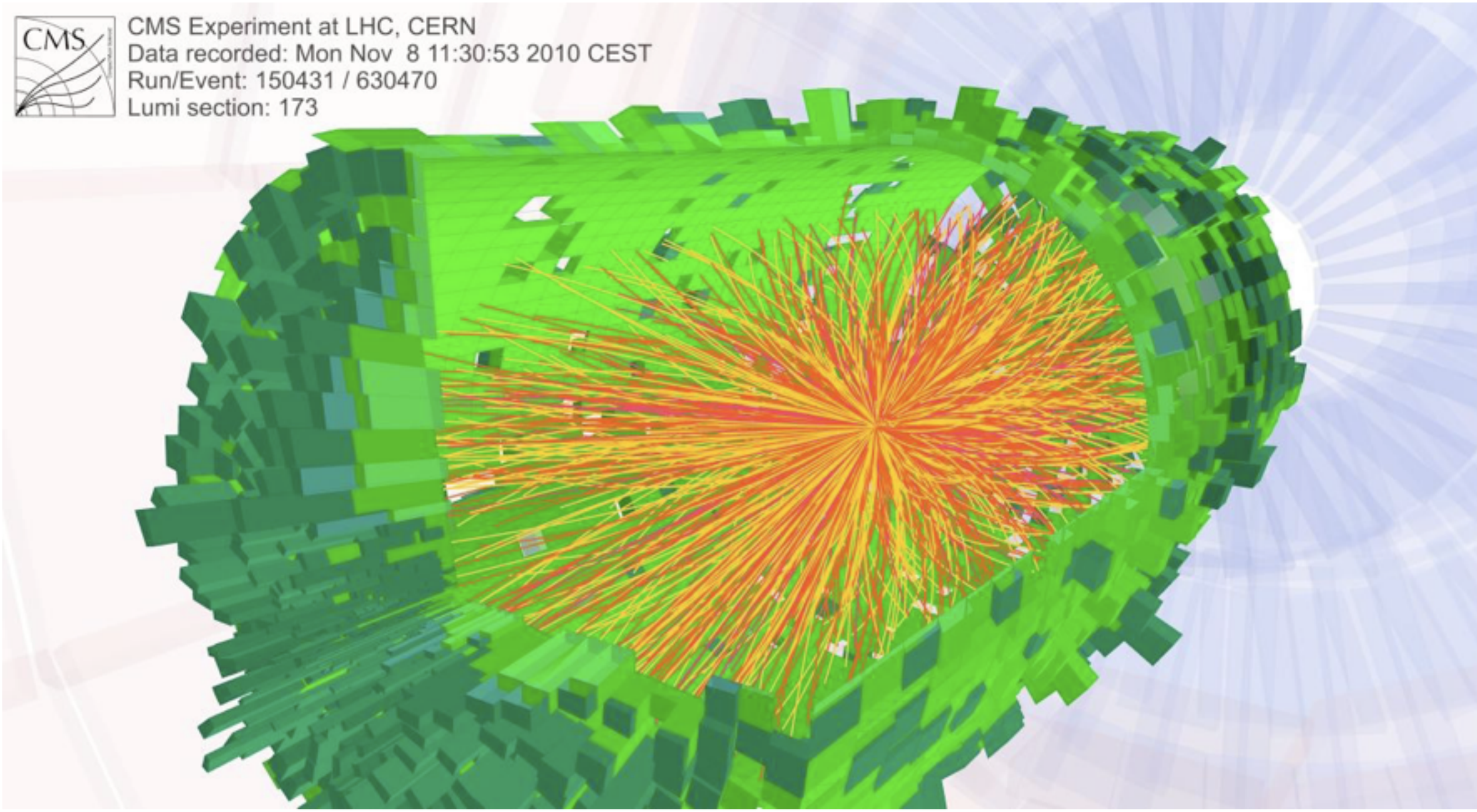
- The only dedicated heavy-ion collider, at Brookhaven (USA), running since 2000.
- Energy per nucleon-nucleon collision:
  RHIC: up to 200 GeV
  LHC: 5.02 TeV

Nuclear collisions at the LHC



- Lorentz contraction factor ~2700: colliding thin pancakes
- *A deconfined quark-gluon plasma* is created and expands into the vacuum
- The best theoretical description of the expansion is a macroscopic one: a small lump of relativistic fluid (v~c), T ~ 200-300 MeV

What we see



Typical Pb-Pb collision “event” at the LHC

The flow paradigm (2010)

- Particles are emitted independently in every event.
- In a hydrodynamic description, the momenta p of outgoing particles are sampled independently from an underlying probability distribution f(p)
- The fluctuations of this single-particle distribution f(p) event to event create the specific correlations seen experimentally.

Alver & Roland arXiv:1003.0194

Consequences of the flow paradigm

- The flow paradigm alone strongly constrains observables, irrespective of any particular hydrodynamic description.

- For instance, the flow paradigm implies that the two-particle correlation is $\langle f(p_1)f(p_2)\rangle - \langle f(p_1)\rangle\langle f(p_2)\rangle$ where $\langle\ldots\rangle$=average over events.

- Viewed as a matrix as a function of p_1 and p_2, it is a covariance matrix, hence its eigenvalues are all positive.

Bhalerao, JYO, Pal, Teaney, arXiv:1410.7739

More consequences

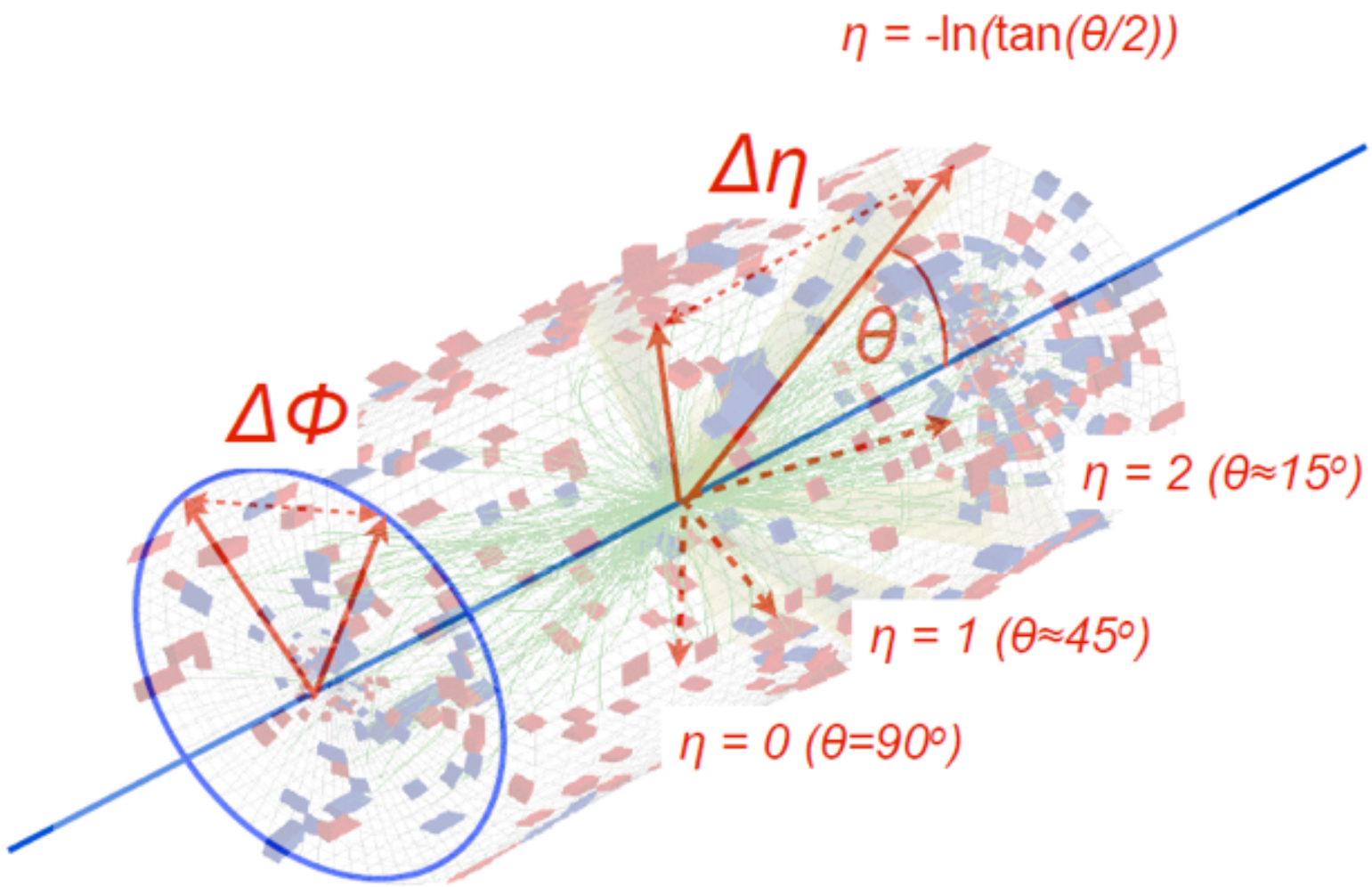
- Experimentally, higher-order correlations are easily measured (8-particle correlations, for instance) because of the large multiplicity & combinatorics.

