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# Exact Sum Rules for Vector Channel at Finite Temperature and its Applications in Lattice QCD Analysis

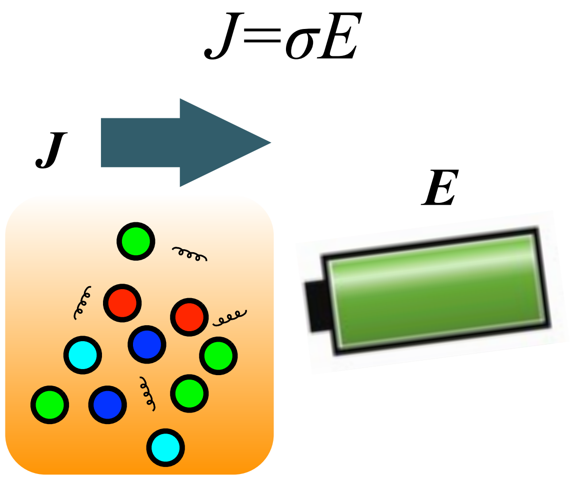
Daisuke Satow (Frankfurt 🇩🇪)

Collaborator: Philipp Gubler (Yonsei 🇰🇷)

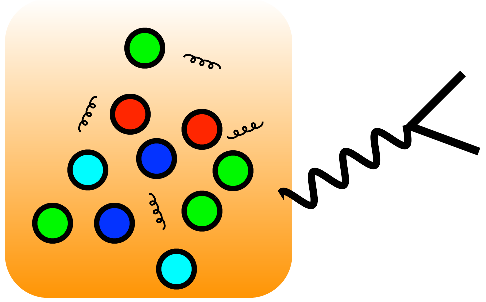
P. Gubler and **D. S.**, arXiv:1602.08265 [hep-ph].

