QCD fixed points: Banks-Zaks scenario or dynamical gluon mass generation?

*J. D. Gómez[1], A. A. Natale[1 2]

[1]Universidade Federal do ABC, Centro de Ciências Naturais e Humanas, Santo André - SP, Brasil.

[2]Instituto de Física Teórica, UNESP, SP, Brasil.

*john.gomez@ufabc.edu.br

Abstract

Fixed points in QCD can appear when the number of quark flavors (N_f) is increased above a certain critical value as proposed by Banks and Zaks (BZ). There is also the possibility that QCD possess an effective charge indicating an infrared frozen coupling constant. In particular, an infrared frozen coupling associated to dynamical gluon mass generation (DGM) does lead to a fixed point even for a small number of quarks. We compare the BZ and DGM mechanisms, their β functions and fixed points, and within the approximations of this work, which rely basically on extrapolations of the dynamical gluon masses at large N_f, we verify that near the so called QCD conformal window both cases exhibit fixed points at similar coupling constant values (g^*). We argue that the states of minimum vacuum energy, as a function of the coupling constant up to g^* and for several N_f values, are related to the dynamical gluon mass generation mechanism.

Introduction

It was observed in the well known work of Refs.[1, 2] that QCD is an asymptotically free theory if the number of quark flavors N_f is smaller than a certain critical value. When $N_f \leq 16$ the one-loop β-function is negative and the coupling constant diminishes as the energy is increased. At two-loop order the β-function receives a contribution with a different signal as observed by Caswell [3], and although at high momentum this contribution is perturbatively small for a small number of flavors, its effect is not trivial if this number is increased, leading to a zero of the β-function, and to the so called Banks-Zaks fixed point [4].

At four loops, as a function of N_f and in the $\overline{MS}$ scheme, the QCD β-function for quarks in the fundamental representation is the following [5]:

$$\beta(a_s) = -b_0 a_s^2 - b_1 a_s^3 - b_2 a_s^4 - b_3 a_s^5 + \mathcal{O}(a_s^6)\,, \qquad (1)$$

where $a_s = \alpha_s/4\pi \equiv (g^2/4\pi)/4\pi$, and a zero of this β function already appears if $N_f \geq 8$.

Parallel to the Banks-Zaks scenario there are other discussions about a possible infrared (IR) freezing (or IR fixed point) of the QCD coupling constant [6]. This discussion has to do with effective charges, where the effective charges naturally have a non-perturbative contribution that eliminates the Landau singularity for small N_f [7].

Fixed points and dynamical gluon masses

The QCD charge effective that we shall consider will not depend on the renormalization point μ but on the dynamical gluon mass $m_g(k^2)$ and, of course, on the QCD characteristic scale $\Lambda_{QCD} \equiv \Lambda$. The work of Ref.[8] was the first one to obtain a gauge invariant SDE for the gluon propagator, verifying the existence of the Schwinger mechanism in QCD.

$$g^2(k^2) = \frac{1}{\beta_0 \ln\left[\frac{k^2+4\cdot m_g^2}{\Lambda_{QCD}^2}\right]} = 4\pi\alpha_s(k^2), \qquad (2)$$

where $\beta_0 = (11N - 2n_q)/48\pi^2$ with n_q quark flavors and $N = 3$. m_g is the IR value of the dynamical gluon mass $m_g(k^2)$, which naturally goes to zero at high energies.

The beta function associated with the effective charge Eq.(2) take the form

$$\beta_{DGM}(k^2) = -\beta_0 g^3 \left(1 - \frac{4m_g^4 e^{-\frac{1}{g^2\beta_0}}}{(m_g^2+k^2)\Lambda^2}\left(1 + \frac{k^2}{m_g^2+k^2}\right)\right), \qquad (3)$$

and, to know how Eq.(3) vary with N_f we must know how $m_g(k^2)$ varies with this quantity. It has been observed in QCD lattice simulations that the dynamical gluon mass increases when N_f increases [9]. Similar results were observed in the solution of the Schwinger-Dyson equation (SDE) for the gluon propagator including dynamical quarks [10]. Therefore, using the lattice data [9], we will assume the following exponential fit to describe the dynamical gluon mass evolution as a function of N_f:

$$m_g^{-1}(N_f) = m_{g_0}^{-1}\, e^{-A_2 N_f}, \qquad (4)$$

where m_{g_0} varies between 373 and 500 MeV and $A_2 = 0.05942$. We will consider in all subsequent calculation $m_{g_0} = 440$ MeV. With these numbers, we can compare the BZ and DGM approaches, verifying that both have exactly opposite behavior. The coupling constant values for each fixed point can be observed in the first table. The DGM coupling moves to higher g_s values as we increase N_f, and at some moment even the non-perturbative method used to obtain this quantity may fail. The renormalization group behavior of the coupling constant is constrained by the analyticity condition as proposed by Krasnikov [11] as

