Heavy-heavy and heavy-light quarks interactions generated by QCD vacuum

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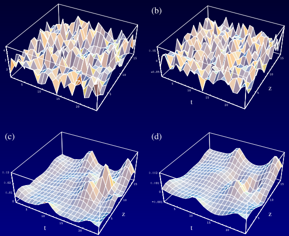
Outline

QCD vacuum. Action and topological charge densities in different configurations on the lattice

(Negele et al 1999).



⇒ liquid instanton model(Shuryak1981, Diakonov-Petrov1983).
Recent progress: BPRST instantons ⇒ KvBLL instantons in terms of the coordinates of the instanton-dyons can describe large instantons!! ⇒ Liquid dyon model(Shuryak et al 2015 also Shuryak-Larsen Conf12),
⇒ can describe confinement at temperature $T < T_c$ and deconfinement at $T > T_c$!!
Small size instantons still can be described in terms of their collective coordinates: positions and color orientations.

Instanton sizes vs heavy quarkonium

Very important: instanton size distribution function!!
KvBLL instanton average size $\rho \sim \frac{\alpha_s N}{2\pi \Lambda_{PV}} \sim 0.5\,\mathrm{fm}$ at $N = 3$, $\alpha_s = 0.5$, $\Lambda_{PV} = 200\,\mathrm{MeV}$ (Diakonov2009).
Instanton liquid model instanton average size $\rho \sim 0.3\,\mathrm{fm}$

State	J/ψ	χ_c	ψ'	Υ	χ_b	Υ'	χ_b'	Υ''
mass [Gev]	3.07	3.53	3.68	9.46	9.99	10.02	10.26	10.36
size r [fm]	0.25	0.36	0.45	0.14	0.22	0.28	0.34	0.39

Table : Quarkonium states and its sizes in non-relativistic potential model (see Satz2012).

Most essential corrections to the quarkonium properties from the instantons with sizes $\sim r =$ small instantons!!
We may safely apply instanton liquid model.

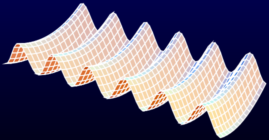
Instanton

Instantons –self-dual classical solutions of Y-M equations in Euclidean space (BPRST1975) = describe the tunneling processes between the C-S states as $N_{CS} \to N_{CS}+1$. Top. charge = 1. In singular gauge

$$A^a_{+,\mu}(x) = \frac{2\rho^2 \bar{\eta}^{\nu}_{\mu a}(x-z)_\nu}{(x-z)^2[\rho^2+(x-z)^2]},\ \ G^a_{\mu\nu} = \tilde{G}^a_{\mu\nu}.$$

collective coordinates ζ:

$$4\ (position)\ +\ 1\ (size)\ +\ (4N_c-5)\ (orientations)\ =\ 4N_c$$



Instanton liquid model and its main parameters

- Sum ansatz $A = \sum_{\pm} A_{\pm}$, $N_+ = N_- = N/2$.
- Average instanton size ρ and inter-instanton distance $R = (V/N)^{1/4}$;
- Estimates:
    - Lattice: $R \approx 0.89\, fm$, $\rho \approx 0.36\, fm$,
    - Phenomenological: $R \approx 1\, fm$, $\rho \approx 0.33\, fm$,
    - Our estimate with account of $1/N_c$ corrections: $R \approx 0.76\, fm$, $\rho \approx 0.32\, fm$, correspond ChPT $F_{\pi, m=0} = 88 MeV, \langle \bar{q} q \rangle_{m=0} = -(255 MeV)^3$

  Thus within $10 - 15\%$ uncertainty different approaches give similar estimates
- Packing parameter $\underline{\pi^2 (\frac{\rho}{R})^4 \sim 0.1 - 0.3}$
  $\Rightarrow$ Independent averaging over instanton positions and orientations.

