Energy loss and diffusion in non-conformal holographic QCD models

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Introduction

Heavy quarks: important probes of deconfined phase.

I will review the holographic description of:

- Energy loss
- Langevin diffusion (*transverse momentum broadening*)

of a heavy quark moving through the (thermalized) plasma.
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Setup: a *non conformal* holographic theory in the deconfined phase (suitable for connection with experiment)

(Review of work with U. Gursoy, E. Kiritsis, L. Mazzanti, G. Michalogiakakis, 2008-2013. Building on previous work by Gubser, Herzog, Son, Casalderrey-Solana, Teaney, Son, Iancu ...
Diffusion of a heavy quark

Quark produced out of equilibrium \((M > T)\): follow diffusion of a single particle. Use Langevin equation:

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\frac{d\vec{p}}{dt} = -\eta D \vec{p}(t) + \vec{\xi}(t)
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Two forces on the r.h.s:

- $\eta_D$: “average” viscous friction force
- $\xi(t)$: Stochastic force with white noise

$$\langle \xi^i(t) \rangle = 0, \quad \langle \xi^i(t)\xi^j(t') \rangle = \kappa^{ij} \delta(t - t')$$
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both forces have the same origin: the integrated effect of stochastic interactions with a medium.
Energy loss

\[ \frac{d\vec{p}}{dt} = -\eta_D \vec{p}(t) + \vec{\xi}(t) \]

Coefficient $\eta_D$ indicates a viscous friction on the quark, and consequent energy loss (if $\eta_D$ constant):

\[ F_{fric} = -\eta_D p, \quad \frac{1}{E_p} \frac{dE_p}{dt} \sim \eta_D \]

The energy is lost in interactions with the plasma.
Transverse Momentum Broadening

Quark momentum obeys a Langevin process with $\langle p^\perp \rangle = 0$, but with an increasing dispersion of $p^\perp$:

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Define the *jet quenching parameter*

$$\hat{q} \equiv \frac{\langle (p^\perp)^2 \rangle}{\text{mean free path}} = \frac{(p^\perp)^2}{vt} = 2\frac{\kappa^\perp}{v}$$
AdS/CFT

The AdS/CFT duality: conjecture that certain quantum field theories are equivalent to theories of strings/gravity in higher dimensions.
**AdS/CFT**

The AdS/CFT duality: conjecture that certain quantum field theories are equivalent to theories of gravity in higher dimensions.

- **Equivalent** means that the two theories contain the same degrees of freedom, but arranged in different ways.
- Depending on the situation, one side or the other may simplify. Gravity description simple at large $N$. 
**AdS/CFT**

The AdS/CFT duality: conjecture that certain quantum field theories are equivalent to theories of gravity in higher dimensions.

- Conformal invariance $\Leftrightarrow$ AdS spacetime $ds^2 = r^{-2}(dr^2 + dx_{\mu}^2)$,
  
  Scaling isometry $r \rightarrow \lambda r$, $x_{\mu} \rightarrow \lambda x_{\mu}$
  
  (e.g. $\mathcal{N} = 4$ SUSY YM at large $N$, large coupling)

- RG scale $\Leftrightarrow$ radial coordinate $r$; UV $\Leftrightarrow$ AdS boundary $r = 0$. 
Field/Coupling correspondence

- A Bulk field $\Phi(x, r)$ corresponds to a running coupling in the QFT
- $\Phi_0(x) = \Phi(x, 0)$ is the UV value of the coupling
Minimal phenomenological setup

- The bulk theory is five-dimensional ($x^\mu + \text{RG coordinate } r$)
- Include only lowest dimension YM operators ($\Delta = 4$)

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- Breaking of conformal symmetry, mass gap, confinement, and all non-perturbative dynamics driven by the dilaton dynamics (aka the Yang-Mills coupling).
5-D Eistein-Dilaton Theory

Bulk dynamics described by a 2-derivative action:

\[ S_E = -M_p^3 N_c^2 \int d^5 x \sqrt{-g} \left[ R - \frac{4}{3} (\partial \Phi)^2 - V(\Phi) \right] \]
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\( \Phi \) constant (conformal invariant)

\( \Phi \) running (non-conformal)
Finite temperature

Deconfined phase described by a 5D black hole geometry

\[ ds^2 = b^2(r) \left[ \frac{dr^2}{f(r)} - f(r) dt^2 + dx^i dx_i \right] \]

For a class of \( V(\Phi) \), theory has confinement and a 1st order phase transition to a deconfined phase (BH geometry).

\[ f(r) = 1 \]
\[ T < T_c \text{ confined} \]

\[ f(r) \neq 1, \ f(r_h) = 0 \]
\[ T > T_c \text{ deconfined (BH)} \]
Holographic description of a heavy quark

Trailing String picture:

- String profile obtained extremizing worldsheet area.
- The moving string equilibrates at a temperature $T_s < T$.
- Drag coefficient $\eta_D$ related to $b(r_s), f(r_s)$. 
Trailing string fluctuations

\[ \vec{X}(t, r) = (vt + \xi(r)) \frac{\vec{v}}{v} + \delta \vec{X}(r, t) \]

Dual to Langevin dynamics on the boundary: fluctuations correlators compute Langevin coefficients.

\[ \dot{\vec{p}} = -\eta_D(p)\vec{p} + \vec{\xi}(t) \quad \langle \xi^i(t)\xi^j(0) \rangle = \kappa^{ij}(p)\delta(t) \]
Langevin diffusion constants

\[ \kappa_{ij} = \kappa_{||}(p)p^i p^j + \kappa_{\perp}(p) \left( \delta^{ij} - \frac{p^i p^j}{p^2} \right) \]
Langevin diffusion constants

\[ \kappa^{ij} = \kappa^\parallel (p)p^i p^j + \kappa^\perp (p) \left( \delta^{ij} - \frac{p^i p^j}{p^2} \right) \]

modified Einstein relations (Relativistic Langevin):

\[ \kappa^\perp = 2\gamma M T_s \eta_D, \quad \kappa^\parallel = 2\gamma^3 M T_s \left[ \eta_D + M \gamma v \frac{\partial \eta_D}{\partial p} \right] \]

(Cfr. non-relativistic, thermal equilibrium E.R. \( \kappa = 2TM\eta_D \))
Explicit Model: Improved Holographic QCD

Gürsoy, Kiritsis, Mazzanti, F.N. ’09

Simple parametrization for $V(\Phi) \Rightarrow$ good qualitative and quantitative agreement with lattice Yang-Mills thermodynamics.
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\[
\frac{s(T)}{T^3}
\]
\text{Fix scale by matching critical temperature and vacuum string tension.}

> lattice data: Panero, 0907.3719
Explicit Model: Improved Holographic QCD

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$\frac{\Delta}{\Delta_{\text{SB}}}$, normalized to the SB limit of SU(3)

Langevin dynamics of heavy quarks in 5D Holographic QCD models

$(\epsilon - 3p)/T^4$ lattice data: Panero, 0907.3719

Fix scale by matching critical temperature and vacuum string tension.
Results: drag coefficients

- Conformal $AdS$/CFT:

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\eta_D \propto \sqrt{\lambda} \frac{T^2}{M_q}
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Results: drag coefficients

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- IHQCD (all free parameters fixed by satic quantities): diffusion time ($\tau = 1/\eta_D$):

\[ \tau_{charm} \simeq 4.5 \text{ fm/c} \]

for $p \approx 10 GeV$, $T \sim T_c$, consistent with size of fireball.
\( \kappa^\perp \) computed numerically given the solution for the background metric \( \Rightarrow \) obtain jet-quenching parameter:

\[
\hat{q} = 2 \frac{\kappa^\perp}{v} \quad \left( \hat{q}_{\text{conf}} \propto \sqrt{\lambda} T^3 \right)
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solid and dotted lines represent different schemes to match the temperature in the holographic YM model to the temperature in deconfined QCD.
Validity of the local approximation

- Langevin equation is an approximation of a more general equation:

\[
\dot{p} = - \int_0^\infty dt' \eta_D(t') p(t - t') + \xi(t), \quad \langle \xi(t)\xi(0) \rangle = G(t)
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*long time* regime \( t \gg \tau_{\text{corr}} \sim 1/T_s \): reduces to Langevin with:

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- $\Delta p^2_\perp$ description in terms of $\kappa_\perp$: short time solution of local Langevin. $t \ll \tau_{relax} \sim 1/\eta_D$
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- \( \Delta p^2_\perp \) description in terms of \( \kappa_\perp \): *short time* solution of *local* Langevin. \( t \ll \tau_{relax} \sim 1/\eta_D \)

- Consistency requires \( 1/\eta_D \gg 1/T_s \) If this fails, need to use the generalized process with memory and “colored” noise kernels. Accessible via Holography.
Validity of the local approximation

Parametrization in terms of $\hat{q}$ justified if:

$$\frac{T_s}{\eta_D} > 1$$

This translates to a bound on quark momentum:

$$p < 1.5M_q \left(\frac{M_q}{T}\right)^2$$
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Conclusion and outlook

- Non-conformal holographic models give realistic results for heavy quark transport in the QGP.

- What happens when the local Langevin description breaks down? Look at $p_\perp$ broadening with a non-trivial kernel and colored noise.

- How does momentum dependence affect the dynamics for large times, $> 1/\eta_D$? Need to extend the analysis to large momentum variations, i.e. non-constant $v$.

- Compare with results for b/c suppression from experiments.