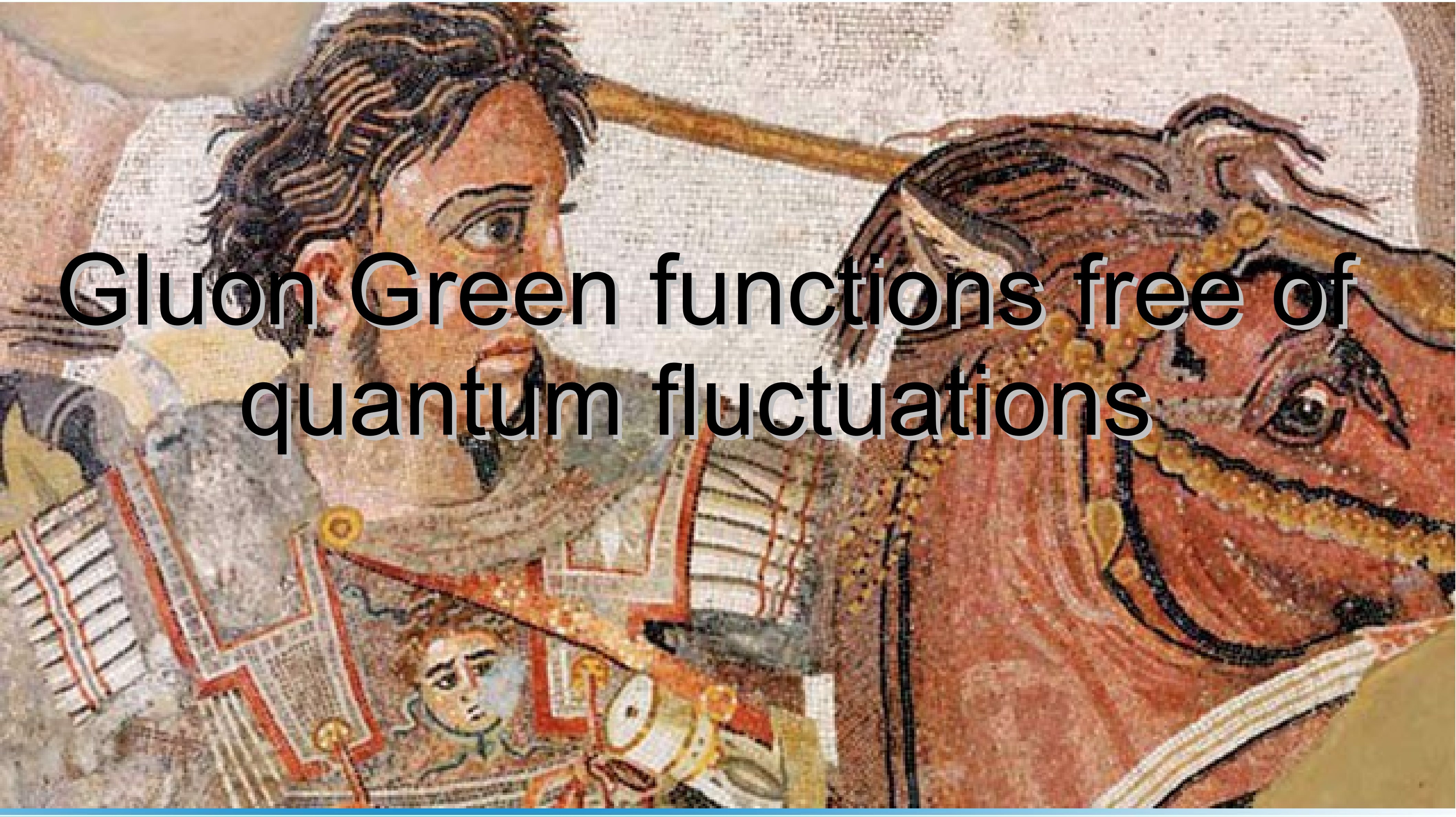
Gluon Green functions free of quantum fluctuations



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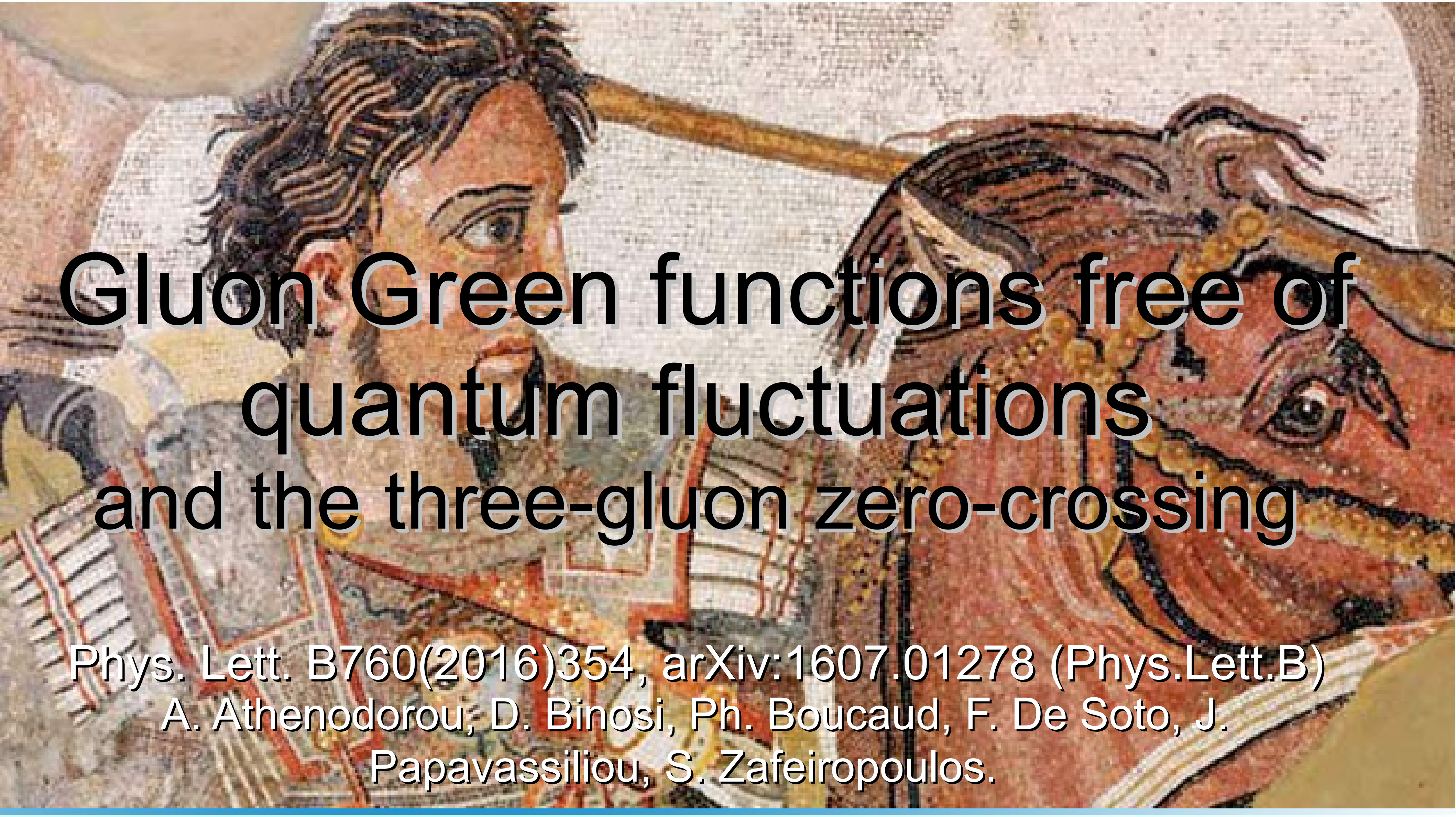
Confinement XII; Thesaloniki, 28 August-3 September



