Energy loss and diffusion in non-conformal holographic QCD models

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Introduction

Heavy quarks: important probes of deconfined phase.

I will review the holographic description of :

- Energy loss
- Langevin diffusion *(transverse momentum broadening)*

of a heavy quark moving through the (thermalized) plasma.

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of a heavy quark moving through the (thermalized) plasma.

Setup: a *non conformal* holographic theory in the deconfined phase (suitable for connection with experiment)

(Review of work with U. Gursoy, E. Kiritsis, L. Mazzanti, G. Michalogeorgiakis, 2008-2013.

Building on previos work by Gubser, Herzog, Son, Casalderrey-Solana, Teaney, Son, Iancu ...)

Diffusion of a heavy quark

Quark produced out of equilibrium ($M > T$): follow diffusion of a single particle. Use Langevin equation:

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- η_D: “average” viscous friction force
- $\xi(t)$: Stochastic force with white noise

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both forces have the same origin: the integrated effect of stochastic interactions with a medium.

Energy loss

$$\frac{d\vec{p}}{dt} = -\eta_D\,\vec{p}(t) + \vec{\xi}(t)$$

Coefficient η_D indicates a viscous friction on the quark, and consequent energy loss (if η_D constant):

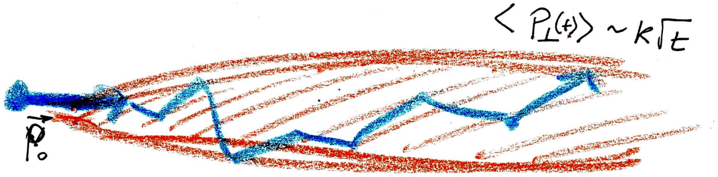
$$F_{fric} = -\eta_D p, \qquad \frac{1}{E_p}\frac{dE_p}{dt} \sim \eta_D$$

The energy is lost in interactions with the plasma.

Transverse Momentum Broadening

Quark momentum obeys a Langevin process with $\langle p^{\perp} \rangle = 0$, but with an increasing dispersion of $p^{\perp}$:

$$\langle (p^{\perp})^2 \rangle \sim 2\kappa^{\perp}\, t$$



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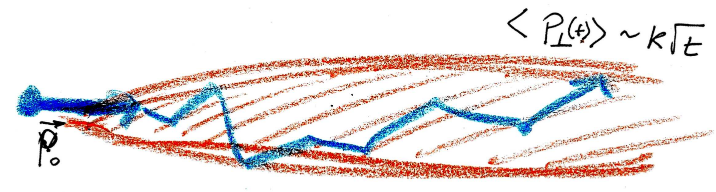
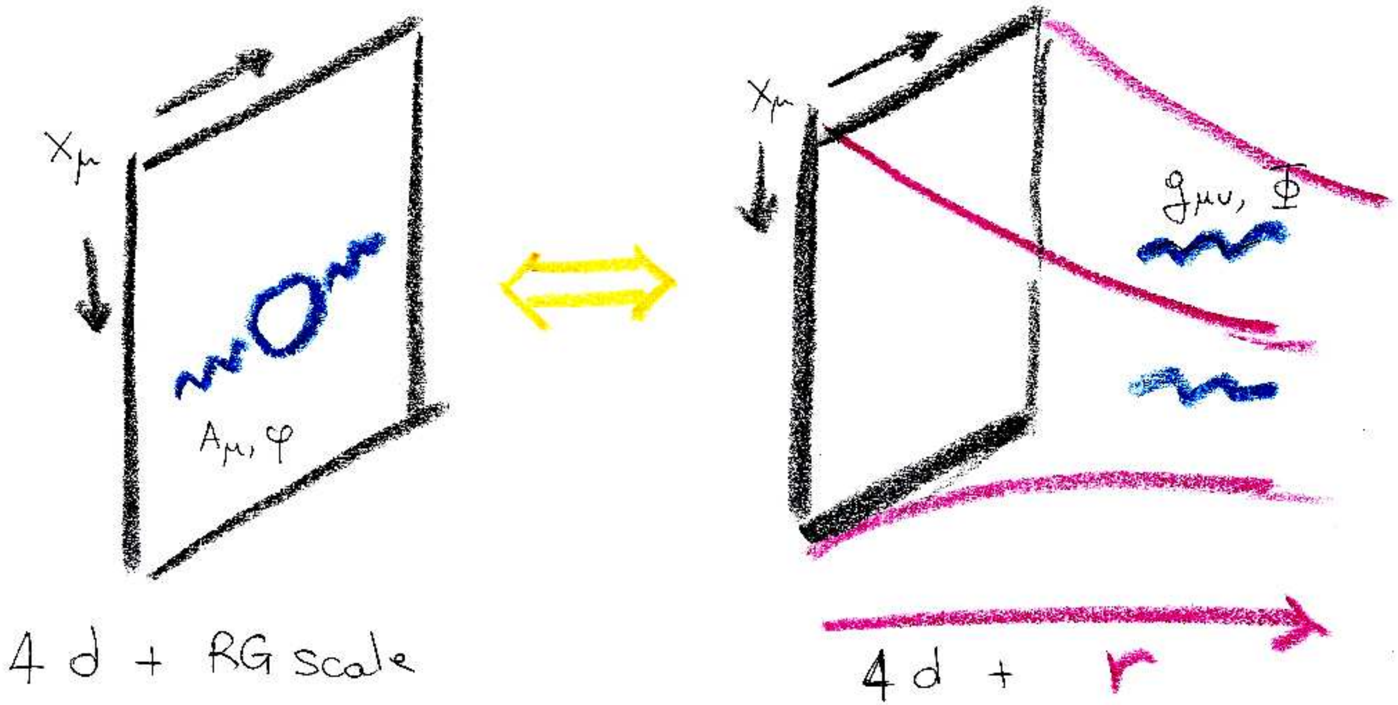


