The axial anomaly and topology in finite temperature QCD

Sayantan Sharma

BROOKHAVEN NATIONAL LABORATORY

August 30, 2016

Work done with:

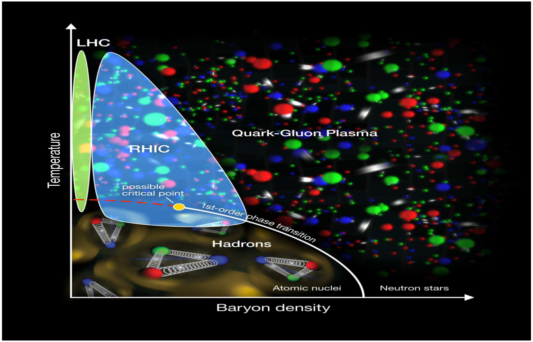
V. Dick, F. Karsch, E. Laermann, S. Mukherjee, P. Petreczky, H-P Schadler

Outline

1. The $U_A(1)$ puzzle in QCD
2. Background
3. Our results: towards solving the puzzle

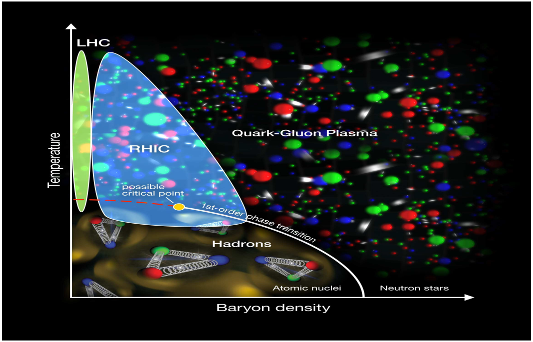
Outline

- The phase diagram of strongly interacting matter is largely unknown



[http://www.bnl.gov/rhic/news]

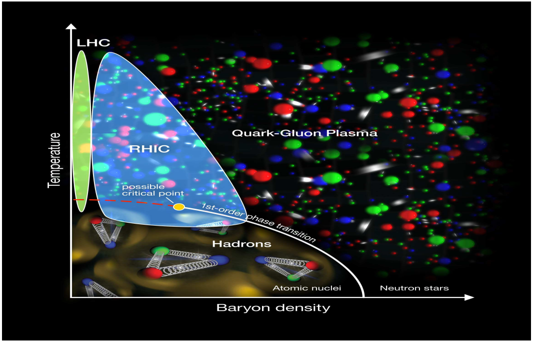
- The phase diagram of strongly interacting matter is largely unknown
- Confinement and chiral symmetry breaking observed, mechanism still unknown. [Schaefer and Shuryak, 96]



[http://www.bnl.gov/rhic/news]

Introduction

- The phase diagram of strongly interacting matter is largely unknown
- Confinement and chiral symmetry breaking observed, mechanism still unknown. [Schaefer and Shuryak, 96]
- At $\mu_B \to 0$, it is now well known from lattice there is a crossover transition.
  [Bielefeld-BNL collaboration 05, MILC collaboration, 05, Budapest-Wuppertal collaboration, 06, HotQCD, 11]
  But can we see hints of the criticality due to the light quarks present.



[http://www.bnl.gov/rhic/news]

Symmetries and order parameters

- $m_f = 0$, $\mathcal{L}_{QCD}$ invariant under
  $U_L(N_f) \times U_R(N_f) \equiv SU(N_f)_V \times SU(N_f)_A \times U_B(1) \times U_A(1)$

Symmetries and order parameters

- $m_f = 0$, $\mathcal{L}_{QCD}$ invariant under
  $U_L(N_f) \times U_R(N_f) \equiv SU(N_f)_V \times SU(N_f)_A \times U_B(1) \times U_A(1)$
- $m_s \sim \Lambda_{QCD}$, $m_u, m_d << \Lambda_{QCD}$,
  The approximate symmetry: $SU(2)_V \times SU(2)_A \times U_B(1) \times U_A(1)$

Symmetries and order parameters

- $m_f = 0$, $\mathcal{L}_{QCD}$ invariant under
  $U_L(N_f) \times U_R(N_f) \equiv SU(N_f)_V \times SU(N_f)_A \times U_B(1) \times U_A(1)$
- $m_s \sim \Lambda_{QCD}$, $m_u, m_d << \Lambda_{QCD}$,
  The approximate symmetry: $SU(2)_V \times SU(2)_A \times U_B(1) \times U_A(1)$
- QGP$\rightarrow$ hadron transition results in light pions
  $\rightarrow$ chiral symmetry breaking
  $SU(2)_V \times SU(2)_A \times U_B(1) \rightarrow SU(2)_V \times U_B(1)$

Symmetries and order parameters

- $m_f = 0$, $\mathcal{L}_{QCD}$ invariant under
  $U_L(N_f) \times U_R(N_f) \equiv SU(N_f)_V \times SU(N_f)_A \times U_B(1) \times U_A(1)$
- $m_s \sim \Lambda_{QCD}$, $m_u, m_d << \Lambda_{QCD}$,
  The approximate symmetry: $SU(2)_V \times SU(2)_A \times U_B(1) \times U_A(1)$
- QGP$\rightarrow$ hadron transition results in light pions
  $\rightarrow$ chiral symmetry breaking
  $SU(2)_V \times SU(2)_A \times U_B(1) \rightarrow SU(2)_V \times U_B(1)$
- Symmetries determine order parameter: $\langle \bar{\psi}_l \psi_l \rangle$.

