Relations among pionic decays of spin-1 mesons from an SU(4)×U(1) emergent symmetry in QCD

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Abstract

[1] M. Denissenya, L. Y. Glozman and C. B. Lang, Phys. Rev. D **91**, 034505 (2015).
[2] L. Y. Glozman and M. Pak, Phys. Rev. D **92**, 016001 (2015).
[3] T. D. Cohen, Phys. Rev. D **93**, 034508 (2016).

Motivated by recent results by lattice analysis[1-3], we assume that the spin-1 mesons of ($\rho, \omega, a_1, \rho', \omega', b_1, f_1, h_1$) make a rep. of **16** of U(4) emergent sym. in two-flavor QCD when the chiral sym. is not broken. We study the decay properties of the spin-1 mesons by using a chiral model with an SU(4)×U(1) hidden local symmetry (HLS). We first show that, since the SU(4) sym. is spontaneously broken together with the chiral symmetry, each coupling of the interaction among one pion and two spin-1 mesons is proportional to the mass difference of the relevant spin-1 mesons similarly to the Goldberger-Treiman (GT) relation. In addition, some of one-pion couplings are related with each other by the SU(4) sym.. We further show that there is a relation among the mass of ρ' meson, the $\rho'\pi\pi$ coupling and the ρ'-photon mixing strength as well as the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation for the ρ meson. From the relations, we give numerical predictions such as ratios of the spin-1 meson decay widths, which are compared with future experiments for testing the existence of the U(4) emergent sym..

SU(4) symmetry ($\neq$ SU(4) flavor sym.)

"Emergent symmetry" in two flavor QCD

Rotation of $\bar\psi\gamma^\mu T^a\psi$

$$\psi^T = \begin{pmatrix} u_l & d_l & u_r & d_r \end{pmatrix}$$

Its chirality $\gamma_5 q_{r,l} = \pm q_{r,l}$

→ SU(4) **15**-plet + U(1) Singlet (in spin-1 sector)

Refs. [1-3] (in lattice QCD cal.)

They found degeneracy of spin-1 mesons ($\rho, \omega, a_1, \rho', \omega', b_1, f_1, h_1$) by removing the smallest eigenvalues of the Dirac operator.

Existence of the SU(4) sym. in the spin-1 meson sector is suggested by Refs. [1-3] via the lattice QCD calculation.

spin-1 mesons ($\rho, \omega, a_1, \rho', \omega', b_1, f_1, h_1$)

Mesons	$I\,(J^{PC})$	mass (MeV)
ρ	$1\,(1^{--})$	775.26 ± 0.25
ω	$0\,(1^{--})$	782.65 ± 0.12
ρ'	$1\,(1^{--})$	1465 ± 25
ω'	$0\,(1^{--})$	$1400 - 1450$
a_1	$1\,(1^{++})$	1230 ± 40
f_1	$0\,(1^{++})$	1281.9 ± 0.5
b_1	$1\,(1^{+-})$	1229.5 ± 3.2
h_1	$0\,(1^{+-})$	1170 ± 20

Chiral Model with SU(4)×U(1) HLS

We construct a chiral model with an SU(4)×U(1) hidden local symmetry (HLS).

<u>The model has the following syms.</u>

Chiral sym. $\quad$ HLS $\quad$ corresponding gauge field $\boxed{V_\mu}$ Including ($\rho, \omega, a_1, \rho', \omega', b_1, f_1, h_1$)

$$\boxed{SU(2)_L \times SU(2)_R} \times \boxed{[SU(4)\times U(1)]_{HLS}}$$

SSB

Remain Isospin sym. $\boxed{SU(2)_V}$

3 + 15 + 1 NG bosons

π Pion $\quad$ Eaten by V_μ → Masses of ($\rho, \omega, a_1, \rho', \omega', b_1, f_1, h_1$)

We introduce the pion through 2 by 2 unitary matrix

$$U = \exp\left(i\frac{\eta}{f_\eta} + i\sum_{a=1}^{3}\frac{\pi^a\sigma^a}{f_\pi}\right)$$