- The consequences of the flow paradigm for higher-order correlations are not yet fully explored.

- I will present two recent applications of the flow paradigm to higher-order correlations.

Giacalone, Yan, Noronha-Hostler, JYO, 1605.08303
1608.01823

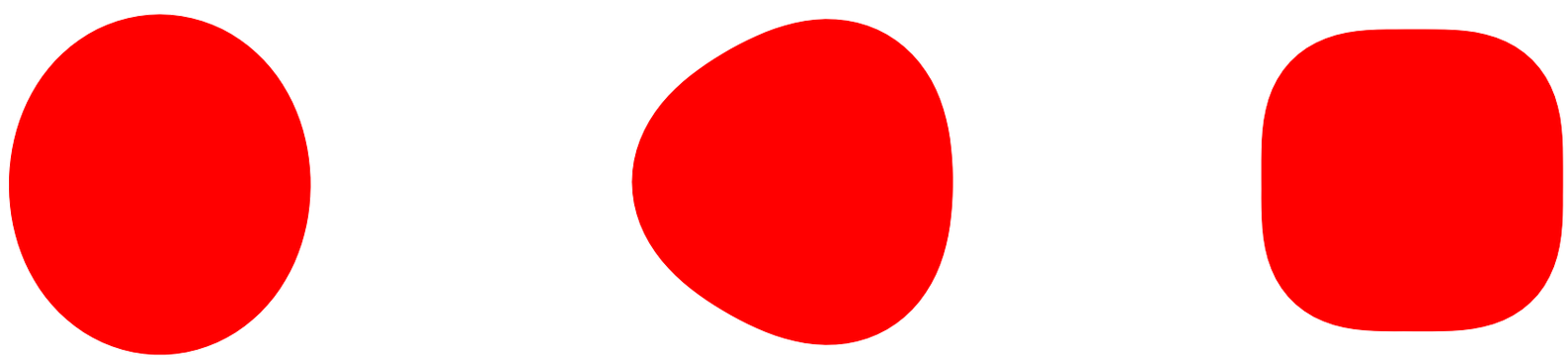
Pseudorapidity, azimuth



- Trajectories of charged particles:
- polar angle θ (or rapidity η=-ln tan θ/2)
- azimuthal angle φ

Anisotropic flow

- The single-particle distribution is essentially independent of rapidity η but depends on azimuthal angle, φ.
- Fourier decomposition : $f(\varphi)=\sum_n V_n e^{-in\varphi}$
- $v_n \equiv |V_n|$=anisotropic flow
- Flow paradigm: all information contained in V_n and its event-to-event fluctuations.



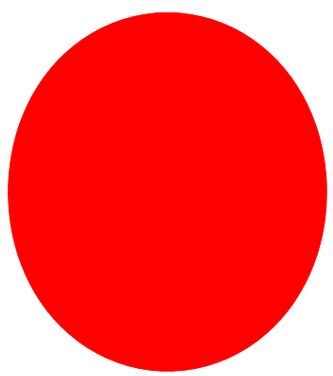
v_2 v_3 v_4

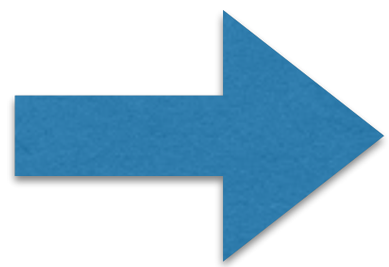
Anisotropic flow and hydrodynamics

Initial transverse density profile

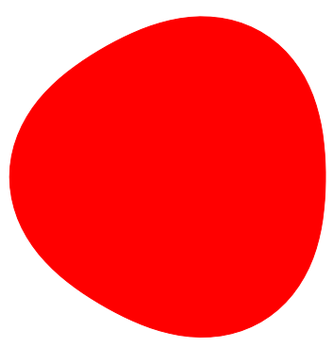
Expansion

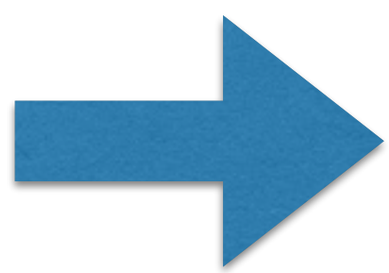
Final distribution





Elliptic flow v_2

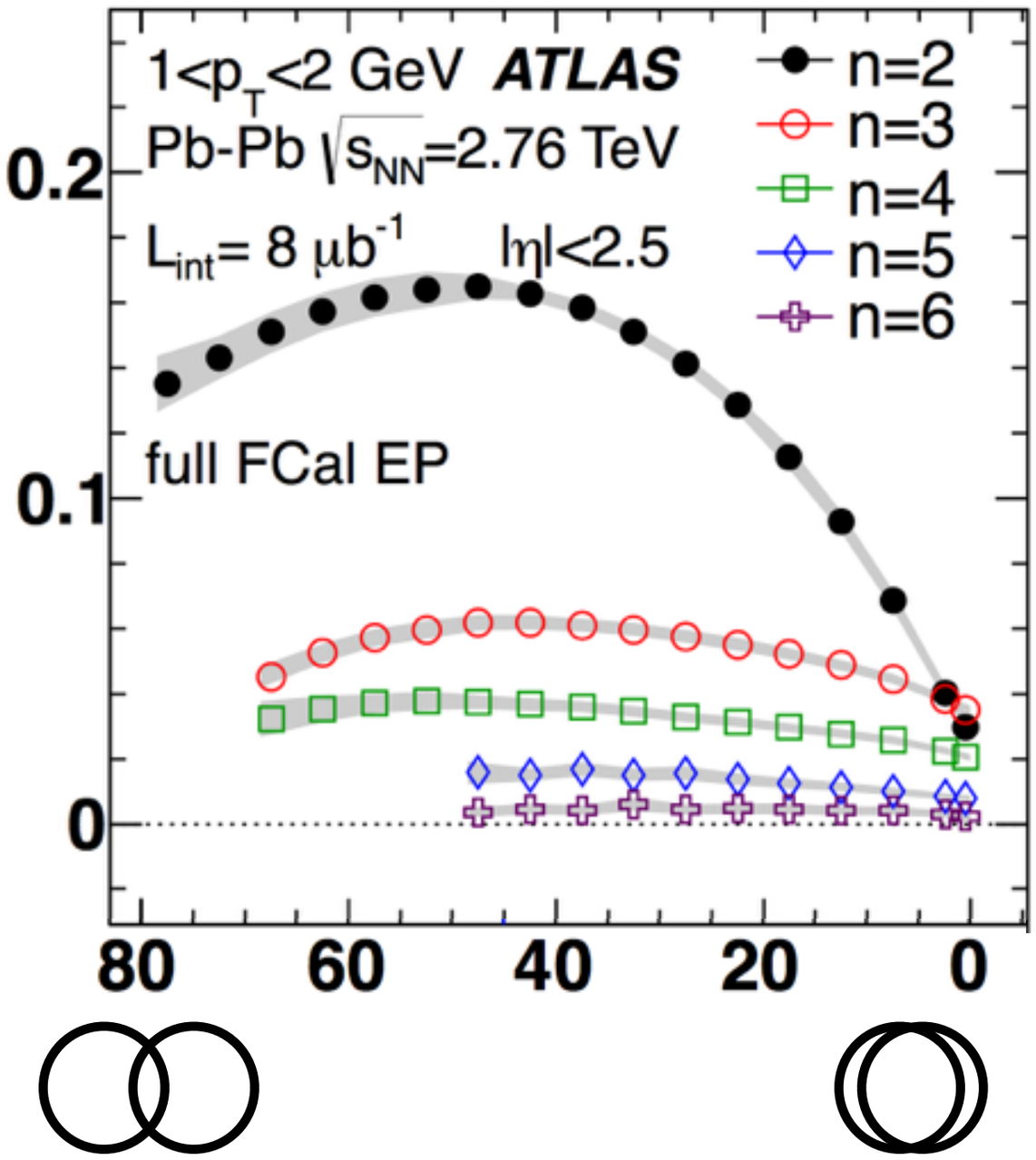




Triangular flow v_3

In hydrodynamics, anisotropic flow is a response to the anisotropy of the initial density profile.