Define the *jet quenching parameter*

$$\hat{q} \equiv \frac{\langle (p^{\perp})^2 \rangle}{mean\, free\, path} = \frac{(p^{\perp})^2}{v\, t} = 2\frac{\kappa^{\perp}}{v}$$

AdS/CFT

The AdS/CFT duality: conjecture that certain quantum field theories are equivalent to theories of strings/gravity in higher dimensions.



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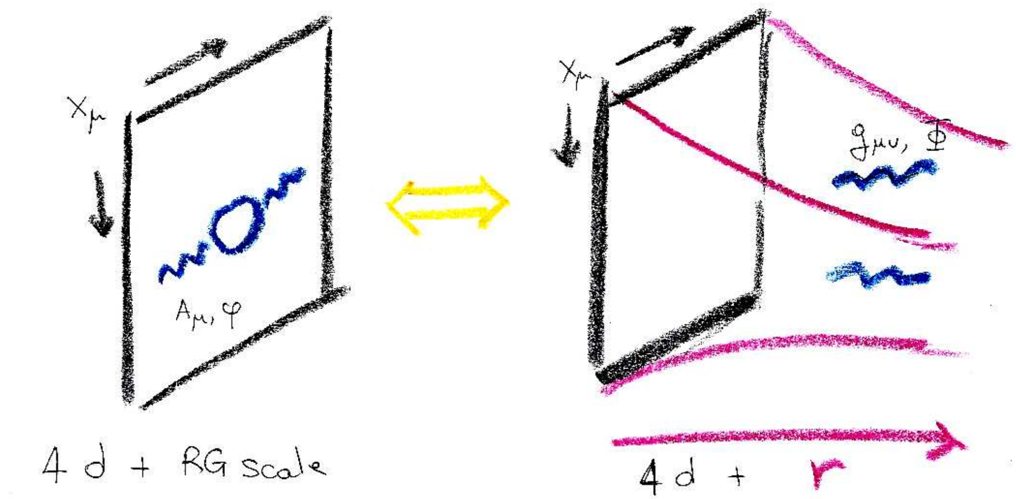

- Equivalent means that the two theories contain the same degrees of freedom, but arranged in differnt ways.
- Depending on the situation, one side or the other may simplify. Gravity description simple at large N.