# Introduction

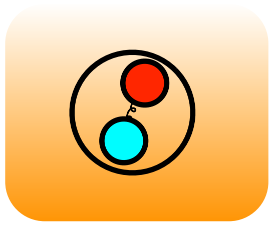
- Electrical conductivity

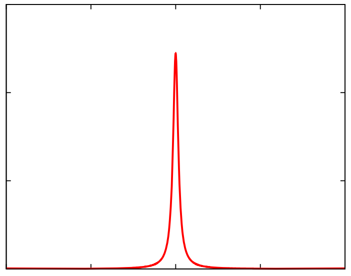



- Dilepton production rate



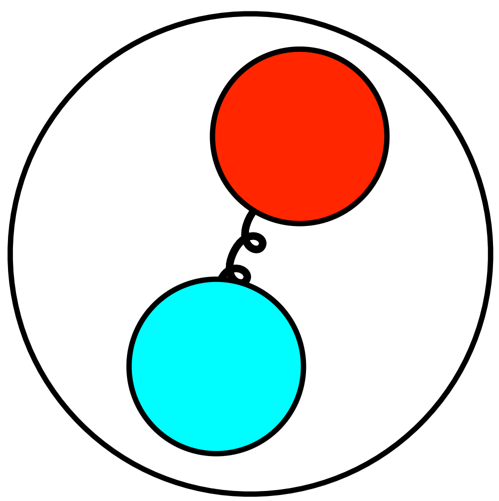
- Vector meson spectrum at finite $T$





# Introduction

## **Vector spectral function contains all information of it.**



$j^\mu \equiv e\sum_f q_f \overline{\psi}_f \gamma^\mu \psi_f$

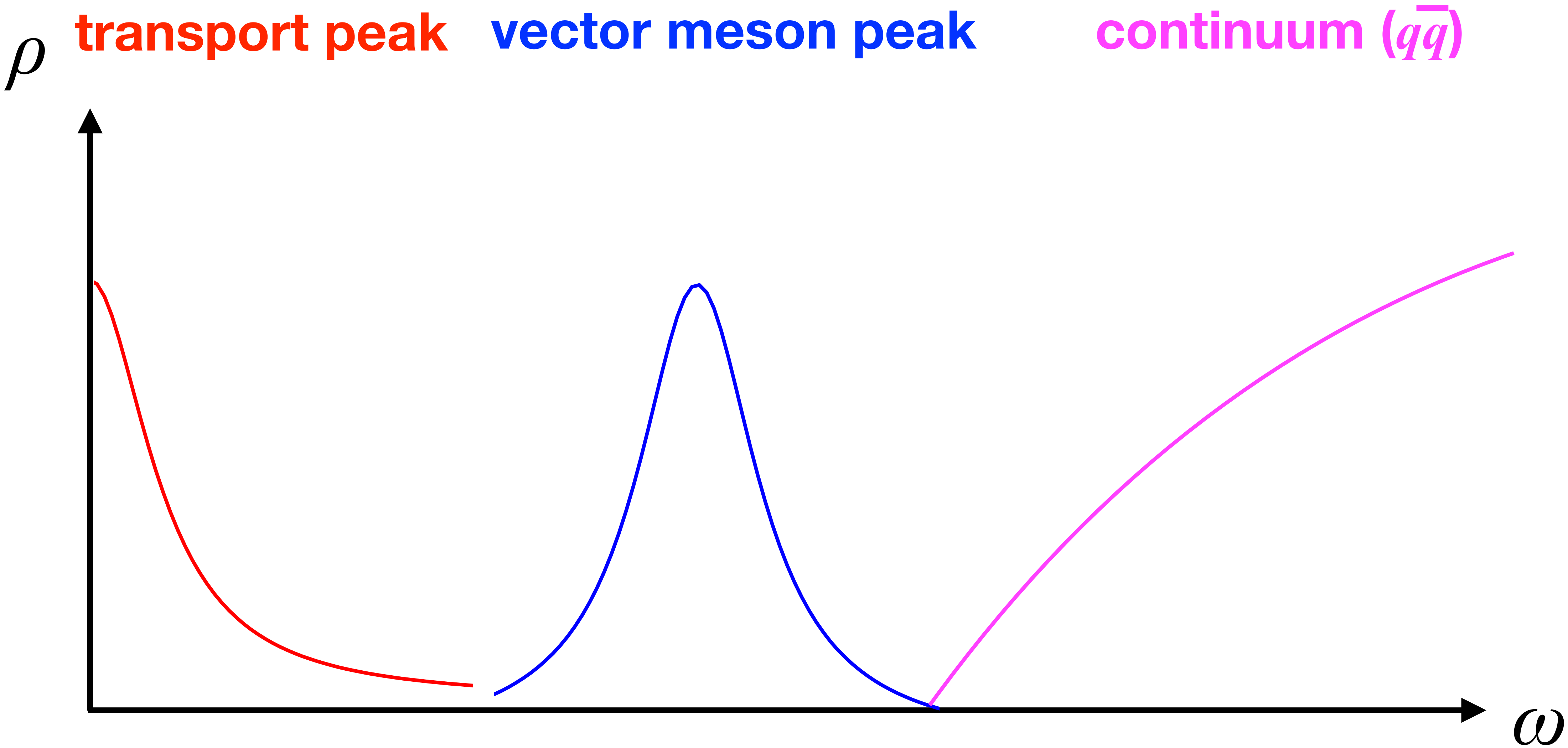
$f$: flavor index, $q_{\mathrm{f}}$: electric charge

$$G^{R\mu\nu}(t,\mathbf{x}) \equiv i\theta(t)\langle[j^\mu(t,\mathbf{x}), j^\nu(0,\mathbf{0})]\rangle$$

$$\rho^{\mu\nu}(p) = \mathrm{Im}G^{R\mu\nu}(p)$$

# Introduction

Possible schematic picture of vector spectral function




**Rich and complicated structure.**

# Motivation

**Is there any exact relation we can use for check?**

**QCD sum rule**

# Sum rule

P. Romatschke, D. T. Son, Phys.Rev. D **80** 065021 (2009).

## Retarded Green function: $G^{R\mu\nu}(\omega, \mathbf{p})$

analyticity in
upper $\omega$ plane

$$\delta G^R(0, \mathbf{p}) - \delta G^R_\infty(\mathbf{p}) = \frac{2}{\pi}\int_0^\infty d\omega \frac{\delta\rho(\omega, \mathbf{p})}{\omega}$$

IR UV

$$\delta G^R(\omega) \equiv G^R(\omega) - G^R_{T=0}(\omega)$$

remove UV divergence

**Sum of spectral function is constrained by the UV/IR behaviors! (sum rule)**

# Sum rule 1

**UV**

$$\delta G^R(\omega) = e^2 \sum_f q_f^2 \frac{1}{\omega^2}\left[2m_f \delta\langle\bar{\psi}_f\psi_f\rangle + \frac{1}{12}\delta\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle + \frac{8}{3}\frac{\delta\langle T^{00}\rangle}{4C_F + N_f}\right].$$

$\Rightarrow$ $\delta G^R(\omega)\rightarrow 0$ irrelevant.