Gluon Green functions free of quantum fluctuations and the three-gluon zero-crossing

Phys. Lett. B760(2016)354, arXiv:1607.01278 (Phys.Lett.B)

A. Athenodorou, D. Binosi, Ph. Boucaud, F. De Soto, J. Papavassiliou, S. Zafeiropoulos.

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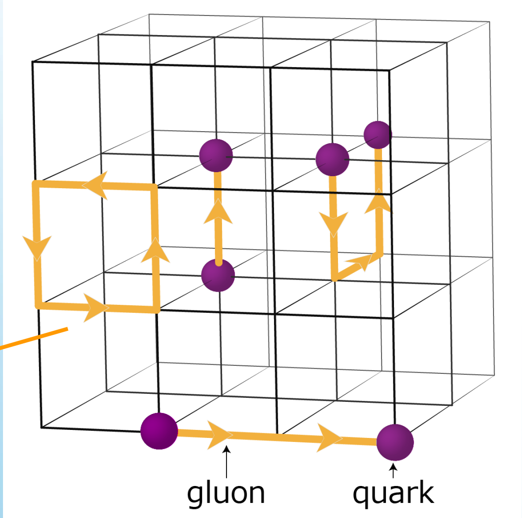
The vertex and the three-gluon Green's function

$$\mathcal{G}^{abc}_{\alpha\mu\nu}(q,r,p) = \langle A^a_\alpha(q) A^b_\mu(r) A^c_\nu(p)\rangle = f^{abc}\mathcal{G}_{\alpha\mu\nu}(q,r,p),$$

$$\widetilde{A}^a_\mu(q) = \frac{1}{2}\mathrm{Tr}\sum_x A_\mu(x+\hat{\mu}/2)\exp[iq\cdot(x+\hat{\mu}/2)]\lambda^a$$

$$A_\mu(x+\hat{\mu}/2) = \frac{U_\mu(x)-U^\dagger_\mu(x)}{2iag_0} - \frac{1}{3}\mathrm{Tr}\,\frac{U_\mu(x)-U^\dagger_\mu(x)}{2iag_0}$$



Tree-level Symanzik gauge action

$$S_g = \frac{\beta}{3}\sum_x\left\{b_0\sum_{\substack{\mu,\nu=1\\1\le\mu<\nu}}^{4}\left[1-\mathrm{Re}\,\mathrm{Tr}\,(U^{1\times1}_{x,\mu,\nu})\right] + b_1\sum_{\substack{\mu,\nu=1\\\mu\neq\nu}}^{4}\left[1-\mathrm{Re}\,\mathrm{Tr}(U^{1\times2}_{x,\mu,\nu})\right]\right\}$$

The gauge fields are to be nonperturbatively obtained from lattice QCD simulations and applied then to get the gluon Green's functions

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Symmetric configuration:

$q^2 = r^2 = p^2$ and $q\cdot r = q\cdot p = r\cdot p = -q^2/2$;

$$\mathcal{G}_{\alpha\mu\nu}(q,r,p) = g\Gamma_{\alpha'\mu'\nu'}(q,r,p)\Delta_{\alpha'\alpha}(q)\Delta_{\mu'\mu}(r)\Delta_{\nu'\nu}(p),$$

$$G_{\alpha\mu\nu}(q,r,p) = T^{sym}(q^2)\,\lambda^{tree}_{\alpha\mu\nu}(q,r,p) + S^{sym}(q^2)\,\lambda^{S}_{\alpha\mu\nu}(q,r,p)$$

$$\Gamma_{\alpha\mu\nu}(q,r,p) = \Gamma^{sym}_{T}(q^2)\,\lambda^{tree}_{\alpha\mu\nu}(q,r,p) + \Gamma^{sym}_{S}(q^2)\,\lambda^{S}_{\alpha\mu\nu}(q,r,p)$$

$$\lambda^{\rm tree}_{\alpha\mu\nu}(q,r,p) = \Gamma^{(0)}_{\alpha'\mu'\nu'}(q,r,p)P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p).$$

$$\lambda^{S}_{\alpha\mu\nu}(q,r,p) = (r-p)_\alpha(p-q)_\mu(q-r)_\nu/r^2.$$

$$\Delta^{ab}_{\mu\nu}(q) = \langle A^a_\mu(q)A^b_\nu(-q)\rangle = \delta^{ab}\Delta(p^2)P_{\mu\nu}(q),$$

where $P_{\mu\nu}(q) = \delta_{\mu\nu} - q_\mu q_\nu/q^2$, implies directly that $\mathcal{G}$ is totally transverse: $q\cdot\mathcal{G} = r\cdot\mathcal{G} = p\cdot\mathcal{G} = 0$.

