

Controlling quark mass determinations non-perturbatively in $N_f = 3$ QCD

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in collaboration with

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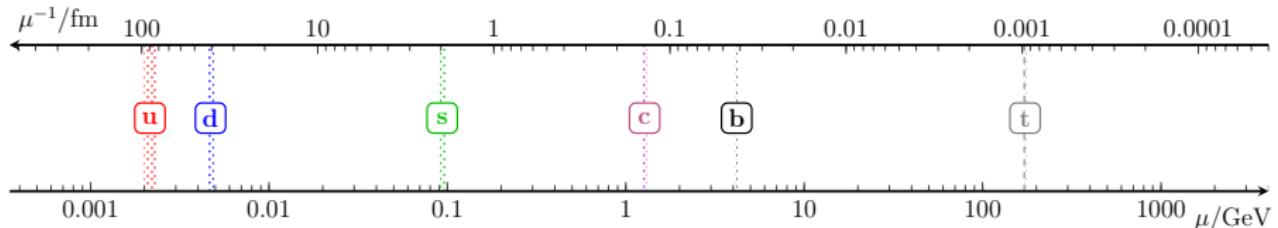


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Makedonia Palace conference centre, Thessaloniki, Greece
Aug 28 – Sep 4, 2016

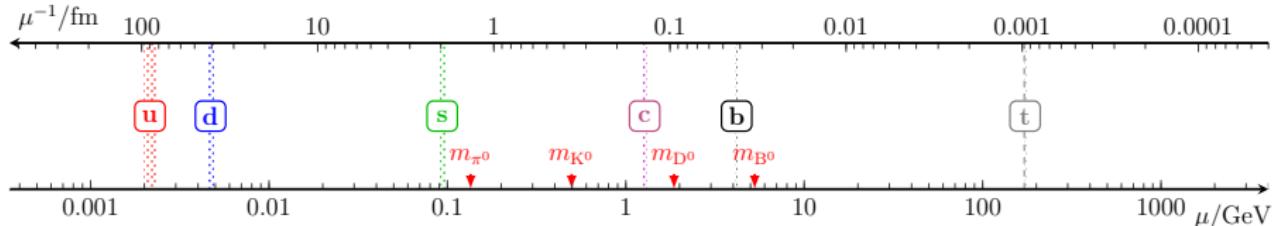
Multiple scales of QCD and Lattice simulations



■ a unique (bare) Lagrangian \mathcal{L}_{QCD} with input (at all scales)

$$g_0^2, m_u, m_d, m_s, m_c, \dots$$

Multiple scales of QCD and Lattice simulations



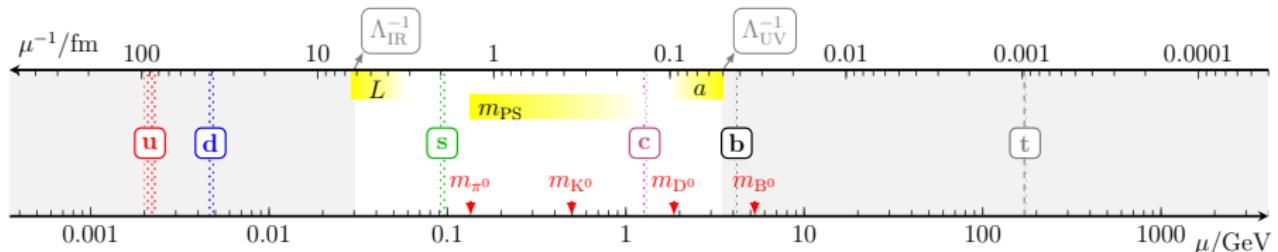
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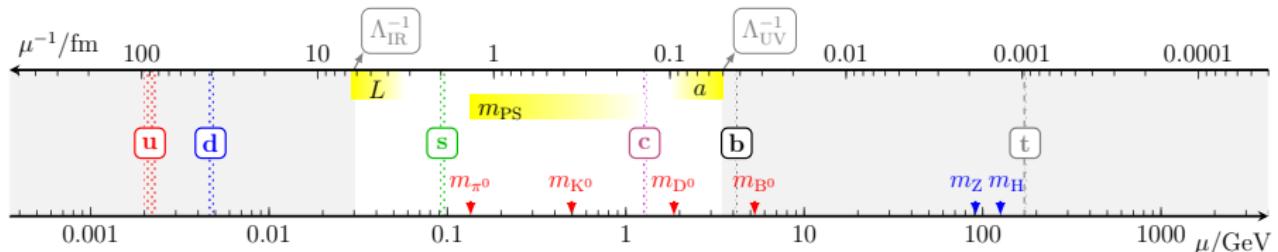
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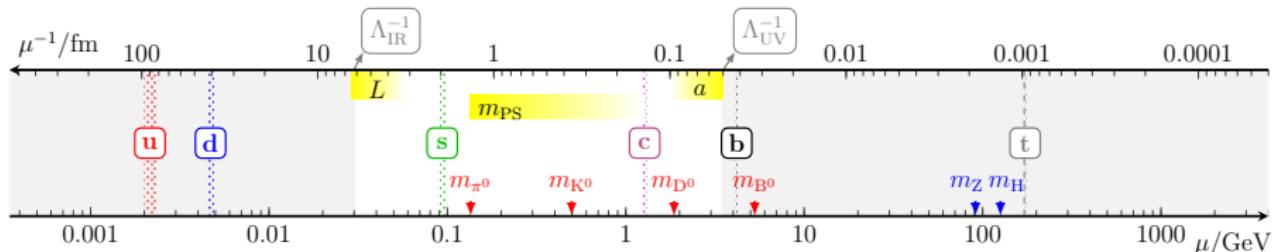
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 $\Lambda_{\text{IR}} \ll \mu_{\text{had}}, m_{\pi^0}, m_{K^0}, m_{D^0}, \dots \ll \Lambda_{\text{UV}}$