Symmetries and order parameters

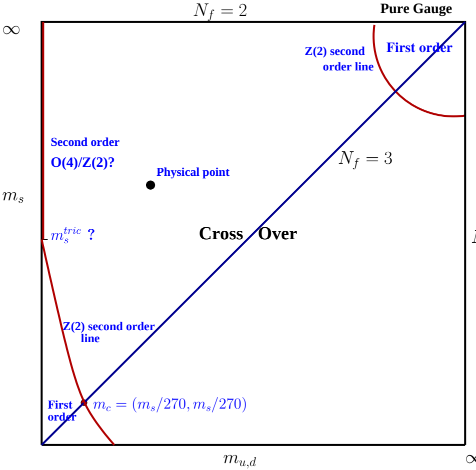
- $m_f = 0$, $\mathcal{L}_{QCD}$ invariant under
  $U_L(N_f) \times U_R(N_f) \equiv SU(N_f)_V \times SU(N_f)_A \times U_B(1) \times U_A(1)$
- $m_s \sim \Lambda_{QCD}$, $m_u, m_d << \Lambda_{QCD}$,
  The approximate symmetry: $SU(2)_V \times SU(2)_A \times U_B(1) \times U_A(1)$
- QGP$\rightarrow$ hadron transition results in light pions
  $\rightarrow$ chiral symmetry breaking
  $SU(2)_V \times SU(2)_A \times U_B(1) \rightarrow SU(2)_V \times U_B(1)$
- Symmetries determine order parameter: $\langle \bar{\psi}_l \psi_l \rangle$.
- $U_A(1) \rightarrow$ not a symmetry yet may affect the order of phase transition for $N_f = 2$ [Pisarski & Wilczek, 84].

Where do we stand now!

- Perturbative RG studies on sigma models with same symmetries as N_f flavour QCD:

  $N_f \geq 3$: 1st order phase transition independent of $U_A(1)$.

  $N_f = 2$: If $U_A(1)$ effectively restored $\Rightarrow$ 1st or 2nd order with $U_L(2) \times U_R(2) \rightarrow U_V(1)$ criticality



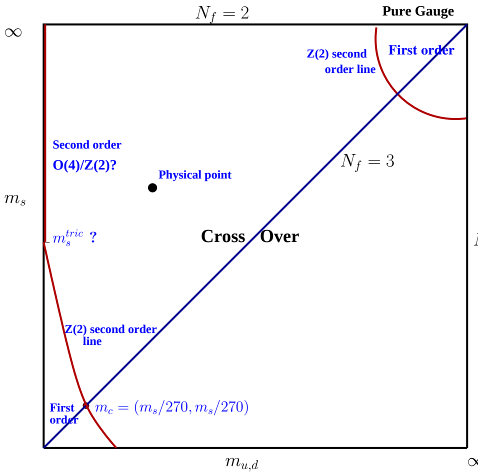
[Pisarski & Wilczek, 84, Butti, Pelissetto & Vicari, 03, 13, Nakayama & Ohtsuki, 15]

Where do we stand now!

- Perturbative RG studies on sigma models with same symmetries as N_f flavour QCD:

  $N_f \geq 3$: 1st order phase transition independent of $U_A(1)$.

  $N_f = 2$: If $U_A(1)$ broken $\Rightarrow$ 2nd order transition with $O(4)$ critical exponents.

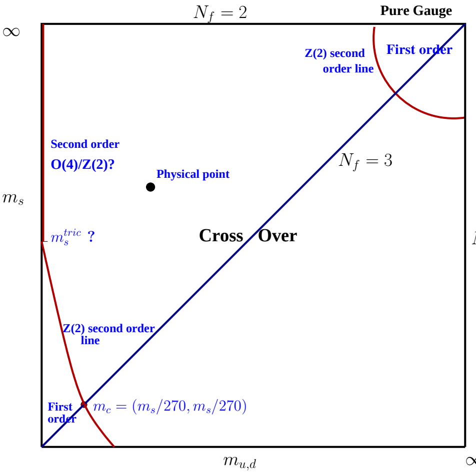


[Pisarski & Wilczek, 84, Butti, Pelissetto & Vicari, 03, 13, Nakayama & Ohtsuki, 15]

- Perturbative RG studies on sigma models with same symmetries as N_f flavour QCD:

  $N_f \geq 3$: 1st order phase transition independent of $U_A(1)$.

  Important to get insight from lattice.

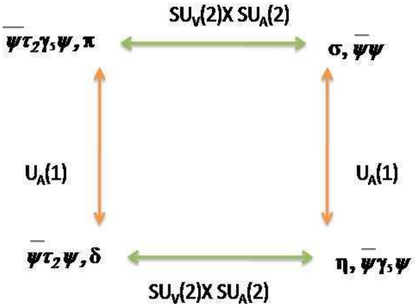


[Pisarski & Wilczek, 84, Butti, Pelissetto & Vicari, 03, 13, Nakayama & Ohtsuki, 15]

Routes to solve the puzzle

Not an exact symmetry $\rightarrow$ what observables to look for?

Degeneracy of the 2-point correlators [Shuryak, 94] $\rightarrow$ higher point correlation functions imp.



Routes to solve the puzzle

- In terms of integrated correlators:

$$\chi_\pi = \int d^4x \, \langle i\pi^+(x) i\pi^-(0) \rangle$$

- Off-Diagonals of the meson spectra: $\chi_\delta - \chi_\sigma = -2\chi_{disc}$
- Furthermore $\chi_\eta - \chi_\pi = -2\chi_{5,disc}$
- When chiral symmetry is restored $\chi_\pi \leftrightarrow \chi_\sigma$, $\chi_\eta \leftrightarrow \chi_\delta$
- Consequently $\chi_{disc} \to \chi_{5,disc}$.
- From definition,
  $\chi_t = \frac{Tm_l^2}{V} \left\langle \left(\mathrm{Tr} D_l^{-1} \gamma^5 \right)^2 \right\rangle = (m_l)^2 \chi_{5,disc}$,
- $\chi_t = m_l^2 \chi_{disc}$
  New fermionic observable when χ_{SB} restored!