Heavy quarks in instanton liquid

Definitions: $w = \int D\zeta(\theta^{-1} - iA_4)^{-1}$, $< t_2|\theta|t_1 >= \theta(t_2 - t_1)$.
Static heavy quark propagator is
$< T|w|0 >= \int D\zeta P \exp(i \int_L dx_4 A_4)$, $L = (\vec{x}, 0, T)$.
A solution of Pobylitca Eq. (DPP1989)

$$w^{-1} = \theta^{-1} - \frac{N}{2}\mathrm{tr}_c \sum_{\pm} \theta^{-1}(w_\pm - \theta)\theta^{-1} + O(N^2/V^2),$$

where $w_\pm = (\theta^{-1} - iA_{\pm,4})^{-1}$.
Instanton media contribution to the heavy quark mass

$$\Delta m_Q = 16\pi i_0(0)(\rho^4/R^4)\rho^{-1}/N_c, \quad i_0(0) = 0.55.$$

At $\rho = 0.33\,\mathrm{fm}, R = 1\,\mathrm{fm}$ $\Delta m_Q \approx 70\,\mathrm{MeV}$,
$\rho = 0.36\,\mathrm{fm}, R = 0.89\,\mathrm{fm}$ $\Delta m_Q \approx 140\,\mathrm{MeV}$
$\sim$ strength of a heavy quark-instanton interaction!!

Heavy quark-antiquark potential from Wilson loop

Averaged Wilson loop

$$< T|W|0 >= \int D\zeta P\exp(i\int_{L_1} dx_4 A_4)P\exp(i\int_{L_2} dx_4 A_4)$$

Here the lines $L_1 = (\vec{x}_1, 0, T), L_2 = (\vec{x}_2, T, 0)$.
In leading order on $\frac{N}{VN_c}$ static central potential (DPP1989)

$$V_C(r) = \frac{N}{2VN_c}\sum_{\pm}\int d^3 z_{\pm}\mathrm{tr}_c\left[1 - P\exp\left(i\int_0^T dx_4 A^{(1)}_{\pm,4}\right)\right.$$

$$\left.\times P\exp\left(-i\int_0^T dx_4 A^{(2)}_{\pm,4}\right)\right]_{z_{\pm,4}=0}$$

Spin-dependent parts of the potential

[U. Yakhshiev, Hyun-Chul Kim, B. Turimov, M.M., E. Hiyama [arXiv:1602.06074 [hep-ph]].

from $1/m_Q^2$ expansion of the heavy quark propagator (Eichten1980).

$$V(\vec{r}) = V_C(r) + V_{SS}(r)(\vec{S}_Q \cdot \vec{S}_{\bar{Q}}) + V_{LS}(r)(\vec{L} \cdot \vec{S})$$

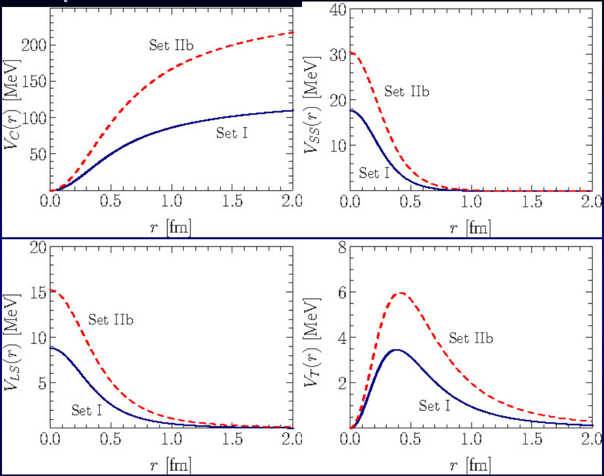
$$+ V_T(r)\left[3(\vec{S}_Q \cdot \vec{n})(\vec{S}_{\bar{Q}} \cdot \vec{n}) - \vec{S}_Q \cdot \vec{S}_{\bar{Q}}\right],$$

where

$$V_{SS}(r) = \frac{1}{3m_Q^2}\nabla^2 V_C(r), V_{LS}(r) = \frac{1}{2m_Q^2}\frac{1}{r}\frac{dV_C(r)}{dr},$$

$$V_T(r) = \frac{1}{3m_Q^2}\left(\frac{1}{r}\frac{dV_C(r)}{dr} - \frac{d^2V_C(r)}{dr^2}\right).$$

The potential



Solid curve ∼ Set I $\rho = 0.33$ fm and $R = 1\,\mathrm{fm}$ ∼ phenomenology (Shuryak1981, Diakonov-Petrov1983), Dashed one ∼ Set II $\rho = 0.36$ fm, $R = 0.89\,\mathrm{fm}$ ∼ lattice (Chu etal1994, Negele1998, DeGrand2001, Faccioli-DeGrand2003), with $1/N_c$ corrections (Goeke etal2007) $m_c = 1275$ MeV.