Embed into 4 by 4 matrix

$$\mathcal{U} = \begin{pmatrix} 0 & U^\dagger \\ U & 0 \end{pmatrix}$$

which transforms $\mathcal{U} \to \underbrace{\mathcal{G}\cdot\mathcal{U}\cdot\mathcal{G}^\dagger}_{\text{Chiral sym.}}$

To introduce HLS, let us decompose $\mathcal{U}$ as

$$\mathcal{U} = \Xi^\dagger(x)\cdot\Xi_m^\dagger(x)\cdot\bar\Sigma\cdot\Xi_m(x)\cdot\Xi(x)$$

where $\bar\Sigma \equiv \begin{pmatrix} 0 & 1_2 \\ 1_2 & 0\end{pmatrix}$ $\quad$ Isospin sym. $\quad$ HLS $\quad$ Chiral sym.

These two variables transform

$$\Xi(x) \to \tilde{\mathcal{G}}(x)\cdot\Xi(x)\cdot\mathcal{G}^\dagger ,$$
$$\Xi_m(x) \to h\cdot\Xi_m(x)\cdot\tilde{\mathcal{G}}^\dagger(x)$$

The covariantized Maurer-Cartan 1-forms are defined as

$$\hat\alpha_\mu(x) \equiv \frac{1}{i}\Xi_m(x)\cdot\left(D_\mu\Xi(x)\cdot\Xi^\dagger(x)\right)\cdot\Xi_m^\dagger(x) ,$$
$$\hat\alpha_\mu^{(m)}(x) \equiv \frac{1}{i}D_\mu\Xi_m(x)\cdot\Xi_m^\dagger(x)$$

Covariant derivatives

$$D_\mu\Xi(x) = \partial_\mu\Xi(x) - iV_\mu\Xi(x) + i\Xi(x)\mathcal{V}_\mu$$
$$D_\mu\Xi_m(x) = \partial_\mu\Xi_m(x) + i\Xi_m(x)V_\mu$$

V_μ : gauge field of HLS

$\mathcal{V}_\mu$: External gauge field (e.g. photon)

The Lagrangian is written as (LO in derivative expansion)

$$\mathcal{L}^{\mathcal{O}(p^2)}_{[SU(4)\times U(1)]_{HLS}} = \mathcal{L}_V + \mathcal{L}_k ,$$

with 16 parameters (14 : $\bar a_{(n)}$ + 2 : g, g_B)

arbitrary real coefficients

$$\mathcal{L}_V = \sum_{n=1}^{14}\bar a_{(n)}\mathcal{L}_n \quad \text{(14 independent operators)}$$
$$\mathcal{L}_k = -\frac{1}{2g^2}\mathrm{Tr}\left[V_{\mu\nu}V^{\mu\nu}\right] - \left(\frac{1}{g_B^2} - \frac{1}{g^2}\right)\mathrm{Tr}\left[V_{\mu\nu}\right]\mathrm{Tr}\left[V^{\mu\nu}\right]$$

Gauge field is classified as

$$V_\mu = \underbrace{V_{\mu\parallel} + V_{\mu\perp(1)}}_{\rho\ \&\ \rho'} + \underbrace{V_{\mu\perp(2)}}_{b_1} + \underbrace{V_{\mu\perp(3)}}_{a_1} + \underbrace{V^{(I=0)}_{\mu\parallel} + V^{(I=0)}_{\mu\perp(1)}}_{\omega\ \&\ \omega'} + \underbrace{V^{(I=0)}_{\mu\perp(2)}}_{h_1} + \underbrace{V^{(I=0)}_{\mu\perp(3)}}_{f_1}$$

Generators of SU(4)

$$S^a \equiv \frac{1}{\sqrt2}\begin{pmatrix} t^a & 0 \\ 0 & t^a\end{pmatrix}, X^a_{(3)} \equiv \frac{1}{\sqrt2}\begin{pmatrix} t^a & 0 \\ 0 & -t^a\end{pmatrix},$$
$$X^a_{(1)} \equiv \frac{1}{\sqrt2}\begin{pmatrix} 0 & t^a \\ t^a & 0\end{pmatrix}, X^a_{(2)} \equiv \frac{1}{\sqrt2}\begin{pmatrix} 0 & -it^a \\ it^a & 0\end{pmatrix},$$
$$X^0_{(3)} \equiv \frac{1}{2\sqrt2}\begin{pmatrix} 1_2 & 0 \\ 0 & -1_2\end{pmatrix}, X^0_{(1)} \equiv \frac{1}{2\sqrt2}\begin{pmatrix} 0 & 1_2 \\ 1_2 & 0\end{pmatrix} = \frac{1}{2\sqrt2}\bar\Sigma,$$
$$X^0_{(2)} \equiv \frac{1}{2\sqrt2}\begin{pmatrix} 0 & -i1_2 \\ i1_2 & 0\end{pmatrix} \quad t^a = \sigma^a/2$$