Routes to solve the puzzle

- Not an exact symmetry$\rightarrow$ what observables to look for?
- When $U_A(1)$ is also restored:

$$\chi_\pi - \chi_\delta = \int d^4x \ \left[\langle i\pi^+(x) i\pi^-(0)\rangle - \langle \delta^+(x)\delta^-(0)\rangle\right] \rightarrow 0$$

- Equivalently study $\rho(\lambda, m_f)$ of the Dirac operator.

$$\chi_\pi - \chi_\delta \overset{V\rightarrow\infty}{\rightarrow} \int_0^\infty d\lambda \frac{4m_f^2\ \rho(\lambda, m_f)}{(\lambda^2 + m_f^2)^2} \ , \ \langle\bar{\psi}\psi\rangle \overset{V\rightarrow\infty}{\rightarrow} \int_0^\infty d\lambda \frac{2m_f\ \rho(\lambda, m_f)}{(\lambda^2 + m_f^2)}$$

- If chiral symmetry restored: $\langle\bar{\psi}\psi\rangle = 0 \Rightarrow \lim_{m_f\rightarrow 0} \lim_{V\rightarrow\infty} \rho(0, m_f) \rightarrow 0$.
- A gap in the infrared spectrum $\Rightarrow U_A(1)$ restored
- chiral symmetry restored $+\ U_A(1)$ broken if:
  $\lim_{\lambda\rightarrow 0} \rho(\lambda, m_f) \rightarrow \delta(\lambda) m_f^\gamma \ , 1 < \gamma \leq 2$

Looking at Spectral density in QCD at $T \neq 0$

- Very little known. Only recently studied in detail [Aoki, Fukaya & Taniguchi, 12].

Looking at Spectral density in QCD at $T \neq 0$

- Very little known. Only recently studied in detail [Aoki, Fukaya & Taniguchi, 12].

- Assuming $\rho(\lambda, m)$ to be analytic in λ, m^2, look at Ward identities of n-point function of scalar & pseudo-scalar currents when chiral symmetry is restored.

Looking at Spectral density in QCD at $T \neq 0$

- Very little known. Only recently studied in detail [Aoki, Fukaya & Taniguchi, 12].

- Assuming $\rho(\lambda, m)$ to be analytic in λ, m^2, look at Ward identities of n-point function of scalar & pseudo-scalar currents when chiral symmetry is restored.

- $\rho(\lambda, m \to 0) \sim \lambda^3 \Rightarrow U_A(1)$ breaking effects invisible in this sector for upto 6-point correlation functions.

Looking at Spectral density in QCD at $T \neq 0$

- Very little known. Only recently studied in detail [Aoki, Fukaya & Taniguchi, 12].

- Assuming $\rho(\lambda, m)$ to be analytic in λ, m^2, look at Ward identities of n-point function of scalar & pseudo-scalar currents when chiral symmetry is restored.

- $\rho(\lambda, m \to 0) \sim \lambda^3 \Rightarrow U_A(1)$ breaking effects invisible in this sector for upto 6-point correlation functions.

- Non-analyticities in the infrared part of the spectrum+analytic form of the bulk.

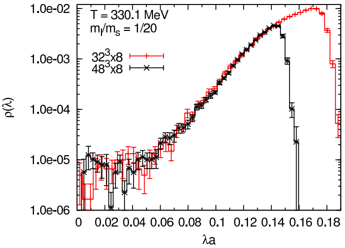
Outline

1. The $U_A(1)$ puzzle in QCD

2. Background

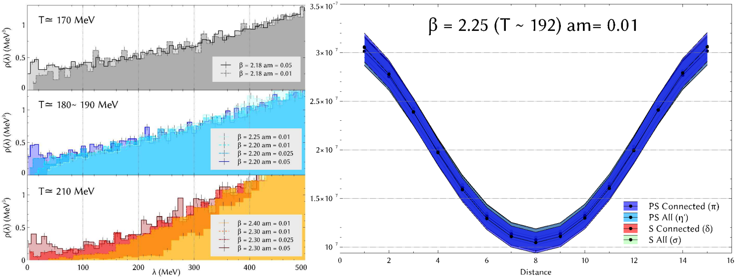
3. Our results: towards solving the puzzle

Lattice studies so far!



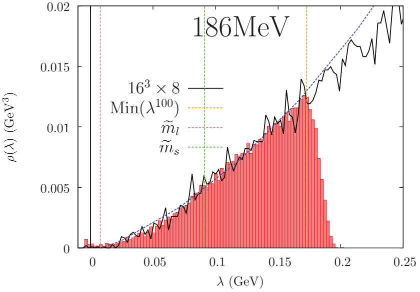
- Improved Staggered fermions: Large $32^3 \times 8$ lattice $\rightarrow$ $U_A(1)$ broken [Ohno et. al. 12]
- Same observation noted earlier for smaller lattice [Chandrasekharan & Christ 96]
- Recent results on screening mass with improved Wilson fermions $M_\eta - M_\sigma = -81(282)$ MeV $\rightarrow$ $U_A(1)$ effectively restored at T_c. Lattice volumes rather small $(2\ fm)^3$ [B. Brandt et. al., 16]
  Issues with lattice artifacts?
  Exact chiral invariance is not maintained

Lattice studies so far!



Dynamical overlap fermions with exact chiral symmetry on lattice $\rightarrow$ $U_A(1)$ restored [Cossu et. al, JLQCD collaboration, 11, 12]
Pion mass 220 MeV. Effects of fixing the topology? Thermodynamic limit?

Lattice studies so far!



- Dynamical domain wall fermions with better chiral symmetry $\rightarrow$ $U_A(1)$ broken [Buchoff et. al. 13] Low statistics in the lower end of spectrum?
- Optimal domain wall fermions: On small lattice $\rightarrow$ $U_A(1)$ restored [Chiu et. al. 13]
- Small eigenvalues in small volumes related to χ_{SB}. Reweighting them $\rightarrow$ $U_A(1)$ restored. [G. Cossu et. al. 15 (JLQCD collaboration.]

Consequence: Constituents of the hot QCD medium

- Near T_c, a medium consisting of interacting instantons can explain chiral symmetry breaking $\Rightarrow$ Instanton Liquid Model

  [Shuryak, 82, Schaefer & Shuryak, 96]

- At $T >> T_c$, medium is like a dilute gas of instantons?

  [Gross, Pisarski & Yaffe, 81].

- What is the medium made up of for $T_c \leq T \leq 2T_c$?
- At what T is DIGA is valid?
- Finite T instantons have substructures instanton-monopoles $\rightarrow$ carry electric and magnetic charges. Can we detect them on lattice.. First signals observed! [Ilgenfritz, Michael Mueller-Preussker et. al 02, 06, 13, 14] .

- Finite volume effects $\rightarrow$ ensure presence of topological objects in a box.

The major issues with the lattice studies so far

- Finite volume effects $\rightarrow$ ensure presence of topological objects in a box.
- Most studies done with lattice fermions with only a remnant of continuum chiral symmetry + absence of flavor singlet $U_A(1)$ [S. Chandrasekharan, 96, H. Ohno et. al 12, V. Dick et. al., 15, B. Brandt et. al., 16].

