Holographic study of the QCD matter under external conditions

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QCD phase diagram studies

The QCD matter at finite values of temperature T and quark chemical potential μ is a problem both for theoretical physics and experimental research:

(I) Experimental studies of HIC
- → LHC (CERN)
- → NA61/SHINE
- → RHIC (BNL)
- → NICA (JINR)
- → FAIR (GSI)

(II) Theoretical methods
- → lattice simulations
- → nonperturbative approaches ⇐ holography
- → perturbative approaches

The AdS/QCD correspondence

Consider that the well-known AdS/CFT correspondence between a string theory on $AdS_5 \times S_5$ and SYM theory on ∂AdS_5 can be viewed as a general correspondence between

weakly coupled theories & strongly coupled theories

⇒ a way to find a holographical dual to QCD in large N_c limit

Thus appeared the AdS/QCD correspondence, which proved to be a fruitful way to study QCD phenomena. In particular, the bottom-up arrroach allows to incorporate the phenomenological features of real QCD.

Methods and properties of the AdS/QCD correspondence:

- operators $\mathcal{O}(x)$ in $4D$ theory ⇔ fields $\phi(x,z)$ in $5D$ dual theory
- canonical dimension Δ of the p-form operator $\mathcal{O}(x)$ ⇔ $5D$ mass of $\phi(x,z)$: $m_5^2 = (\Delta - p)(\Delta + p - 4)$, example: $\bar{q}\gamma^\mu t^a q$ with $p = 1$ and $\Delta = 3$ corresponds to $A_\mu^a(x,z)$ with $m_5^2 = 0$
- source $\phi_{\mathcal{O}}(x)$ of an operator ⇔ value of the $5D$ field on the boundary $\phi(x,\epsilon)$
- the generating functional of the connected correlators in the 4D theory and the effective action of the 5D theory are equivalent: $W_{4D}[\phi_{\mathcal{O}}(x)] = S_{5D,eff}[\phi(x,\epsilon)]$ with $\phi(x,\epsilon) = \phi_{\mathcal{O}}(x)$
- differentiating $S_{5D,eff}$ with respect to $\phi_{\mathcal{O}}$ ⇒ QCD Green's functions
- the deconfinement phase transition ⇔ a Hawking-Page phase transition at T_c between a low temperature thermal AdS space and a high temperature black hole in AdS/QCD models

Examples of the models providing dual description of the vector meson phenomenology:

- Soft-Wall model[a] for vector mesons

$$S = -\frac{1}{4g_5^2}\int d^5x \exp(-az^2)\sqrt{g}g^{MK}g^{LN}F_{MN}F_{KL}$$

provides linear radial Regge spectra $M_n^2 = 4a(n+1)$.

- Generalized Soft-Wall model[b] for vector mesons

$$S = -\frac{1}{4g_5^2}\int d^5x \exp(-az^2)U^2(b,0;az^2)\sqrt{g}g^{MK}g^{LN}F_{MN}F_{KL}$$

where U denotes the Tricomi hypergeometric function ($U(0,0;x) = 1$). The model provides realistic phenomenological spectra with an arbitrary intercept $M_n^2 = 4a(n+1+b)$.

[a] A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. D **74**, 015005 (2006).
[b] S. S. Afonin, Phys. Lett. B **719**, 399 (2013) [arXiv:1210.5210 [hep-ph]].

The critical temperature in AdS/QCD

We assume that the thermodynamics is governed by the gravitational part of the action, which scales as N_c^2 (while the mesonic part $\sim N_c$). In the SW model the gravitational part of the 5D action is:

$$S_{SW} = \frac{1}{2\kappa^2}\int d^4x dz e^{-\Phi(z)}\sqrt{-g}(\mathcal{R} - 2\Lambda)$$

where the dilaton profile $\Phi(z) = az^2$ ($\Phi(z)$ is assumed not to affect the gravitational dynamics of the theory), $\mathcal{R}$ – the scalar curvature.