σ^a :Pauli matrix

1-forms are expanded as (in the unitary gauge) $\boxed{\Xi(\pi) = e^{i\pi/F_\pi}}$

$$\hat\alpha_{\mu\parallel}(x) = -\frac{1}{2iF_\pi}[\pi, \partial_\mu\pi] - V_{\mu\parallel} + 2\mathrm{Tr}\left[\Xi(\pi)\cdot\mathcal{V}_\mu\cdot\Xi^\dagger(\pi)\cdot S^a\right]S^a + \cdots ,$$
$$\hat\alpha_{\mu\perp(3)}(x) = \frac{1}{F_\pi}\partial_\mu\pi - \frac{1}{6F_\pi^3}[\pi,[\pi,\partial_\mu\pi]] - V_{\mu\perp(3)} + 2\mathrm{Tr}\left[\Xi(\pi)\cdot\mathcal{V}_\mu\cdot\Xi^\dagger(\pi)\cdot X^a_{\perp(3)}\right]X^a_{\perp(3)} + \cdots$$
$$\hat\alpha^{(m)}_{\mu\perp(1)}(x) = V_{\mu\perp(1)} ,\quad \hat\alpha^{(m)}_{\mu\perp(2)}(x) = V_{\mu\perp(2)} ,\quad \hat\alpha^{(m)}_{\mu\perp(3)}(x) = V_{\mu\perp(3)}$$

Substituting these into the Lagrangian, we get

<u>Quadratic terms</u>

$$\mathcal{L}_V = \frac12\frac{F^2}{F_\pi^2}\left(\bar a_{(1)} - \frac{\bar a^2_{(7)}}{\bar a_{(5)}}\right)(\partial_\mu\pi^a)^2 + \frac12\frac{F^2}{F_\pi^2}\left(\bar a_{(8)} - \frac{\bar a^2_{(14)}}{\bar a_{(12)}}\right)(\partial_\mu\eta)^2$$

→ Normalize $\quad F^2\left(\bar a_{(1)} - \frac{\bar a^2_{(7)}}{\bar a_{(5)}}\right) = F_\pi^2 ,\quad F^2\left(\bar a_{(8)} - \frac{\bar a^2_{(14)}}{\bar a_{(12)}}\right) = F_\pi^2$

$$+\sqrt2\frac{F^2}{F_\pi}\left(\bar a_{(1)} - \frac{\bar a^2_{(7)}}{\bar a_{(5)}}\right)(\mathcal{A}^a_\mu\partial^\mu\pi^a) + \sqrt2\frac{F^2}{F_\pi}\left(\bar a_{(8)} - \frac{\bar a^2_{(14)}}{\bar a_{(12)}}\right)(\mathcal{A}^0_\mu\partial^\mu\eta)$$

→ Pion decay constant $\quad f_\pi = \sqrt2 F_\pi$

Mass terms

$$+\frac12\left(\bar a_{(4)}F^2g^2\right)\frac{1}{g^2}\left(V^a_{\mu\perp(2)}\right)^2 + \frac12\left(\bar a_{(11)}F^2g^2\right)\frac{1}{g^2}\left(V^0_{\mu\perp(2)}\right)^2$$

→ b_1 & h_1

$$+g^2F^2\mathrm{Tr}\left[\begin{pmatrix}\frac1g V_{\mu\parallel} & \frac1g\bar\Sigma\cdot V_{\mu\perp(1)}\end{pmatrix}\begin{pmatrix}\bar a_{(2)} & -\bar a_{(6)} \\ -\bar a_{(6)} & \bar a_{(3)}\end{pmatrix}\begin{pmatrix}\frac1g V_{\mu\parallel} \\ \frac1g\bar\Sigma\cdot V_{\mu\perp(1)}\end{pmatrix}\right]$$