**IR**

$$G^R(\omega) = i\omega\sigma\,(1 + i\tau_J\omega)$$

$\Rightarrow$ $\delta G^R(\omega)\rightarrow 0$ irrelevant.

$\Rightarrow$ $$0 = \int_0^\infty \frac{d\omega}{\omega}\delta\rho(\omega)$$ **sum rule 1**

$\delta\rho(\omega)=\rho(\omega)-\rho(\omega)_{T=0}$

(Also obtained by current conservation: D. Bernecker and H. B. Meyer, Eur. Phys. J. A **47**, 148 (2011) )

# Ansatz in lattice calculation

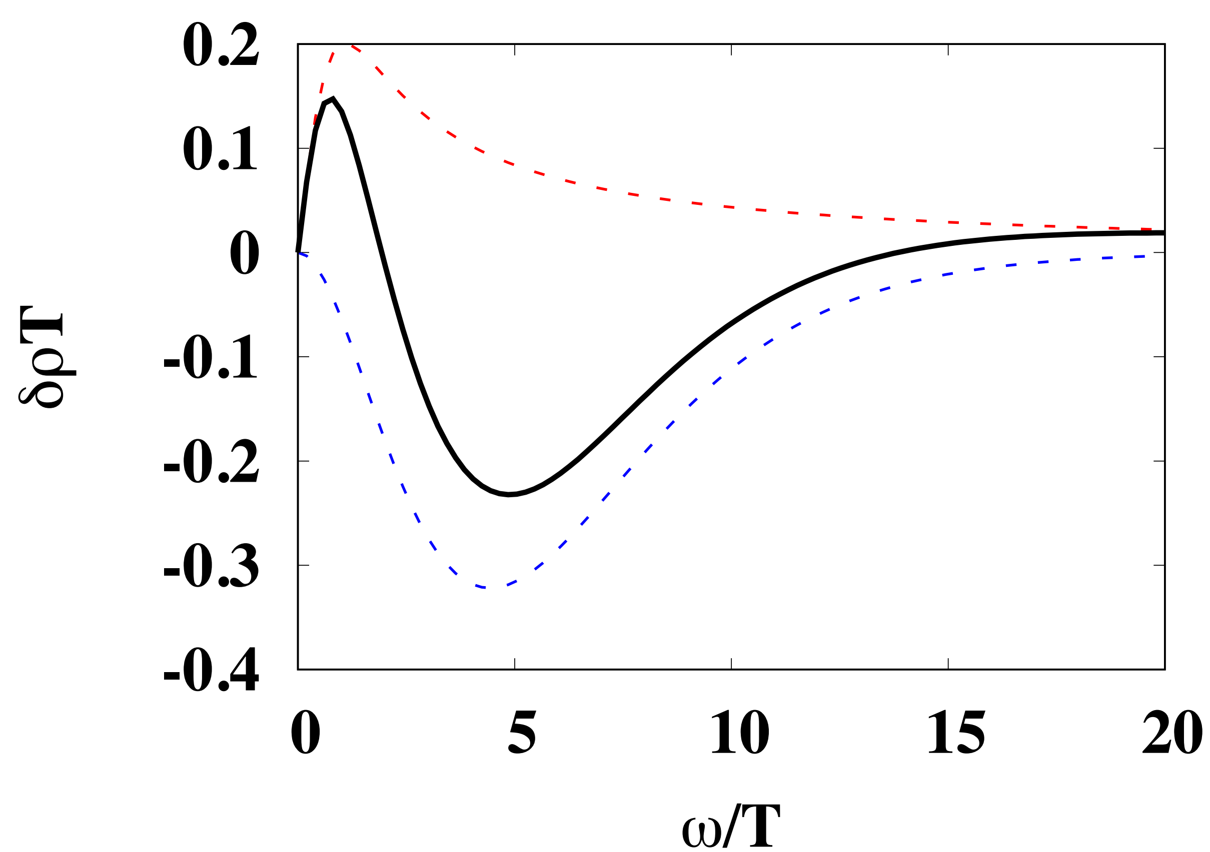
H.-T. Ding, A. Francis, O. Kaczmarek, F. Karsch, E. Laermann, W. Soeldner, Phys.Rev. D **83** 034504 (2011).