The centrality dependence of v_n



- Root-mean-square values of v_n.
- Largest Fourier harmonic is elliptic flow, v_2
- Steep decrease of v_2 for central collisions: reflects the elliptic geometry of the overlap area
- At the qualitative level, the centrality dependence of v_n is naturally explained by hydrodynamics.

1. Symmetric cumulants

Giacalone, Yan, Noronha-Hostler, JYO, 1605.08303

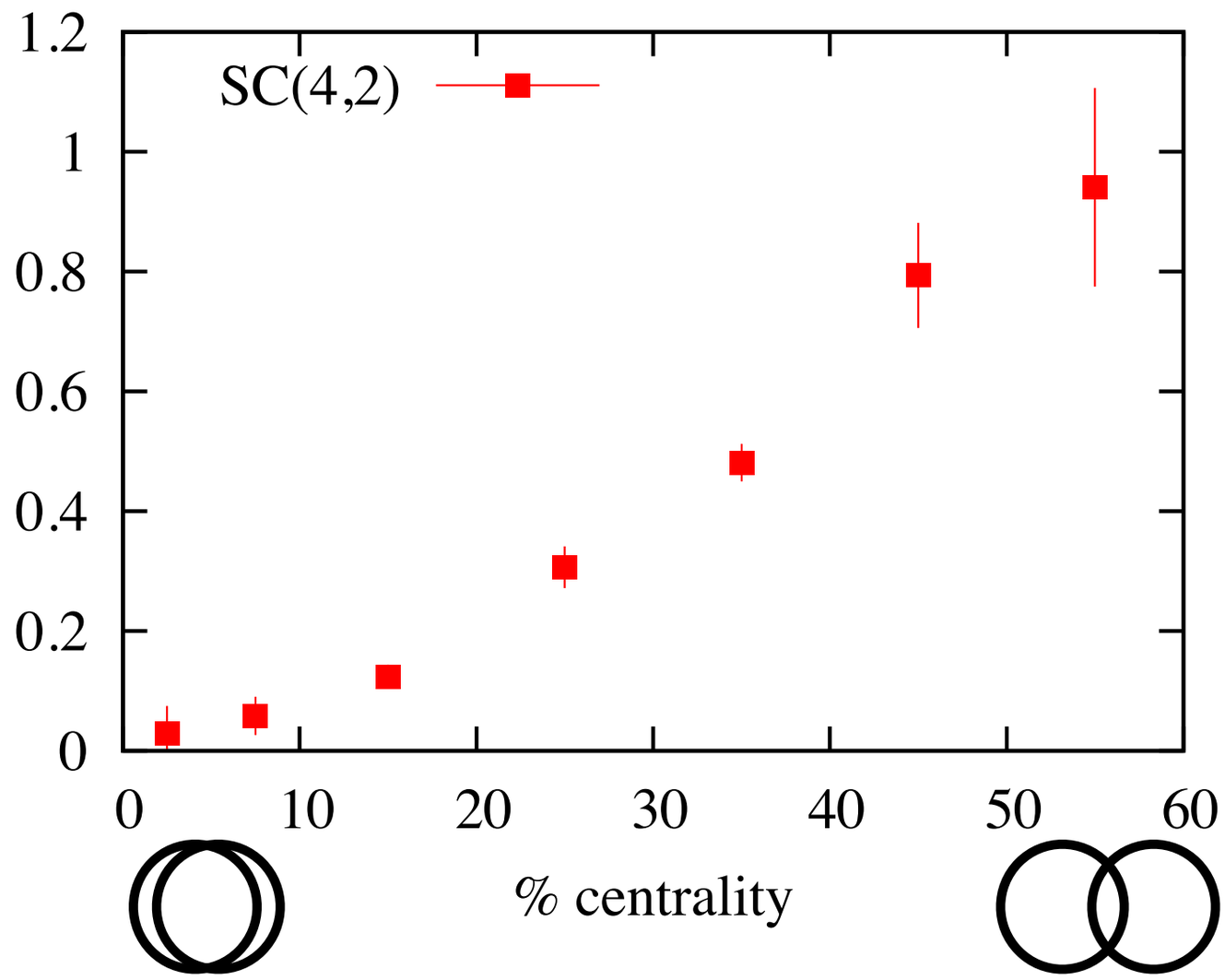
New data from ALICE

- « symmetric cumulants » : specific 4 particle correlations.
- Using the flow paradigm, the symmetric cumulant can be recast as a correlation between the magnitudes of 2 different Fourier harmonics:

$$SC(4,2) \equiv \frac{\langle v_4^2 v_2^2 \rangle - \langle v_4^2 \rangle \langle v_2^2 \rangle}{\langle v_4^2 \rangle \langle v_2^2 \rangle}$$

- ALICE has recently measured SC(4,2) as a function of centrality

New data from ALICE



ALICE Collaboration, arXiv:1604.07663

ATLAS « event-plane correlation »

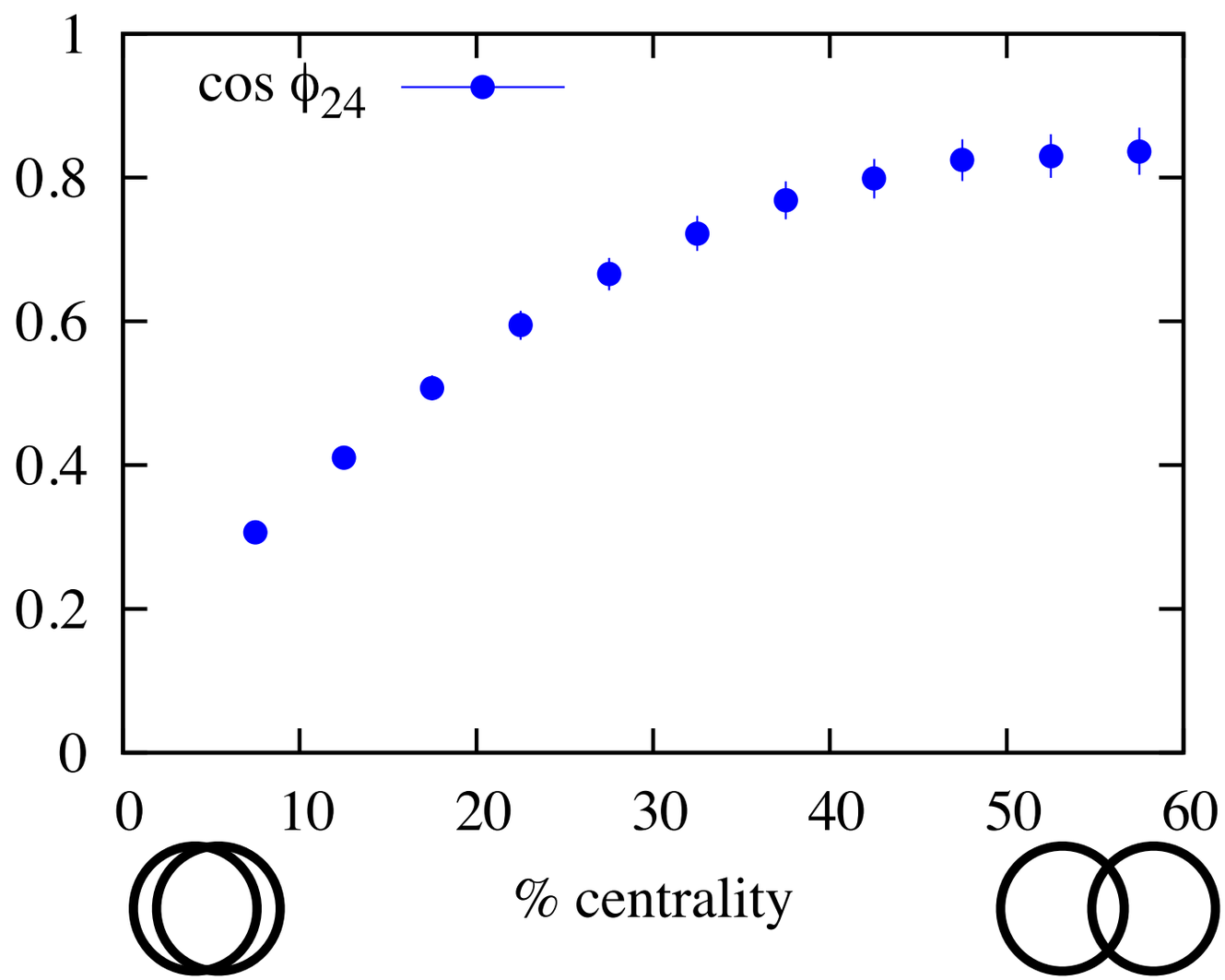
- The *event-plane correlation* measured by ATLAS is in fact a linear (Pearson) correlation between the complex flow coefficients V_4 and $(V_2)^2$

$$\cos \Phi_{24} \equiv \frac{\mathrm{Re}\langle V_4 (V_2^*)^2 \rangle}{\sqrt{\langle v_4^2 \rangle \langle v_2^4 \rangle}}$$

- It is also a measure of the correlation between V_4 and V_2, which involves the relative angle *and* the magnitudes.

