Transverse flow induced by inhomogeneous magnetic fields in the Bjorken expansion

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Abstract

We investigate the magnetohydrodynamics in the presence of an external magnetic field following the power-law decay in proper time and having spatial inhomogeneity characterized by a Gaussian distribution in one of transverse coordinates under the Bjorken expansion. The leading-order solution is obtained in the weak-field approximation, where both energy density and fluid velocity are modified. It is found that the spatial gradient of the magnetic field results in transverse flow, where the flow direction depends on the decay exponents of the magnetic field. We suggest that such a magnetic-field-induced effect might influence anisotropic flow in heavy ion collisions.

Magento-hydrodynamics (MHD)

The ideal and magnetized fluid in the MDH :

$$T^{\mu\nu} = (\epsilon + p + B^2)u^\mu u^\nu + (p + \frac{1}{2}B^2)\eta^{\mu\nu} - B^\mu B^\nu,$$

$$\eta_{\mu\nu} = \mathrm{diag}\{-,+,+,+\}, \qquad B^2 = B^\mu B_\mu,\ B^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu F_{\alpha\beta}.$$

Note $T^\mu{}_\mu = 0$ given that $\epsilon = 3p$.

Conservation equations :

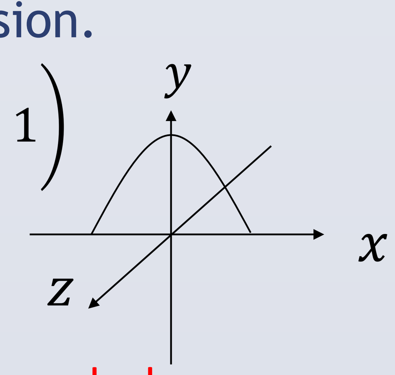
$$\begin{aligned} u_\nu \nabla_\mu T^{\mu\nu} &= -(u\cdot\nabla)(\epsilon + \frac{1}{2}B^2) - (\epsilon + p + B^2)(\nabla\cdot u) - u_\nu\nabla_\mu(B^\mu B^\nu) = 0, \\ \Delta_{\nu\alpha}\nabla_\mu T^{\mu\nu} &= (\epsilon + p + B^2)(u\cdot\nabla)u_\alpha + \Delta_{\nu\alpha}\nabla^\nu(p + \frac{1}{2}B^2) - \Delta_{\nu\alpha}\nabla_\mu(B^\mu B^\nu) = 0 \end{aligned}$$

We discard the induced E fields (assuming $\sigma_c \to \infty$) and consider zero magnetic susceptibility.

Setup

We consider an "external" B field under the Bjorken expansion.

We consider the weak-field approximation : $\left(\frac{B_y^2}{\epsilon_0} \ll 1\right)$



$$u_\mu = (1, \lambda^2 u_x(\tau,x), 0, 0), \qquad \epsilon = \epsilon_0(\tau) + \lambda^2\epsilon_1(\tau,x),$$

$$\mathbf{B} = \lambda B_y(\tau,x)\hat{y}, \qquad \epsilon_0(\tau) = \epsilon_c/\tau^{4/3}.$$

The expansion parameter

we rescale all spacetime coords. by τ_0

Time-dependent solutions

For $B_y = B_y(\tau)$, $u_x = 0$ and one can directly solve for $\epsilon(\tau)$ without making the approximation. S. Pu, V. Roy, L. Rezzolla, D. Rischke , Phys.Rev. D93 (2016) no.7, 074022

Conservation equations :

$$\partial_\tau\epsilon + \frac{4\epsilon}{3\tau} + B_y\partial_\tau B_y + \frac{B_y^2}{\tau} = 0 \implies \epsilon(\tau) = \frac{1}{\tau^{4/3}}\left(\epsilon_c - \int_1^\tau du\, u^{1/3}\left(B_y^2 + uB_y\partial_u B_y\right)\right)$$

Power-law decay : $B_y(\tau) = B_0\tau^{n/2}$ with $n < 0$.

Solutions :

$$\epsilon(\tau) = \frac{\epsilon_c}{\tau^{4/3}} - \frac{3B_0^2(2+n)\tau^n}{8+6n} \quad \text{for } n \neq -4/3 \qquad \epsilon(\tau) = \frac{\epsilon_c}{\tau^{4/3}} - \frac{B_0^2\log\tau}{3\tau^{4/3}} \quad \text{for } n = -4/3$$

When $n = -2$, $\delta\epsilon(\tau) = 0$ due to the "Frozen flux theorem".

$$(u\cdot\nabla)\left(\frac{B^\mu}{s}\right) = \frac{1}{s}\left[(B\cdot\nabla)u^\mu + u^\mu\nabla\cdot B\right]$$

const. entropy density

The medium "does not feel" the presence of the B field.