In Landau gauge and for particular kinematical configurations, transversality and Bose symmetry make possible a simple tensorial decomposition of the gluon Green's function

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$$T^{\text{sym}}(q^2) = g\,\Gamma^{\text{sym}}_T(q^2)\,\Delta^3(q^2),$$
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$$W_{\alpha\mu\nu} = \lambda^{tree}_{\alpha\mu\nu} + \lambda^{S}_{\alpha\mu\nu}/2$$

$$T^{\rm sym}(q^2) = g\,\Gamma^{\rm sym}_T(q^2)\,\Delta^3(q^2),$$
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$$T^{\rm sym}(q^2) = \left.\frac{W_{\alpha\mu\nu}(q,r,p)\,\mathcal{G}_{\alpha\mu\nu}(q,r,p)}{W_{\alpha\mu\nu}(q,r,p)W_{\alpha\mu\nu}(q,r,p)}\right|_{\rm sym},$$

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$$T^{\rm asym}(r^2) = g\,\Gamma^{\rm asym}_T(r^2)\,\Delta(0)\,\Delta^2(r^2),$$

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Symmetric configuration:

$q^2 = r^2 = p^2$ and $q\cdot r = q\cdot p = r\cdot p = -q^2/2;$

$$\Delta_R(q^2;\mu^2) = Z_A^{-1}(\mu^2)\,\Delta(q^2),$$
$$T_R^{\rm sym}(q^2;\mu^2) = Z_A^{-3/2}(\mu^2) T^{\rm sym}(q^2),$$

$$g^{\rm sym}(q^2) = q^3 \frac{T^{\rm sym}(q^2)}{[\Delta(q^2)]^{3/2}} = q^3 \frac{T_R^{\rm sym}(q^2;\mu^2)}{[\Delta_R(q^2;\mu^2)]^{3/2}}.$$

MOM renormalization prescription:

$$\Delta_R(q^2;q^2) = Z_A^{-1}(q^2)\,\Delta(q^2) = 1/q^2,$$
$$T_R^{\rm sym}(q^2;q^2) = Z_A^{-3/2}(q^2)\,T^{\rm sym}(q^2) = g_R^{\rm sym}(q^2)/q^6.$$

$$T^{\rm sym}(q^2) = g\,\Gamma_T^{\rm sym}(q^2)\,\Delta^3(q^2),$$

$$\Delta^{ab}_{\mu\nu}(q) = \langle A^a_\mu(q) A^b_\nu(-q)\rangle = \delta^{ab}\Delta(p^2)P_{\mu\nu}(q),$$

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$$g^{sym}(\mu^2)\,\Gamma^{sym}_{T,R}(q^2;\mu^2) = \frac{g^{sym}(q^2)}{\left[q^2\Delta_R(q^2;\mu^2)\right]^{3/2}}$$

After the required projection and the appropriate renormalization, one can define a QCD coupling from the Green's functions, and relate it to the 1PI vertex form factor, in both symmetric...

The vertex and the three-gluon Green's function

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$$g^{\rm asym}(r^2) = r^3 \frac{T^{\rm asym}(r^2)}{[\Delta(r^2)]^{1/2}\Delta(0)} = r^3 \frac{T_R^{\rm asym}(r^2;\mu^2)}{[\Delta_R(r^2;\mu^2)]^{1/2}\Delta_R(0;\mu^2)}$$

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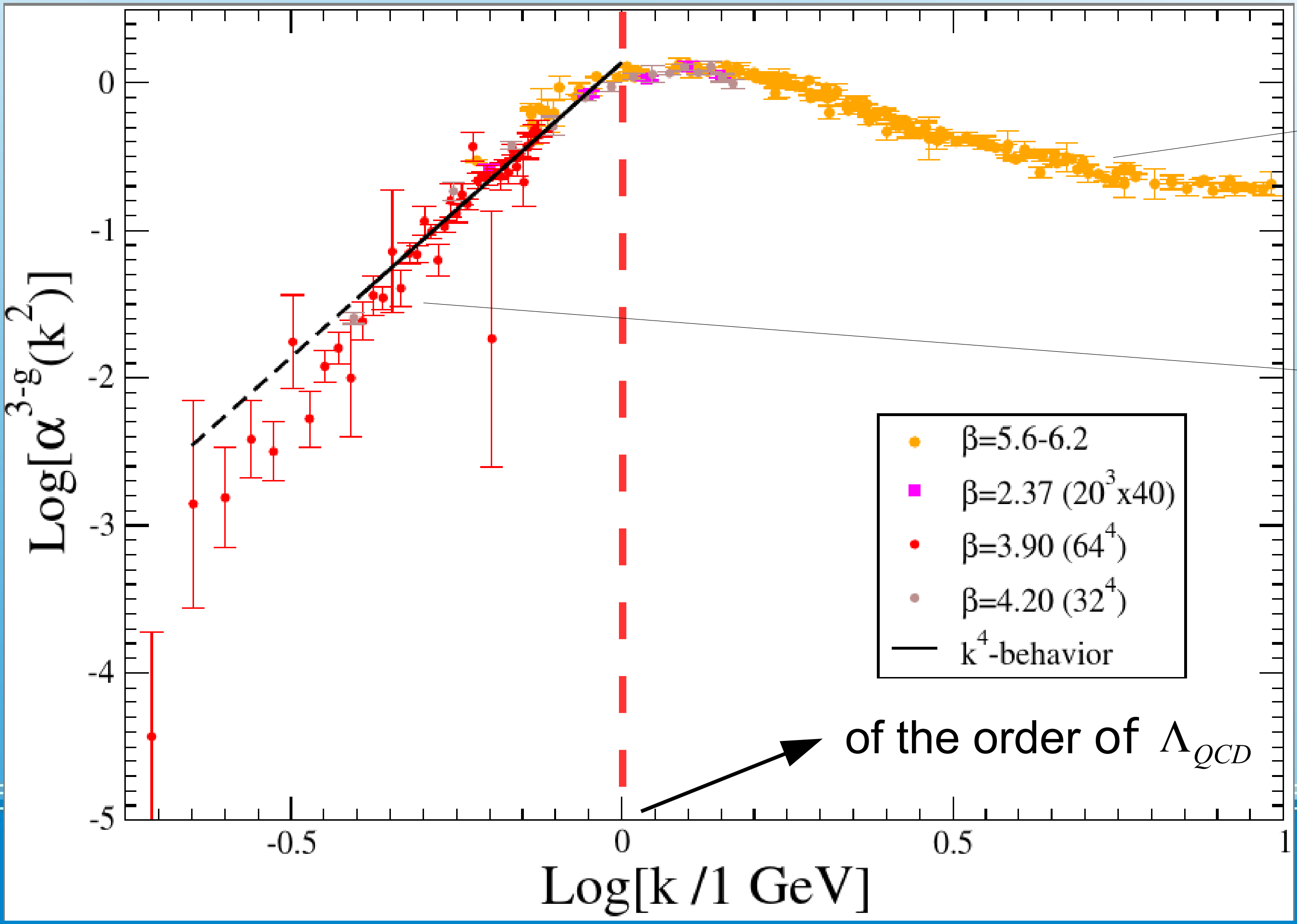
After the required projection and the appropriate renormalization, one can define a QCD coupling from the Green's functions, and relate it to the 1PI vertex form factor, in both symmetric and asymmetric kinematical configurations.

