Emergent Higgsless Superconductivity

M. Cristina Diamantini

Nips laboratory, INFN and Department of Physics and Geology

University of Perugia

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Emergence

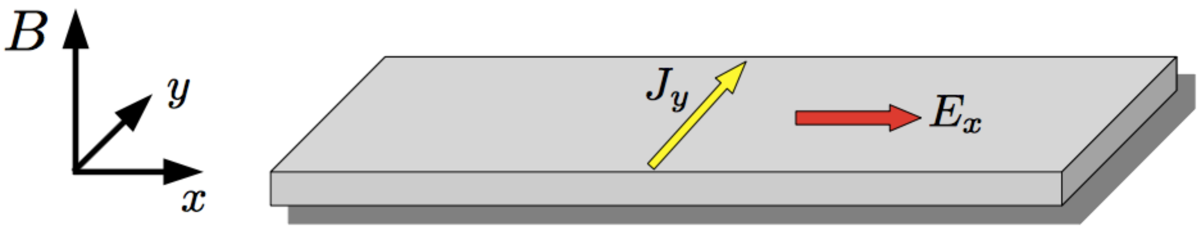
"The emergent is unlike its components insofar as ….it cannot be reduced to their sum or difference" *(G Lewes)*

→ II law of thermodynamics and irreversibility

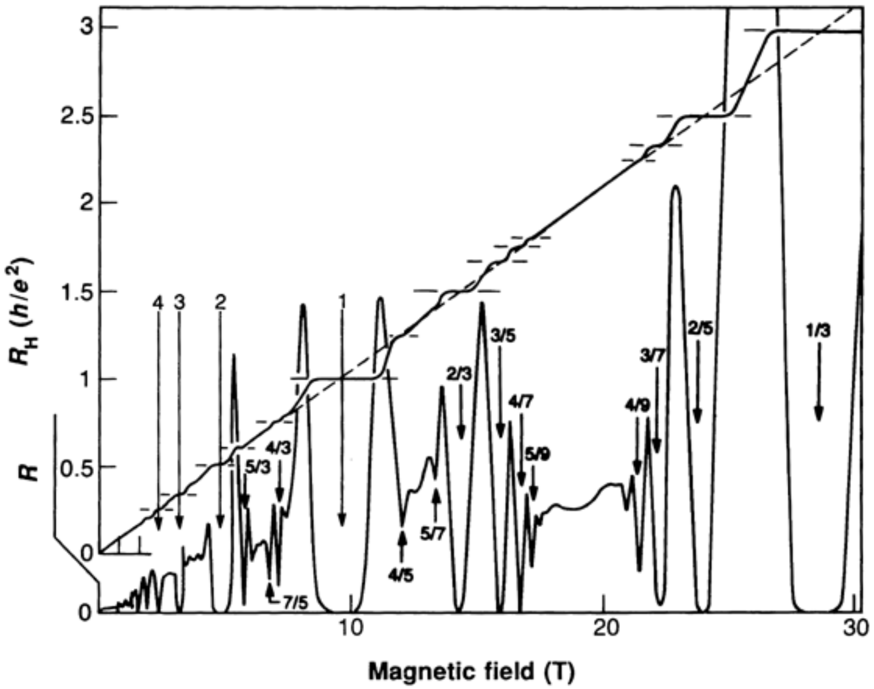
The fundamental laws can be very different from what we observe at low energy
e.g. QCD is very different from hadron physics
→ confinement emergent phenomenon (Maldacena's talk)

Emergent gauge symmetry: FQHE emergent Chern-Simons gauge theory, but the original model does not have the global symmetry associated with the emergent gauge theory
→ topological states of matter, new type of order not based on symmetry breaking
topological order (Wen)

QUANTUM HALL EFFECT: 2 dim electron gas at low temperature T ~ 10 mK and high magnetic field B ~ 10 Tesla



conductance tensor:
$J_i = \sigma_{ij} E_j$



Plateaux:

- $\sigma_{xx} = 0$ no longitudinal conductance → gap
- $\sigma_{xy} = \nu e^2 / h$; ν filling fraction

ν → integer (10^{-8})

ν → $= 1/m$; $m = 2k + 1$, k integer (10^{-6}) ($m = 2k+1/p$ Jain, p integer)

quantization independent of microscopic details:

universality → symmetry

universality → topology

FQE states:

- gapped in the bulk;
- gapless edge excitations.
- low energy effective field theory, CS:

  background independent;

  ground state degeneracy;

  quasiparticles have fractional charge and statistics → fractionalization;

  break P and T symmetries

Recently new topological phases of matter with time-reversal symmetry have been discovered in 2 and 3d (Kane, Mele, Fu, Moore, Balents, Zhang, König):

topological insulators:

- insulating in the bulk
- support conducting edge excitations
- topological BF theory

  ground state degeneracy
- fractionalization (FTI)

- Wen's idea: excitations over top. ground states described by conserved matter currents (2d):

$j_\mu \propto \epsilon_{\mu\nu\alpha}\partial_\nu b_\alpha$ conserved matter current, b_μ U(1) pseudovector gauge field

- topological field theory at low energy:

$S = \frac{k}{4\pi}\int d^3x\, b_\mu \epsilon_{\mu\nu\alpha}\partial_\nu b_\alpha$ → CS, P and T breaking

$S = \frac{k}{2\pi}\int d^3x\, a_\mu \epsilon_{\mu\nu\alpha}\partial_\nu b_\alpha$ → mixed CS, U(1) x U(1) , PT invariant if a_μ is a vector

$\phi_\mu \propto \epsilon_{\mu\nu\alpha}\partial_\nu a_\alpha$ conserved vortex current

- 3d: $S_{BF} = \frac{k}{2\pi}\int d^4x\, b_{\mu\nu}\epsilon_{\mu\nu\alpha\beta}\partial_\alpha a_\beta$ BF action

$j_\mu \propto \epsilon_{\mu\nu\alpha\beta}\partial_\nu b_{\alpha\beta}$ → charge fluctuations

$\phi_{\mu\nu} \propto \epsilon_{\mu\nu\alpha\beta}\partial_\alpha a_\beta$ → vortex fluctuations

- Key feature: existence of a gap

How can we get a mass for a U(1) gauge theory (2d) $\mathcal{L} = -\frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu}$?

Spontaneous Symmetry breaking

- Add $(D_\mu\phi) * (D^\mu\phi) - V(|\phi|)$
- unbroken vacuum: complex scalar field with two real massive degrees of freedom and the gauge field with one massless excitation;
- broken vacuum: massive gauge boson, two degrees of freedom; one massive chargeless Higgs boson
- PT invariant
- phenomenological theory for "standard" superconductivity

Topologically massive gauge theory (Deser, Jackiw, Templeton)

- Add $\frac{k}{4\pi} A_\mu \epsilon^{\mu\nu\alpha} \partial_\nu A_\alpha$
- equation of motion

  $$(\partial_\mu \partial^\mu + (ke^2)^2) F_\mu = 0$$

  which describe the propagation of a single degree of freedom with mass $m=ke^2$,
- gauge symmetry is not broken;
- Gauss law: $\partial_i E^i = mB$

  magnetic field acts as source for electric field;
- P and T breaking.
- leads to anyon superconductivity

SSB valid in any dimension, topological mechanism?

- BF theory

$$S_{TM} = \int_{M_{d+1}} \frac{k}{2\pi} a_1 \wedge db_{d-1} \; (S_{BF})$$

$$\frac{-1}{2e^2} da_1 \wedge *da_1 + \frac{(-1)^{d-1}}{2g^2} db_{d-1} \wedge *db_{d-1}$$

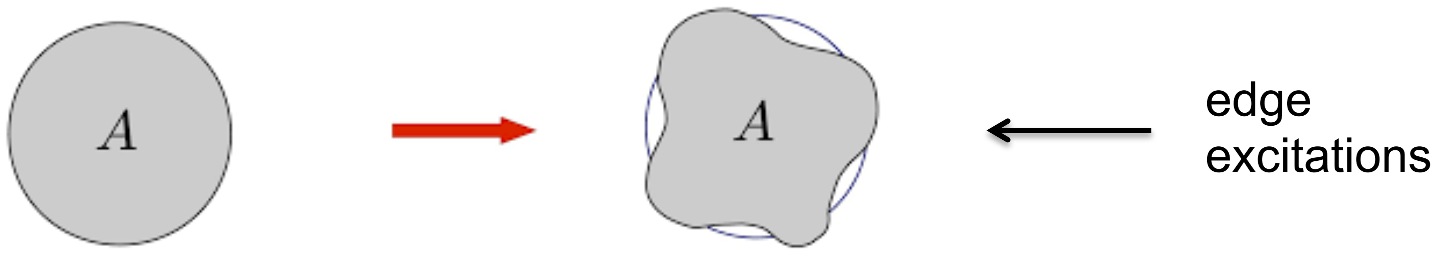
- mass m = (keg) / 2π
- PT invariant

Is ground state degeneracy characteristic only of topologically massive theories?