Spatial dependence

In the presence of spatial inhomogeneity ($B_y = \lambda\, B_s(x)\tau^{n/2}$), we seek for the perturbative solution up to $O(\lambda^2)$.

Conservation equations :

$$\begin{aligned} &\partial_\tau\epsilon_1 + \frac{4\epsilon_1}{3\tau} - \frac{4\epsilon_c\partial_x u_x}{3\tau^{4/3}} + B_y\partial_\tau B_y + \frac{B_y^2}{\tau} = 0, \\ &\partial_x\epsilon_1 - \frac{4\epsilon_c\partial_\tau u_x}{\tau^{4/3}} + \frac{4\epsilon_c u_x}{3\tau^{7/3}} + 3B_y\partial_x B_y = 0. \end{aligned}$$

$$\implies \tau^2\partial_x^2 u_x - u_x - 3\tau^2\partial_\tau^2 u_x + \tau\partial_\tau u_x + \frac{3\tau^{7/3}}{4\epsilon_c}\partial_x\left(B_y^2 + \tau\partial_\tau B_y^2\right) = 0.$$

Perturbative Solutions

Trick : we then approximate $B_s(x)^2$ by a Fourier series.

$$B_y^2(\tau,x) = \sum_k \tilde{B}_k^2(\tau)\cos(kx)$$ sol. of a PDE ~ sum of the sols. of ODEs

An ansatz : $u_x(\tau,x) = \sum_m \left[a_m(\tau)\cos(mx) + b_m(\tau)\sin(mx)\right]$

$$\longrightarrow (3\tau^2\partial_\tau^2 - \tau\partial_\tau + k^2\tau^2 + 1)b_k(\tau) + \frac{3B_k^2}{4\epsilon_c}k(n+1)\tau^{n+7/3} = 0 \qquad \text{solve for } b_k$$

Perturbative solutions are complicated but analytic.

We fix the integrational constants by asymptotic solutions in late times.

$$u_x(\tau,x) \to -\sum_{k\neq0}\frac{3B_k^2(1+n)}{4\epsilon_c k}\tau^{n+1/3}\sin(kx) \quad \text{for } \tau \to \infty$$

Energy density : $$\epsilon_1(\tau,x) = -\frac{3B_0^2(2+n)\tau^n}{8+6n} - \sum_{k\neq0}\frac{\cos(kx)}{k}\left[\frac{4\epsilon_c\partial_\tau b_k(\tau)}{\tau^{4/3}} - \frac{4\epsilon_c b_k(\tau)}{3\tau^{7/3}} + \frac{3k}{2}\tilde{B}_k^2(\tau)\right]$$

Properties of solutions

A special case when $n = -1$: $u_x(\tau,x) = 0$ $\qquad \epsilon_1(\tau,x) = -\frac{3\tilde{B}_y^2(x)}{2\tau} = -\frac{3B_y^2(\tau,x)}{2}$

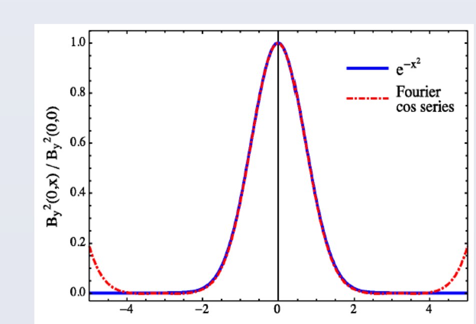
(an exact solution) arbitrary $\longleftarrow$ $B_y^2(\tau,x) = \tilde{B}_y^2(x)\tau^{-1}$

Validity of the perturbative solutions :

The constraint : $B(\tau,x)^2/\epsilon_0(\tau) < 1$ $\quad$ Our setup implies $\sum_k B_k^2\tau^{n+4/3}/\epsilon_c < 1$ with $\sum_k B_k^2/\epsilon_c < 1$.

$\implies$ $\tau \geq 1$ No time constraint for $n < -4/3$.

$1 \leq \tau < \left(\sum_k \epsilon_c B_k^{-2}\right)^{1/(n+4/3)}$ for $n \geq -4/3$.