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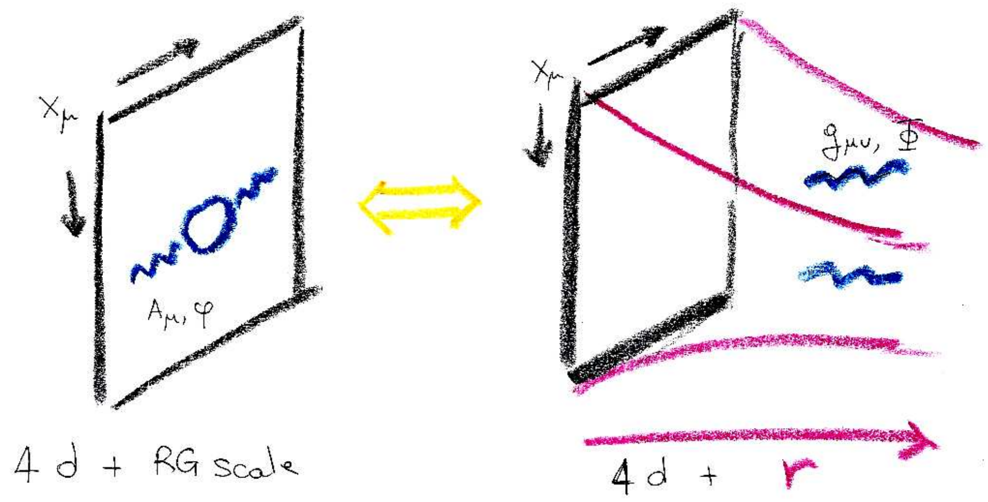
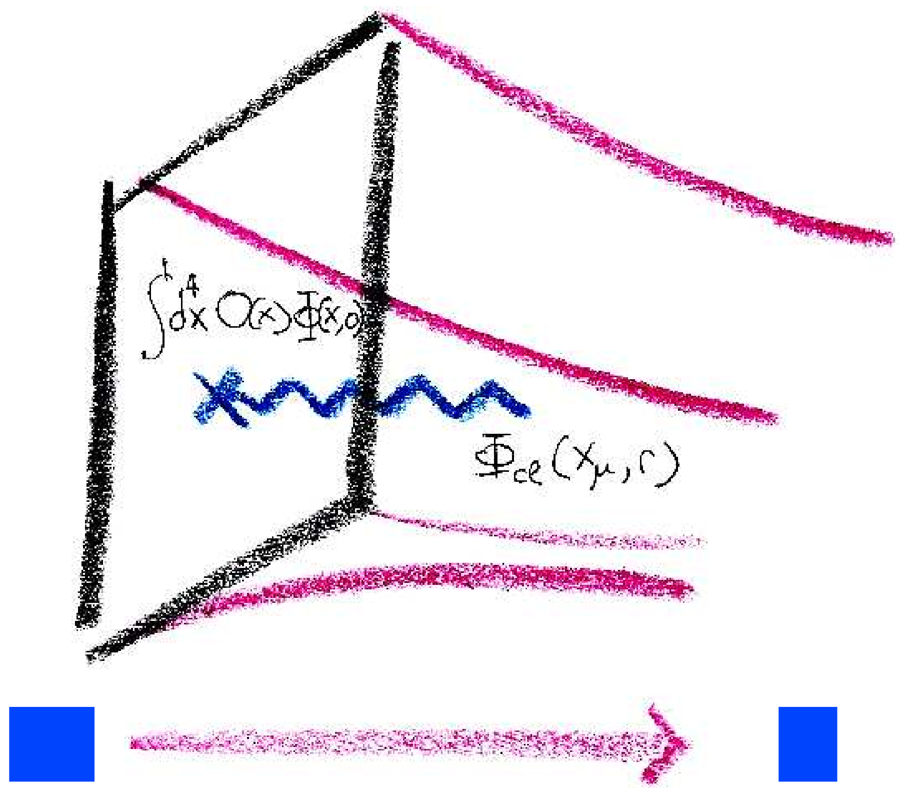


- Conformal invariance $\Leftrightarrow$ AdS spacetime $ds^2 = r^{-2}(dr^2 + dx_\mu^2)$, Scaling isometry $r \to \lambda r$, $x_\mu \to \lambda x_\mu$ (e.g. $\mathcal{N} = 4$ SUSY YM at large N, large coupling)
- RG scale $\Leftrightarrow$ radial coordinate r; UV $\Leftrightarrow$ AdS boundary $r = 0$.

Field/Coupling correspondence

- A Bulk field $\Phi(x,r)$ corresponds to a running coupling in the QFT
- $\Phi_0(x) = \Phi(x,0)$ is the UV value of the coupling



Minimal phenomenological setup

- The bulk theory is five-dimensional (x^μ + RG coordinate r)
- Include only lowest dimension YM operators ($\Delta = 4$)

4D Operator		Bulk field	Coupling
TrF^2	$\Leftrightarrow$	Φ	$N\int e^{-\Phi}\,TrF^2$
$T_{\mu\nu}$	$\Leftrightarrow$	$g_{\mu\nu}$	$\int g_{\mu\nu}T^{\mu\nu}$

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- Breaking of conformal symmetry, mass gap, confinement, and all non-perturbative dynamics driven by the dilaton dynamics (aka the Yang-Mills coupling).