The major issues with the lattice studies so far

- Finite volume effects $\rightarrow$ ensure presence of topological objects in a box.
- Most studies done with lattice fermions with only a remnant of continuum chiral symmetry + absence of flavor singlet $U_A(1)$ [S. Chandrasekharan, 96, H. Ohno et. al 12, V. Dick et. al., 15, B. Brandt et. al., 16].
- Studies done with chiral fermions are in a fixed topological sector+ small volume [JLQCD collaboration, 13].

The major issues with the lattice studies so far

- Finite volume effects $\rightarrow$ ensure presence of topological objects in a box.
- Most studies done with lattice fermions with only a remnant of continuum chiral symmetry + absence of flavor singlet $U_A(1)$ [S. Chandrasekharan, 96, H. Ohno et. al 12, V. Dick et. al., 15, B. Brandt et. al., 16].
- Studies done with chiral fermions are in a fixed topological sector+ small volume [JLQCD collaboration, 13].
- Lattice cut-off effects need careful consideration for near-zero eigenvalues [G. Cossu et. al, 15]

Techniques used on lattice

- To measure the localized topological structures $\rightarrow$ remove the ultraviolet fluctuations
- Two known methods: Gauge operator & Fermionic operator
- Cooling to the classical action or smearing UV modes and then measure $F\tilde{F}$ [Ilgenfritz, Michael Mueller-Preussker et. al 06, 13, Bonati, M. D'Elia et. al., 13, 14]
- Controlled smearing method $\rightarrow$ Wilson flow [M. Luscher, 09, 10].
- May cause disappearance of small instantons, density $\propto \rho^{-5}$.

Techniques used on lattice

- To measure the localized topological structures $\rightarrow$ remove the ultraviolet fluctuations
- Two known methods: Gauge operator & Fermionic operator

- Cooling to the classical action or smearing UV modes and then measure $F\tilde{F}$ [Ilgenfritz, Michael Mueller-Preussker et. al 06, 13, Bonati, M. D'Elia et. al., 13, 14]
- Controlled smearing method $\rightarrow$ Wilson flow [M. Luscher, 09, 10].
- May cause disappearance of small instantons, density $\propto \rho^{-5}$.

- Alternative: Use the index theorem from fermion zero modes

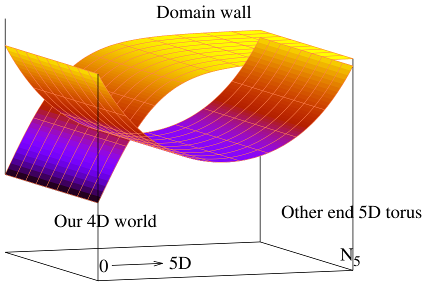
Index theorem on the lattice

- It is impossible to define chiral fermions on lattice which are (ultra)local. [Nielsen & Ninomiya, 82]
- Overlap fermions [Narayanan & Neuberger, 94, Neuberger, 98] have exact chiral symmetry on the lattice.

$$D_{ov} = M(1+\gamma_5\, \mathbf{sgn}(\gamma_5 D_W(-M)))\ ,\ \mathbf{sgn}(A) = A/\sqrt{A.A}.$$

- It satisfies the Ginsparg-Wilson relation $\{\gamma_5, D_{ov}\} = aD_{ov}\gamma_5 D_{ov}$ [Ginsparg & Wilson, 82]
- D_{ov} has an exact index theorem like in the continuum $\Rightarrow$ the zero modes of D_{ov} related to topological structures of the underlying gauge field. [Hasenfratz, Laliena & Niedermeyer, 98]

Index theorem on the lattice



- One can also start from 5D world+ put a defect to localize chiral fermions on the 4D brane.
- Domain wall fermions [Kaplan 92, Shamir 95] in the limit $N_5 \to \infty$
$D_{DW} = M(1 - \gamma_5\ \mathbf{sgn}(\ln|T|))\ ,\ \ T = (1 + a_5\gamma_5 D_W P_+)^{-1}(1 - a_5\gamma_5 D_W P_-)$.

The technique we use

- **Method I:** Use overlap as valence operator to probe the infrared spectrum.
- Sea quarks in background $\rightarrow$ HISQ and domain wall fermions
- We look at the eigenvalue distribution of D_{ov} on the ensembles.
- Zero modes of D_{ov} related to topological structures of sea quarks.
- Infrared part of eigenvalue distribution gives us idea about the χ_{SB} , $U_A(1)$ and the topological structures that contribute to them.

The technique we use

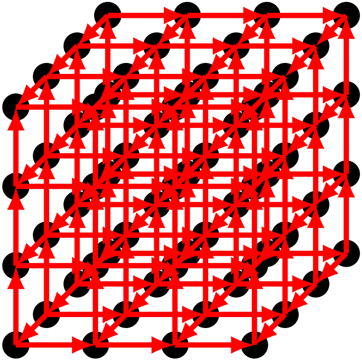
- **Method I:** Use overlap as valence operator to probe the infrared spectrum.
- Sea quarks in background $\rightarrow$ HISQ and domain wall fermions
- We look at the eigenvalue distribution of D_{ov} on the ensembles.
- Zero modes of D_{ov} related to topological structures of sea quarks.
- Infrared part of eigenvalue distribution gives us idea about the χ_{SB} , $U_A(1)$ and the topological structures that contribute to them.
- **Method II** Use Wilson flow on staggered fermions $\rightarrow$ continuum limit

The technique we use

- **Method I:** Use overlap as valence operator to probe the infrared spectrum.
- Sea quarks in background $\rightarrow$ HISQ and domain wall fermions
- We look at the eigenvalue distribution of D_{ov} on the ensembles.
- Zero modes of D_{ov} related to topological structures of sea quarks.
- Infrared part of eigenvalue distribution gives us idea about the χ_{SB} , $U_A(1)$ and the topological structures that contribute to them.
- **Method II** Use Wilson flow on staggered fermions $\rightarrow$ continuum limit
- First we present results with staggered (HISQ) quarks and then with "chiral" domain wall fermions.