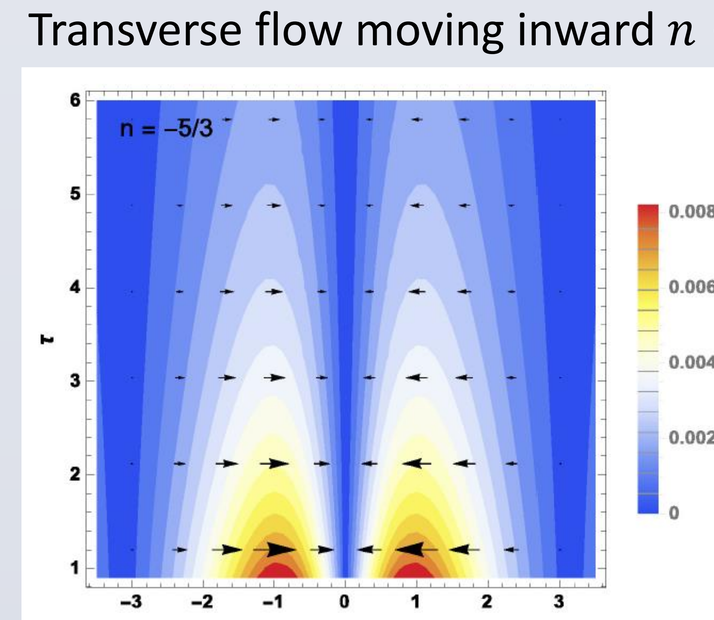
A concrete example

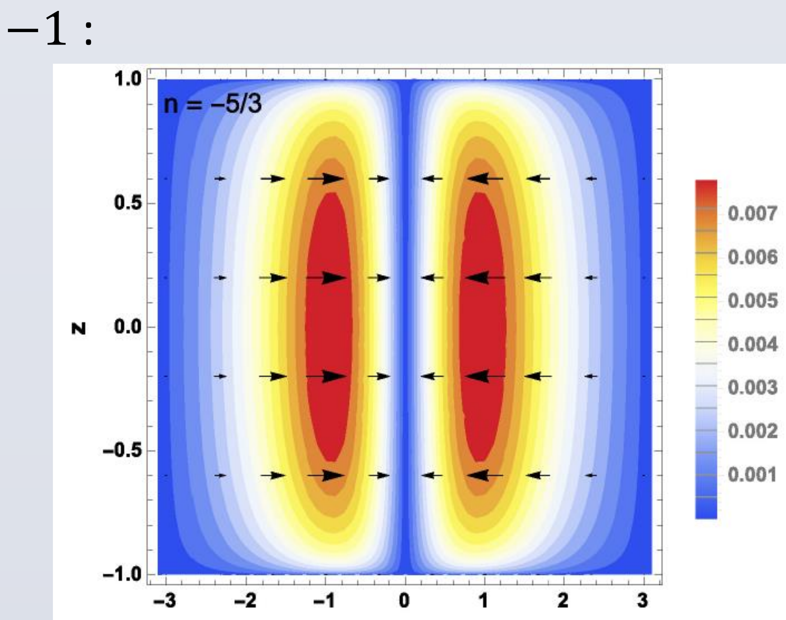
A Gaussian distribution : $\mathbf{B} = B_y(\tau,x)\hat{y} = B_c\tau^{n/2}e^{-x^2/2}\hat{y}$

The Fourier series : $B_y(\tau,x)^2 = B_c^2\tau^n\,(0.28, +0.44\cos x + 0.21\cos 2x + 0.06\cos 3x + 0.01\cos 4x)$

(For larger spatial widths, one simply needs to rescale x.)

Transverse flow moving inward $n < -1$:

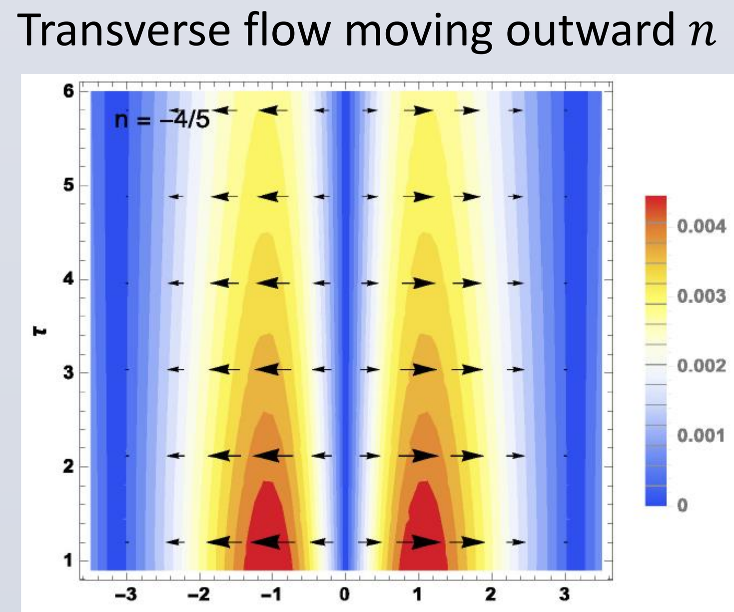


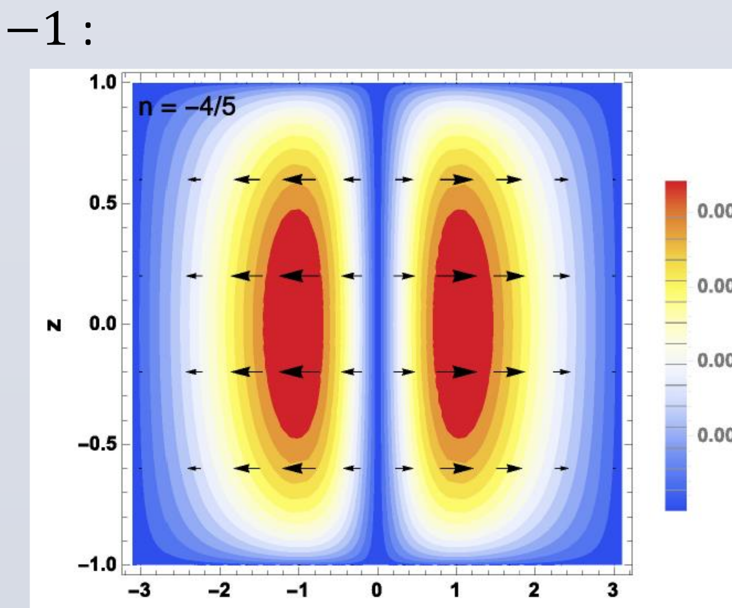


$B_c^2/\epsilon_c = 0.1$

$t = 1$

Transverse flow moving outward $n > -1$:



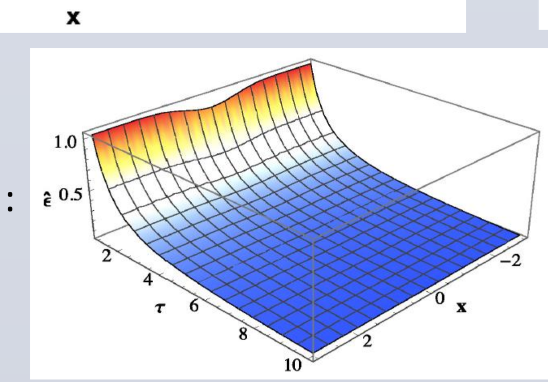


$B_c^2/\epsilon_c = 0.1$

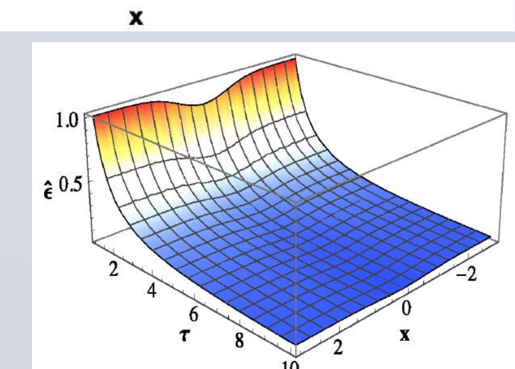
$t = 1$

$\hat{\epsilon} = \epsilon/\epsilon_c$

Energy density :



$n = -4/5$



$n = -5/3$

Conservation of the magnetic flux :

$n \gg -1 (n \to 0)$: A static magnetic field $B_y(x)$ with longitudinal expansion.

- Magnetic flux increases with respect to time.
- To reduce the flux, the medium expands transversely.

$n \ll -1$: A time-decreasing magnetic field $B_y(\tau,x)$ in a static medium

- Magnetic flux decreases with respect to time.
- To increase the flux, the medium is compressed transversely.

$n = -1$: the decreasing B field compensates the expansion of the medium.