The vertex and the three-gluon Green's function

Let's focus on the symmetric coupling:

$$\alpha^{sym}(q^2) = \frac{\left(g^{sym}(q^2)\right)^2}{4\pi} = \frac{q^6}{4\pi} \frac{\left[T^{sym}(q^2)\right]^2}{\left[\Delta(q^2)\right]^3}$$



Logarithmic running accounted for by perturbation theory

A k^4 power law clearly appears to rise up from data within the IR domain

Can we somehow interpret this feature?

Two domains, wherein very different running behaviors appear to dominate each, lie separated by a momentum scale of the order of Λ_{QCD}

Multi-instanton background

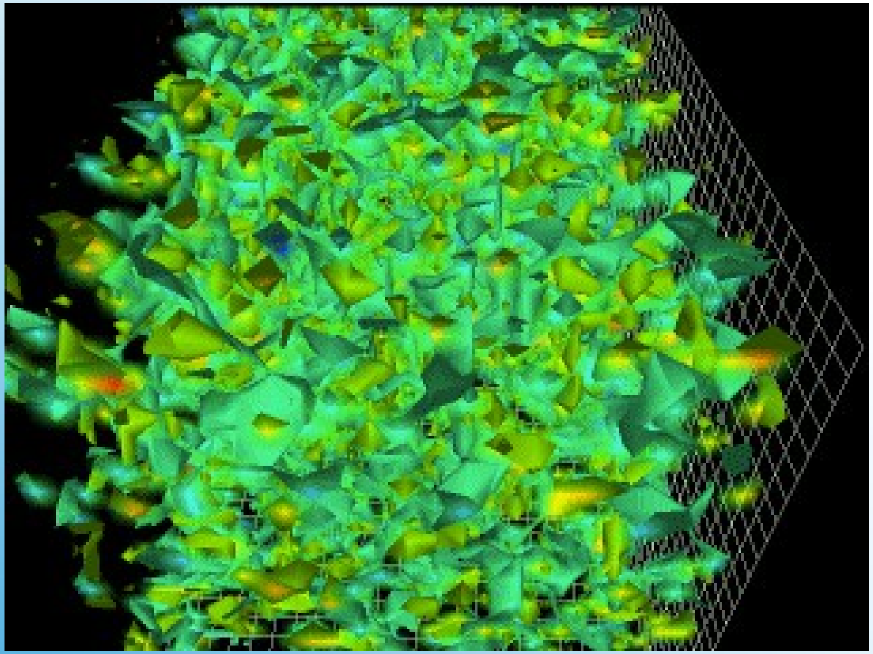
The classical gauge field solution from a multi-instanton ensemble can be cast as the so-called *ratio ansatz* [E.V. Shuryak; Nucl.Phys.B302(1988)574]

$$g_0 B^a_\mu(x) = \frac{2 \sum\limits_{i=I,A} R^{a\alpha}_{(i)} \bar{\eta}^{\alpha}_{\mu\nu} \frac{y_i^\nu}{y_i^2} \rho_i^2 \frac{f(|y_i|)}{y_i^2}}{1 + \sum\limits_{i=I,A} \rho_i^2 \frac{f(|y_i|)}{y_i^2}},$$

$$y_i = x - z_i$$

$\bar{\eta}_{\mu\nu}, R^{a\alpha}_{(i)}$ `t Hooft symbols and color rotation matrices

ρ_i instanton radius



http://www.physics.adelaide.edu.au/theory/staff/leinweber/VisualQCD/QCDvacuum/
"Visualizations of QCD" by Derek B. Leinweber

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$y_i = x - z_i$

$$\sim 2 \sum_{i=I,A} R^{a\alpha}_{(i)} \overline{\eta}^\alpha_{\mu\nu} \frac{y_i^\nu}{y_i^2} \rho_i^2 \frac{f(|y_i|)}{y_i^2} \qquad y_i >> \rho_i \text{ for all } i,$$

$$\sim 2 \sum_{i=I,A} R^{a\alpha}_{(i)} \overline{\eta}^\alpha_{\mu\nu} \frac{y_i^\nu}{y_i^2} \frac{f(|y_i|)}{f(|y_i|) + \frac{y_i^2}{\rho_i^2}} \qquad y_j << \rho_j, \; y_i >> \rho_i \text{ for any } i \neq j,$$

$\overline{\eta}_{\mu\nu}$, $R^{a\alpha}_{(i)}$ \`t Hooft symbols and color rotation matrices

ρ_i instanton radius

f(z) is a shape function [f(0)=1] that might be eventually obtained by minimization of the action per particle for some statistical ensemble of instantons (classical background).

Then:

$$g_0 B^a_\mu(\mathbf{x}) = 2 \sum_i R^{a\alpha}_{(i)} \overline{\eta}^\alpha_{\mu\nu} \frac{y_i^\nu}{y_i^2} \phi_{\rho_i}\left(\frac{|y_i|}{\rho_i}\right)$$

D. Diakonov, V. Petrov; Nucl.Phys.B45386(1992)236

Boucaud et al.; Phys.Rev.D70(2004)114503

$$\phi_\rho(z) = \begin{cases} \frac{f(\rho z)}{f(\rho z) + z^2} \simeq \frac{1}{1+z^2} & z \ll 1 \\ \frac{f(\rho z)}{z^2} & z \gg 1 \end{cases}$$

The classical gauge field can be effectively accounted for by an independent pseudo-particule sum ansatz approach in both large- and low-distance regimes.

