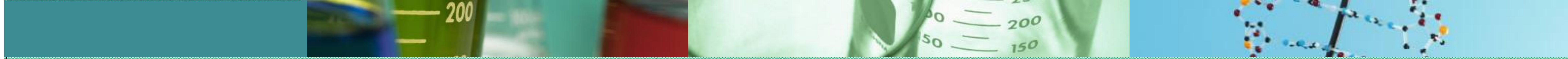

Confinement in F_4 Exceptional Group Using Domain Structure

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ABSTRACT

We calculate the potential between static quarks in the fundamental representation of the F_4 exceptional gauge group using domain structures of the thick center vortex model . As non-trivial center elements are absent, the asymptotic string tension is lost while an intermediate linear potential is Observed. $SU(3)$ is a subgroup of F_4 .

Investigating the decomposition the 26 dimensional representation of F_4 to the $SU(3)$ representations, might explain what accounts for the intermediate linear potential, in the exceptional groups with no center element.

Thick Center Vortex Model

Center vortex model has been a successful mechanism for explaining quark confinement. Thickening the vortices leads to the correct N-ality dependence of the potentials of the color sources at large distances. In Casimir Scaling regime, the string tension is proportional to the quadratic Casimir of the representation. In this model the vortices which carry color magnetic flux and fill the QCD vacuum , interact with the Wilson Loop. This interaction affects the Wilson Loop by an element of the $SU(N)$ gauge group center :

$$W_r(C) \rightarrow Z^{K_r}\ W_r(C)$$

N – ality of the representation

Dimension of the representation

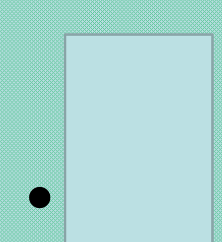
And Z is group center element that we can depict it as following formula :

$$Z = exp\left(\frac{2\pi i n}{N}\right) \quad ; \quad n = 1, \dots, n-1$$

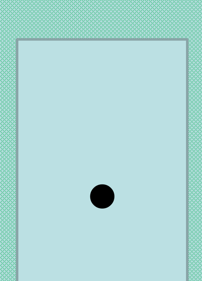
When the vortex is thin, two scenarios may occur :

I) Vortices and the Wilson loop do not link. In this case, the Wilson Loop is unaffected.

$$W(C) \downarrow Tr[UU(+1)\dots U]$$



II) Vortices may pierce the minimal area of the Wilson Loop. The effect of this interaction is inserting a center element z between link operators :

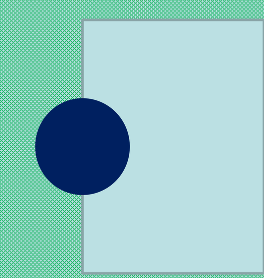


$$W(C) \downarrow Tr[UU(Z)\dots U]$$

On the other hand, Thickening the vortex leads to a new scenario:

The vortex may overlap the perimeter of the Wilson Loop and a part of the vortex flux enters the loop. In this case, the center element is replaced by a group element G which is a unitary matrix:

$$W(C) = Tr\,[UU\ \dots\ U] \downarrow Tr[UU\ \dots\ G\ \dots\ U]$$



Based on the assumptions of the model, one has to average G over all orientations in the group manifold :

$$\overline{G}(x,S) = \int S\, exp[i\alpha_c(x).\vec{H}]\, S^{\dagger}\ ds = g_r[\vec{\alpha}]$$

g_r gives the information about the flux distribution and is given by :

$$g_r[\vec{\alpha}] = \frac{1}{d_r} Tr\, exp[i\vec{\alpha}.\vec{H}]$$

Vortex Flux

Cartan Generators *of the gauge group*

Dimension of the representation

$$\textit{Flux Profile :}\quad \alpha_i^0 = \frac{\alpha_i^{0(Max)}}{2}\left[1 - \tanh\left(ay(x) + \frac{b}{R}\right)\right]$$

Zero , indicates the trivial domain of F_4 .

