Universal scaling of gluon and ghost propagators in the infrared

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Abstract

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A universal behaviour is predicted for ghost and gluon propagators in the infrared. The universal behaviour is shown to be a signature of a one-loop approximation and emerges naturally by the massive expansion that predicts universal analytical functions for the inverse dressing functions that do not depend on any parameter or color number.

By a scaling of units and by adding an integration constant, all lattice data, for different color numbers (and even quark content for the ghosts), collapse on the same universal curves predicted by the massive expansion.

1. Introduction

In the last years, substantial progresses have been made in the knowledge of the elementary two-point functions of QCD.
Deep in the infrared, by a massive expansion for the exact Lagrangian, explicit analytical expressions have been found for the propagators[1, 2], from first principles. At one-loop, the optimized expansion is in very good agreement with the lattice data in the Euclidean space[1] and provides a direct and simple way to explore the analytic properties of the propagators in Minkowski space[2, 3] (See also Talk on Thursday 2 September - Parallel A, 18:20).
Here some universal properties of the propagators are discussed, as emerging from the one-loop expansion in the infrared.

2. Dependence on Color Number

Even in the case of pure $SU(N)$ Yang-Mills theory, the role played by the color number N has not been fully clarified yet. A comparison of the propagators was made in Ref.[4] for different values of N and a quantitatively different result was found on the lattice for $SU(2)$ and $SU(3)$, indicating a similar behaviour but only qualitative agreement for different values of N[4].

Here, the agreement is made *quantitative* by scaling the inverse dressing functions and adding an integration constant. In the infrared, all data collapse on the same curve by tuning the additive constant and by scaling the energy units, confirming a universal behaviour predicted by the massive expansion at one-loop[1, 2].

Since higher loops would spoil the universal behaviour, the universal scaling is a proof that the neglegted higher order terms are very small in the optimized expansion.

Moreover, at one-loop the ghosts are decoupled from the quarks in the loop expansion, predicting the same universal behaviour even for unquenched data, irrespective of the number of quarks[2].

3. The Universal scaling as a test of the one-loop approximation

ASSUMPTIONS (all satisfied in the massive expansion [1]):

- A one-loop approximation can be used deep in the infrared
- There is no gluon mass at tree level
- The color number N appears only as an argument of the effective coupling $\alpha = \alpha(N)$

The assumptions are not satisfied if the Lagrangian is changed by inclusion of spurious terms or masses, as it happens for some massive models where the gluon mass is added by hand to the Lagrangian.
Moreover, deviations are expected in the UV where the RG running of the coupling cannot be neglected.
By a fixed coupling, the massive expansion provides a very good description of the data below 2 GeV, so that the hypotheses are satisfied in the infrared.

Denoting by α the effective coupling, assumed to be a function of N, the gluon and ghost self energies can be written in powers of α as

$$\Sigma(p) = \alpha\Sigma^{(1)}(p) + \alpha^2\Sigma^{(2)}(p,N) + \cdots \tag{1}$$

Any acceptable theory must also predict a finite gluon propagator in the IR, giving a mass scale $m^2 = \Delta(0)^{-1}$ which is arbitrary because of the arbitrary renormalization of the propagator. Since the Lagrangian is scaleless, the mass m can only be determined by the phenomenology. For instance, in the massive expansion the mass scale is provided by an arbitrary gluon mass in the loops. Then, on general grounds, the self energy can be written as

$$\frac{\Sigma(p)}{\alpha p^2} = -F(p^2/m^2) + \mathcal{O}(\alpha) \tag{2}$$

where the adimensional function $F(s)$ is given by the one-loop self energy

$$F(p^2/m^2) = -\frac{\Sigma^{(1)}}{p^2} \tag{3}$$

and can only depend on the ratio $s = p^2/m^2$. The same argument applies to gluons and ghosts so that we can denote by Σ the generic self energy and by Δ the generic propagator.