Multi-instanton background

$$g_0^m G^{(m)}(k^2) = \frac{1}{N} W^{\mu_1 \ldots \mu_m}_{a_1 \ldots a_m} \langle g_0 A^{a_1}_{\mu_1}(k_1) \ldots g_0 A^{a_m}_{\mu_m}(k_m) \rangle$$

$$G^{(2)}(k^2) = \Delta(k^2);\; G^{(3)}(k^2) = T^{sym}(k^2)$$

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Multi-instanton background

Instanton density

$$g_0^m G^{(m)}(k^2) = \frac{1}{N} W^{\mu_1 \ldots \mu_m}_{a_1 \ldots a_m} \langle g_0 A^{a_1}_{\mu_1}(k_1) \ldots g_0 A^{a_m}_{\mu_m}(k_m) \rangle = \frac{k^{2-m}}{m 4^{m-1}} n \langle \rho^{3m} I^m(k\rho) \rangle$$

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$$I(s) = \frac{8\pi^2}{s} \int_0^\infty z dz \, J_2(sz) \phi(z)$$

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$$\alpha^{sym}(k^2) = \frac{k^6}{4\pi} \frac{\left[G^{(3)}(k^2)\right]^2}{\left[G^{(2)}(k^2)\right]^3} = \frac{k^4}{18\pi n} \frac{\langle \rho^9 I^3(k\rho) \rangle^2}{\langle \rho^6 I^2(k\rho) \rangle^3}$$

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$$I(s) = \frac{8\pi^2}{s} \int_0^\infty z dz \, J_2(sz) \phi(z)$$

$$\phi_\rho(z) = \begin{cases} \Phi_\rho(0) = 1 & z \ll 1 \\ \frac{f(\rho z)}{z^2} & z \gg 1 \end{cases}$$

$$\alpha^{sym}(k^2) = \frac{k^6}{4\pi} \frac{\left[G^{(3)}(k^2)\right]^2}{\left[G^{(2)}(k^2)\right]^3} = \frac{k^4}{18 \pi n} \frac{\langle \rho^9 I^3(k\rho) \rangle^2}{\langle \rho^6 I^2(k\rho) \rangle^3}$$

$$1 + \mathcal{O}\left(\frac{\delta\rho^2}{k^2 \bar{\rho}^4}\right)$$

where $\bar{\rho} = \sqrt{\langle \rho^2 \rangle}$ and $\delta\rho^2 = \langle (\rho - \bar{\rho})^2 \rangle$

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$$g_0^m G^{(m)}(k^2) = \frac{1}{N} W^{\mu_1 \dots \mu_m}_{a_1 \dots a_m} \langle g_0 A^{a_1}_{\mu_1}(k_1) \dots g_0 A^{a_m}_{\mu_m}(k_m) \rangle = \frac{k^{2-m}}{m 4^{m-1}} n \langle \rho^{3m} I^m(k\rho) \rangle$$

$$G^{(2)}(k^2) = \Delta(k^2); \; G^{(3)}(k^2) = T^{sym}(k^2)$$

$$g_0 B^a_\mu(\mathbf{x}) = 2 \sum_i R^{a\alpha}_{(i)} \overline{\eta}^\alpha_{\mu\nu} \frac{y_i^\nu}{y_i^2} \phi_{\rho_i}\left(\frac{|y_i|}{\rho_i} \right)$$

$$I(s) = \frac{8\pi^2}{s} \int_0^\infty z dz J_2(sz) \phi(z)$$

$$\phi_\rho(z) = \begin{cases} \frac{f(\dots}{f(\rho z) \dots} & z \ll 1 \\ \frac{f(\rho z \dots}{z^2} & z \gg 1 \end{cases}$$

$$\Phi_\rho(0) = 1$$

$$o(z^\infty)$$

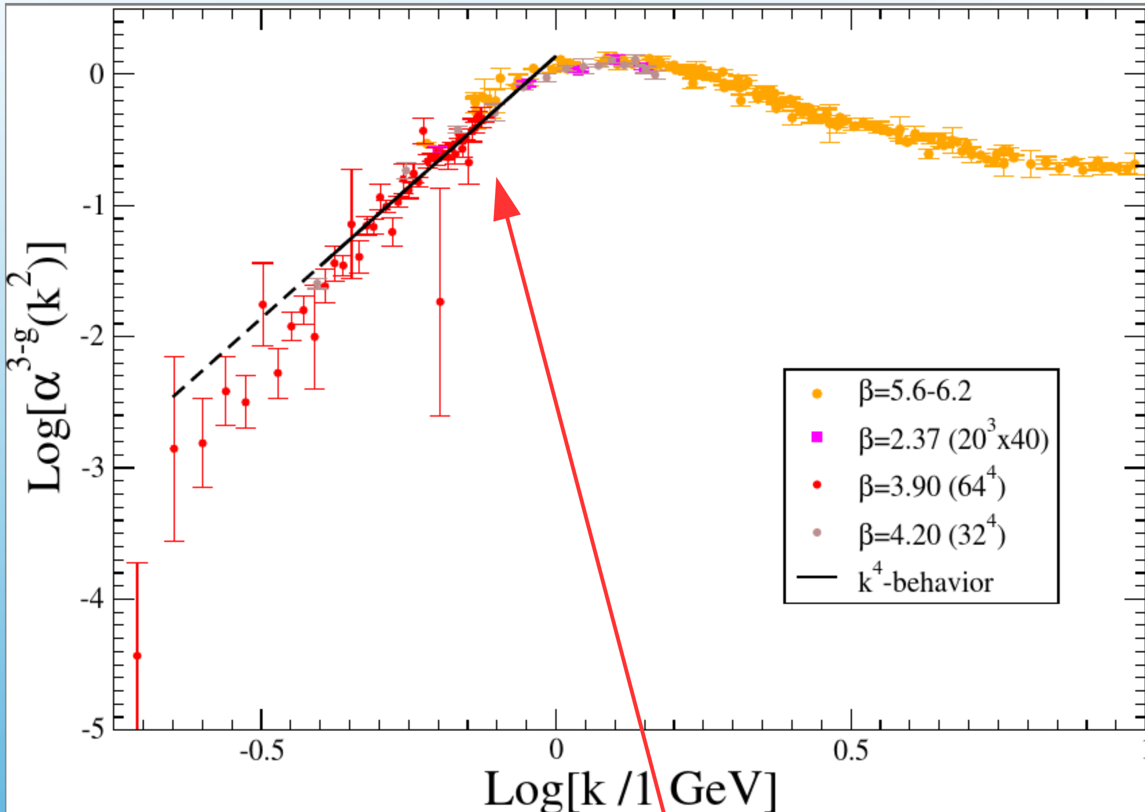
$$\alpha^{sym}(k^2) = \frac{k^6}{4\pi} \frac{\left[G^{(3)}(k^2)\right]^2}{\left[G^{(2)}(k^2)\right]^3} = \frac{k^4}{18\pi n} \frac{\langle \rho^9 I^3(k\rho) \rangle^2}{\langle \rho^6 I^2(k\rho) \rangle^3}$$

$$\begin{cases} 1 + \mathcal{O}\left(\frac{\delta\rho^2}{k^2 \bar{\rho}^4} \right) \\ 1 + 48 \frac{\delta\rho^2}{\bar{\rho}^2} + \mathcal{O}\left(k^2 \delta\rho^2, \frac{\delta\rho^4}{\bar{\rho}^4} \right) \end{cases}$$

where $\bar{\rho} = \sqrt{\langle \rho^2 \rangle}$ and $\delta\rho^2 = \langle (\rho - \bar{\rho})^2 \rangle$

The asymptotic behavior at both the large- and low-momentum limits appears to be driven by the fourth power of the momentum, the result relying on a very general ground, irrespective of the details of the profile and its breaking of the scale independence.