The potential between a quark and an anti-quark induced by Vortices in the $SU(N)$ gauge group is as the following :
[M.Faber, J.Greensite, S.Olejnik, Phys.Rev, D 57 (1998) 2603]

$$V = -\sum_{n=-\infty}^{+\infty} ln\left\{1 - \sum_{n=1}^{N-1} f_n\left(1 - \mathcal{R}e\, g_j[\vec{\alpha}_c^n(x_n)]\right)\right\}$$

The probability that any given unit area is pierced by a vortex

Location of the center of the Vortex

For the F_4 case with only one trivial center element, $V(R)$ is given by :

$$V(R) = \sum_x ln\left\{1 - f_0\left(1 - \mathcal{R}e\, g_r[\vec{\alpha}_c^0(x)]\right)\right\}$$

[S.Deldar, H.lookzadeh, S.M.Hosseini Nejad, Phys.Rev.D85, 054501 (2012)]

The Exceptional Group F_4

In general, there are five exceptional groups, G_2 , F_4 , E_6 , E_7 and E_8. We note here that the subscripts in the case of Exceptional groups correspond to their ranks or equivalently, the Number of simple roots. Thus the rank of F_4 is 4 and it has four Cartan generators.

Its fundamental and adjoint representations are 26 and 52-Dimensional respectively. By investigating the extended Dynkin diagram of this group we can find its subgroups. One of them is $SU(3) \times SU(3)$.

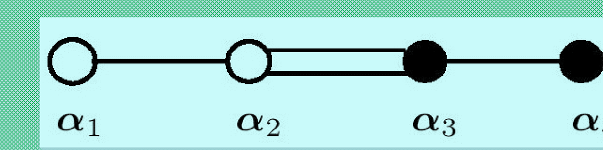


Figure 1 : The Dynkin diagram for the simple roots of F_4

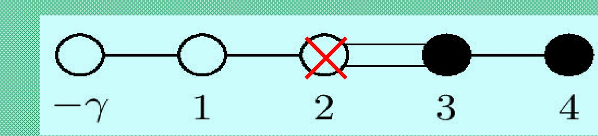


Figure 2 : The Extended Dynkin diagram of the F_4

By omitting root number 2 we will have two $SU(3)$ Dynkin diagram. [LieArt-R.Feger, T.kephart; arXiv: 1206.6379V2 (math-ph) 2014]

Through branching rules we can decompose representations of the F_4 into representations of its subgroups. Hence the decomposition of 26 and 52 dimensional of the F_4 to its $SU(3)$ subgroup is:

[R. Slansky, Group Theory for Unified Model Building, Phys.Rept. 79 (1981)]

$$26 = (8,1) + (3,3) + (3,\overline{3})$$

$$52 = (8,1) + (1,8) + (6,\overline{3}) + (\overline{6},3)$$

As we mentioned before the group has 4 Cartan generators and for simplification we can depict the Cartan generators of its fundamental, 26-dimensional representation by the 26×26 matrices as follows:

$$H_1 = D_5^5 + D_6^6 - D_7^7 + D_8^8 - D_9^9 - D_{10}^{10}$$

$$H_2 = D_3^3 + D_4^4 - D_5^5 - D_6^6 + D_{10}^{10} - D_{11}^{11}$$

$$H_3 = \frac{1}{2}\left(D_2^2 - 2D_3^3 - D_4^4 + D_6^6 - D_8^8 + D_9^9 - D_{10}^{10} + D_{11}^{11} - D_{12}^{12}\right)$$

$$H_4 = \frac{1}{2}\left(-2D_2^2 + D_3^3 - D_4^4 + D_5^5 - D_6^6 + D_7^7 - D_9^9 + D_{12}^{12} - D_{13}^{13}\right)$$

[A.M.Bincer; arXiv: hep-th/9312148v1 (1993)]

The matrices D_b^a are given in the defining 26-dimensional representation as the following 26×26 matrices:

$$D_a^b = I_{ab} - I_{\overline{ba}}$$

Where I_{ab} is the 26×26 matrix with matrix elements:

$$(I_{ab})_{jk} = \delta_{aj}\delta_{bk}$$

With the labels j,k taking on the same values as a,b: $-13 \leq j,k \leq 13$,zero excluded.