The exact gluon or ghost propagator $\Delta(p)$ can be written as

$$\Delta(p) = \frac{Z}{p^2 - \Sigma(p)} = \frac{J(p)}{p^2} \tag{4}$$

As usual, at one loop, we can write Z as the product of a finite renormalization constant z times a diverging factor $1 + \alpha\,\delta Z$, so that the dressing function reads

$$z\,J(p)^{-1} = 1 + \alpha\left[F(p^2/m^2) - \delta Z\right] + \mathcal{O}(\alpha^2). \tag{5}$$

Here, the divergent part of δZ cancels the divergence of the one-loop self energy, yielding a finite result. We can divide by α and absorb the coupling in the arbitrary factor z yielding

$$z\,J(p)^{-1} = F(p^2/m^2) + F_0 + \mathcal{O}(\alpha). \tag{6}$$

where the new constant F_0 is the sum of all the constant terms. If the higher order terms can be neglected, the dressing functions are determined by the adimensional universal function $F(s)$ that does not depend on any parameter. For any data set $\{J(p)\}$ we can always find a set of constants x, y, z:

$$\boxed{z\,J(p/x)^{-1} + y = F(p^2/m^2) + F_0 + \mathcal{O}(\alpha)} \tag{7}$$

OPTIMIZED RENORMALIZATION CONSTANTS

Data set	N	N_f	x	y	z	y'	z'
Bogolubsky et al.	3	0	1	0	3.33	0	1.57
Duarte et al.	3	0	1.1	-0.146	2.65	0.097	1.08
Cucchieri-Mendes	2	0	0.858	-0.254	1.69	0.196	1.09
Ayala et al.	3	0	0.933	-	-	0.045	1.17
Ayala et al.	3	2	1.04	-	-	0.045	1.28
Ayala et al.	3	4	1.04	-	-	0.045	1.28

Table: Scaling constants x, y, z (gluon) and y', z' (ghost). The constant shifts $F_0 = -1.05$ (gluon), $G_0 = 0.24$ (ghost) and the mass $m = 0.73$ GeV are optimized by requiring that $x = 1$ and $y = y' = 0$ for the lattice data of Bogolubsky et al.[5]. The other data are from Duarte et al.[6], Cucchieri and Mendes[7, 8], Ayala et al.[9].