Multi-instanton background



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This is consistent with the low-momentum behavior obtained from the lattice gluon correlation functions!!!

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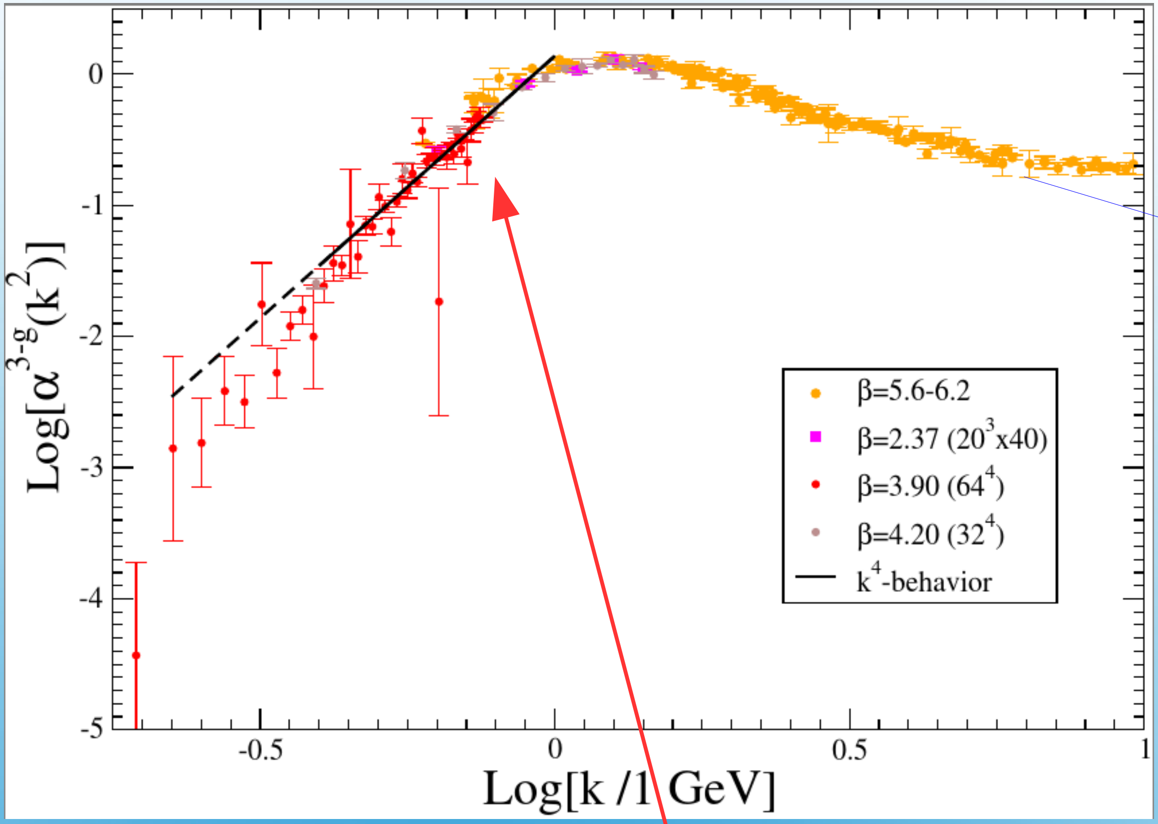

The large-momentum limit in the field of a multi-instanton solution appears here hidden by the quantum UV fluctuations!!!

$$\alpha^{sym}(k^2) = \frac{k^6}{4\pi} \frac{\left[G^{(3)}(k^2)\right]^2}{\left[G^{(2)}(k^2)\right]^3} = \frac{k^4}{18\pi n} \frac{\langle \rho^9 I^3(k\rho)\rangle^2}{\langle \rho^6 I^2(k\rho)\rangle^3}$$

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The Wilson flow smoothing procedure

The Wilson flow $B_\mu(t,x)$ of an SU(N) gauge field is defined by [M. Luescher; JHEP02(2010)071]

$$\partial_t B_\mu = D_\nu G_{\nu\mu}$$

where $t = a^2 \tau$ is the so-called flow time and

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu]$$
$$D_\mu = \partial_\mu + [B_\mu, \cdot\,]$$

with the initial condition $B_\mu(0,x) = A_\mu(x)$.

Then, the expansion in terms of $A_\mu(x)$ gives at tree-level:

$$B_\mu(t,x) = \int d^4 y\, K(t; x-y)\, A_\mu(x)$$

$$K(t;x) = \frac{e^{-x^2/4t}}{(4\pi t)^2}$$

The Wilson flow has been proven to be an useful tool to deprive the lattice gauge fields from their short-distance (UV) quantum fluctuations.