→ ρ & ρ'

Mixing angle : θ_ρ $\quad \tan2\theta_\rho \equiv \frac{2\bar a_{(6)}}{\bar a_{(2)} - \bar a_{(3)}}$

$$\rho_\mu = \rho^a_\mu S^a \equiv \frac1g\left(\cos\theta_\rho V_{\mu\parallel} - \sin\theta_\rho\bar\Sigma\cdot V_{\mu\perp(1)}\right) ,$$
$$(\rho')_\mu = (\rho')^a_\mu S^a \equiv \frac1g\left(\cos\theta_\rho\bar\Sigma\cdot V_{\mu\perp(1)} + \sin\theta_\rho V_{\mu\parallel}\right)$$

$$+g^2F^2\mathrm{Tr}\left[\begin{pmatrix}\frac{1}{g_B}V^{(I=0)}_{\mu\parallel} & \frac1g\bar\Sigma\cdot V^{(I=0)}_{\mu\perp(1)}\end{pmatrix}\begin{pmatrix}\frac{g_B^2}{g^2}\bar a_{(9)} & -\frac{g_B}{g}\bar a_{(13)} \\ -\frac{g_B}{g}\bar a_{(13)} & \bar a_{(10)}\end{pmatrix}\begin{pmatrix}\frac{1}{g_B}V^{(I=0)}_{\mu\parallel} \\ \frac1g\bar\Sigma\cdot V^{(I=0)}_{\mu\perp(1)}\end{pmatrix}\right]$$

→ ω & ω'

Mixing angle : θ_ω $\quad \tan2\theta_\omega \equiv \frac{2g_Bg\bar a_{(13)}}{g_B^2\bar a_{(9)} - g^2\bar a_{(10)}}$

$$\hat\omega_\mu = \omega_\mu S^0 \equiv \frac{1}{g_B}\cos\theta_\omega V^{(I=0)}_{\mu\parallel} - \frac1g\sin\theta_\omega\bar\Sigma\cdot V^{(I=0)}_{\mu\perp(1)} ,$$
$$(\hat\omega')_\mu = (\omega')_\mu S^0 \equiv \frac1g\cos\theta_\omega\bar\Sigma\cdot V^{(I=0)}_{\mu\perp(1)} + \frac{1}{g_B}\sin\theta_\omega V^{(I=0)}_{\mu\parallel}$$

$$+\frac12\left(\bar a_{(5)}F^2g^2\right)\frac{1}{g^2}\left(V^a_{\mu\perp(3)} - \frac{\bar a_{(7)}}{\bar a_{(5)}}\frac{1}{F_\pi}\partial_\mu\pi^a\right)^2 + \frac12\left(\bar a_{(12)}F^2g^2\right)\frac{1}{g^2}\left(V^0_{\mu\perp(3)} - \frac{\bar a_{(14)}}{\bar a_{(12)}}\frac{1}{F_\pi}\partial_\mu\eta\right)^2 + \cdots$$

→ a_1 & f_1

Mixing of a_1 & π (f_1 & η)

$$\boxed{(a_1)_\mu \equiv \frac1g\left(V^a_{\mu\perp(3)} - \frac{r_{a_1}}{f_\pi}\partial_\mu\pi^a\right)X^a_{(3)} ,\quad (f_1)_\mu \equiv \frac1g\left(V^0_{\mu\perp(3)} - \frac{r_{f_1}}{f_\pi}\partial_\mu\eta\right)X^0_{(3)}}$$

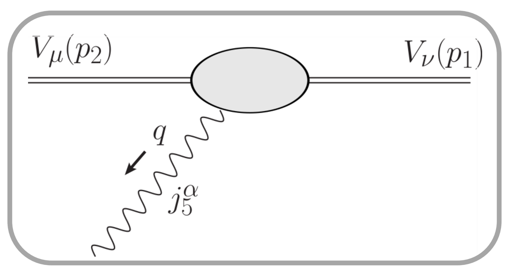
Strength of mixing $\quad r_{a_1} = \sqrt2\frac{\bar a_{(7)}}{\bar a_{(5)}} ,\quad r_{f_1} = \sqrt2\frac{\bar a_{(14)}}{\bar a_{(12)}}$

Extended GT Relations

Extended Goldberger-Treiman (GT) relation is obtained.