- BCS: gap arises from SSB $U(1) \rightarrow Z_N$, Z_2 for Cooper pairs
- residual Aharonov-Bohm interaction between charges and vortices (Bais et al.)
- effective BF theory with k = N , k=2 for Cooper pairs
- degeneracy 4 on the torus in (2+1) d (Hansson et al.)
  (same as topological model in (2+1) d)

gap → ground state has universal properties of an incompressible fluids
disc:

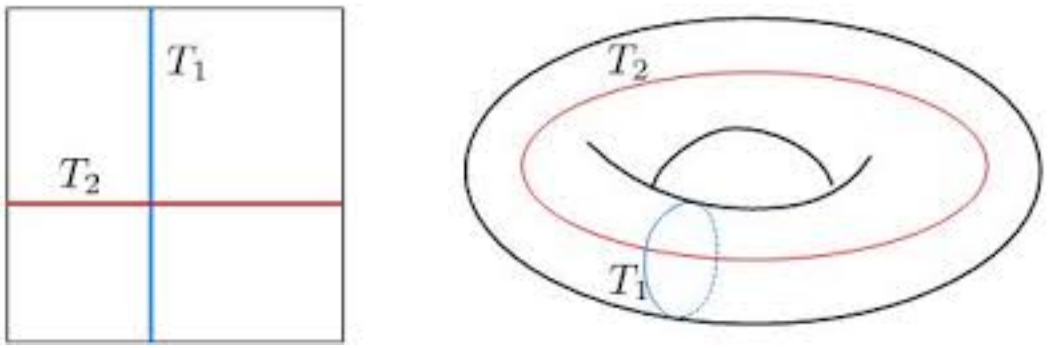
- edge excitations carries rep. of area preserving diffeomorphisms $W_{1+\infty}$



- ∞ dimensional algebra
- hierarchical minimal models as in CFT, characterized by c = m
- fractional charge and statistics
- c = 1 abelian excitations, Laughlin fluids
- Jain hierarchy of FQHE (Cappelli, Trugenberger, Zemba)
- c > 1 possible quark-like excitations with quantum numbers

U(1) x SU(m) ⟶ isospin
↓
charge

torus:



- Fairlie, Fletcher, Zachos   algebra on the torus (finite dimensional)
- incompressible fluids must fall into representation of this algebra
  → ground state degeneracy
- degeneracy is a kinematical consequence of incompressibility
- CS theory: generators of large gauge transformations can be identified with generators of $W_{1+\infty}$

Anyon Superconductivity:

$$\mathcal{L} = \psi^\dagger i\left(\partial_0 + i\alpha_0\right)\psi + \frac{1}{2m}\psi^\dagger\left(\partial_i + i\alpha_i\right)^2\psi + \frac{k}{4\pi}\alpha_\mu\epsilon^{\mu\nu\sigma}\partial_\nu\alpha_\sigma$$

$\rho \equiv \psi^\dagger\psi = \frac{k}{2\pi}\epsilon^{ij}\partial_i\alpha_j = \frac{k}{2\pi}\mathcal{B}$, eq. of motion for α_0

α_μ statistical gauge field

- average field approximation, role external magnetic field played by the statistical magnetic field, QHE
- allow the statistical gauge fields to fluctuate: gapless mode: anyon superfluid mode
- couple to e.m. field → mass to the photon via the mixed CS term, no Higgs boson
- pairing (BCS) → statistical transmutation
- PT breaking ⇒ different from Higgs mechanism

can we find an analogous mechanism with P and T symmetries and that also works in (3+1) dimensions?

how can we distinguish it from Higgs mechanism?

what is the origin of the emergent gauge field?