Luzum JYO arXiv:1209.2323

ATLAS « event-plane correlation »



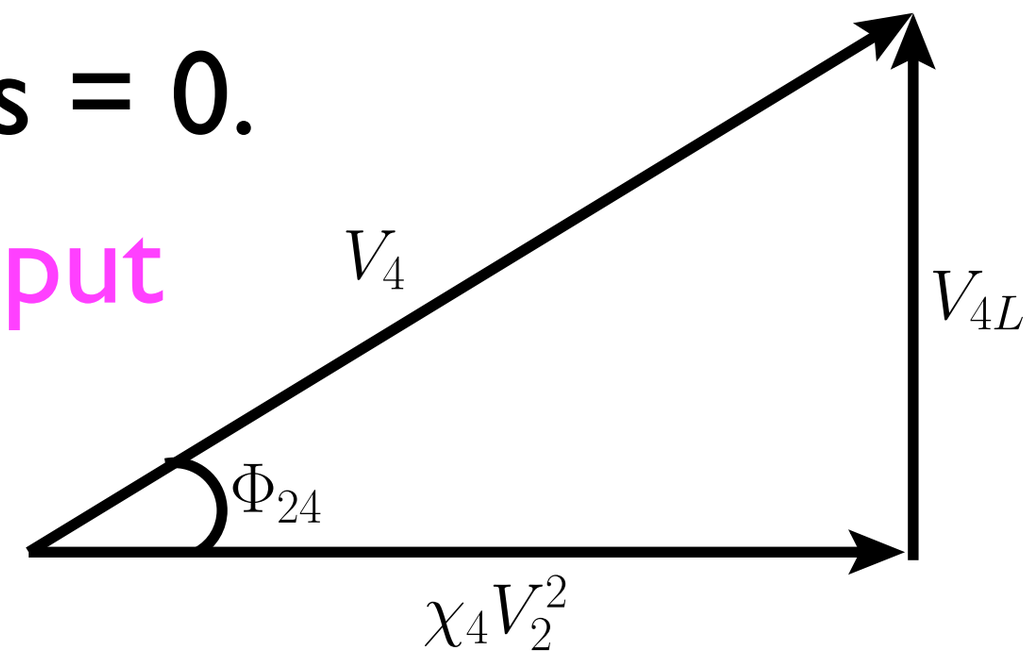
ATLAS Collaboration, arXiv:1403.0489

Can we compare ALICE and ATLAS?

- We have two measures of the correlation between V_4 and V_2, the *symmetric cumulant (ALICE)* and the *event-plane correlation (ATLAS)*.
- I derive a *quantitative relation* between these two measures, test it on hydro calculations and then on data.

Modeling the correlation

- Decompose $V_4 = V_{4L} + \chi_4 (V_2)^2$, with χ_4 = constant fixed so that linear correlation between the two terms = 0.
- Just math, no physics input



- Then Φ_{24} measures the relative magnitude of the 2 terms: $\chi_4^2 \langle v_2^4 \rangle = \langle v_4^2 \rangle \cos^2 \Phi_{24}$

Modeling the correlation

- $v_4 = v_{4L} + \chi_4 (v_2)^2$
- We assume that the two terms are *independent* (stronger than *uncorrelated*)
- Then: correlation between $(v_4)^2$ and $(v_2)^2$ is only from the nonlinear part:

$$\langle v_4^2 v_2^2 \rangle - \langle v_4^2 \rangle \langle v_2^2 \rangle = \chi_4^2 \left(\langle v_2^6 \rangle - \langle v_2^4 \rangle \langle v_2^2 \rangle \right)$$

Result

Expressing χ_4 as a function of the event-plane correlation, we obtain:

$$SC(4,2) = \left(\frac{\langle v_2^6 \rangle}{\langle v_2^4 \rangle \langle v_2^2 \rangle} - 1 \right) \cos^2 \Phi_{24}$$

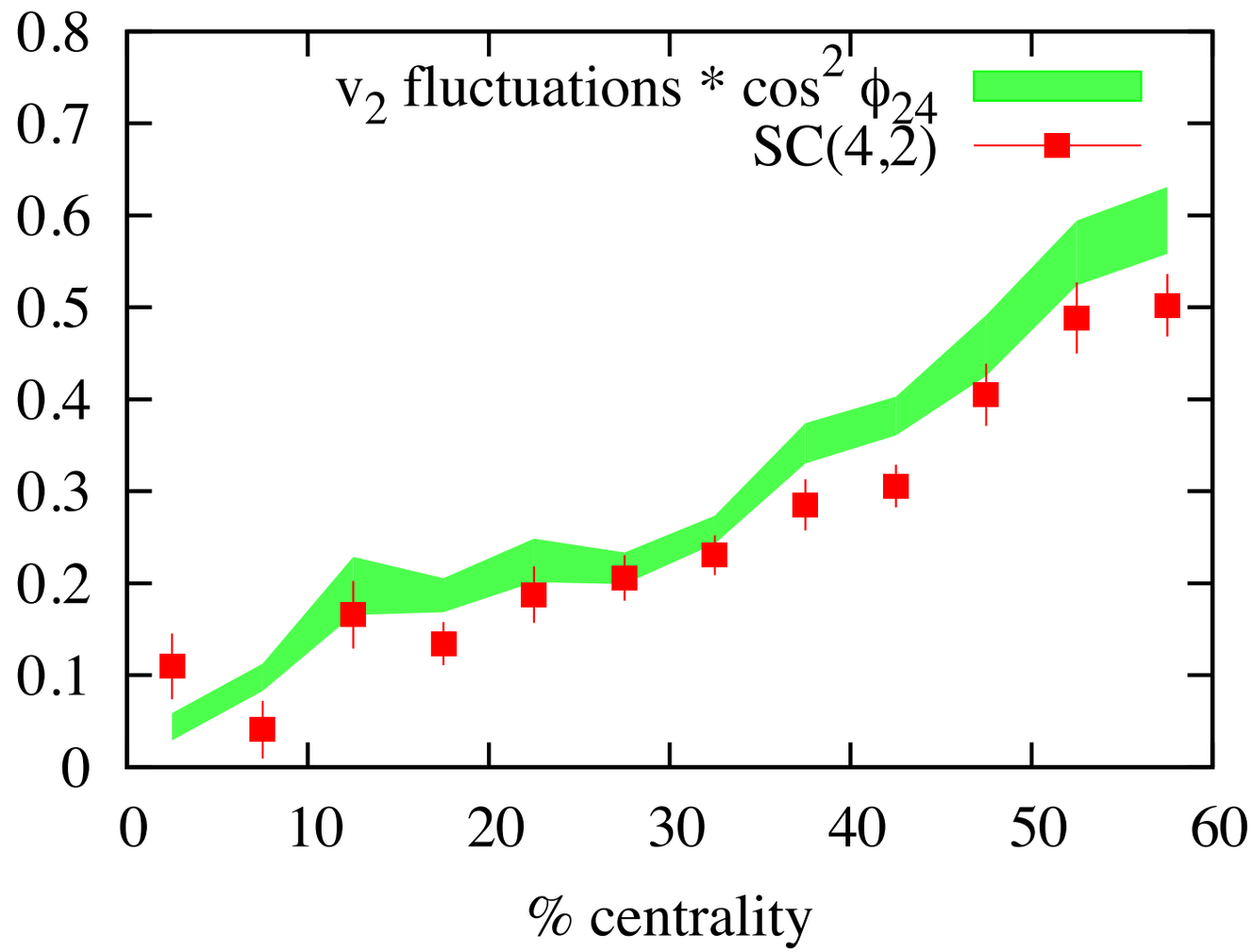
symmetric cumulant

elliptic flow fluctuations

event-plane correlation

Event-by-event hydrodynamics

We compute both sides of the equation independently in event-by-event viscous hydro with Glauber initial conditions