The amplitude is given as

$$\mathcal{M}^\alpha = \int d^4x e^{-iqx}\langle V_\mu(p_2)|\,j_5^\alpha(x)\,|V_\nu(p_1)\rangle$$
$$= \epsilon^*_\mu(p_2)\left[g_1(q^2)g^{\mu\nu}ip^\alpha + g_2(q^2)\frac{q^\mu q^\nu}{m_1^2 + m_2^2}ip^\alpha + g_3(q^2)\left(iq^\mu g^{\nu\alpha} + iq^\nu g^{\mu\alpha}\right) + g_4(q^2)\left(iq^\mu g^{\nu\alpha} - iq^\nu g^{\mu\alpha}\right)\right.$$
$$\left. + h_1(q^2)g^{\mu\nu}iq^\alpha + h_2(q^2)\frac{q^\mu q^\nu}{m_1^2 + m_2^2}iq^\alpha\right]\epsilon_\nu(p_1)$$



The longitudinal parts include a massless pole: $h_n(q^2) = \frac{f_\pi}{q^2}G^{(n)}_{V_1V_2\pi} + \cdots$ (Pion)

- WT-identity $\quad q_\alpha\mathcal{M}^\alpha = 0$
- Soft momentum limit $\quad q_\alpha \to 0$

$$\boxed{G^{(1)}_{V_1V_2\pi} = -\frac{m_1^2 - m_2^2}{2f_\pi}g_1(0)}$$

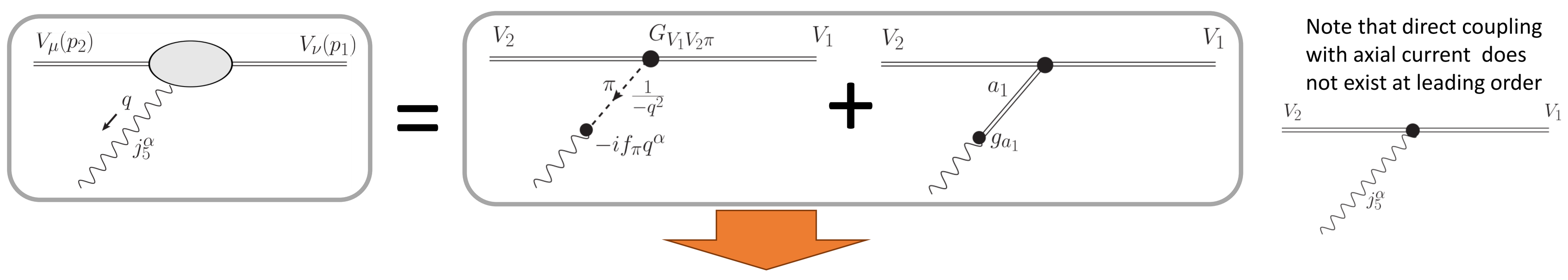
<u>Extended GT relation</u>

Note

SU(4) sym. exists → $m_1^2 - m_2^2$ is not large → $q_\alpha \to 0$ is reasonable

The SU(4) sym. together with the chiral sym. ensures low energy theorems.

Next, we turn to make an analysis based on the present model.