Outline

Our Set-up

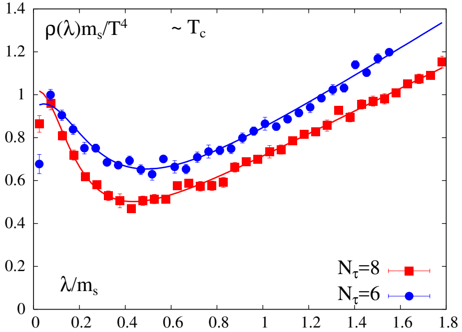


- $V = N^3 a^3$, $T = \frac{1}{N_\tau a}$.
- Box size: $m_\pi \ V^{1/3} > 4$
- Input m_s physical ≈ 100 MeV and $m_\pi = 160, 110$ MeV (staggered) $m_\pi = 200, 135$ MeV (domain wall fermions)

Eigenvalue distribution near T_c

- General features: Near zero mode peak +bulk.
- We fit the $\rho(\lambda)$ to ansatz: $\rho(\lambda) = \frac{A\epsilon}{\lambda^2+A} + B\lambda^\gamma$

[V. Dick, F. Karsch, E. Laermann, S. Mukherjee and S.S, PRD 91, 15].

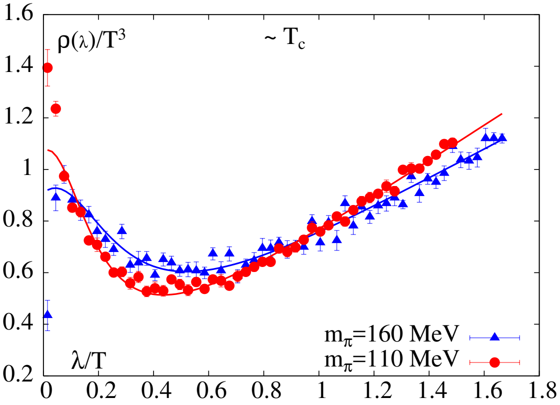


Bulk rises linearly as λ, consistent with χ_{PT} predictions. No gap seen.

Eigenvalue distribution near T_c

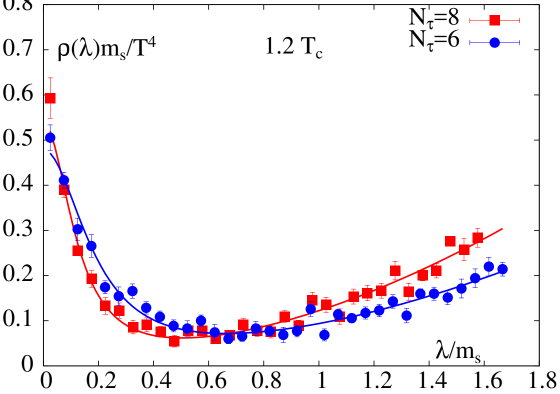
- General features: Near zero mode peak +bulk.
- We fit the $\rho(\lambda)$ to ansatz: $\rho(\lambda) = \frac{A\epsilon}{\lambda^2+A} + B\lambda^\gamma$

[V. Dick, F. Karsch, E. Laermann, S. Mukherjee and S.S, PRD 91, 15].



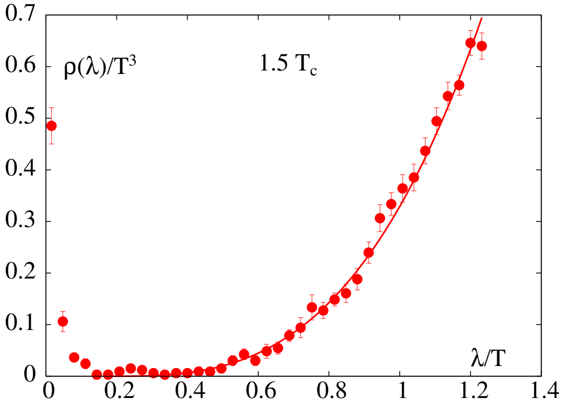
No gap even when quark mass reduced! Peak height increases as mass lowered.

At higher temperatures..



Near zero mode peak shows little cut-off dependence. Bulk rises as λ^2.

At higher temperatures..



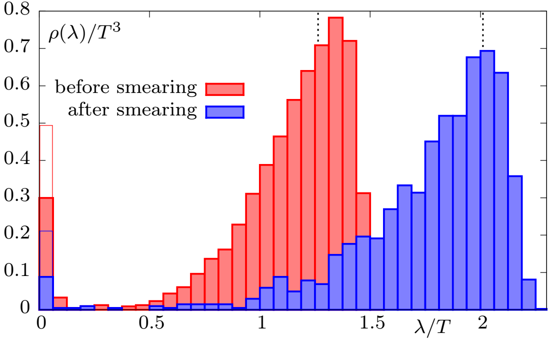
At $1.5\,T_c$, the bulk and the near zero peak decouples completely

Bulk rises as λ^3.

Breaking of $U_A(1)$ from the near-zero modes? Are these physical?

Robustness of near zero modes

- The infrared part could be affected by unphysical dislocations. $\rightarrow$ partial quenching + rough configurations.
- These have smaller classical action than instantons. $\rightarrow$ lattice cut-off effect.
- HYP smearing [Hasenfratz & Knechtli, 02] expected to eliminate such structures.



- Smearing does not eliminate the near zero modes.
- At this temperature, rooting is insignificant.
- Difference? Smearing may suppress small instantons.

What we found till now...

- The near-zero modes survive till $1.5\,T_c$.

What we found till now...

- The near-zero modes survive till $1.5\,T_c$.
- The bulk modes have very distinct rise..changes from $|\lambda|$ to λ^2 as $T_c \rightarrow 1.2\,T_c$

What we found till now...

- The near-zero modes survive till $1.5\,T_c$.
- The bulk modes have very distinct rise..changes from $|\lambda|$ to λ^2 as $T_c \to 1.2\,T_c$
- both conspire to break $U_A(1)$.

What we found till now...

- The near-zero modes survive till $1.5\,T_c$.
- The bulk modes have very distinct rise..changes from $|\lambda|$ to λ^2 as $T_c \rightarrow 1.2\,T_c$
- both conspire to break $U_A(1)$.
- Near zero modes are physical $\rightarrow$ not artifacts even at $1.5\,T_c$.