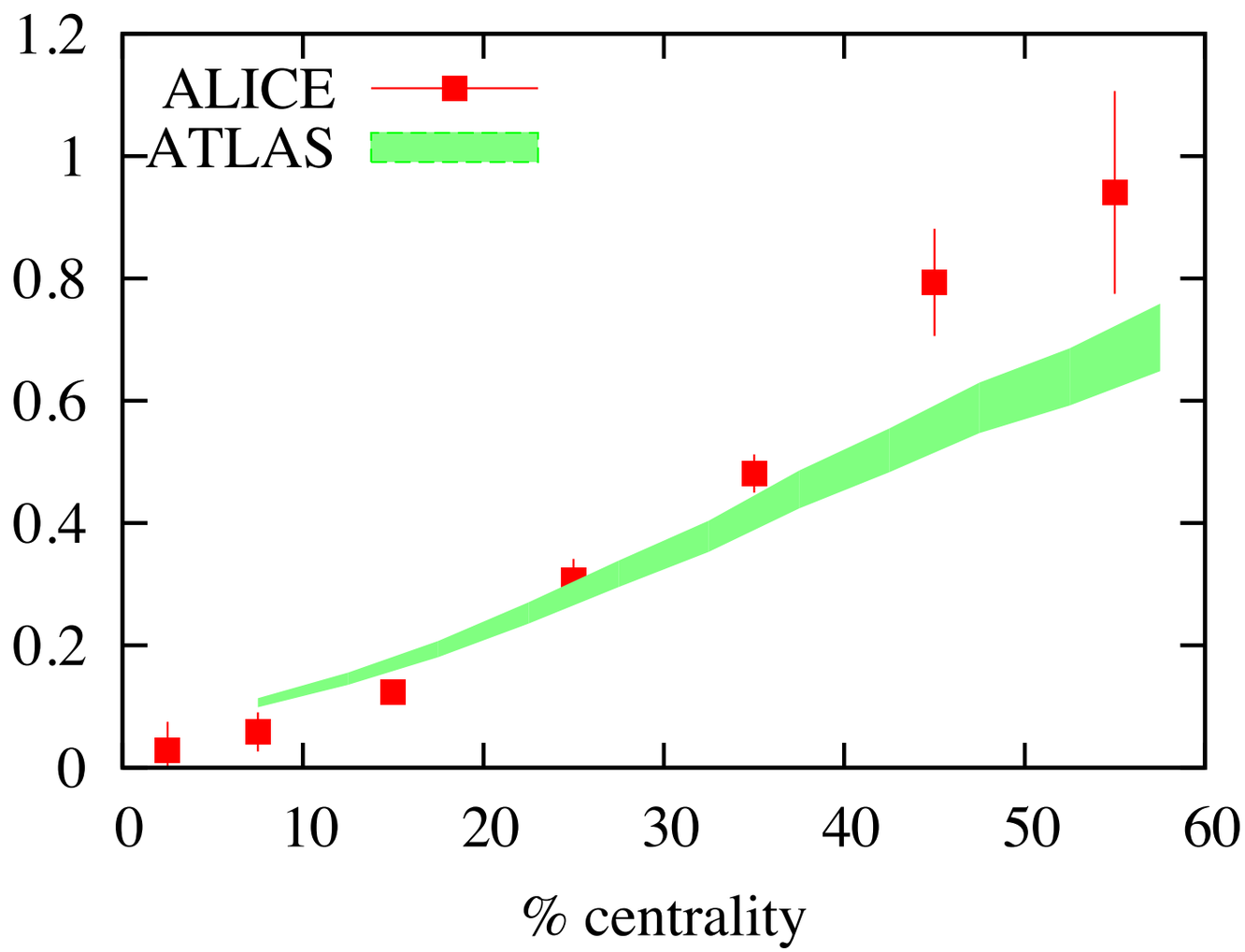


The relation is satisfied to a good approximation for all centralities

ALICE versus ATLAS

Using elliptic flow fluctuations (cumulants) and event plane correlations from ATLAS:

SC(4,2)



Predictions

Same methodology applied to different orders:

$$SC(4,3) = \left(\frac{\langle v_2^4 v_3^2 \rangle}{\langle v_2^4 \rangle \langle v_3^2 \rangle} - 1 \right) \cos^2 \Phi_{24}$$

$$SC(5,2) = \left(\frac{\langle v_2^4 v_3^2 \rangle}{\langle v_2^2 v_3^2 \rangle \langle v_2^2 \rangle} - 1 \right) \cos^2 \Phi_{235}$$

$$SC(5,3) = \left(\frac{\langle v_2^2 v_3^4 \rangle}{\langle v_2^2 v_3^2 \rangle \langle v_3^2 \rangle} - 1 \right) \cos^2 \Phi_{235}$$

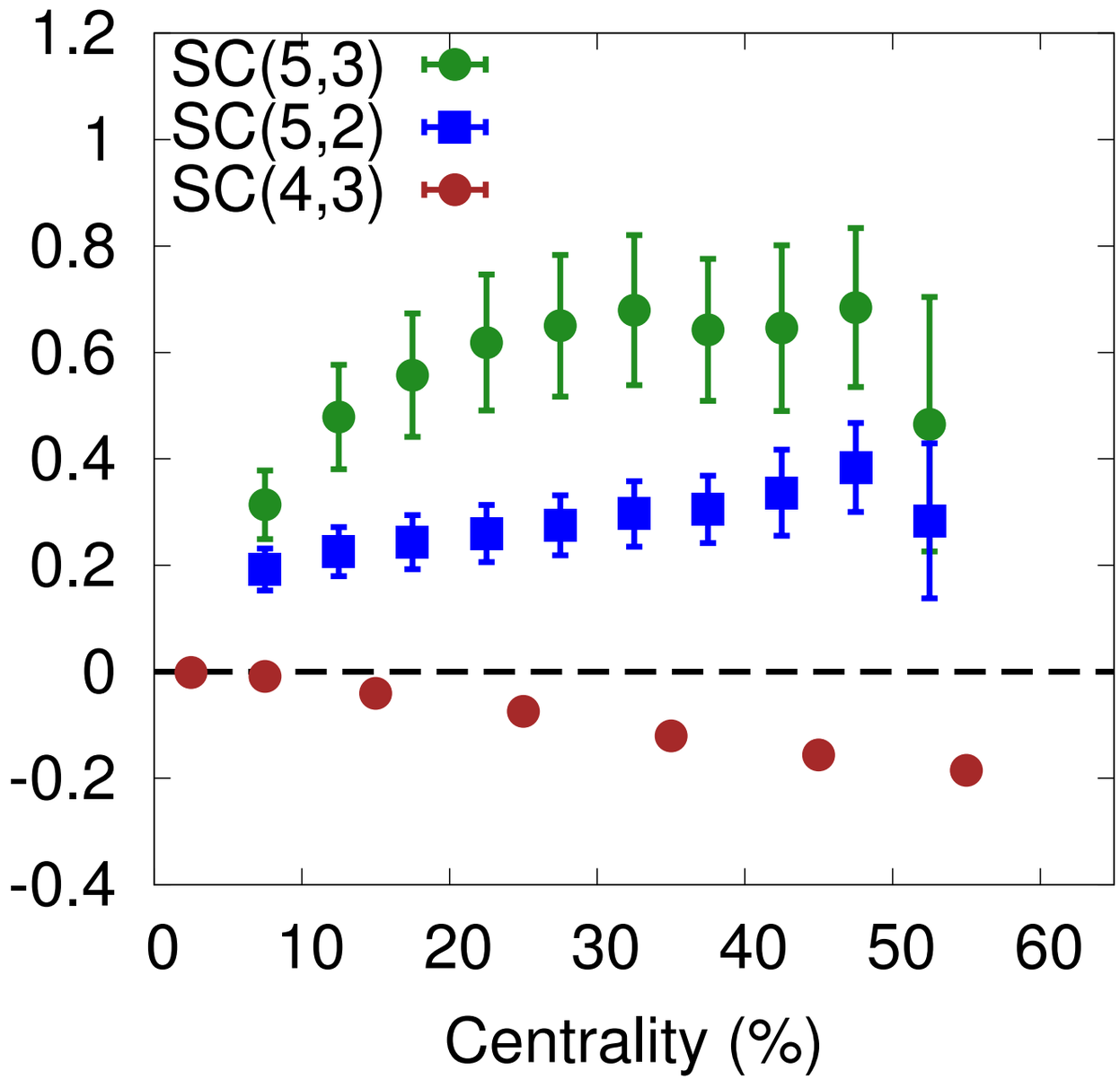
symmetric cumulants

flow fluctuations

event-plane correlations

Predictions

Data-driven predictions (no hydro calculation!) using ATLAS results on v_n fluctuations and event-plane correlations.