Charmonium states in [MeV]. $\Delta M_{c\bar{c}} = M_{c\bar{c}} - 2m_c$.

$\Delta M_{c\bar{c}}(J^P)$	Set I	Set IIb	Exp.
$\Delta M_{\eta_c}(0^-)$	118,81	203,64	$433,6 \pm 0.6$
$\Delta M_{J/\psi}(1^-)$	119,57	205,36	$546,916 \pm 0.11$
$\Delta M_{\chi_{c0}}(0^+)$	142,43	250,86	$864,75 \pm 0.31$

One can see that the instanton effects are not small $\sim 30 - 40\,\%$ in comparison with the experimental data and strongly depend on instanton liquid parameters. Since dyon liquid model pretend to describe QCD vacuum on the large distances too, it is natural to extend these calculations to the whole range of distances.

Light quarks in instanton liquid

Zero modes

$(\hat{p} + g\hat{A}_\pm)\Phi_{\pm,0}(x,\zeta_\pm) = 0$

$\Rightarrow Z[\xi^+,\xi] =$

$\int D\zeta \mathrm{Det}_{low}(\hat{p} + g\hat{A} + im) \exp\left(-\xi^+(\hat{p} + g\hat{A} + im)^{-1}\xi\right) =$

$$= \int D\zeta \prod_f D\psi_f D\psi_f^\dagger \exp \int \left(\psi_f^\dagger(\hat{p} + im_f)\psi_f + \psi_f^\dagger\xi_f + \xi_f^+\psi_f\right)$$

$$\times \prod_f \left\{ \prod_+^{N_+} V_{+,f}[\psi^\dagger,\psi] \prod_-^{N_-} V_{-,f}[\psi^\dagger,\psi] \right\}, \quad V_{\pm,f}[\psi^\dagger,\psi] =$$

$$= i \int dx \left(\psi_f^\dagger(x)\, \hat{p}\Phi_{\pm,0}(x;\zeta_\pm)\right) \int dy \left(\Phi^\dagger_{\pm,0}(y;\zeta_\pm)(\hat{p}\,\psi_f(y)\right).$$

$\psi^\dagger, \psi$ are constituent quarks.

Small packing parameter $\Rightarrow$ independent averaging:

$\overline{V_\pm[\psi^\dagger,\psi]} = \int d\zeta_\pm \prod_f V_{\pm,f}[\psi^\dagger,\psi]$

$\Rightarrow$ non-local ($\sim \rho$) t'Hooft-like vertex with $2N_f$-legs.

Spontaneous Breaking of the Chiral Symmetry (SBCS)

Partition function Z in saddle-point approximation (leading order on $1/N_c$) $\rightarrow$ effective coupling λ and vacuum average of scalar field σ_0 (SBCS) $\rightarrow$ Dynamical quark mass $M(p)$, where p-dependence from zero-mode Φ_0.
At $\rho = 0.33\,fm, R = 1\,fm$ $M(0) \approx 360$ MeV
$\sim$ strength of light quark-instanton interaction!!
Successful reproducing of quark condensate, pion and nucleon properties etc!! (see eg Diakonov2002).
Next to leading order $1/N_c$ corrections: most important are meson loops. Successful reproducing of Low Energy Constants of Chiral Perturbation Theory (Goeke etal 2007).

Heavy-light quarks interactions from instanton liquid [M.M. arXiv:1412.4472 [hep-ph]].

Account of light quarks: $D\zeta \Rightarrow D\zeta\,\mathrm{Det}_{low}(\hat{p} + g\hat{A} + im)$.