Gluon dressing function

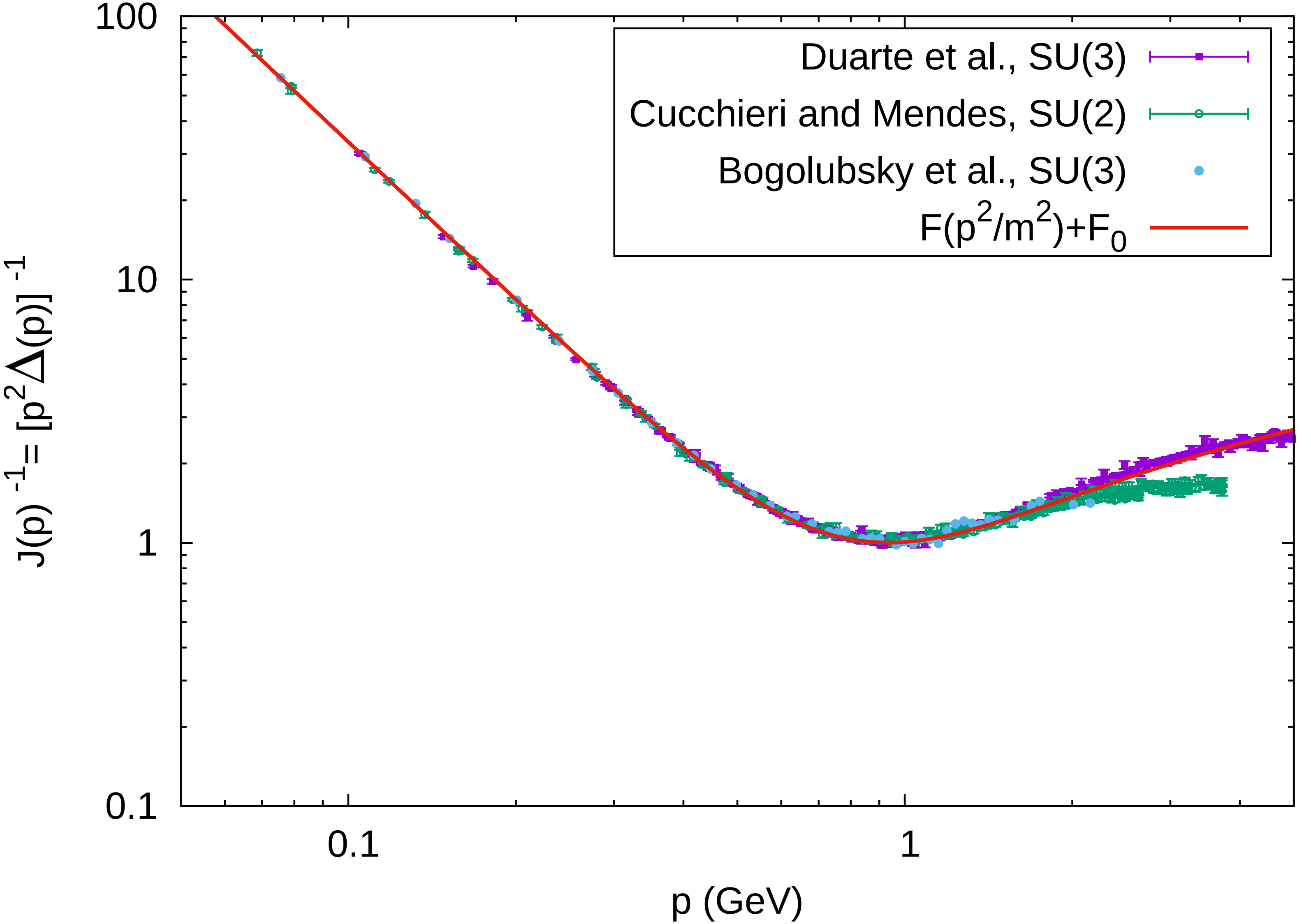

Figure 1. The solid curve (red line) is the one-loop universal function $F(s)$ of Eq.(7), evaluated by the massive expansion of Ref.[1] for $s = p^2/m^2$, $m = 0.73$ GeV and shifted by the constant $F_0 = -1.05$. The data are scaled by the constants of the table.