<u>Table : One pion and axial couplings in the SU(4) HLS model.</u>

V_1	V_2	$G^{(1)}_{V_1V_2\pi}$	$g_1(0)$
a_1	ρ	$(m^2_{a_1} - m^2_\rho)\left(\frac{r_{a_1}}{\sqrt2f_\pi}\cos\theta_\rho\right)$	$\sqrt2g\cos\theta_\rho\frac{g_{a_1}}{m^2_{a_1}}$
ρ'	a_1	$(m^2_{\rho'} - m^2_{a_1})\left(\frac{r_{a_1}}{\sqrt2f_\pi}\sin\theta_\rho\right)$	$\sqrt2g\sin\theta_\rho\frac{g_{a_1}}{m^2_{a_1}}$
h_1	ρ	$(m^2_{h_1} - m^2_\rho)\left(\frac{r_{a_1}}{\sqrt2f_\pi}\sin\theta_\rho\right)$	$\sqrt2g\sin\theta_\rho\frac{g_{a_1}}{m^2_{a_1}}$
ρ'	h_1	$(m^2_{\rho'} - m^2_{h_1})\left(\frac{r_{a_1}}{\sqrt2f_\pi}\cos\theta_\rho\right)$	$\sqrt2g\cos\theta_\rho\frac{g_{a_1}}{m^2_{a_1}}$
b_1	ω	$(m^2_{b_1} - m^2_\omega)\left(\frac{r_{a_1}}{\sqrt2f_\pi}\sin\theta_\omega\right)$	$\sqrt2g\sin\theta_\omega\frac{g_{a_1}}{m^2_{a_1}}$
ω'	b_1	$(m^2_{\omega'} - m^2_{b_1})\left(\frac{r_{a_1}}{\sqrt2f_\pi}\cos\theta_\omega\right)$	$\sqrt2g\cos\theta_\omega\frac{g_{a_1}}{m^2_{a_1}}$

We can easily confirm that these actually satisfy the extended GT relation.

<u>Extended GT relation</u>

$$G^{(1)}_{V_1V_2\pi} = -\frac{m_1^2 - m_2^2}{2f_\pi}g_1(0)$$

where $g_{a_1} = -\frac{r_{a_1}m^2_{a_1}}{g}$

Other Relations

Due to **a_1- π mixing,** $\boxed{V_{\mu\perp(3)} = g\,(a_1)_\mu + \frac{r_{a_1}}{f_\pi}\partial_\mu\pi}$

interactions among $VV\pi$ are obtained from $\mathcal{L}^{(3)}_{int} = -\frac{1}{ig^2}\mathrm{Tr}\left[(\partial_\mu V_\nu - \partial_\nu V_\mu)\left[V^\mu, V^\nu\right]\right]$ included in $\mathcal{L}_k$

$VV\pi$ ints. are controlled by only three : r_{a_1}, θ_ρ, θ_ω

Thanks to existence of the SU(4) sym.

<u>Relations among $VV\pi$ coups.</u>

$$g_{\rho a_1\pi} = g_{\rho' h_1\pi} = \frac{r_{a_1}}{\sqrt2f_\pi}\cos\theta_\rho ,$$
$$g_{\rho' a_1\pi} = -g_{\rho h_1\pi} = \frac{r_{a_1}}{\sqrt2f_\pi}\sin\theta_\rho ,$$
$$g_{b_1\omega\pi} = -\frac{r_{a_1}}{\sqrt2f_\pi}\sin\theta_\omega ,\quad g_{b_1\omega'\pi} = \frac{r_{a_1}}{\sqrt2f_\pi}\cos\theta_\omega$$

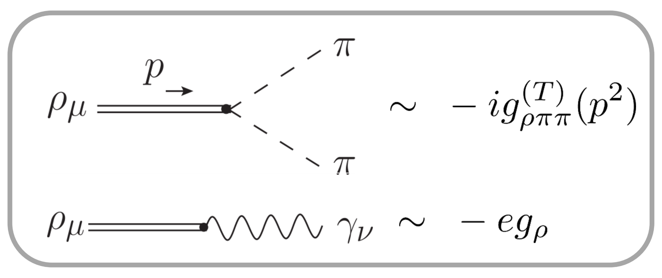
Vertex function

$$\Gamma^{\mu\nu}\left[(V_1)_\mu(p_1), (V_2)_\nu(p_2), \pi\right] = g_{V_1V_2\pi}\left(p_1^2P^{\mu\nu}(p_1) - p_2^2P^{\mu\nu}(p_2)\right)$$

where $P^{\mu\nu}(p) \equiv g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}$

KSRF-I relation is extended to **ρ' version**

KSRF-I relation : $\boxed{g_\rho = 2g^{(T)}_{\rho\pi\pi}(p^2 = 0)f_\pi^2}$



Since ρ & ρ' are given from one field,

HLS gauge field $\quad V_\mu = \underbrace{V_{\mu\parallel} + V_{\mu\perp(1)}}_{\rho\ \&\ \rho'} + \cdots$

$$g^{(T)}_{\rho\pi\pi}(p^2) = \frac{m_\rho^2 - r^2_{a_1}p^2}{\sqrt2gf_\pi^2}\cos\theta_\rho \quad g_\rho = \frac{\sqrt2m_\rho^2\cos\theta_\rho}{g}$$
$$g^{(T)}_{\rho'\pi\pi}(p^2) = \frac{m_{\rho'}^2 - r^2_{a_1}p^2}{\sqrt2gf_\pi^2}\sin\theta_\rho \quad g_{\rho'} = \frac{\sqrt2m_{\rho'}^2\sin\theta_\rho}{g}$$