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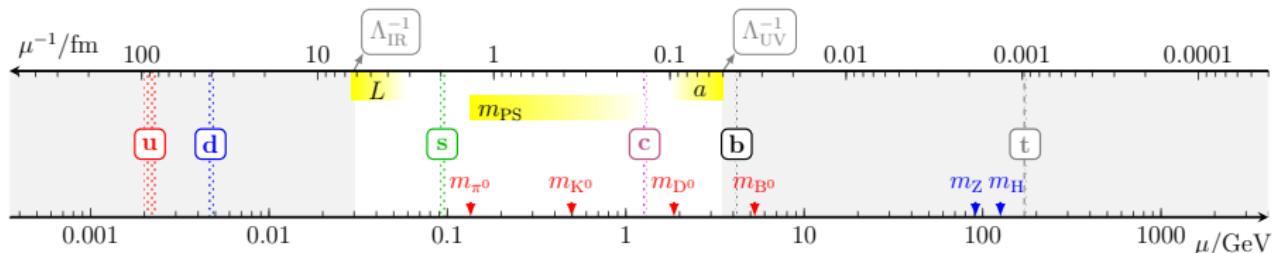
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- Non-perturbative renormalization: $\Rightarrow \bar{g}^2(\mu_{\text{had}}), \bar{m}_{u,d,s,\dots}(\mu_{\text{had}})$ with $\mu_{\text{had}} \ll m_Z$
- Issue: series truncation & asymptotic nature of pQCD !

QCD parameters and the renormalization group

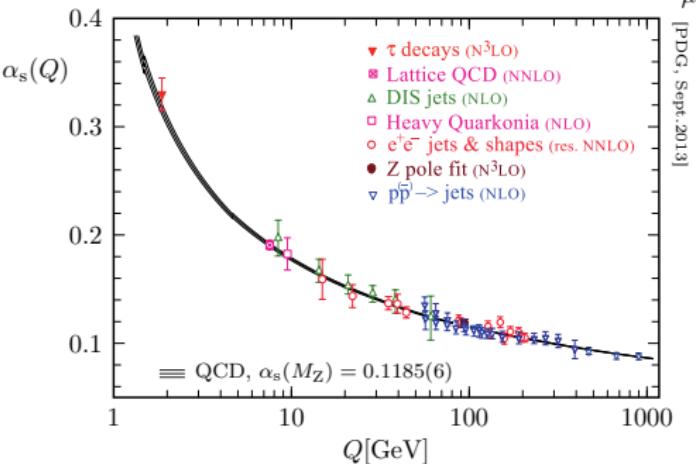


RG equations

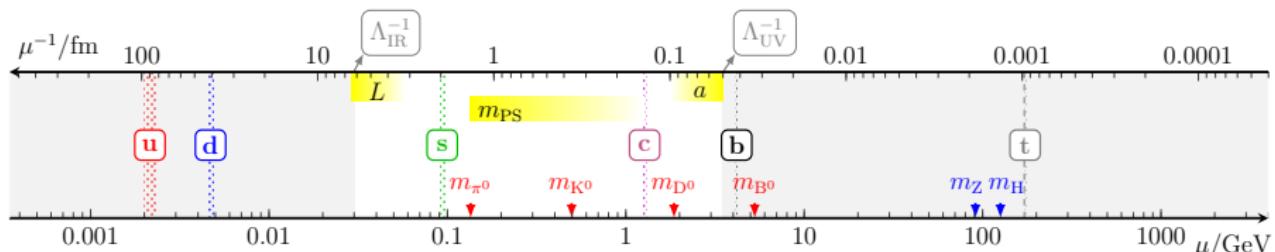
$$Q \frac{\partial}{\partial Q} \bar{g}(Q) \equiv \beta(\bar{g})$$