  [V. Dick, F. Karsch, E. Laermann, S. Mukherjee and S.S, PRD 91, 15].

What we found till now...

- The near-zero modes survive till $1.5\,T_c$.
- The bulk modes have very distinct rise..changes from $|\lambda|$ to λ^2 as $T_c \to 1.2\,T_c$
- both conspire to break $U_A(1)$.
- Near zero modes are physical $\to$ not artifacts even at $1.5\,T_c$.

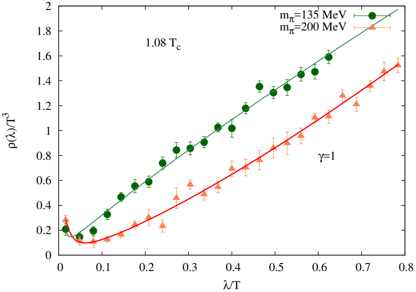
  [V. Dick, F. Karsch, E. Laermann, S. Mukherjee and S.S, PRD 91, 15].

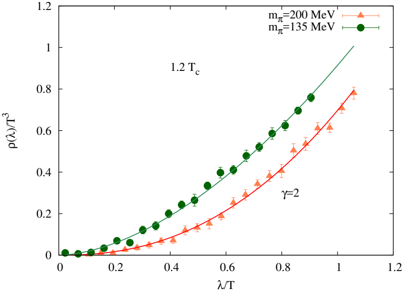
- How robust are these characteristics? Do fermions with exact chiral symmetry show similar trends?

Sensitivity to fermion discretization

- Use of fermions exact chiral symmetry and $U_A(1) \rightarrow$ much clear interpretation of the topology issues.
- First results for domain wall fermion spectrum exciting!

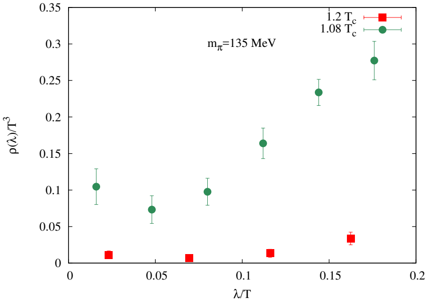
  [V. Dick et. al., 1602.02197, in prep]
- Near zero peak persists at $T > T_c$+bulk shows the expected rise
  Insensitive to quark mass effects.

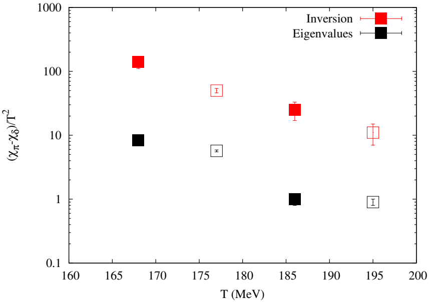




Sensitivity to fermion discretization

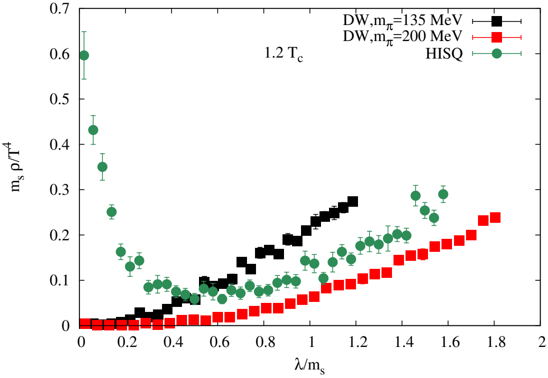
- The near-zero mode falls by more than a third at $1.2T_c$.
- First 50 eigenvalues of the Dirac operator contribute significantly to $U_A(1)$ breaking. [V. Dick et. al., 1602.02197, in prep]





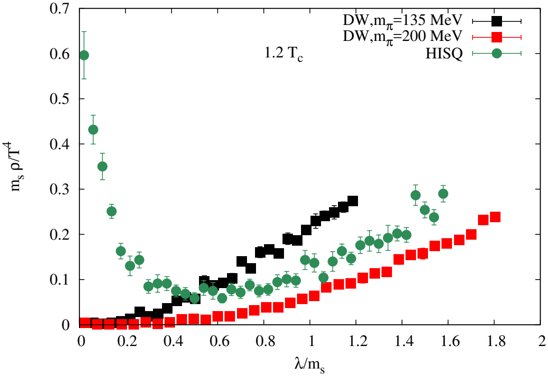
Eigenvalue spectra of different lattice fermions

- How do the HISQ and Domain wall spectra compare?



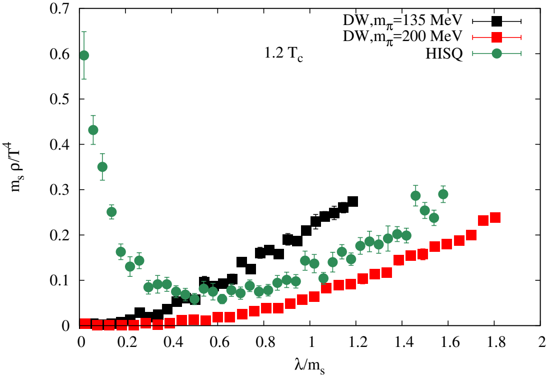
Eigenvalue spectra of different lattice fermions

- How do the HISQ and Domain wall spectra compare?
- The bulk HISQ spectra with Goldstone pion mass 160 MeV consistent with DW with $m_\pi = 200$ MeV at $1.2\,T_c$.



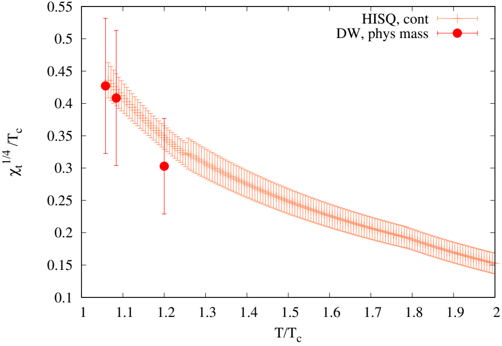
Eigenvalue spectra of different lattice fermions

- How do the HISQ and Domain wall spectra compare?
- The bulk HISQ spectra with Goldstone pion mass 160 MeV consistent with DW with $m_\pi = 200$ MeV at $1.2\,T_c$.
- More near-zero states in HISQ than domain wall..redundant symmetries due to rooting?