Heavy quark propagator is

$\int \prod_f D\psi_f D\psi_f^\dagger \exp \int \left(\psi_f^\dagger(\hat{p} + im_f)\psi_f\right) \prod_\pm \left(\overline{V_\pm[\psi^\dagger,\psi]}\right)^{N_\pm}$

$< T|w[\psi,\psi^\dagger]|0 >$,

where $w[\psi,\psi^\dagger] =$

$\prod_\pm \left(\overline{V_\pm[\psi^\dagger,\psi]}\right)^{-N_\pm} \int D\zeta\,(\theta^{-1} - iA_4)^{-1} \prod_f \prod_\pm^{N_\pm} V_{\pm,f}[\psi^\dagger,\psi]$

Solution of extended Pobilitca Eq. is $w^{-1}[\psi,\psi^\dagger] =$

$$= \theta^{-1} - \frac{N}{2}\sum_\pm \frac{1}{\overline{V_\pm[\psi^\dagger,\psi]}}\Delta_{H,\pm}[\psi^\dagger,\psi] + O(N^2/V^2),$$

$$\Delta_{H,\pm}[\psi^\dagger,\psi] = \int d\zeta_\pm \prod_f V_{\pm,f}[\psi^\dagger,\psi]\theta^{-1}(w_\pm - \theta)\theta^{-1}.$$

$\Rightarrow$ heavy (Q)-light quarks(ψ) interaction term

$$S_{Q\psi} = -\lambda \sum_\pm Q^\dagger \Delta_{H,\pm}[\psi^\dagger,\psi]Q$$

Heavy–light quarks interactions at light quark number $N_f = 1$

is $S_{Q\psi} =$

$$= i \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \frac{d^3 q}{(2\pi)^3} (2\pi)^4 \delta^3(\vec{k}_2 + \vec{k}_1 - \vec{q}) \delta(k_{2,4} - k_{1,4})$$

$$(M(k_1)M(k_2))^{1/2} \Delta m_Q R^4 \frac{i_0(q\rho)}{i_0(0)} \left[\frac{2N_c^2 - N_c}{2N_c^2 - 2} \psi^+(k_1)\psi(k_2) Q^+ Q \right.$$

$$\left. + \frac{N_c^2 - 2N_c}{2N_c^2 - 2} (\psi^+(k_1) Q Q^+ \psi(k_2) + \psi^+(k_1)\gamma_5 Q Q^+ \gamma_5 \psi(k_2)) \right]$$

First term is heavy quark–light meson interaction term, while second and third terms – Qq mesons degenerated on parity. It is similar to (Chernyshev etal94,95).

Heavy quark-antiquark – light quarks interactions

Now averaged Wilson loop is given by

$\int \prod_f D\psi_f D\psi_f^\dagger \exp \int \left(\psi_f^\dagger(\hat{p} + im_f)\psi_f\right) \prod_\pm \left(\overline{V_\pm[\psi^\dagger,\psi]}\right)^{N_\pm}$

$\mathrm{tr} < T|W[\psi,\psi^\dagger]|0 >$

where

$< T|W[\psi,\psi^\dagger]|0 >=$

$\prod_\pm \left(\overline{V_\pm[\psi^\dagger,\psi]}\right)^{-N_\pm} \int D\zeta \prod_f \prod_\pm^{N_\pm} V_{\pm,f}[\psi^\dagger,\psi]$

$P\exp(i\int_{L_1} dx_4 A_4) P\exp(i\int_{L_2} dx_4 A_4)$

Solution of extended Pobilitca Eq.

$$W^{-1}[\psi,\psi^\dagger] = w_1^{-1}[\psi,\psi^\dagger](\times) w_2^{-1,T}[\psi,\psi^\dagger]$$
$$-\frac{N}{2}\sum_\pm \left(\overline{V_\pm[\psi^\dagger,\psi]}\right)^{-1} \int d\zeta_\pm \prod_f V_{\pm,f}[\psi_f^\dagger,\psi_f]$$
$$\left(\theta^{-1}\left(w_\pm^{(1)} - \theta\right)\theta^{-1}\right)(\times)\left(\theta^{-1}\left(w_\pm^{(2)} - \theta\right)\theta^{-1}\right)^T + O(\frac{N^2}{V^2})$$

where, superscript T means the transposition, $(\times)$ – tensor product

Heavy quark–antiquark potential V_{lq}, generated by light quarks ($N_f = 1$)