$$\frac{Q}{\bar{m}} \frac{\partial}{\partial Q} \bar{m}(Q) \equiv \tau(\bar{g})$$

■ are valid beyond PT



QCD parameters and the renormalization group

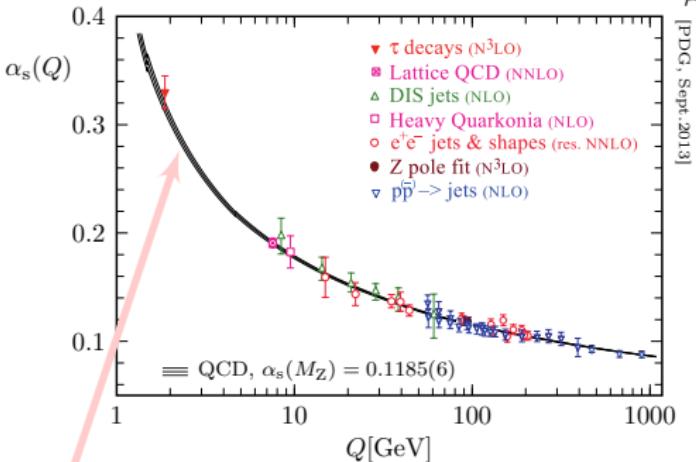


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■ are valid **beyond PT**



cannot control PT truncation, $\beta(g) \stackrel{\bar{g} \rightarrow 0}{\sim} -\bar{g}^3(b_0 + b_1\bar{g}^2 + b_2\bar{g}^4 + \dots)$, at low- Q
 ← non-perturbative effects (instantons, renormalons, ..., you name it)

QCD parameters and the renormalization group

$$\Lambda \equiv \mu [b_0 \bar{g}^2(\mu)]^{-b_1/(2b_0^2)} e^{-1/(2b_0 \bar{g}^2(\mu))} \exp \left\{ - \int_0^{\bar{g}(\mu)} dg \left[\frac{1}{\beta(g)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g} \right] \right\}$$

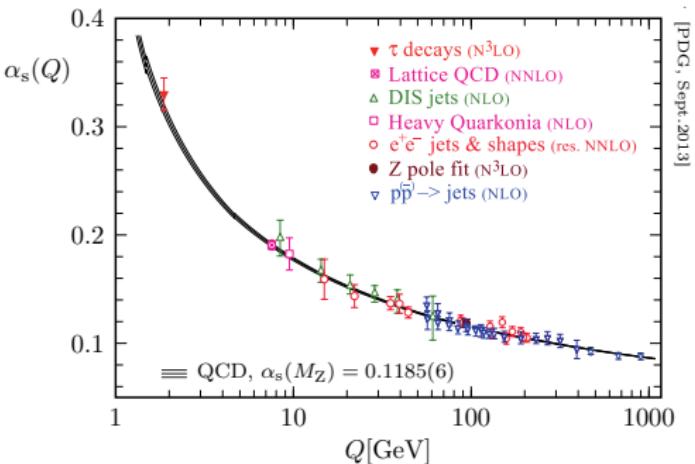
$$M_i \equiv \bar{m}_i(\mu) [2b_0 \bar{g}^2(\mu)]^{-d_0/(2b_0)} \exp \left\{ - \int_0^{\bar{g}(\mu)} dg \left[\frac{\tau(g)}{\beta(g)} - \frac{d_0}{b_0 g} \right] \right\}$$