Topological susceptibility: different lattice fermions

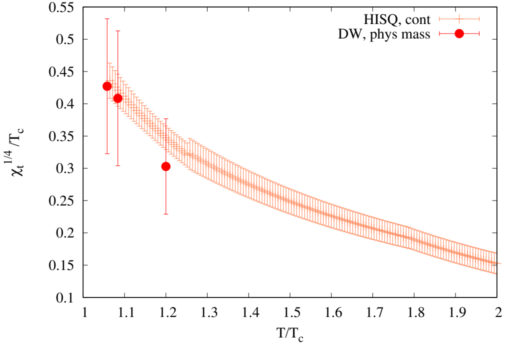
- Are continuum symmetries recovered for staggered fermions for $a \to 0$?



Topological susceptibility: different lattice fermions

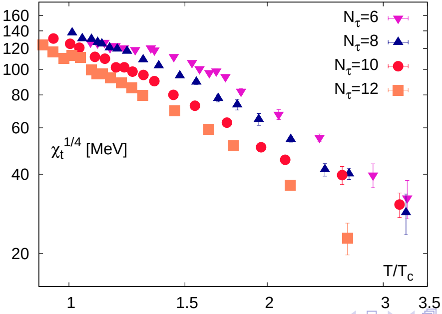
- Are continuum symmetries recovered for staggered fermions for $a \to 0$?
- Results from QCD with "chiral" fermions agree well with continuum extrapolated results using staggered fermions.

  [P. Petreczky, H-P Schadler, SS, 1606.03145]



Topological susceptibility: Staggered fermions using Wilson flow

- Wilson flow on improved staggered fermion (HISQ) ensembles $\rightarrow$ remove ultra-violet fluctuations.
- Q measured using lattice definition of $F\tilde{F}$.
- Using $\chi_t^{1/4} = AT^{-B}$. Fit to the data shows very distinct slopes.
- $B = 0.9 - 1.2$ for $T < 250$ MeV. Agrees well with another independent work [Bonati et. al, 1512.06746]

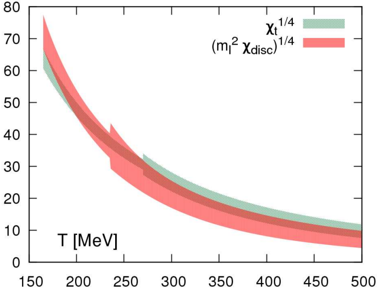


Topological susceptibility at high T

- Using the fit ansatz:

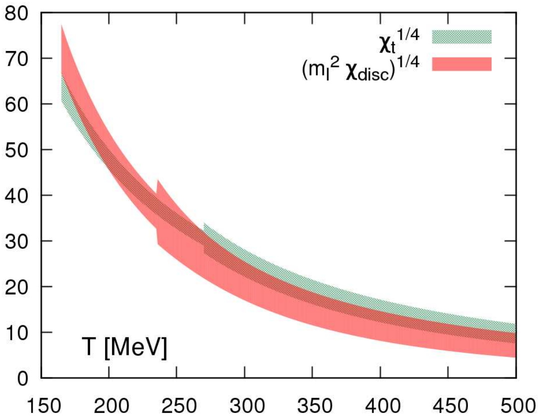
$$\chi_t^{1/4}(T, N_\tau) = (a_0 + a_2/N_\tau^2 + a_4/N_\tau^4)\cdot(T_c/T)^{b+b_2/N_\tau^2+b_4/N_\tau^4+b_6/N_\tau^6}$$

- $T > 300$ MeV: Continuum extrapolated $b = 1.85(15)$ in agreement with Dilute instanton gas.
Also in agreement with an independent lattice work [Borsanyi et. al, 1606.07494]
For details see talk by Sandor Katz, Monday 19:30, Session G2



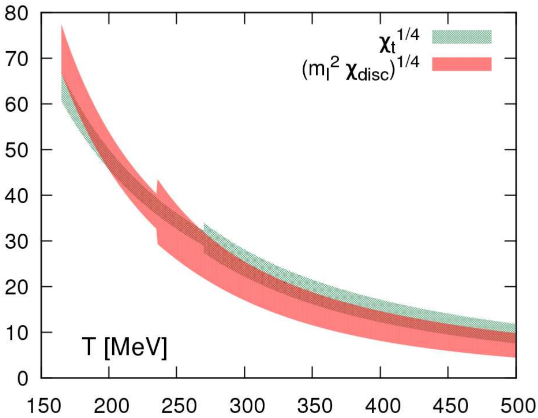
Topological susceptibility at high T

- At $T > 300$ MeV topological fluctuations are rare.



Topological susceptibility at high T

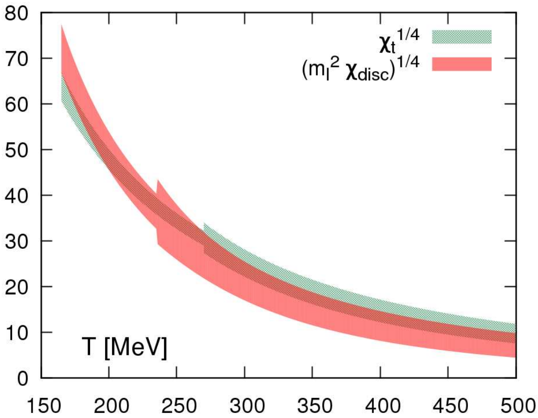
- At $T > 300$ MeV topological fluctuations are rare.
- Another check: $m_l^2 \chi_{disc} = \chi_t$.