5-D Eistein-Dilaton Theory

Bulk dynamics described by a 2-derivative action:

$$S_E = -M_p^3 N_c^2 \int d^5x \sqrt{-g} \left[R - \frac{4}{3}(\partial\Phi)^2 - V(\Phi) \right]$$

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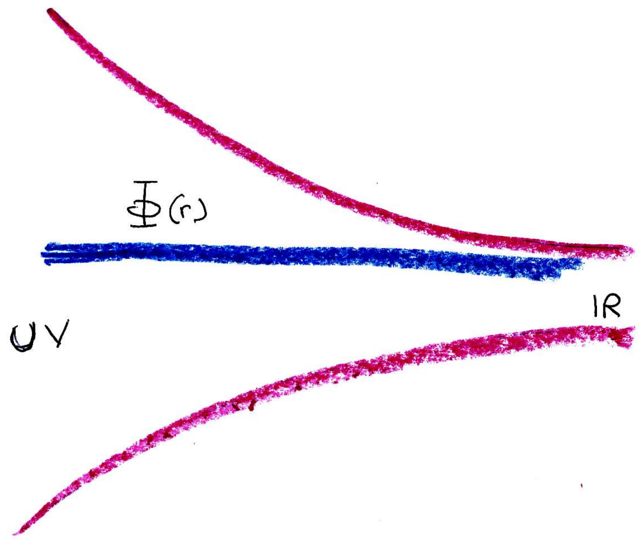
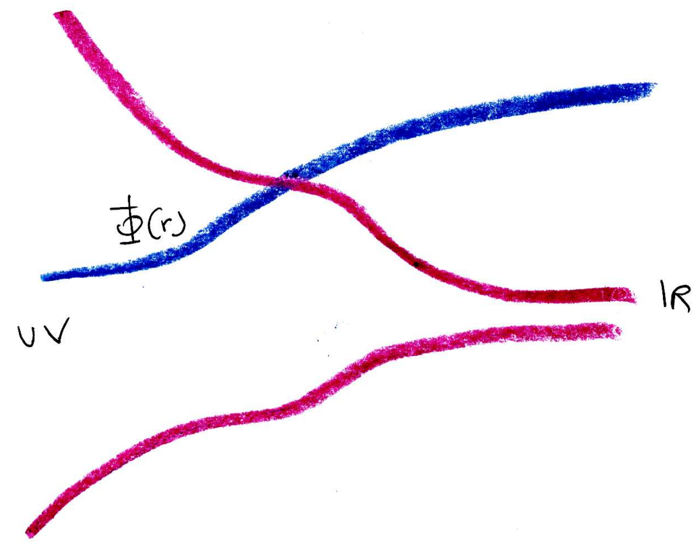


Φ constant
(conformal invariant)



Φ running
(non-conformal)

Finite temperature

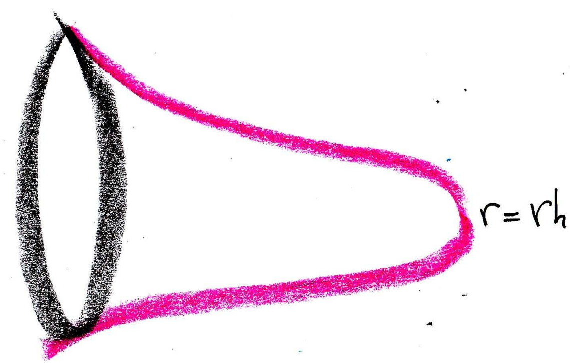
Deconfined phase described by a 5D black hole geometry

$$ds^2 = b^2(r)\left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^i dx_i\right]$$

For a class of $V(\Phi)$, theory has confinement and a 1st order phase transition to a deconfined phase (BH geometry).