The on-shell gravitational action is the same ($S_{SW}|_{on-shell} = \frac{4}{\kappa^2 L^2}\int d^4x dz e^{-\Phi(z)}\sqrt{g}$) for:

(1) thermal AdS metric: $ds^2 = \frac{L^2}{z^2}\left(dt^2 - d\vec{x}^2 - dz^2\right)$,

(2) metric of AdS with a black hole: $ds^2 = \frac{L^2}{z^2}\left(f(z)dt^2 - d\vec{x}^2 - \frac{dz^2}{f(z)}\right)$,

where $f(z) = 1 - (z/z_h)^4$ and L denotes the AdS radius.

Transition between two backgrounds ⟺ (De)confinement transition

The Hawking temperature is related to the black hole horizon z_h via the relation $T_c = 1/(\pi z_h)$.

The free action densities V identified with the regularized action $S_{SW}|_{on-shell}$ are:

$$V_{\text{Th}}(\epsilon) = \frac{4L^3}{\kappa^2}\int_0^\beta dt \int_\epsilon^\infty e^{-\Phi(z)}z^{-5}dz, \quad V_{\text{BH}}(\epsilon) = \frac{4L^3}{\kappa^2}\int_0^{\pi z_h} dt \int_\epsilon^{z_h} e^{-\Phi(z)}z^{-5}dz.$$

The two geometries are compared at a radius $z = \epsilon$ where the periodicity in the time direction is locally the same ⇒ $\beta = \pi z_h \sqrt{f(\epsilon)}$.

The order parameter in the phase transition is defined by ΔV:

$$\Delta V = \lim_{\epsilon\to\infty}(V_{\text{BH}}(\epsilon) - V_{\text{Th}}(\epsilon)) = \frac{\pi\kappa L^5}{4z_h^3}\left[e^{-az_h^2}(az_h^2 - 1) + \frac{1}{2} - (az_h^2)^2\int_{az_h^2}^{\infty}\frac{dt}{t}e^{-t}\right].$$

The procedure for GSW is the same, the resulting expression being more complicated with the Tricomi functions.

The Hawking-Page phase transition occurs at a point where $\Delta V = 0$.

$\Delta V = 0$ yields az_h^2 (at a given b in GSW) ⇒ T_c depends on the slope parameter a and the intercept parameter b.

In the table[a] we calculate T_c with input parameters from different vector meson spectra ⟹

Particle	Radial states	m_n^2, GeV^2	T_c, MeV
ρ	$n = 0,1,2$	$1.18(n+0.61)$	143
ω	$n = 0,1,2$	$1.09(n+0.66)$	149
ρ	$n = 0,1,2,3,4$	$0.99(n+0.89)$	207
ω	$n = 0,1,2,3,4$	$1.03(n+0.74)$	166
ρ	$n = 0,1,2,4,5$	$0.88(n+1.12)$	270
ω	$n = 1,2,3,4$	$0.95(n+1.04)$	255
-	"universal" slope[b]	$1.14(n+1)$	263

[a] S. S. Afonin, A.D.Katanaeva, Eur. Phys. J. C, 74, 3124 (2014)
[b] D. V. Bugg, Phys. Rept. **397**, 257 (2004).

Quark chemical potential inclusion

In QCD the quark chemical potential μ is intoduced as $\mu_q\bar{\psi}\gamma_0\psi$ ⇒ we need to include a U(1) field in AdS_5 – A_M, with μ_q being its boundary value.

Consider the GSW Euclidean action consisting of the gravitational part and the U(1) kinetic term:

$$S = \int d^4x dz\sqrt{g}e^{-az^2}U^2(b,0;az^2)\left(-\frac{1}{2\kappa^2}(\mathcal{R} + 12/L^2) + \frac{1}{4g_5^2}F_{MN}F^{MN}\right)$$

Solution of the Maxwell equations: $A_t = i(\mu - Qz^2)$, $A_i = A_z = 0$

Solutions of the Einstein equations correspond to different phases:

(1) thermal charged AdS – confining phase

$ds^2 = \frac{L^2}{z^2}\left(f_{tc}(z)dt^2 + d\vec{x}^2 + \frac{1}{f_{tc}(z)}dz^2\right)$,

where $f_{tc}(z) = 1 + q'^2 z^6$

(2) Reissner-Nordstrem black hole in AdS – deconfined phase

$ds^2 = \frac{L^2}{z^2}\left(f_{RN}(z)dt^2 + d\vec{x}^2 + \frac{1}{f_{RN}(z)}dz^2\right)$,

where $f_{RN}(z) = 1 - m^2 z^4 + q^2 z^6$

The BH charge and quark number density are connected via: $Q = \sqrt{\frac{3g_5^2 L^2}{2\kappa^2}}q$.