Gluon dressing function

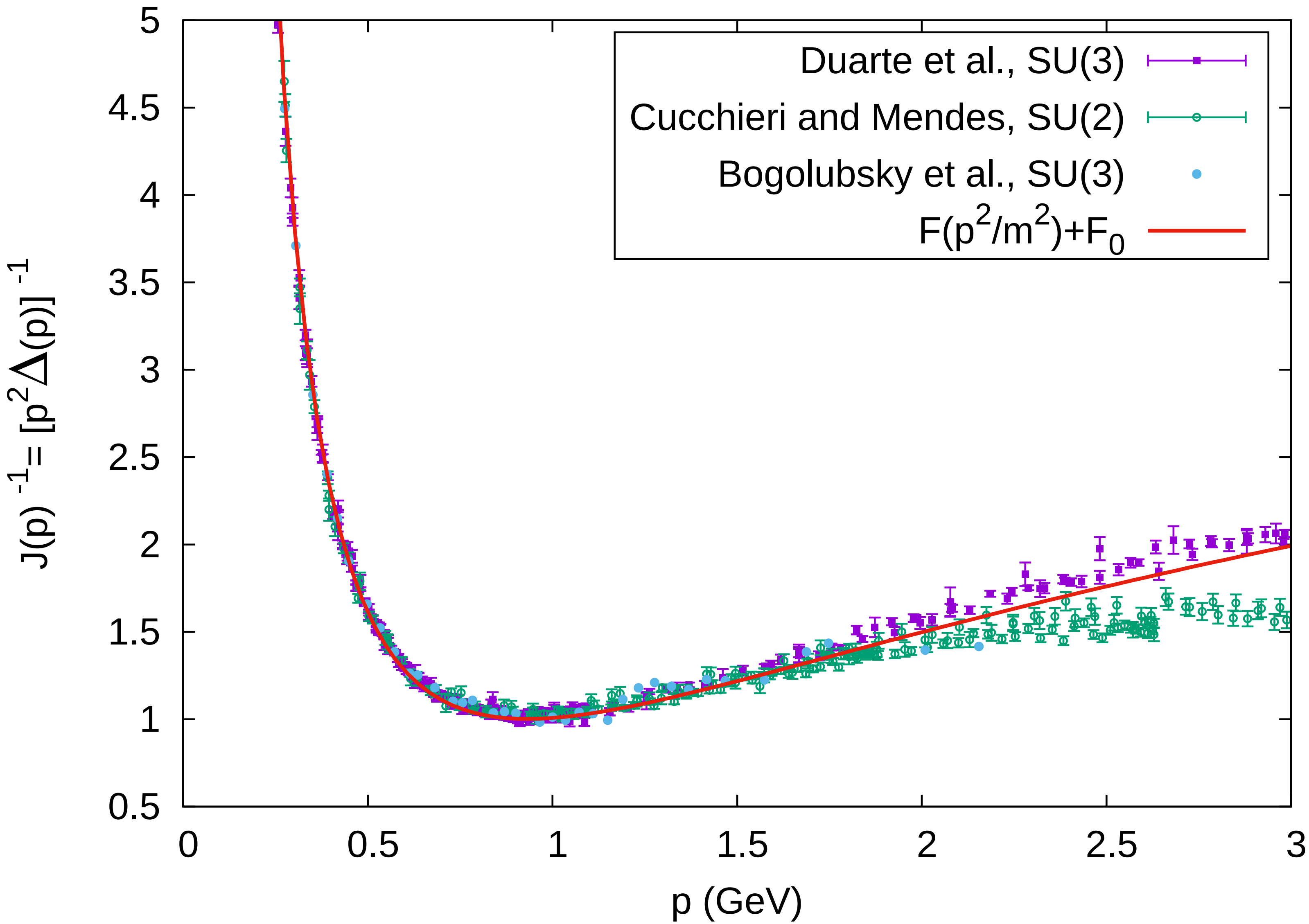

Figure 2. Inverse gluon dressing function. The same content of Fig. 1 is shown at a larger linear scale.

Ghost dressing function

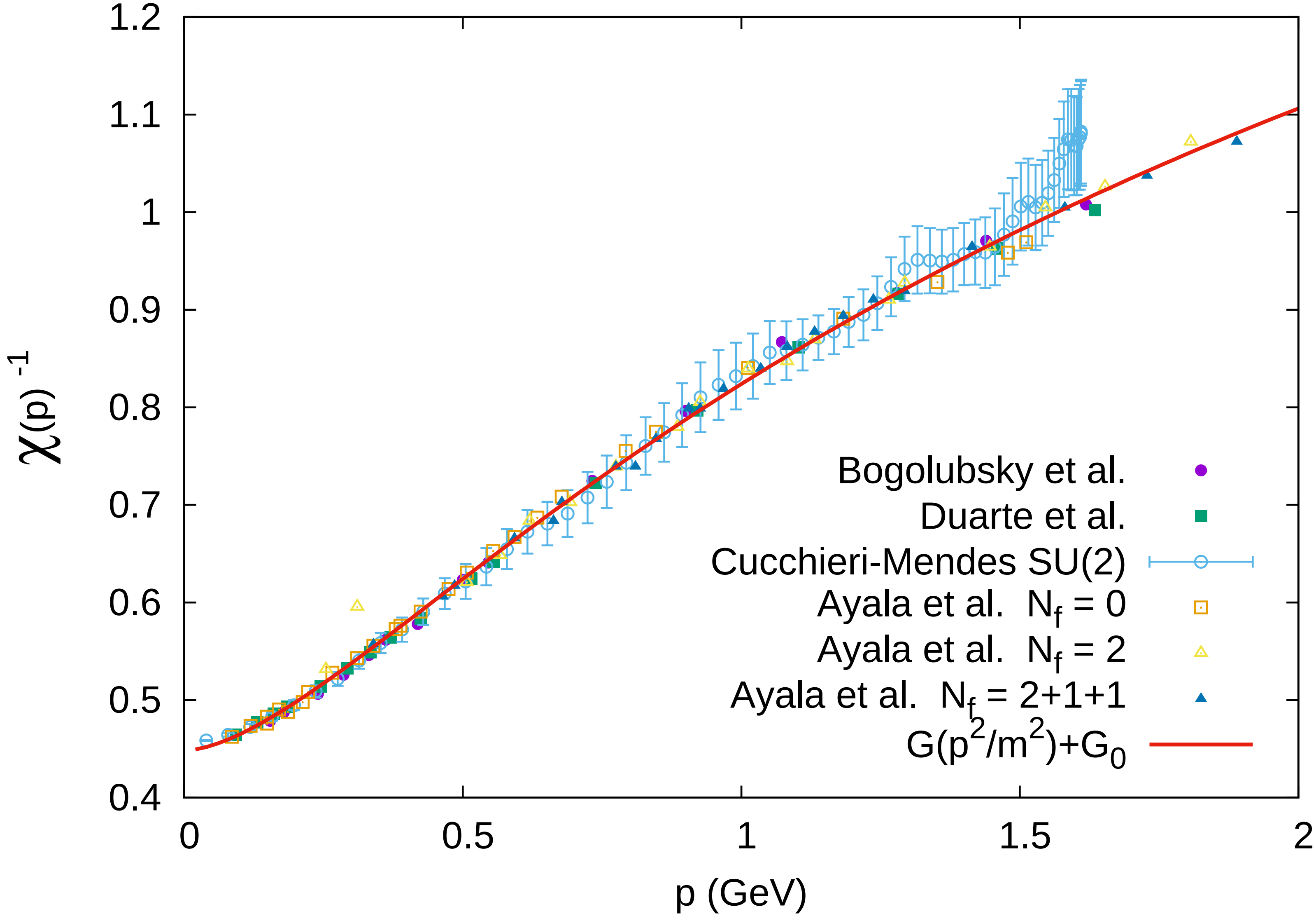

Figure 3. The solid curve (red line) is the ghost universal function $G(s)$, replacing the function $F(s)$ in Eq.(7), evaluated by the massive expansion of Ref.[1] for $s = p^2/m^2$, $m = 0.73$ GeV and shifted by the constant $G_0 = 0.24$. The data are scaled by the constants of the table.