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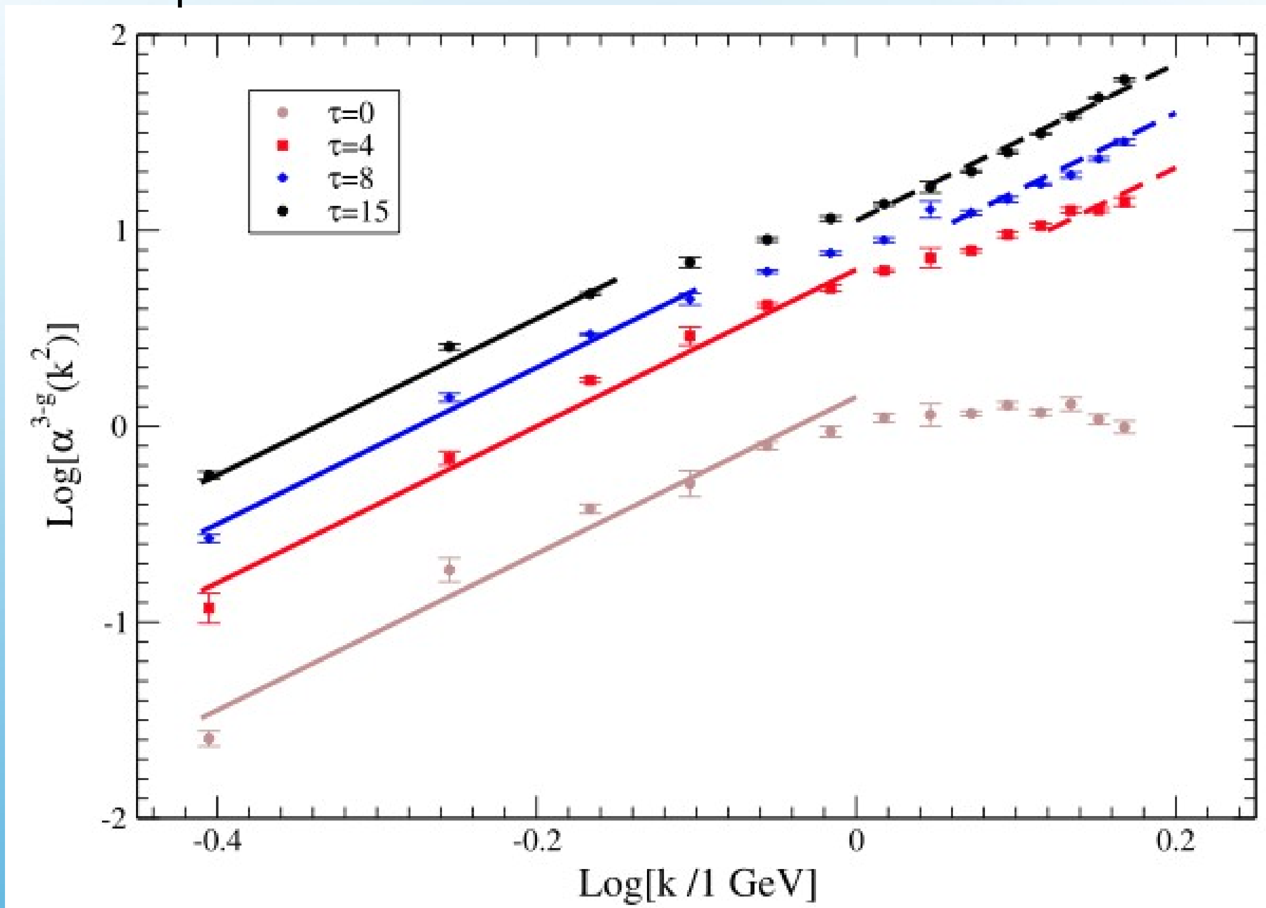
Table 1
Estimates for the densities, obtained as explained in the text, for the different flow times, also expressed in physical units. For this to be done, according to [27], we have defined $\sqrt{8t_0} = 0.3$ fm, whence $t_0 = a^2\tau_0 = 0.0113$ fm^2 and $t = \frac{\tau}{\tau_0} t_0$. At $\tau = 4$, in the unquenched case, the characteristic diffusion length is so small that quantum fluctuations have not been properly removed yet.

	τ	t/t_0	n (fm^{-4})
Quenched	4	6.84	
	8	13.7	
	15	25.6	
Unquenched	4	2.34	
	8	4.70	
	15	8.84	

The Wilson flow has been proven to be an useful tool to deprive the lattice gauge fields from their short-distance (UV) quantum fluctuations.

The Wilson flow smoothing procedure

$\beta = 4.20$



$$\alpha(k^2) = \frac{k^4}{18\pi n} \times \begin{cases} 1 + \mathcal{O}\left(\frac{\delta\rho^2}{k^2\bar{\rho}^4}\right) \\ 1 + 48\frac{\delta\rho^2}{\bar{\rho}^2} + \mathcal{O}\left(k^2\delta\rho^2, \frac{\delta\rho^4}{\bar{\rho}^4}\right) \end{cases}$$

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	τ	t/t_0	n (fm^{-4})
Quenched	4	6.84	3.5(1)
	8	13.7	1.75(4)
	15	25.6	0.98(5)
Unquenched	4	2.34	
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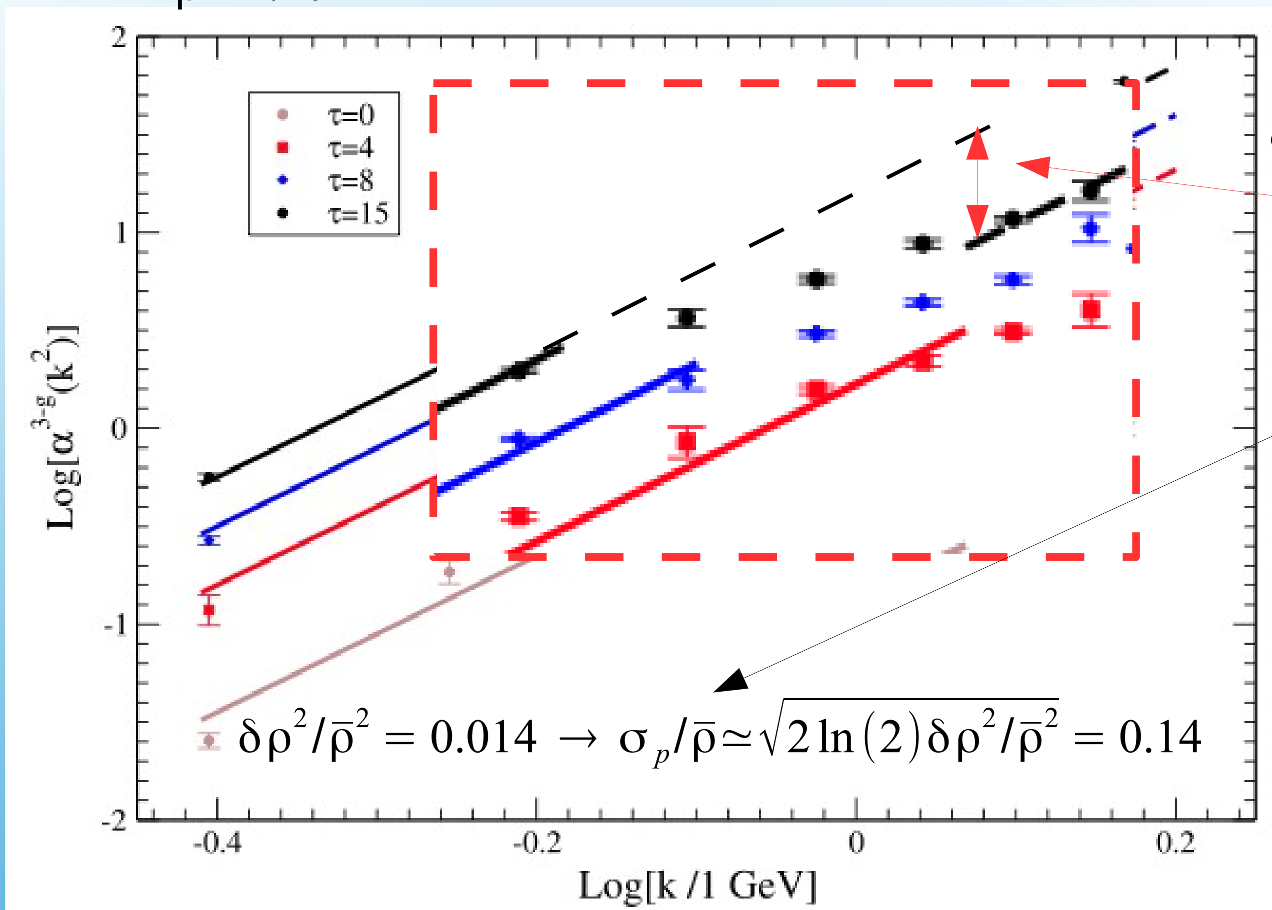