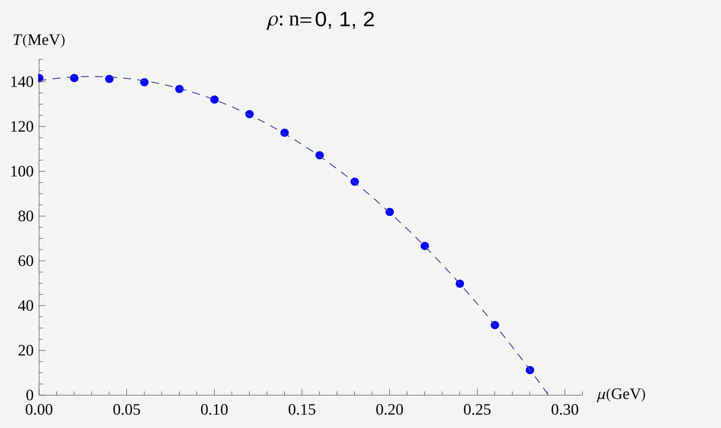
The position of the phase transition can be found from $\Delta\bar{S} = \bar{S}_{RN} - \bar{S}_{tc} = 0$

Setting a, b ⇒ position of z_h

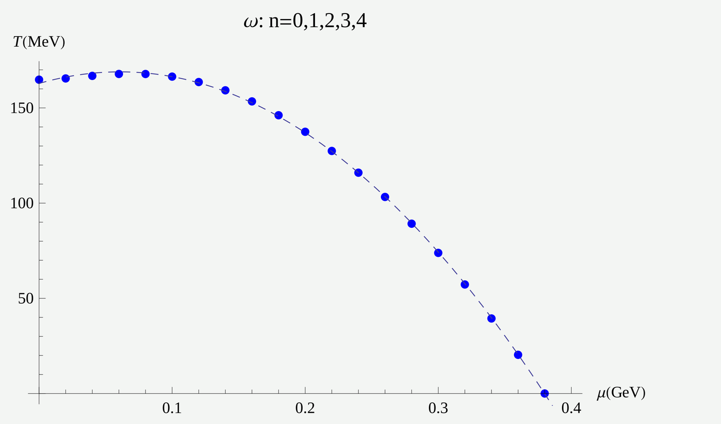
⇒ curve on the (T, μ) plane:

$$T = -\frac{1}{4\pi}\frac{\partial f_{RN}}{\partial z}\bigg|_{z=z_h} = \frac{1}{\pi z_h} - \frac{1}{3\pi}\frac{\kappa^2}{g_5^2 L^2}\mu^2 z_h$$

Phase diagrams on the (T, μ_B) plane with their endpoints ⟹



ρ meson, $n = 0,1,2$ – ($T_c = 143$ MeV, $\mu_B = 0$) and ($T = 0$, $\mu_B \simeq 0.9$ GeV).



ω meson, $n = 0,1,2,3,4$ – ($T_c = 166$ MeV, $\mu_B = 0$) and ($T = 0$, $\mu_B \simeq 1.1$ GeV).

Phase diagram endpoints: expectations

(I) T_c when T finite, $\mu_B = 0$
- → lattice with physical quarks[a]: 150 – 170 MeV;
- → lattice with non-dynamical quarks and $N_c \to \infty$[b]: ~ 250 MeV;
- → lattice for $SU(3)$ theory[c]: 260 – 270 MeV;
- → experimental results favour the range of 150 – 160 MeV.