$f(r) = 1$

$T < T_c$ confined

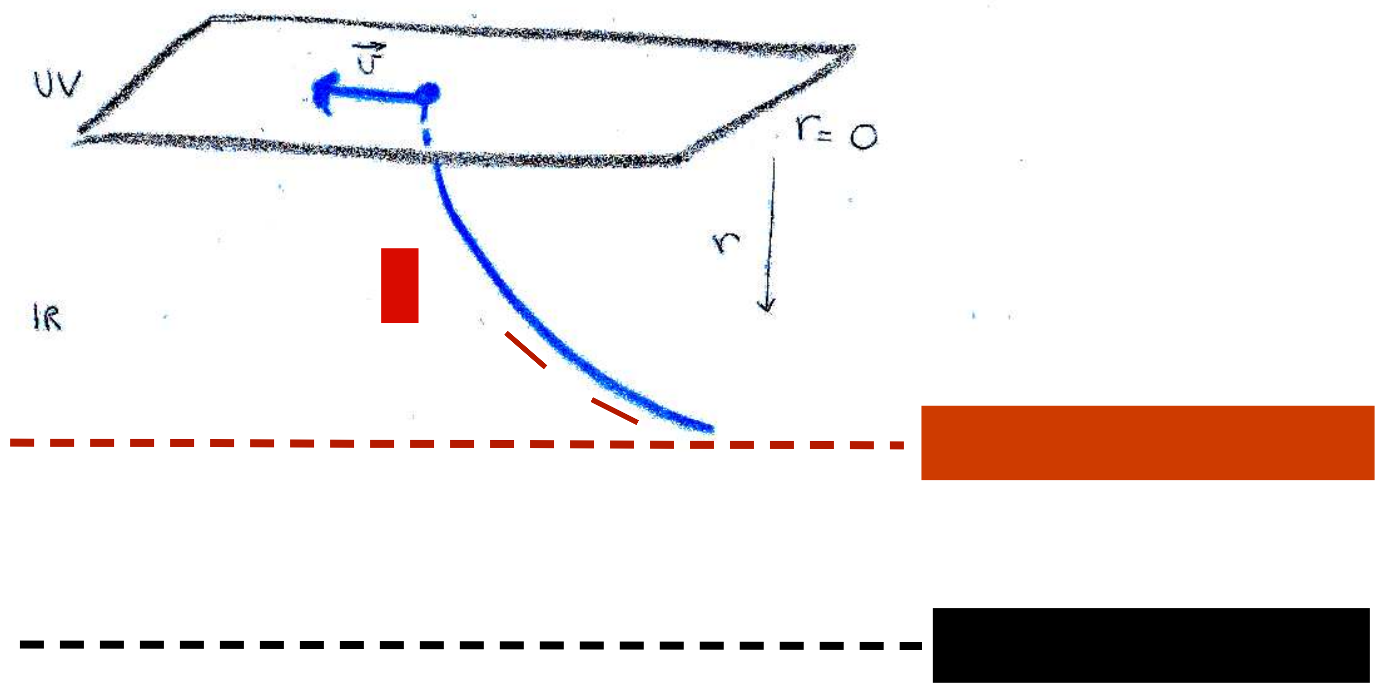


$f(r) \neq 1,\ f(r_h) = 0$

$T > T_c$ deconfined (BH)

Holographic description of a heavy quark

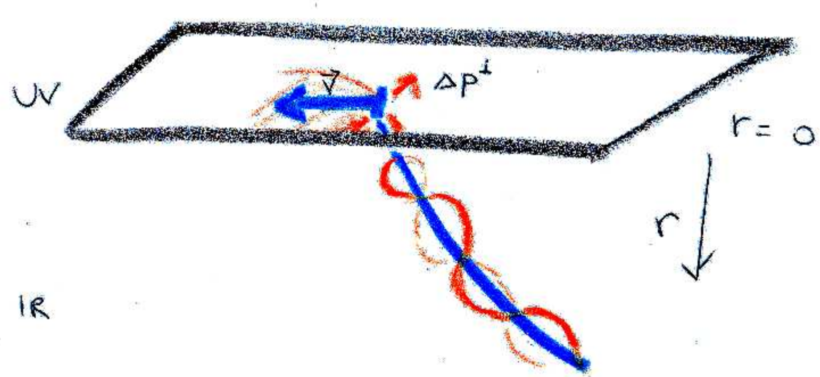
Trailing String picture:



- String profile obtained extremizing worldsheet area.
- The moving string equilbrates at a temperature $T_s < T$.
- Drag coefficient η_D related to $b(r_s), f(r_s)$.

Trailing string fluctuations

$$\vec{X}(t,r) = \big(vt + \xi(r)\big)\frac{\vec{v}}{v} + \delta\vec{X}(r,t)$$



Dual to Langevin dynamics on the boundary: fluctuations correlators compute Langevin coefficients.

$$\dot{\vec{p}} = -\eta_D(p)\vec{p} + \vec{\xi}(t) \qquad \langle \xi^i(t)\xi^j(0)\rangle = \kappa^{ij}(p)\delta(t)$$

Langevin diffusion constants

$$\kappa^{ij} = \kappa^{\|}(p)p^i p^j + \kappa^{\perp}(p)\left(\delta^{ij} - \frac{p^i p^j}{p^2}\right)$$

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modified Einstein relations (Relativistic Langevin):

$$\kappa^{\perp} = 2\gamma M\, T_s \eta_D, \qquad \kappa^{\parallel} = 2\gamma^3 M T_s \left[\eta_D + M\gamma v \frac{\partial \eta_D}{\partial p}\right]$$