ansatz:

**transport peak** **continuum**

$$\rho(\omega) = C_{\rm em}\left[3\chi c_{BW}\frac{\omega\Gamma/2}{\omega^2+(\Gamma/2)^2} + \frac{9}{4\pi}(1+k)\omega^2\left(1-2n_F\left(\frac{\omega}{2}\right)\right)\right]$$

3 parameters.



# Ansatz in lattice calculation

Sum rule 1 $$0 = \int_0^\infty \frac{d\omega}{\omega}\delta\rho(\omega)$$

**constraint:** $\chi c_{BW} = (1+k)T^2$

**We can reduce independent parameters.**

Also done in
B. B. Brandt, A. Francis, B. Jäger and H. B. Meyer, Phys. Rev. D **93**, 054510 (2016).

# Sum rule 2

$\times\omega^2$ ➡ **Sum rule for $\omega\rho$, not $\rho/\omega$**

**UV**

$$\delta G^R(\omega) = e^2 \sum_f q_f^2 \frac{1}{\omega^2}\left[2m_f \delta\langle\bar{\psi}_f\psi_f\rangle + \frac{1}{12}\delta\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle + \frac{8}{3}\frac{\delta\langle T^{00}\rangle}{4C_F+N_f}\right]. \times\omega^2$$

relevant.

**IR** irrelevant.

➡

$$\frac{2}{\pi}\int_0^\infty d\omega\omega\delta\rho(\omega) = -e^2\sum_f q_f^2\left[2m_f\delta\langle\bar{\psi}_f\psi_f\rangle + \frac{1}{12}\delta\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle + \frac{8}{3(4C_F+N_f)}\delta\langle T^{00}\rangle\right].$$

**sum rule 2**

Expectation values of operators appear.

# Sum rule 3

$/\omega^2$ ➡ **Sum rule for $\rho/\omega^3$, not $\rho/\omega$**

**UV** irrelevant.

**IR** $G^R(\omega) = i\omega\sigma\left(1 + i\tau_J\omega\right)/\omega^2$

➡ $$-\sigma\tau_J = \frac{2}{\pi}\int_0^\infty \frac{d\omega}{\omega^3}\left[\delta\rho(\omega) - \sigma\omega\right]$$ **sum rule 3**

Transport coefficients appear.

# Ansatz in lattice calculation

$\sim\omega^2$

ansatz: $$\rho(\omega) = C_{\mathrm{em}}\left[c_{BW}\rho_{\mathrm{peak}}(\omega) + (1+k)\rho_{\mathrm{cont}}(\omega)\right]$$

H.-T. Ding, A. Francis, O. Kaczmarek, F. Karsch, E. Laermann, W. Soeldner, Phys.Rev. D **83** 034504 (2011).

sum rule 3

$$-\sigma\tau_J = \frac{2}{\pi}\int_0^\infty \frac{d\omega}{\omega^3}\left[\delta\rho(\omega) - \sigma\omega\right]$$