2. The statistics of v_2 fluctuations

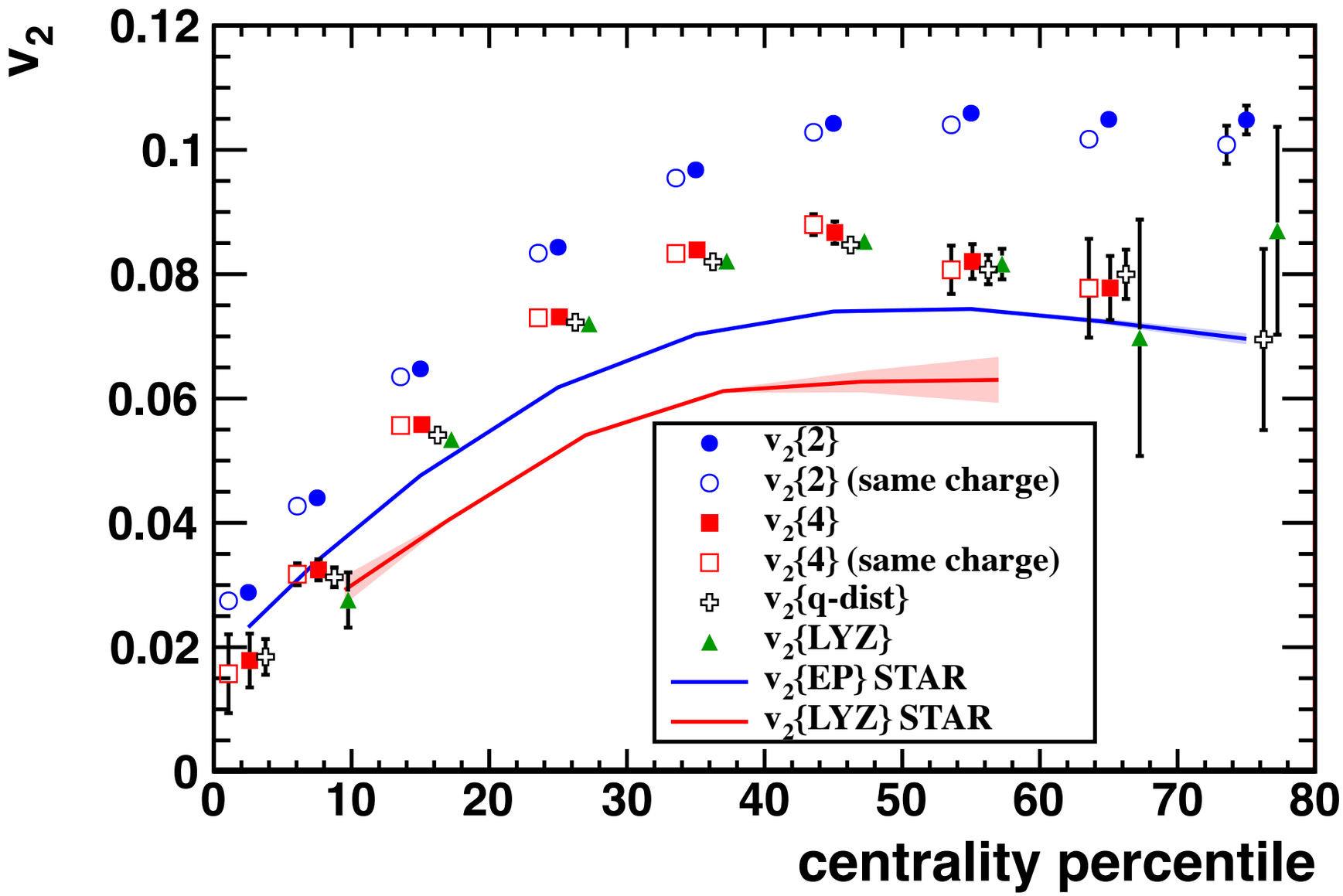
Giacalone, Yan, Noronha-Hostler, JYO, 1608.01823

The fluctuations of elliptic flow

- One can measure much more than the rms value of v_2. Also higher order moments and cumulants

$$v_2\{2\}=(\langle v_2^2\rangle)^{1/2}$$
$$v_2\{4\}=(2\langle v_2^2\rangle^2-\langle v_2^4\rangle)^{1/4}$$
$$v_2\{6\}=((\langle v_2^6\rangle-9\langle v_2^4\rangle\langle v_2^2\rangle+12\langle v_2^2\rangle^3)/4)^{1/6}$$

- $v_2\{4\} < v_2\{2\}$ if v_2 fluctuates
- $v_2\{4\}=v_2\{6\}$ if fluctuations are 2-dim. Gaussian.



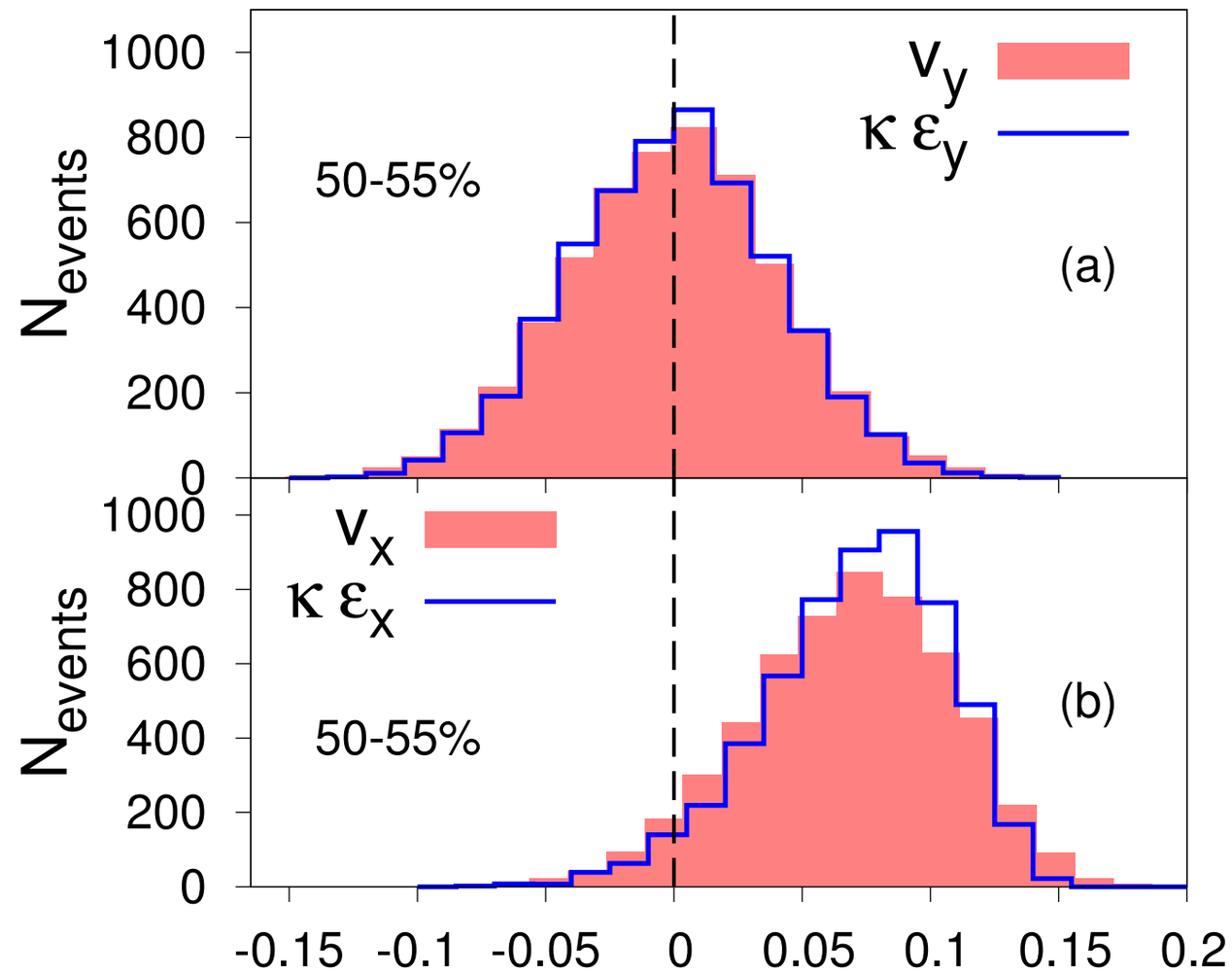
ALICE 1011.3916

The fluctuations of elliptic flow

Decompose V_2 into real (cos) and imaginary (sin) parts

$$V_2 = v_x + i\, v_y$$

Probability distribution in a hydrodynamic calculation:



Fluctuations of v_x are not symmetric: they have negative skew

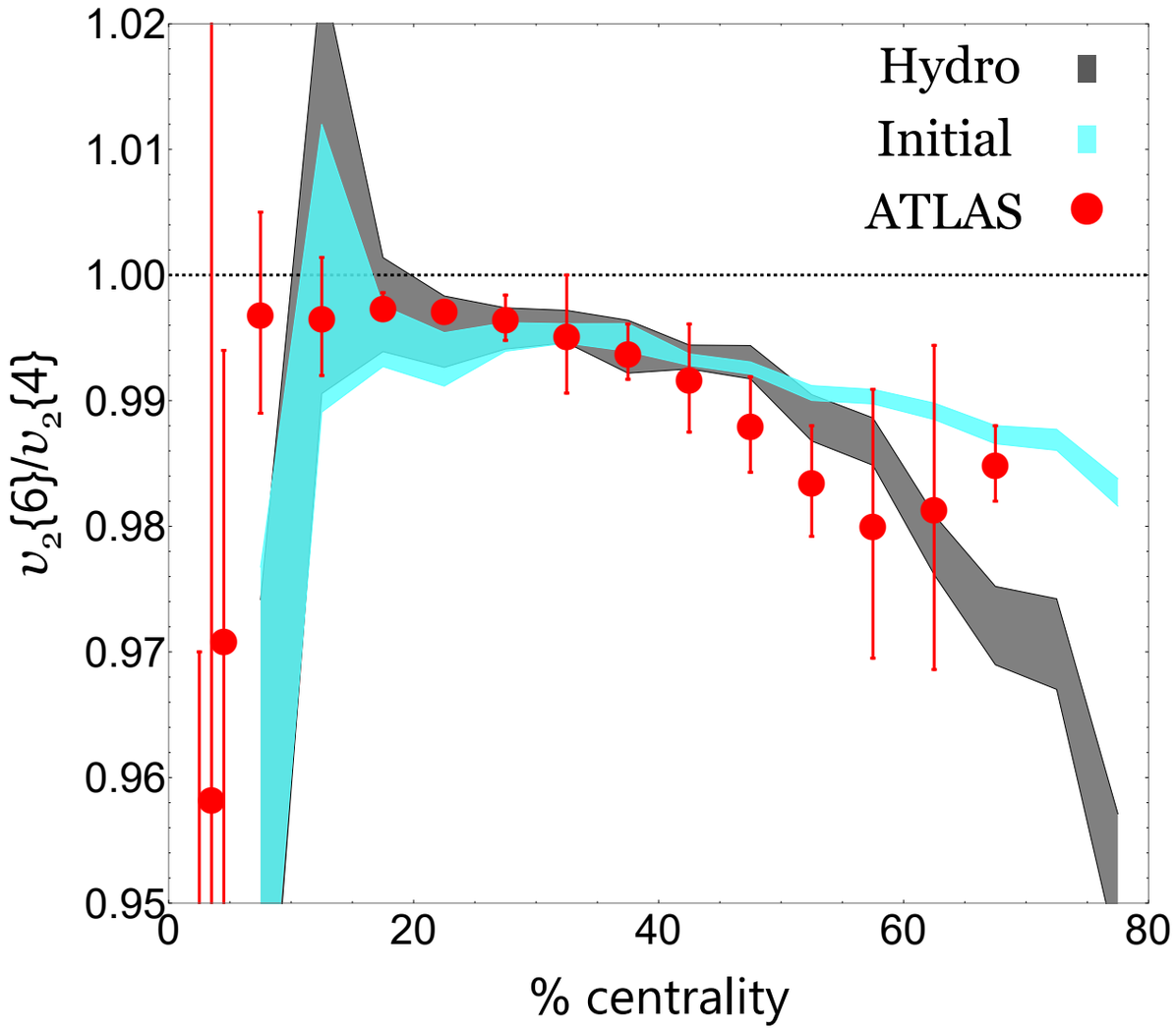
skewness=3rd centered moment : $s \equiv \langle (v_x - \langle v_x \rangle)^3 \rangle$

Non-Gaussian fluctuations

We have shown by an expansion in powers of the fluctuations that

$v_2\{6\}-v_2\{4\} = s/(3\langle v_x\rangle^2)$

Negative skewness, $s<0$, naturally explains the small lifting of degeneracy seen by ATLAS:



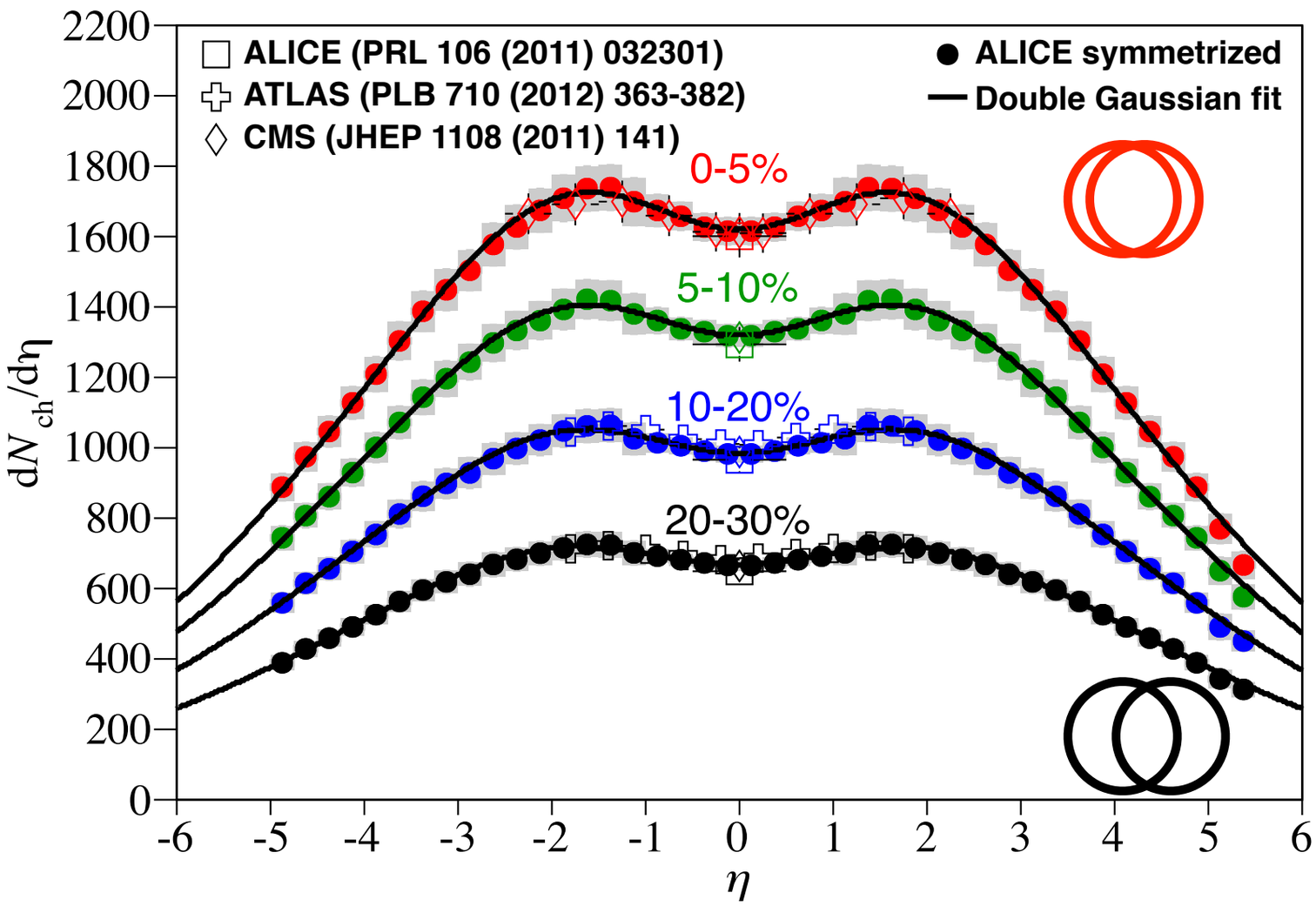
ATLAS arXiv:1408.4342

Conclusions

- Bulk (soft particle) correlations of arbitrary order are, at the qualitative level, naturally explained by the flow paradigm and minimal assumptions from hydrodynamics (e.g., eccentricity scaling of v_2).
- I have illustrated the predictive power of this approach on two examples: symmetric cumulants and elliptic flow fluctuations.
- The consequences of the flow paradigm for higher-order correlations have not yet been fully explored.