(Cfr. non-relativistic, thermal equilibrium E.R. $\kappa = 2TM\eta_D$)

Explicit Model: Improved Holographic QCD

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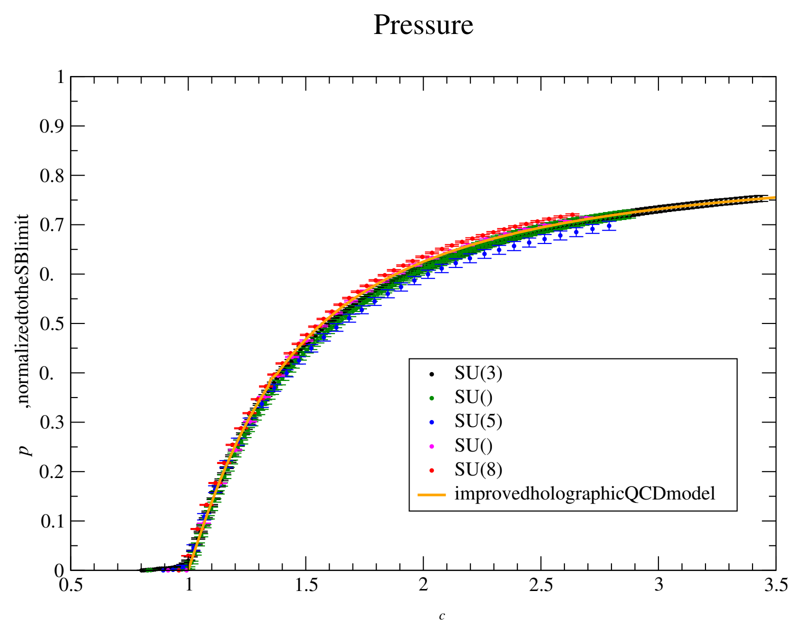

$s(T)/T^3$ lattice data: Panero, 0907.3719

Fix scale by matching critical temperature and vacuum string tension.

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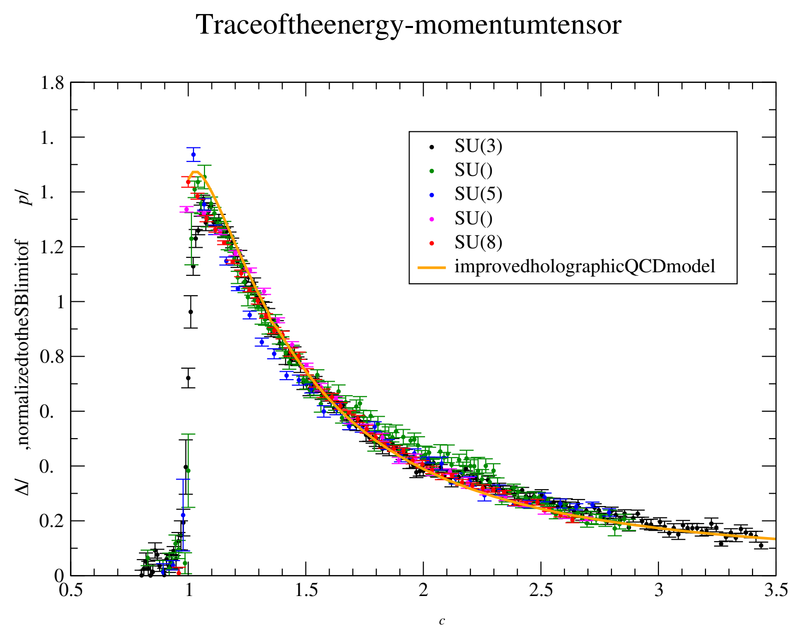

$(\epsilon - 3p)/T^4$ lattice data: Panero, 0907.3719

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Results: drag coefficients

- Conformal AdS/CFT:

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- Non-conformal model:$\eta_D = \eta_D(p)$; More complicated T-dependence; no need to fix λ;
- IHQCD (all free parameters fixed by satic quantities): diffusion time ($\tau = 1/\eta_D$):

$$\tau_{charm} \simeq 4.5 fm/c$$

for $p \approx 10\,GeV$, $T \sim T_c$, consistent with size of fireball.