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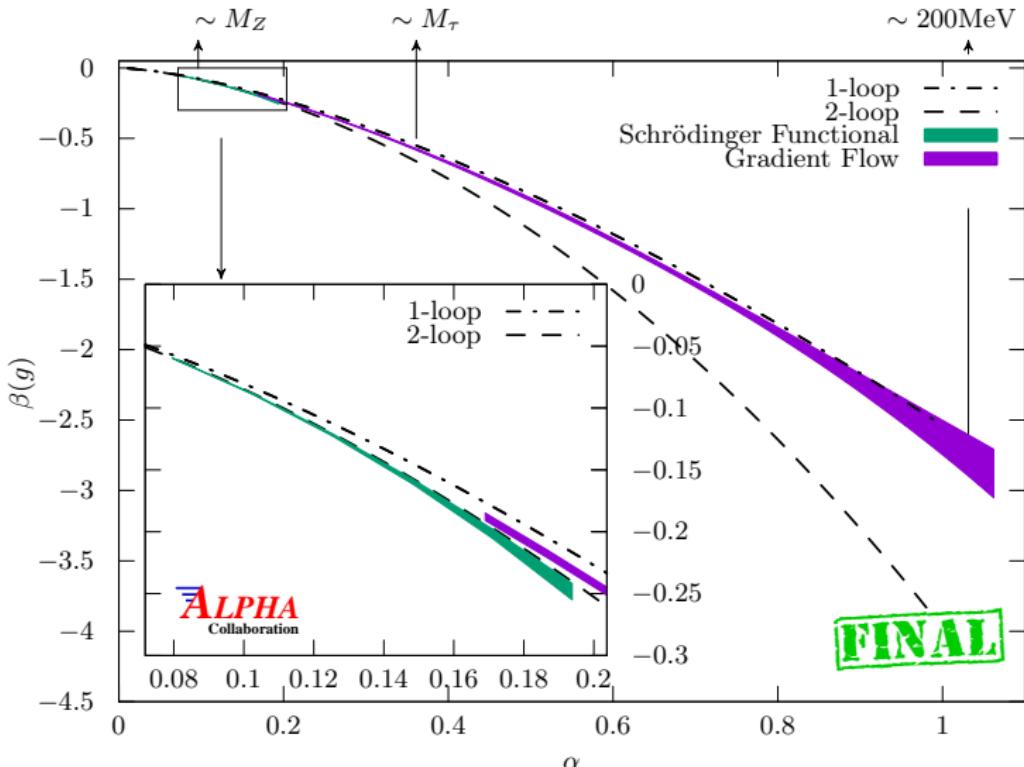
- are valid beyond PT
- formal solution: RGIs Λ, M



GOAL: fully non-perturbative determination of M_i

$\leftarrow \mu \geq \mu_{\text{had}}$

NON-PERTURBATIVE β -function(s)



ALPHA'2016 [1]

↪ talk by S. Sint

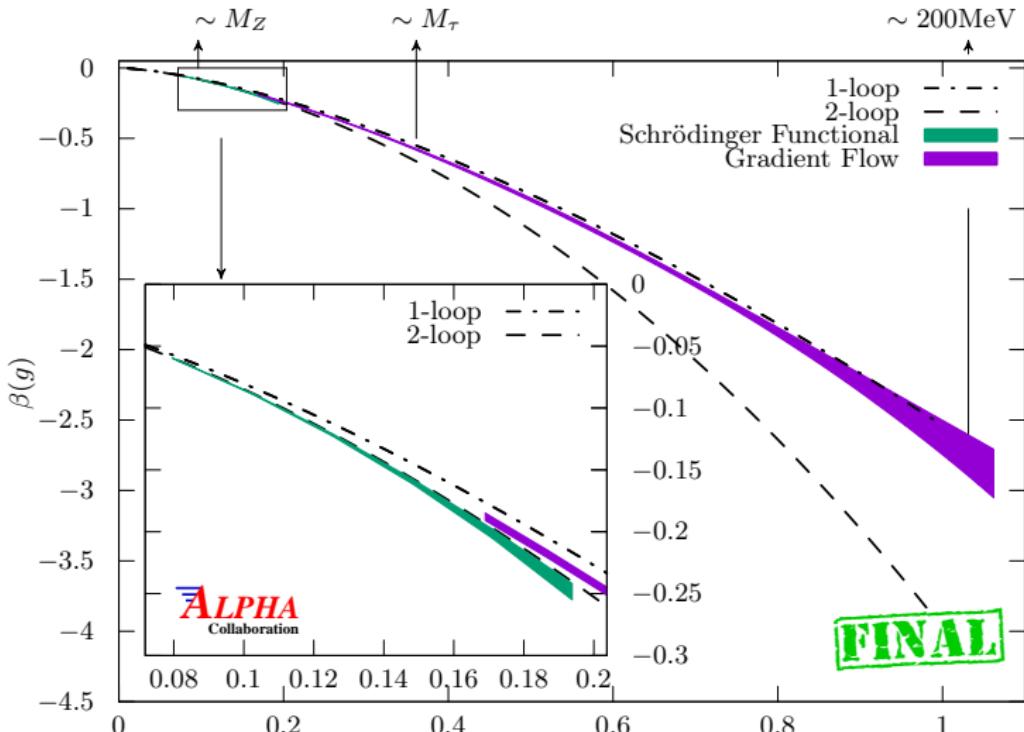
[Thu/Sect.E/16:00]