- Irreducible two-component spinors in 2d carry a pseudoscalar (vortex) degrees of freedom $S_z = \pm 1/2 \rightarrow$ spin up (down) electrons can be described by a statistical gauge field

$$a^{\pm}: (\phi^{\mu})^{\pm} = \frac{1}{2\pi}\epsilon^{\mu\nu\sigma}\partial_{\nu}a_{\sigma}^{\pm}$$

→ combine two spin states to get a PT invariant model

$$\mathcal{L} = \frac{1}{2\pi}a_{\mu}\epsilon^{\mu\nu\sigma}\partial_{\nu}b_{\sigma} + \frac{1}{2\pi}a_{\mu}\epsilon^{\mu\nu\sigma}\partial_{\nu}\alpha_{\sigma} + 2A_{\mu}j^{\mu}$$

- $\phi^{\mu} = \frac{1}{2\pi}\epsilon^{\mu\nu\sigma}\partial_{\nu}a_{\sigma}$: conserved vortex current
- $j^{\mu} = \frac{1}{2\pi}\epsilon^{\mu\nu\sigma}\partial_{\nu}b_{\sigma}$: conserved charge current

mutual CS between vortices and charges (unit 2) + mutual CS between charges and statistical gauge field :

- vortices fall in Landau level (dual, statistical field provides an emergent charge for vortices) and are gapped;
- intrinsic charges are gaped due to the mutual CS term
- combination corresponding to superfluid mode can move freely

gapped average statistical fields state:

- anyon superconductivity → quantum hall state (charges are gapped)
- doubled anyon superconductivity → topological insulator (both charges and vortices are gapped)

physical content:

- gapless mode gauge field, charge 2 (like BCS)
- charged massive vector, 2 degrees of freedom, no Higgs boson
- photon acquires a mass trough CS mechanism → Meissner effects
- ground state degeneracy 4 on a torus ≡ BCS on a torus

is this mechanism genuinely different from SSB?

- same phenomenological low-energy structure
- only the origin of the gap (high-energy physics) can distinguish them
- here gapless mode intimately tied to the emergent statistical gauge field.

- 3d:

$$\mathcal{L} = \frac{1}{2\pi} a_\mu \epsilon^{\mu\nu\sigma\rho} \partial_\nu b_{\sigma\rho} + \frac{1}{2\pi} a_\mu \epsilon^{\mu\nu\sigma\rho} \partial_\nu \alpha_{\sigma\rho}$$

- $b_{\mu\nu}$ and the fictitious gauge $\alpha_{\mu\nu}$ are Kalb-Ramond antisymmetric (two-form) gauge field;

$j^\mu = \frac{1}{2\pi} \epsilon^{\mu\nu\sigma\rho} \partial_\nu b_{\sigma\rho}$ charge current;

$\phi^{\mu\nu} = \frac{1}{2\pi} \epsilon^{\mu\nu\sigma\rho} \partial_\sigma a_\rho$ current of vortex strings;

- $\alpha_{\mu\nu}$ provides a uniform Kalb-Ramond emergent charge attached to physical charges

- one gapless mode
- massive vector spin 1 particle
- no Higgs boson

What is the origin of the statistical emergent gauge field?

$$S_{BF} = \frac{k}{2\pi} \int_{M_{d+1}} a_1 \wedge db_{d-1}$$

gauge symmetry are compact ⇒presence of topological defects

- Q_1 closed electric loops
- M_{d-1} closed (d-1) magnetic branes (loops in (2+1) d)

electric condensation:

- Q_1 is promoted to a dynamical field over which one has to sum in the partition function

$$Z = \int \mathcal{D}a_1 \mathcal{D}b_{d-1} \mathcal{D}Q_1 \exp i \frac{k}{2\pi} \int_{M_{d+1}} (a_1 \wedge db_{d-1} + a_1 \wedge *Q_1)$$

- Q_1 is closed → $Q_1 = * d\alpha_{d-1}$
- summation over Q_1 → summation over α_{d-1}

$$S = \frac{k}{2\pi} \int_{M_{d+1}} \left(a_1 \wedge db_{d-1} + a_1 \wedge d\alpha_{d-1} \right)$$

- generalization to any dimensions of the mutual CS
- the origin of the fictitious emergent gauge field lies in the condensation of topological defects
- gap opened trough topological mechanism, no Higgs boson we call this superconductivity model "topological”