Qhat

$\kappa^{\perp}$ computed numerically given the solution for the background metric $\Rightarrow$ obtain jet-quenching parameter:

$$\hat{q} = 2\frac{\kappa^{\perp}}{v} \qquad \left(\hat{q}_{conf} \propto \sqrt{\lambda} T^3\right)$$

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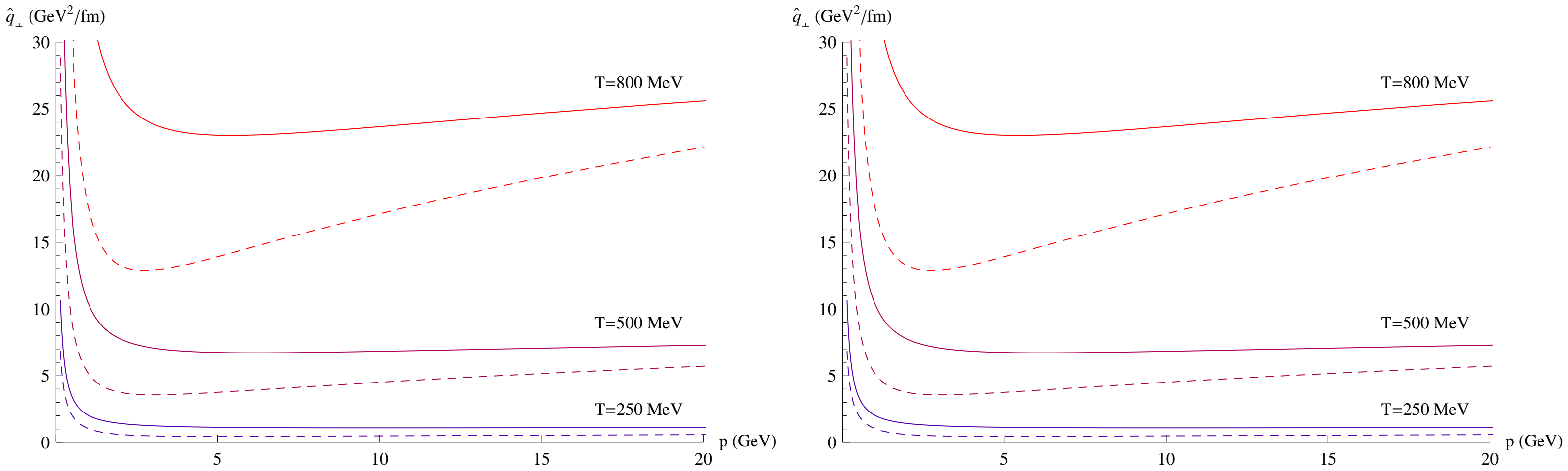

solid and dotted lines represent different schemes to match the temperature in the holographic YM model to the temperature in deconfined QCD.

Validity of the local approximation

- Langevin eqation is an approximation of a more general equation:

$$\dot{p} = -\int_0^\infty dt'\, \eta_D(t') p(t-t') + \xi(t), \qquad \langle \xi(t)\xi(0)\rangle = G(t)$$

long time regime $t \gg \tau_{corr} \sim 1/T_s$: reduces to Langevin with:

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- Consistency requires $1/\eta_D \gg 1/T_s$ If this fails, need to use the generalized process with memory and "colored" noise kernels. Accessible via Holography.

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Parametrization in terms of $\hat{q}$ justified if:

$$T_s/\eta_D > 1$$

This translates to a bound on quark momentum:

$$p < 1.5 M_q \left(M_q / T \right)^2$$

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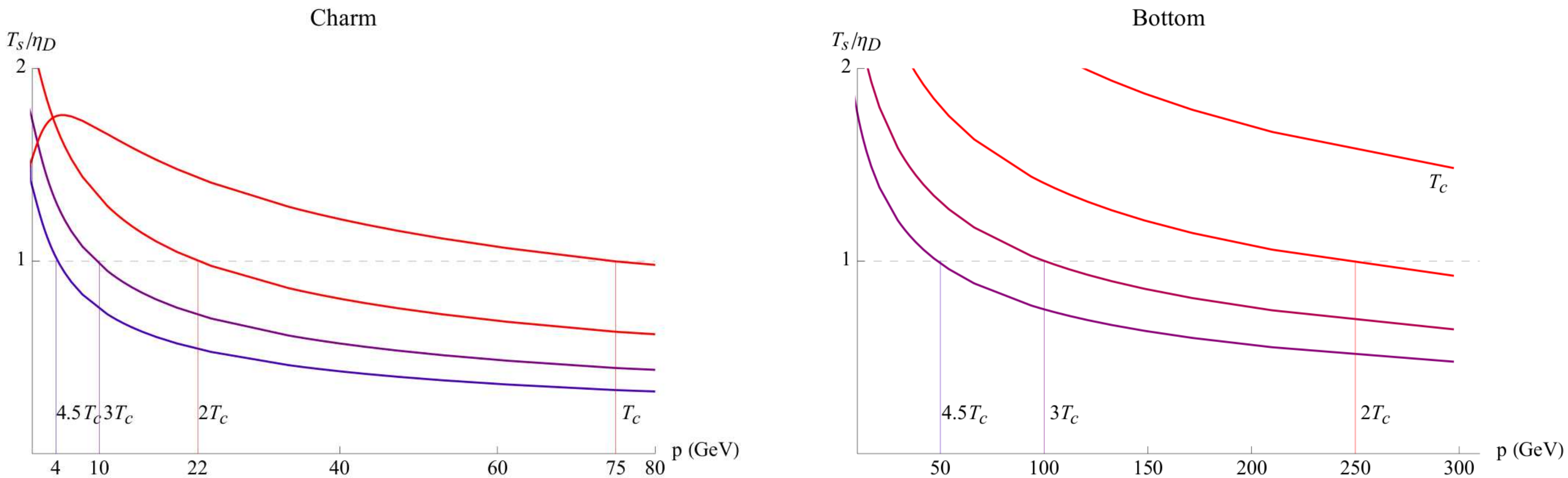

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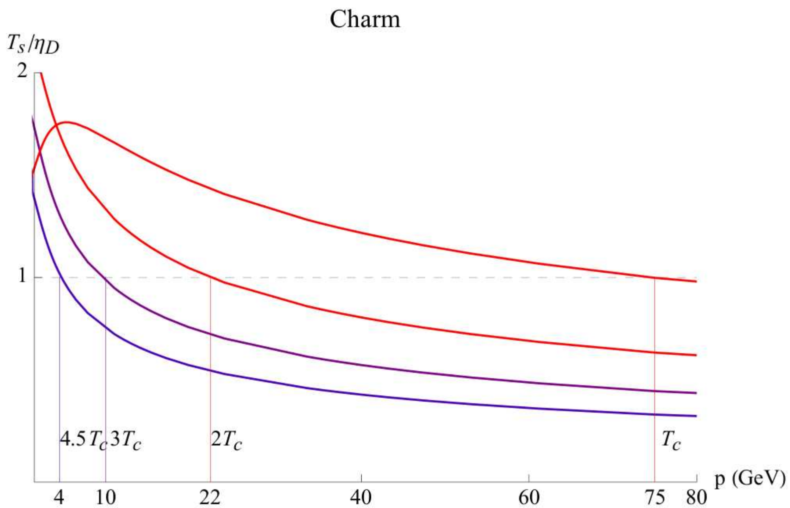

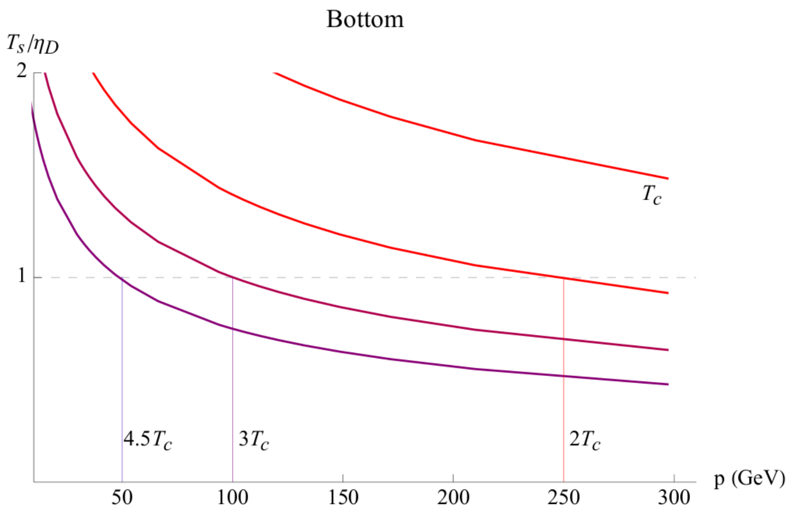

Conclusion and outlook

- Non-conformal holographic models give realistic results for heavy quark transport in the QGP.
- What happens when the local Langevin description breaks down?
  Look at $p_\perp$ broadening with a non-trivial kernel and colored noise.
- How does momentum dependence affect the dynamics for large times, $> 1/\eta_D$? Need to extend the analysis to large momentum variations, i.e. non-constant v.
- Compare with results for b/c suppression from experiments.