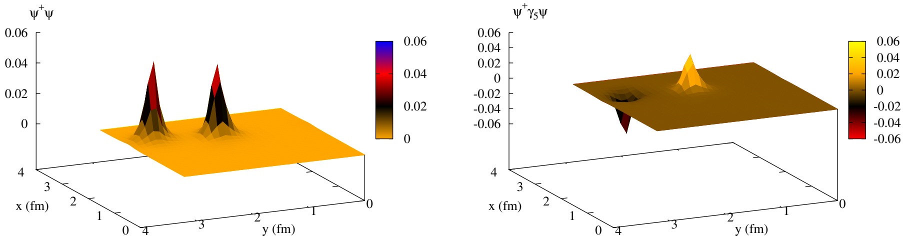
Topological susceptibility at high T

- At $T > 300$ MeV topological fluctuations are rare.
- Another check: $m_l^2 \chi_{disc} = \chi_t$.
- Cut-off effects are very different. But have the same continuum limit.

[P. Petreczky, H-P Schadler, SS, 1606.03145]



Independent confirmation: near-zero modes



Near-zero modes of QCD Dirac operator due to a weakly interacting instanton-antiinstanton pair!

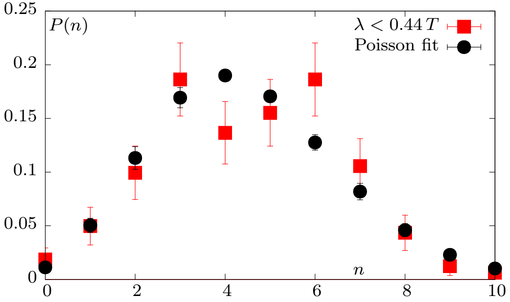
The nature of the infrared modes at $1.5\,T_c$

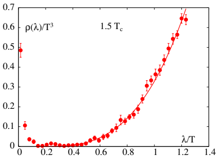
- If n=total no. of instantons+antiinstantons and form a dilute gas,

$$P(n,\langle n\rangle) = \langle n\rangle^n \mathrm{e}^{-\langle \mathrm{n}\rangle}/\mathrm{n}!$$

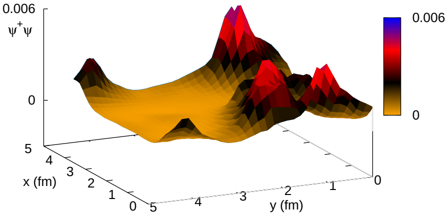
- For $\lambda/T < 0.44$, the value of $\langle n\rangle = 4.2 = \langle n^2\rangle \Rightarrow$ density $\simeq 0.147(7) fm^{-4}$. This is much more dilute than an instanton liquid with density $1 fm^{-4}$.

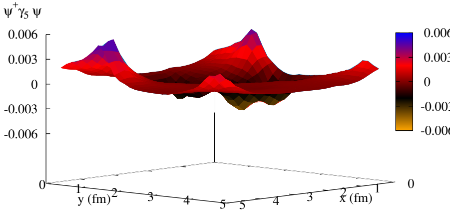
  [V. Dick, F. Karsch, E. Laermann, S. Mukherjee and S.S, PRD 91, 15].





Localization properties of near zero modes near T_c





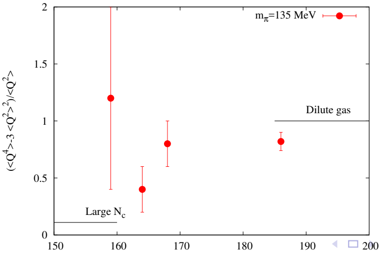
Combination of L and M instanton-monopoles of different chiralities?

Topological susceptibility and its fluctuations

- Higher order fluctuations:

$$\frac{< Q^4 > -3 < Q^2 >^2}{< Q^2 >}$$

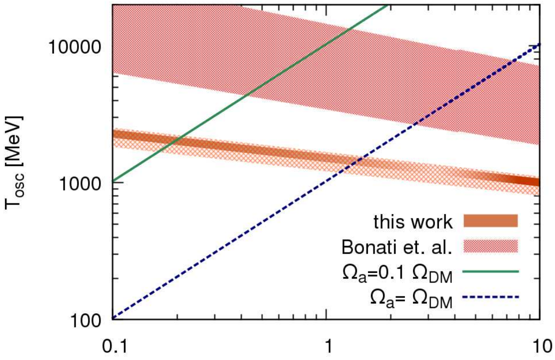
- At $T = 0$ QCD consistent with large N_c expansion of χ_t [M. Unsal, 08].
- Departure from large N_c expectations but a slow rise towards DIG $\gtrsim T_c \rightarrow$ effects of residual interactions? [Work in progress].
  For an independent measurement and conclusions see talk by Massimo D'Elia, Monday, 18:45 Session E2.



DIGA and Axion dark matter

- Slow roll of axion field at the bottom of the potential: $\chi_t = 9f_a^2 H^2 = m_a^2$
- Assuming axion density $\leq$ dark matter density $\Rightarrow$ $f_a \lesssim 1.2 \times 10^{12}$ GeV.
- Need a scaling factor to match lattice results with 1-loop DIGA predictions $\rightarrow$ major uncertainty from estimates of m_{Debye}.

  [P. Petreczky, H-P Schadler, SS, 1606.03145]



Outlook

- The $U_A(1)$ is not effectively restored near T_c.

Outlook

- The $U_A(1)$ is not effectively restored near T_c.
- Both near-zero+ bulk eigenvalues of QCD break $U_A(1)$ explicitly.

Outlook

- The $U_A(1)$ is not effectively restored near T_c.
- Both near-zero+ bulk eigenvalues of QCD break $U_A(1)$ explicitly.
- Topological susceptibility is measured for a large temperature range in the continuum...agreement with the fermions with good "chiral" properties on the lattice.

Outlook

- The $U_A(1)$ is not effectively restored near T_c.
- Both near-zero+ bulk eigenvalues of QCD break $U_A(1)$ explicitly.
- Topological susceptibility is measured for a large temperature range in the continuum...agreement with the fermions with good "chiral" properties on the lattice.
- Dilute gas scenario for instantons sets in already at $T \sim 300$ MeV.

Outlook

- The $U_A(1)$ is not effectively restored near T_c.
- Both near-zero+ bulk eigenvalues of QCD break $U_A(1)$ explicitly.
- Topological susceptibility is measured for a large temperature range in the continuum...agreement with the fermions with good "chiral" properties on the lattice.
- Dilute gas scenario for instantons sets in already at $T \sim 300$ MeV.
- Topological properties in QCD near T_c still needs to be understood.