Backup slides

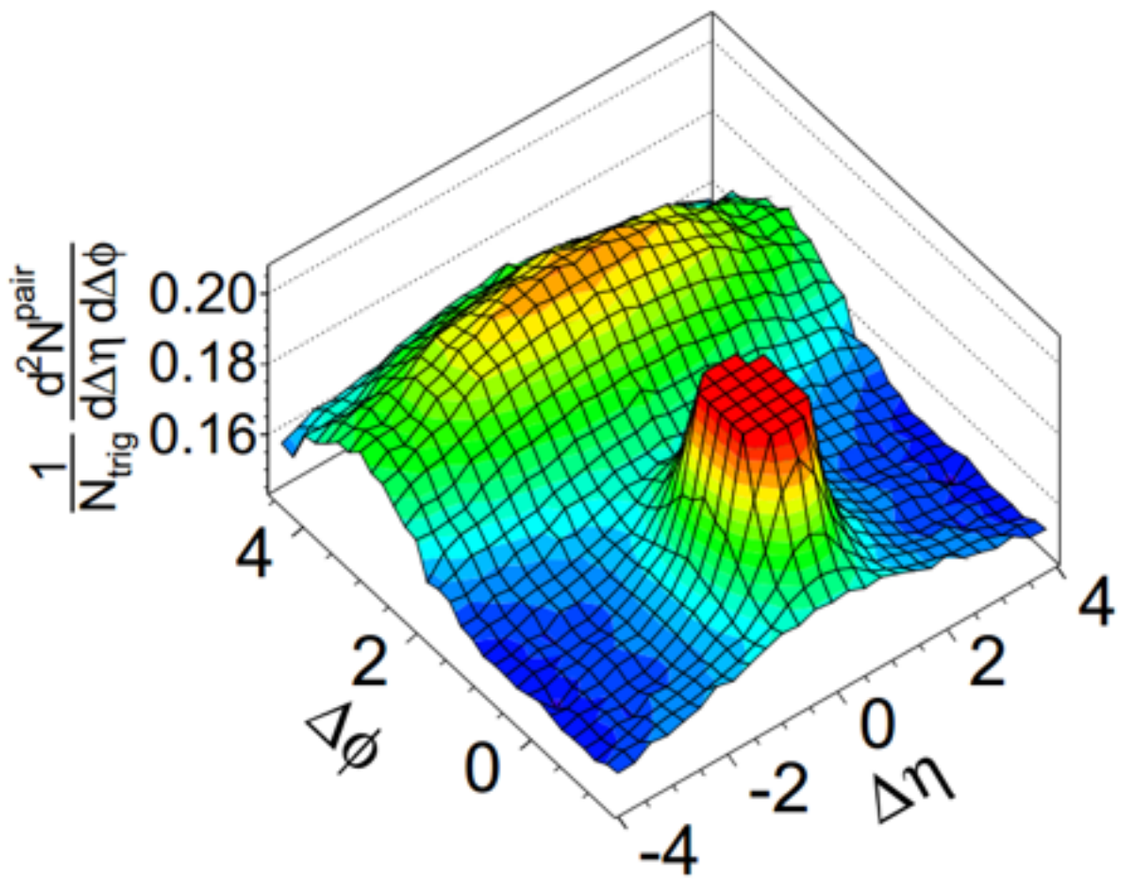
Multiplicity and centrality



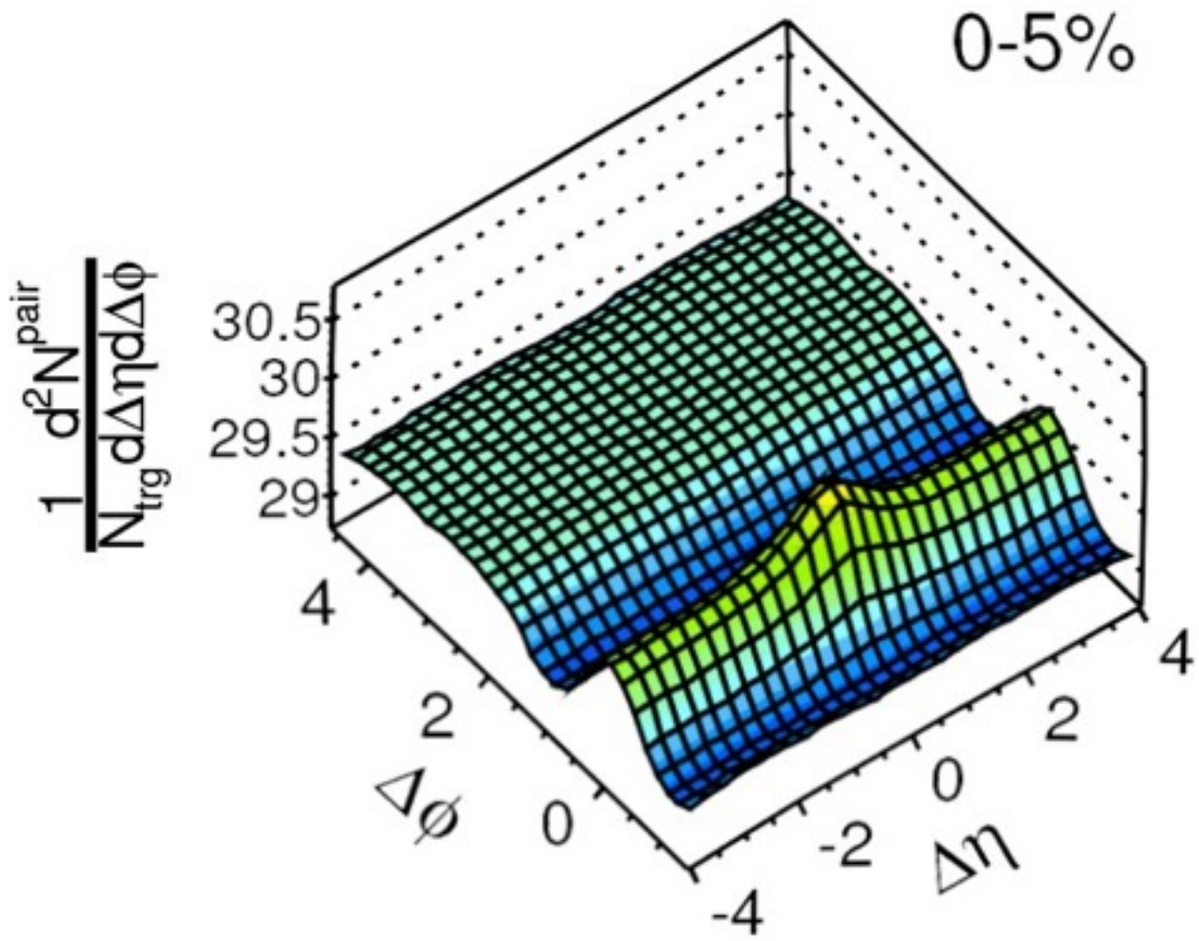
- Collisions classified from more central to less central in 5% bins
- More central creates more particles
- A central collision (b=0) typically produces 25000 particles.

ALICE arxiv:1304.0347

Two-particle correlations



proton-proton collision



central Pb+Pb collision

CMS 1201.3158

Linear and nonlinear hydro response

- Hydrodynamics also predicts the decomposition $V_4 = V_{4L} + \chi_4 (V_2)^2$, where
- V_{4L} = response to initial fluctuations in 4th harmonic
- $\chi_4 (V_2)^2$= nonlinear response induced by hydrodynamic evolution

Teaney & Yan arXiv:1206.1905

Yan & JYO arXiv:1502.02502

ALICE versus ATLAS

- Agreement not as good as in hydro. why?
- ATLAS event-plane correlations are measured with a large pseudorapidity gap and over a wide interval -4.8 to 4.8
- ALICE SC(4,2) is measured without any gap and over the interval -0.8 to 0.8
- Longitudinal flow fluctuations induce a decoherence which may explain why the ATLAS result is smaller.