ALPHA'2016 [2]

↪ talk by M. Dalla Brida

[Thu/Sect.E/16:15]

NON-PERTURBATIVE β -function(s)



NOTE: • 2 non-perturbative couplings \Leftrightarrow 2 renormalization schemes
 • GF and SF schemes matched non-perturbatively at $\approx 4\text{ GeV}$

- 1 compute current quark mass at some hadronic scale μ_{had} , renormalize & take CL

$$\bar{m}_i(\mu_{\text{had}}) = \lim_{a \rightarrow 0} \left[\frac{Z_A(g_0^2)}{Z_P(g_0^2, a\mu_{\text{had}})} \frac{m_i}{f_{\text{had}}} \right] \times f_{\text{had}}^{\text{phys}}$$

using some scale-setting observable $f_{\text{had}} \in \{f_K, \dots\}$

- 2 connect to RGI mass

$$M_i = \frac{M}{\bar{m}(\mu_{\text{had}})} \times \bar{m}_i(\mu_{\text{had}})$$

- RG running factor to $\mu = \infty$ (continuum, flavour-independent)
- hadronic computation

- 3 convert to any scheme you want, say $\overline{\text{MS}}$

Tools:

- massless finite-volume renormal. scheme $\mu = 1/L$ (Schrödinger functional, SF)
- non-perturbative couplings: SF [3] and gradient flow (GF) [4]
- (continuum) recursive finite size scaling: $L \rightarrow sL \rightarrow s^2L \rightarrow \dots$ ($s = 2$)

Computing $M/\bar{m}(\mu_{\text{had}})$

Impose a **renormalization condition** for the *non-singlet pseudoscalar current* in L and $2L$:

- *Lattice step-scaling function:*

$$\Sigma_P(u, a/L) = \frac{Z_P(g_0, 2L/a)}{Z_P(g_0, L/a)}, \quad Z_P \propto Z_m^{-1}, \quad \begin{cases} \mu = 1/L & \text{fixed} \\ L/a & \text{varies} \end{cases}$$

- *Step-scaling function:*

$$\sigma_P(u) = \exp \left[- \int_{\bar{g}(\mu)}^{\bar{g}(\mu/2)} dg \frac{\tau(g)}{\beta(g)} \right]_{\bar{g}^2(\mu)=u} = \lim_{a \rightarrow 0} \Sigma_P(u, a/L)$$

- For the two schemes determine

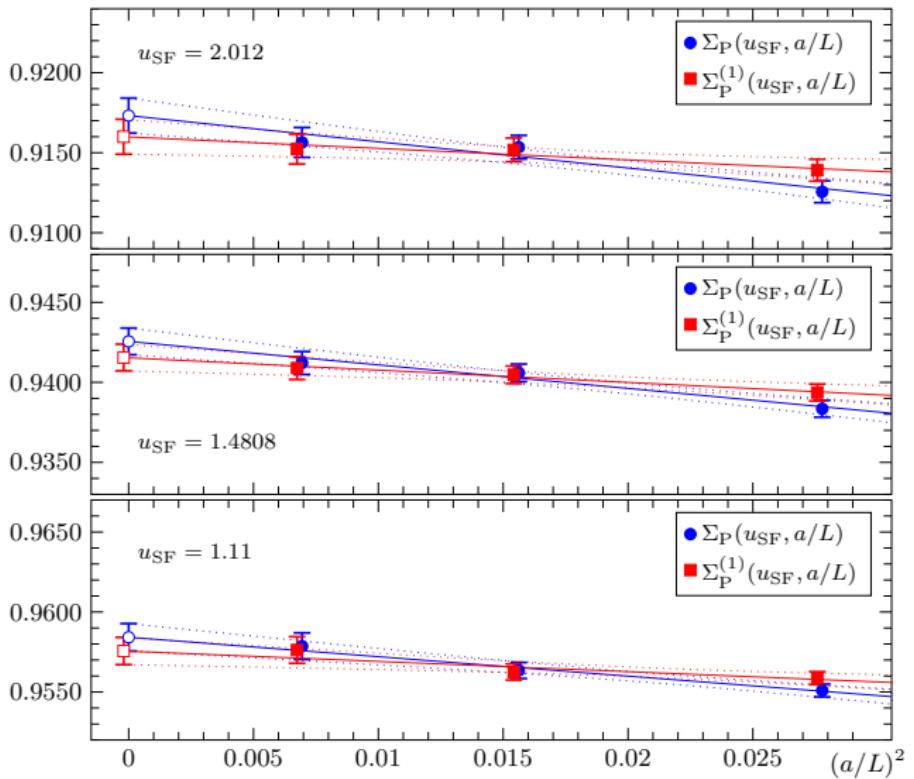
$$\frac{\bar{m}(\mu_{\text{swi}})}{\bar{m}(\mu_{\text{had}})} \propto \prod_i^N \sigma_P(u_i), \quad \frac{\bar{m}(\mu_{\text{pert}})}{\bar{m}(\mu_{\text{swi}})} \propto \prod_j^M \sigma_P(u_j)$$