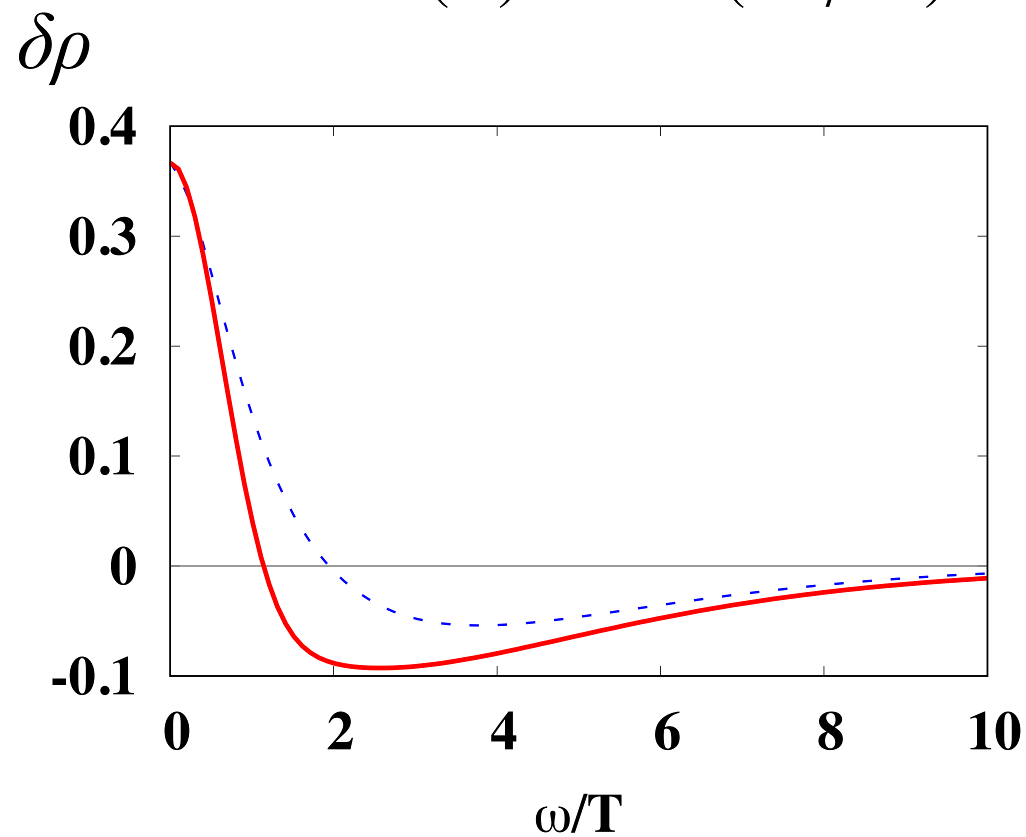
**Contribution from the continuum contains IR divergence.**
**More sophisticated ansatz is necessary.**

# Ansatz in lattice calculation

**suggestion for improved ansatz:**

$$\rho(\omega) = C_{\rm em}\left[c_{BW}\rho_{\rm peak}(\omega)[1 - A(\omega)] + A(\omega)(1 + k)\rho_{\rm cont}(\omega)\right]$$

$$A(\omega) \equiv \tanh(\omega^2/\Delta^2).$$




**Connect the two regions smoothly.**

# Ansatz in lattice calculation

Spectral function obtained by fit of lattice data

$$\rho(\omega) = C_{\rm em}\left[c_{BW}\rho_{\rm peak}(\omega)[1 - A(\omega)] + A(\omega)(1+k)\rho_{\rm cont}(\omega)\right]$$

Sum rule 3

$$-\sigma\tau_J = \frac{2}{\pi}\int_0^\infty \frac{d\omega}{\omega^3}\left[\delta\rho(\omega) - \sigma\omega\right]$$

$$\tau_J = 0.067 C_{\rm em}/T$$

**$\tau_J$ is evaluated non-perturbatively for the first time.**

$\tau_J$ does not appear in $\rho$ directly, so this is not circular reasoning.

# Summary

- We **derived three exact sum rules** in vector channel **at finite temperature** by using **OPE (UV)** and **hydrodynamics (IR)**

$$0 = \int_0^\infty \frac{d\omega}{\omega}\delta\rho(\omega)$$

$$\frac{2}{\pi}\int_0^\infty d\omega\omega\delta\rho(\omega) = -e^2\sum_f q_f^2\Big[2m_f\delta\langle\bar{\psi}_f\psi_f\rangle + \frac{1}{12}\delta\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle + \frac{8}{3(4C_F+N_f)}\delta\langle T^{00}\rangle\Big].$$

$$-\sigma\tau_J = \frac{2}{\pi}\int_0^\infty \frac{d\omega}{\omega^3}\left[\delta\rho(\omega) - \sigma\omega\right]$$

- **These sum rules are satisfied at least in weak coupling case.**

- We used our sum rules to **improve the ansatz used in the lattice calculation, and evaluate $\tau_J$.**