<u>Extended KSRF-I relation</u>

$$\boxed{g_{\rho'} = 2g^{(T)}_{\rho'\pi\pi}(p^2 = 0)f_\pi^2}$$

Using the above relations and experimental values, we get predictions of decay widths

<u>Table : Experimental values</u>

Mesons	mass (MeV)
ρ	775.26 ± 0.25
ω	782.65 ± 0.12
ρ'	1465 ± 25
ω'	$1400 - 1450$
a_1	1230 ± 40
f_1	1281.9 ± 0.5
b_1	1229.5 ± 3.2
h_1	1170 ± 20

$\Gamma(\rho\to\pi\pi)$	147.8 ± 0.9 MeV
$\Gamma(\rho^0\to e^+e^-)$	7.04 ± 0.06 keV
$\Gamma(\omega\to e^+e^-)$	0.60 ± 0.02 keV
$m_{\pi^\pm}$	139.57018 ± 0.00035 MeV
m_e	$548.57990946 \pm 0.00000022$ keV
$\alpha \equiv \frac{e^2}{4\pi}$	$\frac{1}{137}$
f_π	92.21 ± 0.14 MeV
$\langle r^2\rangle_V^{\pi^\pm}$	0.452 ± 0.011 fm ^{2}

<u>Table : Predictions</u>

Independent of the parameters				
$\frac{\Gamma(\rho'\to h_1\pi)}{\Gamma(a_1\to\rho\pi)}$	=	0.16 ± 0.07		
$\frac{\Gamma(h_1\to\rho\pi)}{\Gamma(\rho'\to a_1\pi)}$	=	6.4 ± 4.3		
Dependent on θ_ρ or θ_ω				
$\frac{\Gamma(\rho'\to a_1\pi)}{\Gamma(a_1\to\rho\pi)}$	=	$(0.15 \pm 0.11)\tan^2\theta_\rho$	$\lesssim$	0.015
$\frac{\Gamma(h_1\to\rho\pi)}{\Gamma(a_1\to\rho\pi)}$	=	$(1.0 \pm 0.3)\tan^2\theta_\rho$	$\lesssim$	0.10
$\frac{\Gamma(\rho'\to\pi\pi)}{\Gamma(\rho\to\pi\pi)}$	=	$(28 \pm 2)\tan^2\theta_\rho$		(Input)
$\frac{\Gamma(\rho'^0\to e^+e^-)}{\Gamma(\rho^0\to e^+e^-)}$	=	$(1.89 \pm 0.03)\tan^2\theta_\rho$	$\lesssim$	0.2
$\frac{\Gamma(\omega\to e^+e^-)}{\Gamma(\rho^0\to e^+e^-)}$	=	$(0.11219 \pm 0.00004)\frac{g^2}{g_B^2}\frac{\cos^2\theta_\omega}{\cos^2\theta_\rho}$		(Input)
$\frac{\Gamma(b_1\to\omega\pi)}{\Gamma(\omega'\to b_1\pi)}$	=	$(3.9 \pm 1.9)\tan^2\theta_\omega$		
$\frac{\Gamma(\omega'\to e^+e^-)}{\Gamma(\omega\to e^+e^-)}$	=	$(1.01 \pm 0.03)\tan^2\theta_\omega$		

Summary

- We constructed a chiral model with SU(4)×U(1) HLS.
- The following relations are obtained.
  1) **<u>Extended GT relation</u>** , 2) **<u>Relations among the $VV\pi$ couplings</u>** , 3) **<u>KSRF-I relations for ρ and ρ'</u>**
- We gave some predictions.

<u>Future work ...</u>

○ Clarifying the correspondence between the Lattice QCD result and our calculation.

e.g. Partial linearization