- Combine everything

	GF-NP	SF-NP	SF-PT	
$\frac{M}{\bar{m}(\mu_{\text{had}})}$	$\frac{\bar{m}(\mu_{\text{swi}})}{\bar{m}(\mu_{\text{had}})}$	$\times \frac{\bar{m}(\mu_{\text{pert}})}{\bar{m}(\mu_{\text{swi}})} \times \frac{\textcolor{teal}{M}}{\bar{m}(\mu_{\text{pert}})}$		$\begin{cases} \mu_{\text{had}} & \approx 200 \text{ MeV} \\ \mu_{\text{swi}} & \approx 4 \text{ GeV} \\ \mu_{\text{pert}} & \approx 60 \text{ GeV} \end{cases}$

Step-scaling function I

PRELIMINARY



$$\sigma_P = 0.9173(11)(19)_{\text{sys}}$$

$$\sigma_P = 0.9160(11)(8)_{\text{sys}}$$

$$\sigma_P = 0.9426(8)(17)_{\text{sys}}$$

$$\sigma_P = 0.9416(8)(9)_{\text{sys}}$$

$$\sigma_P = 0.9584(9)(17)_{\text{sys}}$$

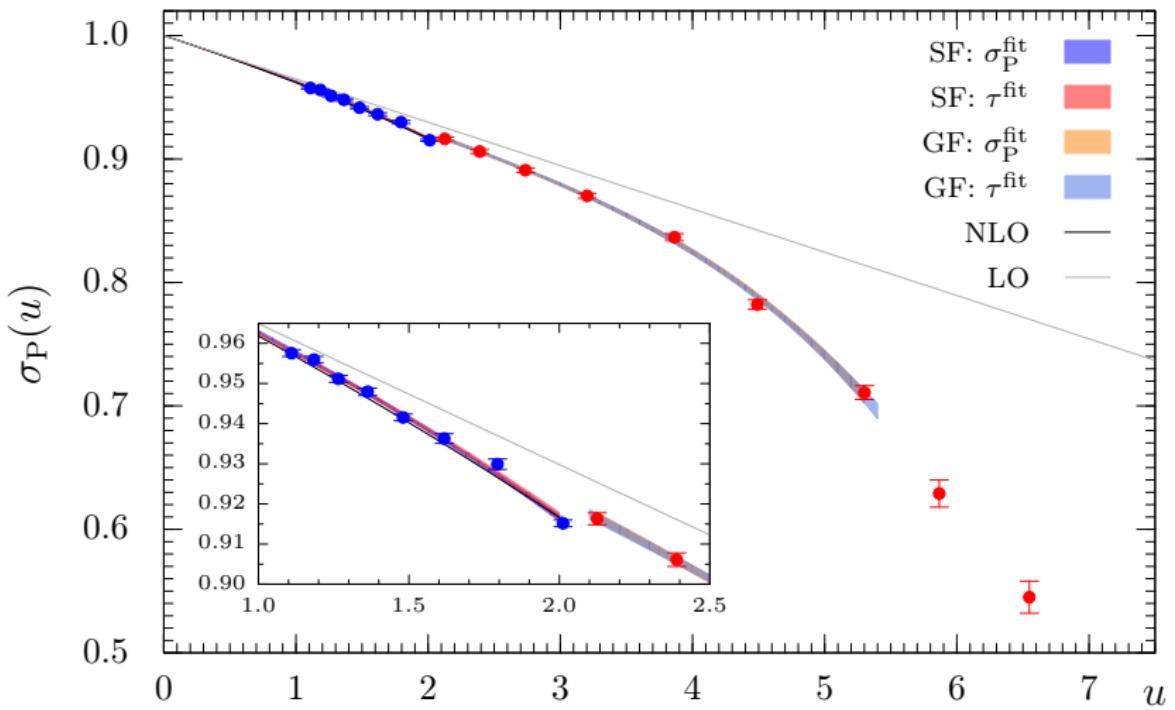
$$\sigma_P = 0.9576(9)(9)_{\text{sys}}$$