2d: this mechanism is explicitly realized in Josephson junction arrays (Sodano, Trugenberger, MCD)

3d:

- BF terms is the low-energy effective action for topological insulators, charges and vortices are completely screened in the bulk, metallic edge states

- if topological defects condense, topological insulators could turn into Higgsless topological superconductors in “conventional” superconductivity the photon acquires a mass due to SSB, what is the fate of the photon in this model?

effective action induced by the condensation of topological defects:

- couple charges and fluxes to em field
- Julia-Toulouse mechanism (Quevedo and Trugenberger): the condensation of topological defects in solid state media generates new hydrodynamical modes for the low-energy effective theory; these new modes are the long wavelength fluctuations of the continuous distribution of topological defects
- the Julia-Toulouse prescription is sufficient to fully determine the low-energy action due to the condensation of topological defects in generic compact antisymmetric field theories

dilute phase: integration over a_μ and $b_{\mu\nu}$

$$S(A) = \int d^4x \frac{1}{2}\left[\frac{1}{\lambda}\mathbf{E}^2 + \frac{1}{\eta}\mathbf{B}^2\right]$$

- the model describes a topological insulator (Moore)

- $1/\lambda$ electric permittivity; $\mu = \eta$ magnetic permeability

electric condensation:

magnetic topological excitations are dilute, the form that describes electric topological defects get promoted to a continuous two-form antisymmetric field $B_{\mu\nu}$

$$S_{\text{eff}}^{TS} = \int d^4x \; i\pi k \; B_{\mu\nu}\epsilon_{\mu\nu\alpha\beta}F_{\alpha\beta} + \frac{1}{4}F_{\mu\nu}F_{\mu\nu}$$

$$+ \int d^4x \; \frac{1}{12\Lambda^2} H_{\mu\nu\alpha}H_{\mu\nu\alpha}$$

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu} \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- Λ new mass scale ≈ average density of topological defects
- the Kalb-Ramond field embodies a single scalar degree of freedom that is "eaten" by the original photon, no SSB
- mass arises as a consequence of quantum mechanical condensation of topological excitations

magnetic condensation:

- electric topological excitations are dilute, the form that describes magnetic topological defects get promoted to a continuous two-form antisymmetric field $B_{\mu\nu}$

$$S_{\rm eff}^{TC} = \int d^4x \, \frac{1}{4} B_{\mu\nu} B_{\mu\nu} + \frac{1}{12\Lambda^2} H_{\mu\nu\alpha} H_{\mu\nu\alpha}$$

- $F_{\mu\nu}$ have been reabsorbed into the new field $B_{\mu\nu}$ by a gauge transformation: $B_{\mu\nu} \rightarrow B_{\mu\nu} + F_{\mu\nu}$
- the new degrees of freedom arising from the condensation of topological defect "eat" the original photon to become a massive ($m = \Lambda$) two-form Kalb-Ramond field: this is the Stückelberg mechanism which is dual to the Higgs mechanism, no SSB

- the Stückelberg mechanism implies that the massive Kalb-Ramond field couples to a $T_{\mu\nu}$:

$$\partial_\nu T_{\mu\nu} = j_\mu$$

$$S_{\text{eff}}^{TC} = \int d^4x \, \frac{1}{4} B_{\mu\nu} B_{\mu\nu} + \frac{1}{12\Lambda^2} H_{\mu\nu\alpha} H_{\mu\nu\alpha} + B_{\mu\nu} T_{\mu\nu}$$ ≈ confining string action

- Wilson loop order parameter $$W(C) = < \exp i \int_C dx^\mu A_\mu >$$

- Wilson loop become a Wilson surface order parameter:

$$W(C) = < \exp i \int_C dx^\mu A_\mu > \quad \rightarrow \quad W(S) = < \exp i \int_S dx^\mu \wedge dx^\nu B_{\mu\nu} >$$

- the mass term for the Kalb-Ramond fields gives

$$W(C) = \exp\left(-T A(S)\right)$$

A(S) = area of the surface S string tension T determined by the new mass scale $T \propto \Lambda^2$

- area-law for the Wilson loop order parameter → linear potential between charges → confinement
- topological matter with a compact BF term can realize U(1) confinement via the Stückelberg mechanism

(Trugenberger, MCD)

Thank you!!!!