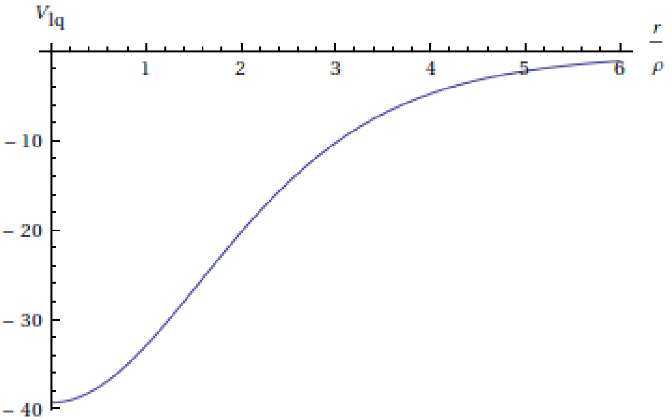
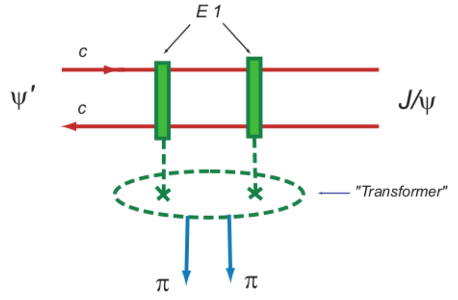


Figure : Heavy quark–antiquark potential $V_{lq}(r/\rho)$ (in MeV), generated by light quarks, at Set II $\rho = 0.36$ fm, $R = 0.89\,\mathrm{fm}$.

Quarkonium light hadron transitions.

Charmonium sizes $r_c \sim 0.4\,fm$, bottomonium sizes $r_b \sim 0.2\,fm$.
Hadronic transitions at the assumption $\lambda_g >> r_c, r_b \Rightarrow$ multipole expansion (see eg Voloshin12):



But $\lambda_g \approx \rho = 0.33\,fm \sim r_c, r_b$.
What are an instanton corrections?

Quarkonium pion transitions $(Q^+Q)' \to (Q^+Q)\ \pi\pi$

.

Essential part of heavy–light quarks interactions at $N_f = 2$ –the co-product of colorless heavy Q^+Q and light $\psi^+\psi$ quarks factors $\Rightarrow$ heavy quark–pion interaction action:

$$\begin{aligned} S_{Q\pi} &= -\frac{i}{2}\Delta m_Q R^4 f_{\pi Q}^2 \int d^4x \,\mathrm{tr}\, \partial_\mu U^\dagger(x)\partial_\mu U(x) \\ &\times \int e^{-ipx}\frac{d^4p_1}{(2\pi)^4}\frac{d^4p_2}{(2\pi)^4} i_0(p\rho)/i_0(0)Q^\dagger(p_2)Q(p_1) \end{aligned}$$

where $f_{\pi Q}^2 \simeq 0.3 f_\pi^2$ and pions $\vec{\phi}$ are given by $U = \exp(i\vec{\tau}\vec{\phi})$. Similar heavy quark-antiquark – pion interaction term $S_{QQ\pi}$. Both of these terms give a contribution to the quarkonium pion transitions and needs detailed investigations.

Discussion

- Instantons with sizes $\rho \sim$ heavy quarkonium sizes r give most essential contribution to their properties. In this case instanton liquid model is applicable.
- The strength of a heavy quark-instanton interaction is defined by $\Delta m_Q \sim packing\ \ parameter\ \ \rho^{-1}$,
  at $\rho = 0.33\, fm$, $R = 1\, fm$ $\Delta m_Q \approx 70\, MeV$.
- The analogous quantity for light quarks is
  $M \sim (packing\ \ parameter)^{1/2}\ \ \rho^{-1}$,
  at the same ρ, R $M \approx 360\, MeV$.
  Light quarks much more strongly interact with instantons due to zero-modes.
- Instantons naturally generate also heavy-light quarks interaction, which might be important for the heavy quarkonium and heavy-light quarks systems properties.
  It can be responsible for the SBCS effects in heavy quarks physics.

Future work

- Extend the calculations of heavy-heavy quarks potential to all distances within dyon model.
- Take into account light quarks in the observables of heavy quark physics:
  - in the heavy-heavy quarks potential;
  - quarkonium pion transitions;
  - heavy-light mesons etc.

Thank you for the attention.