Step-scaling function II

PRELIMINARY

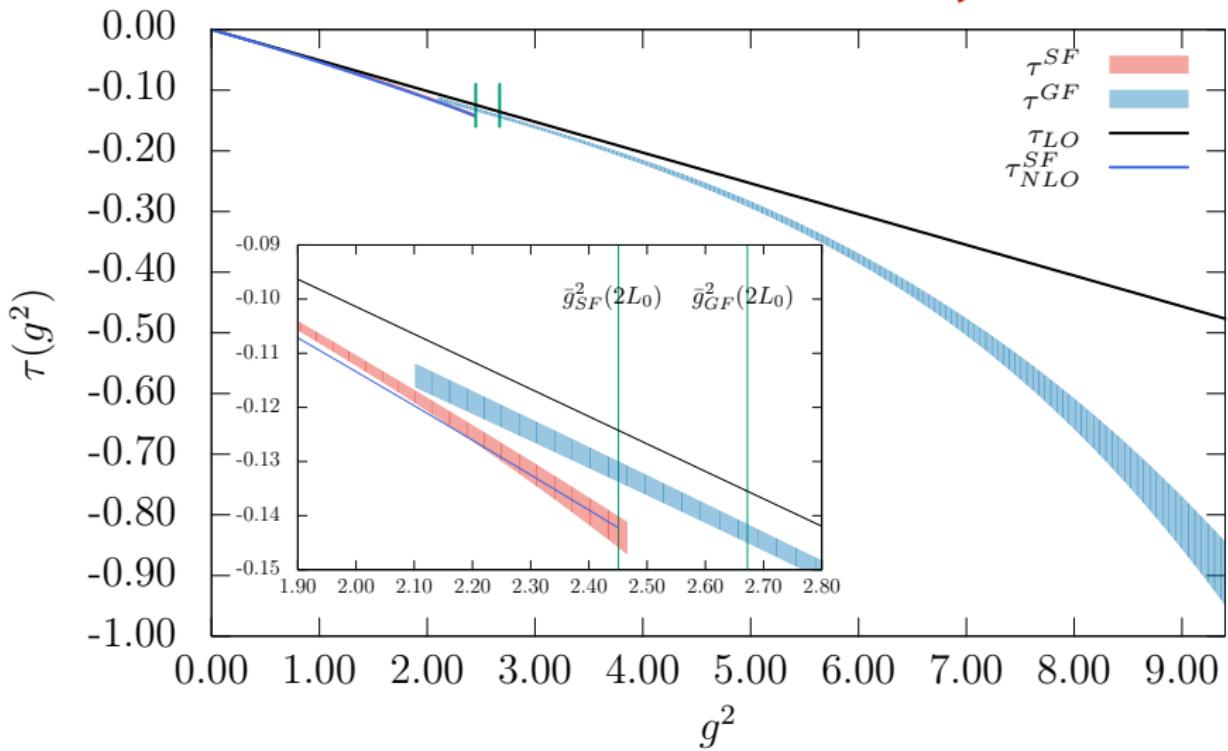
- statistical accuracy in continuum limit:
- current systematic uncertainties:

$$\Delta[\sigma_P] \lesssim 1\%$$
$$\delta_{\text{sys}} \lesssim 1\%$$



Non-perturbative mass anomalous dimension(s)

on(s)
PRELIMINARY ift



$$\mu_{\text{swi}} = 1/(2L_0)$$

Summary & Outlook

- We have computed the $N_f = 3$ running quark mass for $200 \text{ MeV} \lesssim \mu \lesssim 60 \text{ GeV}$ with overall non-perturbative uncertainty $\lesssim 1\%$.
- 1st time: running in 2 schemes, matched non-perturbatively (at $\mu \approx 4 \text{ GeV}$)
- 1st time: “effective” mass anomalous dimension extracted non-perturbatively, thanks to *high accuracy* of SF- and GF-based schemes in the high and low energy regime, respectively
- **Missing**: final analysis (more data at 2 strongest couplings + global fits)
- **Missing**: connection to spectrum (m_π, m_K) to determine quark masses m_{ud}, m_s

Future:

- renormalization group running of the non-singlet tensor current in $N_f = 3$ QCD along the same lines [6]

Take home message:

- we can assign a **trustworthy uncertainty** to the RG running from *first principles calculations*

$$\frac{M}{\bar{m}(\mu)} \Big|_{\mu \approx 213 \text{ MeV}} = 0.9088(78)$$

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THANK YOU FOR
YOUR ATTENTION!