After calculations of normalization coefficients by using the following normalization condition:

$$Tr[H_a H_b] = \frac{1}{2}\delta_{ab}$$

Now we have to put the results into the below formula. as F_4 has only one trivial center element and no other nontrivial ones, the group factor equals to 1.

$$exp(i\vec{\alpha}^n.\vec{H}) = \mathbb{I}$$

Therefore;

$$exp(\alpha_1^{max}H_1 + \alpha_2^{max}H_2 + \alpha_3^{max}H_3 + \alpha_4^{max}H_4) = \mathbb{I} = exp(2\pi i)$$

Then we have 26 equations :

$$\frac{\alpha_4}{\sqrt{48}} = 2\pi \quad ; \quad \frac{\alpha_3}{\sqrt{48}} - \frac{\alpha_4}{\sqrt{48}} = 2\pi \quad ; \quad \frac{\alpha_2}{\sqrt{24}} - \frac{\alpha_3}{\sqrt{48}} = 2\pi$$

$$\frac{\alpha_1}{\sqrt{24}} - \frac{\alpha_2}{\sqrt{24}} + \frac{\alpha_3}{\sqrt{48}} = 2\pi \quad ; \quad \frac{\alpha_1}{\sqrt{24}} - \frac{\alpha_3}{\sqrt{48}} + \frac{\alpha_4}{\sqrt{48}} = 2\pi \quad ; \quad \frac{-\alpha_1}{\sqrt{24}} + \frac{\alpha_3}{\sqrt{48}} = 2\pi$$

$$\frac{\alpha_1}{\sqrt{24}} - \frac{\alpha_4}{\sqrt{48}} = 2\pi \quad ; \quad \frac{-\alpha_1}{\sqrt{24}} + \frac{\alpha_2}{\sqrt{24}} - \frac{\alpha_3}{\sqrt{48}} + \frac{\alpha_4}{\sqrt{48}} = 2\pi \quad ; \quad \frac{-\alpha_1}{\sqrt{24}} + \frac{\alpha_2}{\sqrt{24}} - \frac{\alpha_4}{\sqrt{48}} = 2\pi$$

$$\frac{-\alpha_2}{\sqrt{24}} + \frac{\alpha_3}{\sqrt{48}} + \frac{\alpha_4}{\sqrt{48}} = 2\pi \quad ; \quad \frac{-\alpha_2}{\sqrt{24}} + \frac{\alpha_3}{\sqrt{12}} - \frac{\alpha_4}{\sqrt{48}} = 2\pi \quad ; \quad \frac{-\alpha_3}{\sqrt{48}} + \frac{\alpha_4}{\sqrt{12}} = 2\pi$$

Two of the equations are equal to zero and the left part of the rest twelve, are minus of the above equations.

There can be different appropriate sets of answers. For instance one sets of answers are :

$$\alpha_1^{max} = 2\pi\sqrt{24} \quad \& \quad \alpha_2^{max} = 3\pi\sqrt{48}$$

$$\alpha_3^{max} = 4\pi\sqrt{48} \quad \& \quad \alpha_4^{max} = 2\pi\sqrt{48}$$

Conclusion

By using the mentioned sets of answers in the previous section and putting them in the potential equation $V(R)$ we can plot the potential between two static quark in the F_4 as the following. The free parameters of the model have been chosen as :