N_f	BZ	DGM
6	$*$	2.64
7	$*$	2.73
8	4.41	2.83
10	3.20	3.13
11	2.80	3.36
12	2.43	3.65
13	2.06	4.08

$$\left|\alpha_s \frac{d}{d\alpha_s}\left(\frac{\beta(\alpha_s)}{\alpha_s}\right)\right| \leq 1. \qquad (5)$$

This is a perfect condition to test if the different fixed points, or critical coupling constants, discussed in the previous sections can be considered still small enough to be reliable, even if they were obtained with a non-perturbative method as in the DGM case.

Since we are particularly interested in what happens at the fixed point, in the side table, we show the value of the left-hand side of inequality (5) evaluated exactly at the fixed points for both cases: BZ and DGM.

This means that we are inside, within our approximations, of the analytic region. In particular, the DGM β function and the respective fixed point seems to be at the border of the analytic region. Since the derivative of Eq.(3) is linear in α_s it is easy to understand why Eq.(5) is saturated at the fixed point, where the left hand side of Eq.(5) is proportional to $d\beta\alpha_s/d\alpha_s$.

N_f	BZ	DGM
3	$*$	0.9998
6	$*$	1.0000
8	0.1053	0.9999
9	0.0583	1.0000
10	0.0340	0.9997
12	0.0112	0.9998

Minimum of energy: BZ or DGM?

If the β functions in these two approaches are comparable and lead to approximately the same fixed points for some N_f values, can we determine which one leads to the actual minimum of energy? The β function can be related to the trace of the energy momentum tensor [14, 15]

$$\langle\theta_{\mu\mu}\rangle = \frac{\beta(g)}{g}\langle G_{\mu\nu}G^{\mu\nu}\rangle\,, \qquad (6)$$

which is proportional to the vacuum energy $\langle\Omega\rangle$ as

$$\langle\Omega\rangle = \frac{1}{4}\langle\theta_{\mu\mu}\rangle\,. \qquad (7)$$

The minimum of the vacuum energy is a scheme independent quantity, and, in principle, this quantity could be used to discriminate which β function leads to the deepest minimum of energy.

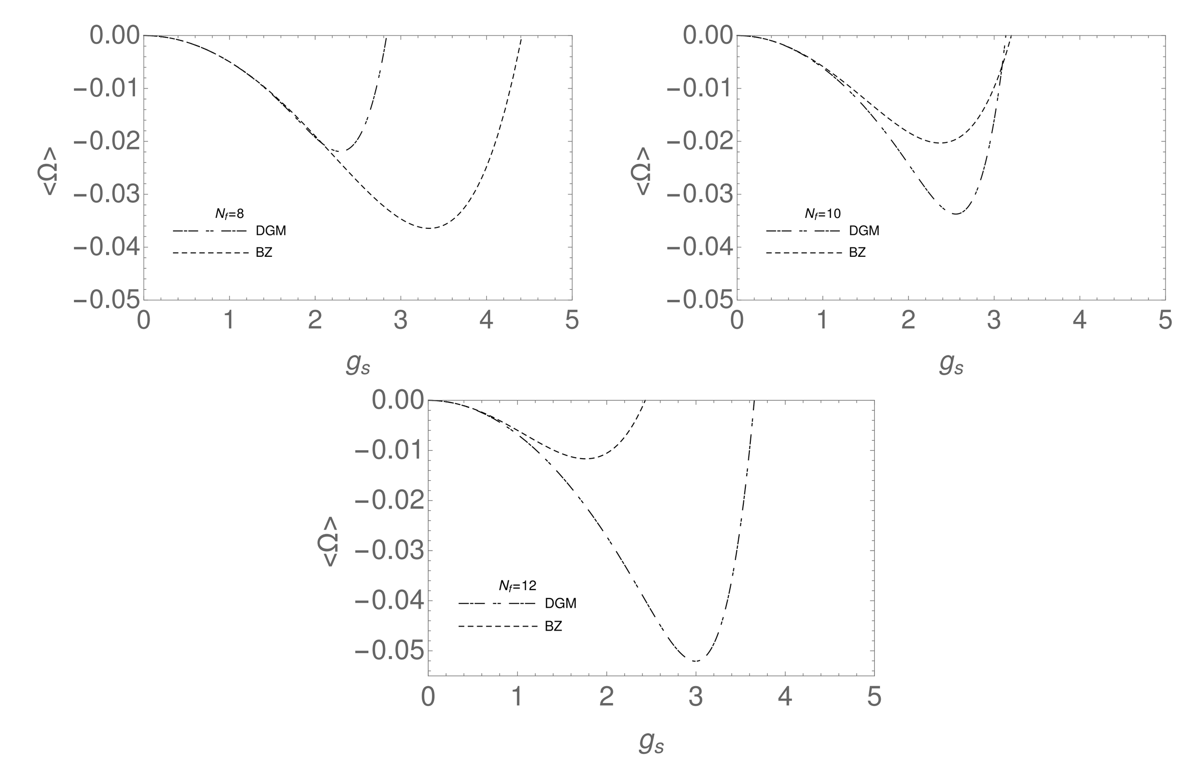
In order to calculate the vacuum energy we must know how the gluon condensate is modified as we change the number of flavors. One expression for the gluon condensate as a function of the dynamical gluon mass was determined in Ref.[8] and also studied in Ref.[13]:

$$\left\langle\frac{\alpha_s}{\pi}G_{\mu\nu}G^{\mu\nu}\right\rangle = A\frac{3m_g^4}{4\pi^4\beta_0\ln\left(\frac{4m_g^2}{\Lambda^2}\right)}, \qquad (8)$$

where we are going to assume that m_g is a function of N_f, as described by the exponential behavior of Eq.(4) with $m_{g_0} = 440$ MeV, and A is a constant value such that $\left\langle\frac{\alpha_s}{\pi}G_{\mu\nu}G^{\mu\nu}\right\rangle = 0.012$ GeV^4 when $N_f = 0$ [12]. So, $\langle\Omega\rangle$ is given by

$$\langle\Omega\rangle = \frac{3A}{4\pi^2}\frac{\beta(g)}{g}m_g^4(N_f). \qquad (9)$$

Our results for the vacuum energy as a function of the coupling constant g_s are shown in next figure for $N_f \approx 8 - 12$, defining the so called conformal window.



Conclusions

Our results are surprising in the following sense: For $N_f = 8$ it seems that the BZ β function is the one that leads to the deepest minimum of energy as a function of the coupling constant up to the critical g^* value, although $N_f = 8$ is at the border of the conformal window and below this value the "perturbative" vacuum becomes unstable. Above this N_f value it is the DGM β function that leads to the deepest state of minimum energy. As we increase N_f above 12 it is the coupling constant in the DGM approach that increases at one point that we cannot be sure how much the SDE truncation, leading to this solution, is still reliable. It is possible that the claimed fixed points at the confomal window are related to the DGM mechanism.

Acknowledgements

The authors thank to Fundação de Amparo à Pesquisa do Estado de São Paulo FAPESP (grant 2013/24065-4) for supporting this work.

References

[1] D. J. Gross and F. J. Wilczek, Phys. Rev. Lett. **30**, 1343 (1973).

[2] H. D. Politzer, Phys. Rev. Lett. **30**, 1346 (1973).

[3] W. E. Caswell, Phys. Rev. Lett. **33**, 244 (1974).

[4] Tom Banks, A. Zaks, Nucl. Phys. B **196**, 189 (1982).

[5] T. van Ritbergen, J. A. Vermaseren, and S. A. Larin, Phys. Lett. B **400**, 379 (1997).

[6] G. M. Prosperi, M. Raciti and C. Simolo, Prog. Part. Nucl. Phys. **58**, 387 (2007).

[7] G. Grunberg, JHEP **08**, 019 (2001).

[8] J. M. Cornwall, Phys. Rev. D **26**, 1453 (1982).

[9] A. Ayala et al., Phys. Rev. D **86**, 074512 (2012).

[10] A. C. Aguilar, D. Binosi and J. Papavassiliou, Phys. Rev. D **88**, 074010 (2013).

[11] N.V. Krasnikov, Nucl. Phys. B **192**, 497 (1981).

[12] M.A. Shifman, A.I. Vainshten, and V.I. Zakharov, Nucl. Phys. B **163**, 46 (1980).

[13] E. V. Gorbar and A. A. Natale, Phys. Rev. D **61**, 054012 (2000).

[14] R. Crewter, Phys. Rev. Lett. **28**, 1421 (1972).

[15] M. Chanowitz and J. Ellis, Phys. Lett. B **40**, 397 (1972).