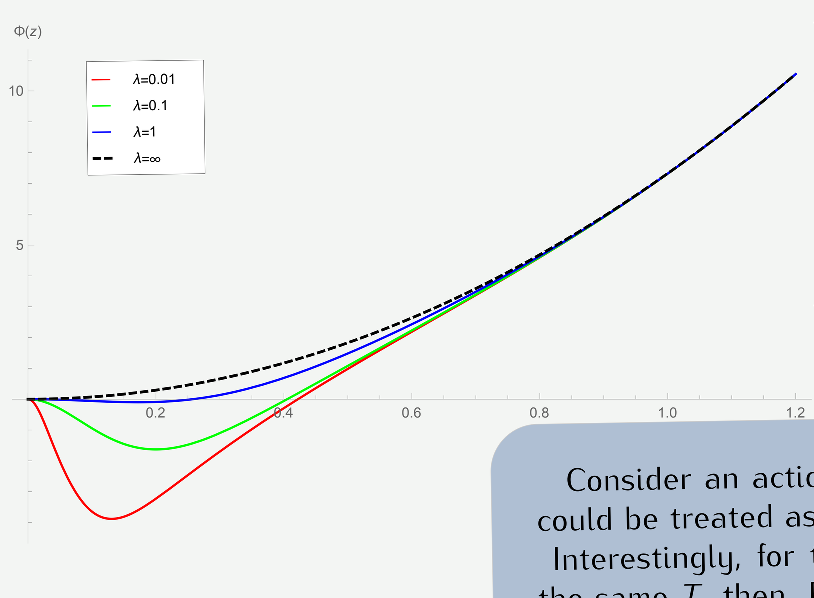
(II) μ_{Bc} when $T = 0, \mu_B$ finite
- → lattice simulations fail for medium and large values of μ_B;
- → μ_{Bc} about the nucleon mass;
- → experimental data converge to $\mu_{Bc} \simeq 1.1 \div 1.2$ GeV;
- → the Nambu–Jona-Lasinio model[d]: $\mu_{Bc} \simeq 1.05$ GeV.

[a] S. Borsanyi *et al.* [Wuppertal-Budapest Collab.], JHEP **1009**, 073 (2010).
[b] B. Lucini, A. Rago and E. Rinaldi, Phys. Lett. B **712**, 279 (2012)
[c] G. Boyd *et al.*, Nucl. Phys. B **469**, 419 (1996); Y. Iwasaki *et al.*, Nucl. Phys. Proc. Suppl. **53**, 429 (1997)
[d] S. P. Klevansky, Rev. Mod. Phys. **64**, no. 3 (1992).

Isospectral models

Consider an idea from SUSY quantum mechanics ⇒ a family of strictly isospectral potentials associated with a given potential for Schrödinger-type equations. The family members are distinguished through a parameter λ, with $\lambda = \infty$ corresponding to the original spectrum.

We can use this to modify the dilaton profile $\Phi(z)$, thus constructing new models while keeping the spectrum fixed.



These modifications affect the critical temperature greatly in AdS/QCD models for vector mesons.

For instance, for the "universal" slope fit: T_c changes from 263 MeV at $\lambda = \infty$ to 195 MeV at $\lambda = 1$ and 154 MeV at $\lambda = 0.01$.

The general tendency: as λ decreases T_c decreases also.

Consider an action for the scalar fields in this framework. This model could be treated as a holographic description of the scalar glueballs. Interestingly, for the whole isospectral family we'll get approximately the same T_c then. For example, take the lattice scalar glueball spectrum $M_n^2 = 4.5(n + 0.56)\text{GeV}^2$, then the predicted $T_c \simeq 139 \div 140$ MeV.

Conclusions

This work is devoted to the determination of properties of some AdS/QCD models on the (T, μ_B) plane within the bottom-up holographic approach.

- ▼ We have analysed SW and GSW model for the value of T_c. We wish to emphasize that from the point of view of the AdS/QCD ideology the predicted T_c must refer to the deconfinement phase transition in the pure gluodynamics (~ 250 MeV). The best prediction comes from the most general "universal" slope fit;
- ▼ In GSW the curve of phase transition becomes ambiguous mostly because of lack of sufficient amount of reliable experimental data on the radially excited light mesons. The use of well established states gives the phase structure similar to the experimental and lattice (with dynamical quarks) results.

Work in progress: isospectral AdS/QCD models.

Would they just help to better adjust to phenomenology or give deeper insightŇŃ?