APPENDIX

Renormalization Group Invariant (RGI) parameters



RG equations for running coupling and quark mass (mass-independent scheme)

$$q \frac{\partial}{\partial q} \bar{g}(q) = \beta(\bar{g}) \quad \bar{g} \xrightarrow{\sim 0} -\bar{g}^3(b_0 + b_1 \bar{g}^2 + b_2 \bar{g}^4 + \dots)$$

$$\frac{q}{\bar{m}} \frac{\partial}{\partial q} \bar{m}(q) = \tau(\bar{g}) \quad \bar{g} \xrightarrow{\sim 0} -\bar{g}^2(d_0 + d_1 \bar{g}^2 + \dots)$$

b_0, b_1, d_0 are universal and $b_{i>1}, d_{j>0}$ scheme-dependent coeff.s

integrated over renormalization scale $q \in [\mu, \infty]$

$$\Lambda \equiv \mu [b_0 \bar{g}^2(\mu)]^{-b_1/(2b_0^2)} e^{-1/(2b_0 \bar{g}^2(\mu))} \exp \left\{ - \int_0^{\bar{g}(\mu)} dg \left[\frac{1}{\beta(g)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g} \right] \right\}$$

$$M_i \equiv \bar{m}_i(\mu) [2b_0 \bar{g}^2(\mu)]^{-d_0/(2b_0)} \exp \left\{ - \int_0^{\bar{g}(\mu)} dg \left[\frac{\tau(g)}{\beta(g)} - \frac{d_0}{b_0 g} \right] \right\}$$

encode information about fundamental parameters of QCD

- defined without relying on perturbation theory
- RGI masses M_i independent of renormalization scheme, (Λ trivial dep.)
- allow for easy conversion (at high μ) between renorm. mass/coupling in diff. schemes

Finite renormalizations between massless schemes

Sint,Weisz [7]

Assuming a diagonal quark mass matrix in two massless schemes, one can write

$$\begin{aligned}\mu' &= c\mu, \quad c > 0, & g'_R &= g_R \sqrt{\mathcal{X}_g(g_R)}, \\ m'_{R,s} &= m_{R,s} \mathcal{X}_m(g_R), & s &= 1, \dots, N_f\end{aligned}$$

Invariance of a physical observable P under this change of variables implies

$$P'(\mu'(g_R), g'_R, \{m'_{R,s}(g_R, m_{R,s})\}) = P(\mu, g_R, \{m_{R,s}\}),$$

where P' satisfies the Callan-Symanzik equation in the primed scheme w.r.t.

$$\beta'(g'_R) = \left\{ \beta(g_R) \frac{\partial g'_R}{\partial g_R} \right\}_{g_R=g_R(g'_R)}$$

$$\tau'(g'_R) = \left\{ \tau(g_R) + \beta(g_R) \frac{\partial}{\partial g_R} \ln \mathcal{X}_m(g_R) \right\}_{g_R=g_R(g'_R)}$$

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- [3] M. Lüscher, R. Narayanan, P. Weisz and U. Wolff, *The Schrödinger functional: A Renormalizable probe for non-Abelian gauge theories*, *Nucl.Phys.* **B384** (1992) 168–228, [[hep-lat/9207009](#)].
- [4] P. Fritzsch and A. Ramos, *The gradient flow coupling in the Schrödinger Functional*, *JHEP* **1310** (2013) 008, [[1301.4388](#)].
- [5] S. Capitani, M. Lüscher, R. Sommer and H. Wittig, *Nonperturbative quark mass renormalization in quenched lattice QCD*, *Nucl.Phys.* **B544** (1999) 669–698, [[hep-lat/9810063](#)].
- [6] P. Fritzsch, C. Pena and D. Preti, *Non-perturbative renormalization of tensor bilinears in Schrödinger Functional schemes*, in *Proceedings, 33rd International Symposium on Lattice Field Theory (Lattice 2015): Kobe, Japan, July 14–18, 2015*, vol. LATTICE2015, p. 250, 2016. [1511.05024](#).
- [7] S. Sint and P. Weisz, *The Running quark mass in the SF scheme and its two loop anomalous dimension*, *Nucl.Phys.* **B545** (1999) 529–542, [[hep-lat/9808013](#)].