Ghost dressing function

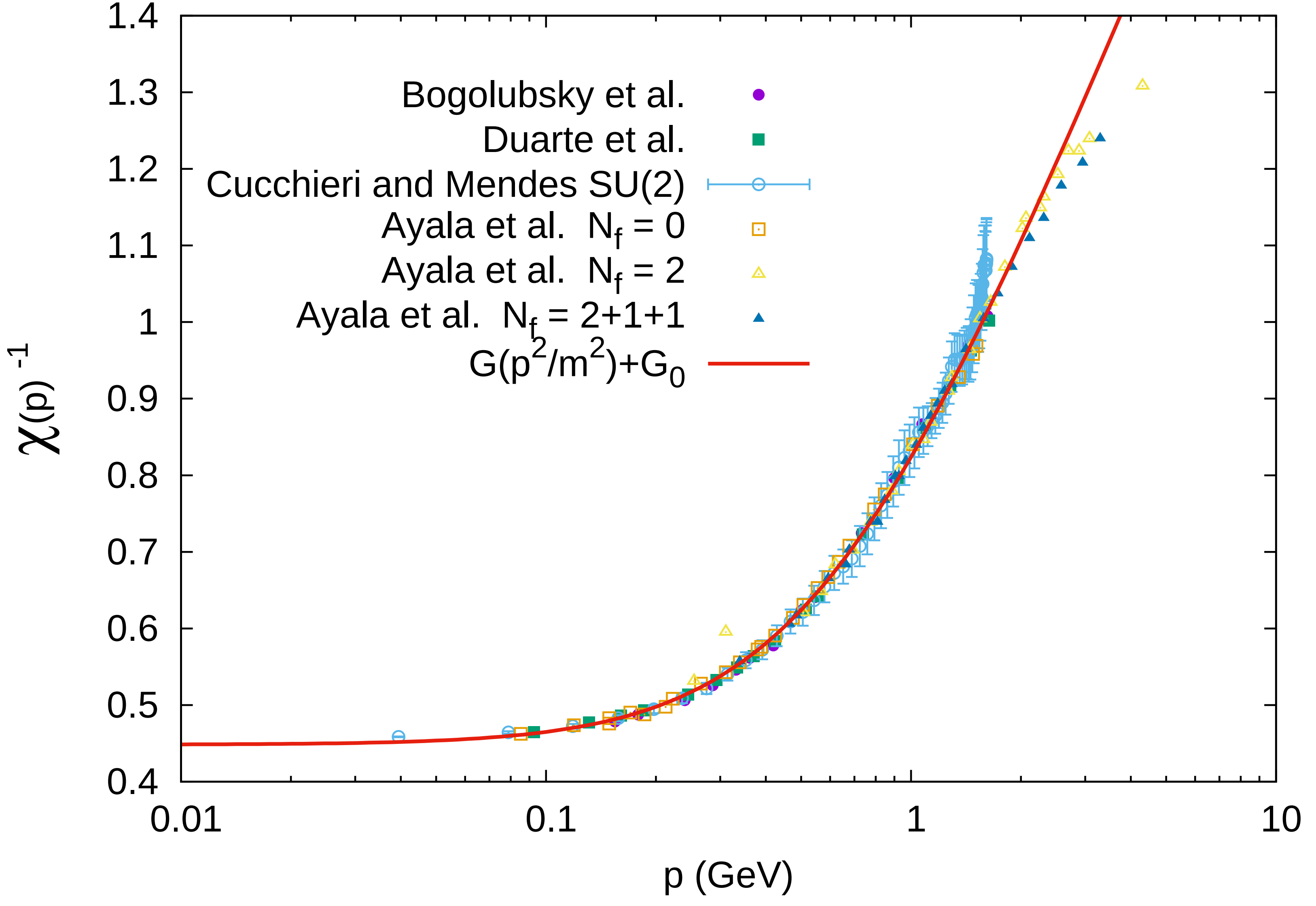

Figure 4. Inverse ghost dressing function. The same content of Fig. 3 is shown on a wider range by a logarithmic scale.

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