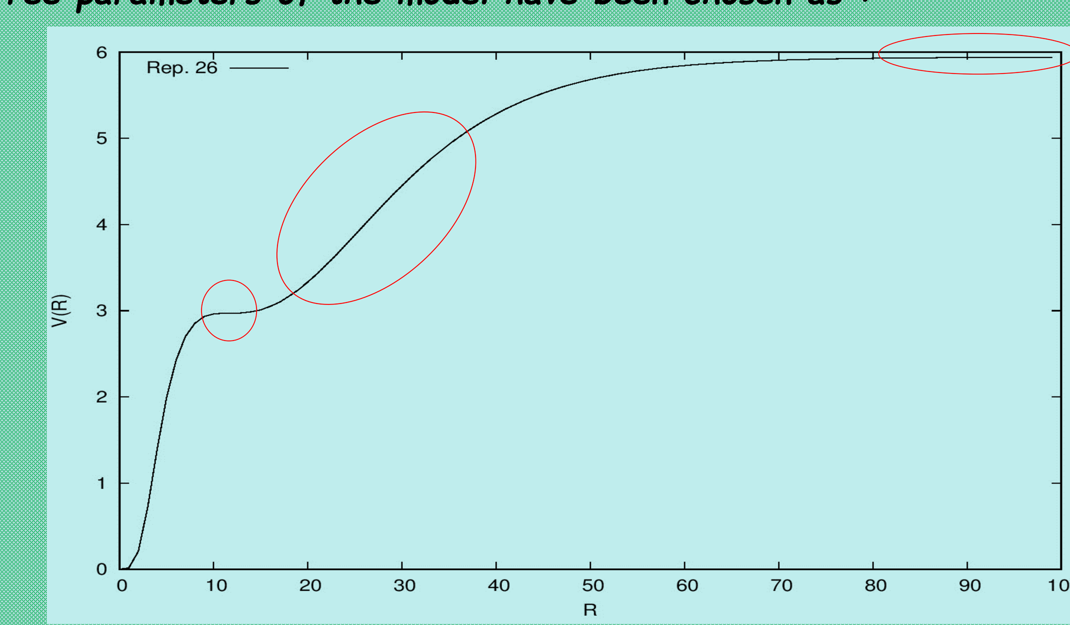


Figure 3 : potentials between two F_4 static quarks obtained from the domain structure model.

The potential is screened at large distances as expected. At intermediate distances we can observe two linear regimes. For this regime some fraction of the trivial domain is located inside the Wilson Loop.

Now we can draw the $\mathcal{R}e[g_r]$ by choosing $R = 100$:

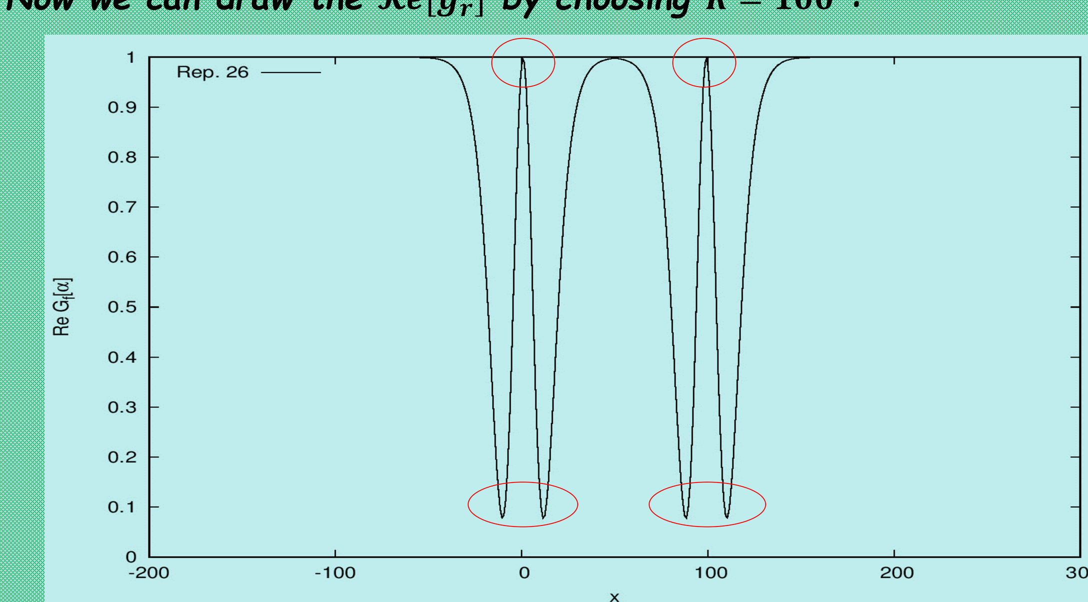


Figure 4 : $\mathcal{R}e\left(g_r[\alpha_f^0(x)]\right)$ versus x, plotted for the fundamental representation of the F_4 group.

$\mathcal{R}e(g_r)$ has a maximum value of 1 and two extremums 0.8 and 1. When the vacuum domain locates completely inside the Wilson Loop, $\mathcal{R}e(g_r)$ reaches to 1 .