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$$\delta\rho^2/\bar{\rho}^2 = 0.014 \rightarrow \sigma_p/\bar{\rho} \simeq \sqrt{2\ln(2)\delta\rho^2/\bar{\rho}^2} = 0.14$$

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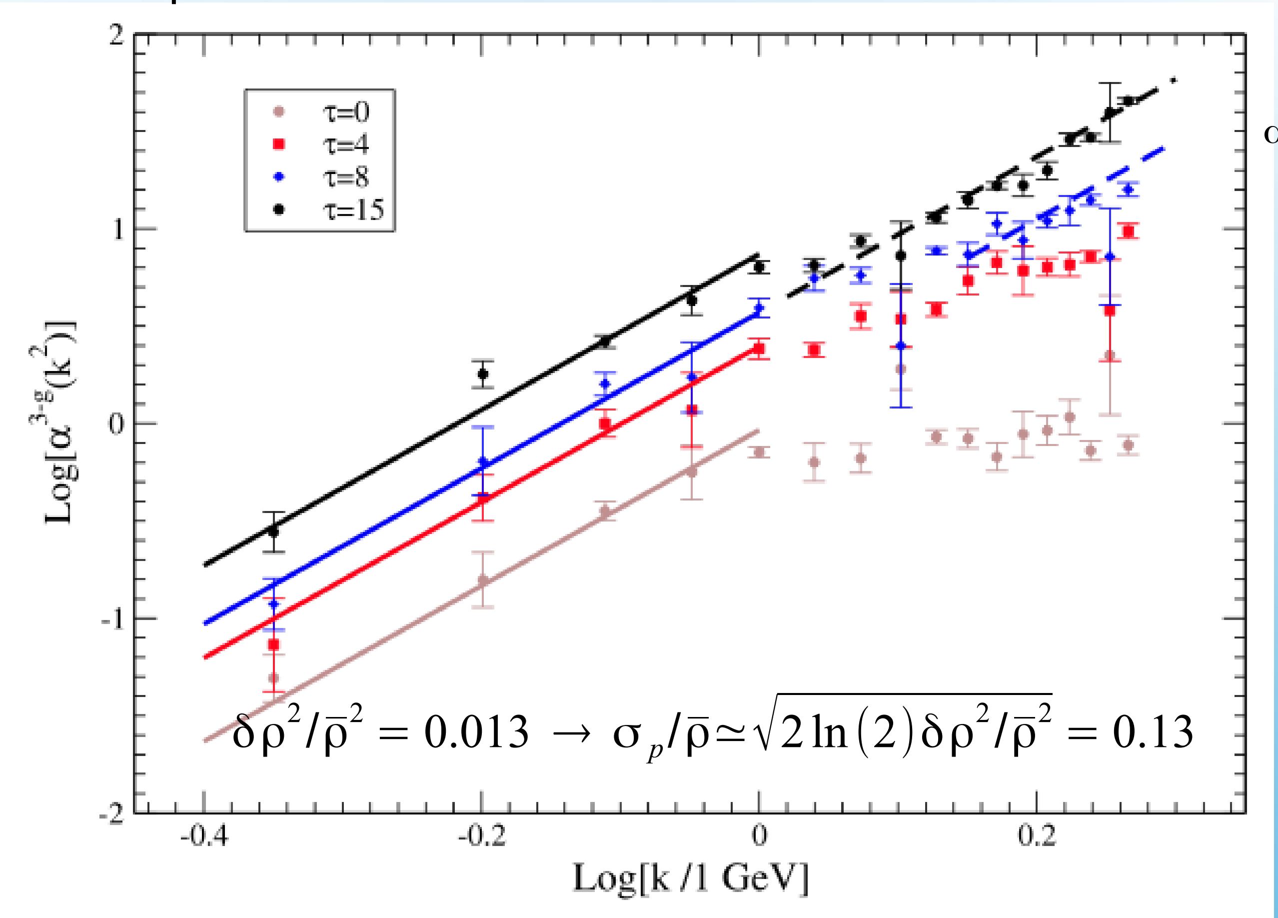
Unquenched	4	2.34	
	8	4.70	
	15	8.84	

Fairly consistent with lattice estimates made by applying direct instanton detection: $\sigma_p/\bar{\rho} \sim 0.17 - 0.22$ [D.A. Smith, M.J. Teper; PRD58(1998)014505]

The Wilson flow has been proven to be an useful tool to deprive the lattice gauge fields from their short-distance (UV) quantum fluctuations.
The main features observed in the gluon correlations obtained with lattice flown gauge fields can be well described within the multi-instanton approach framework.

The Wilson flow smoothing procedure

$\beta = 1.95$



$$\delta\rho^2/\bar{\rho}^2 = 0.013 \rightarrow \sigma_p/\bar{\rho} \simeq \sqrt{2\ln(2)\delta\rho^2/\bar{\rho}^2} = 0.13$$

$$\alpha(k^2) = \frac{k^4}{18\pi n} \times \begin{cases} 1 + \mathcal{O}\left(\frac{\delta\rho^2}{k^2\bar{\rho}^4}\right) \\ 1 + 48\frac{\delta\rho^2}{\bar{\rho}^2} + \mathcal{O}\left(k^2\delta\rho^2, \frac{\delta\rho^4}{\bar{\rho}^4}\right) \end{cases}$$

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Unquenched	4	2.34	–
	8	4.70	6.8(5)
	15	8.84	3.0(2)

The Wilson flow has been proven to be an useful tool to deprive the lattice gauge fields from their short-distance (UV) quantum fluctuations.
The main features observed in the gluon correlations obtained with lattice flown gauge fields can be well described within the multi-instanton approach framework.

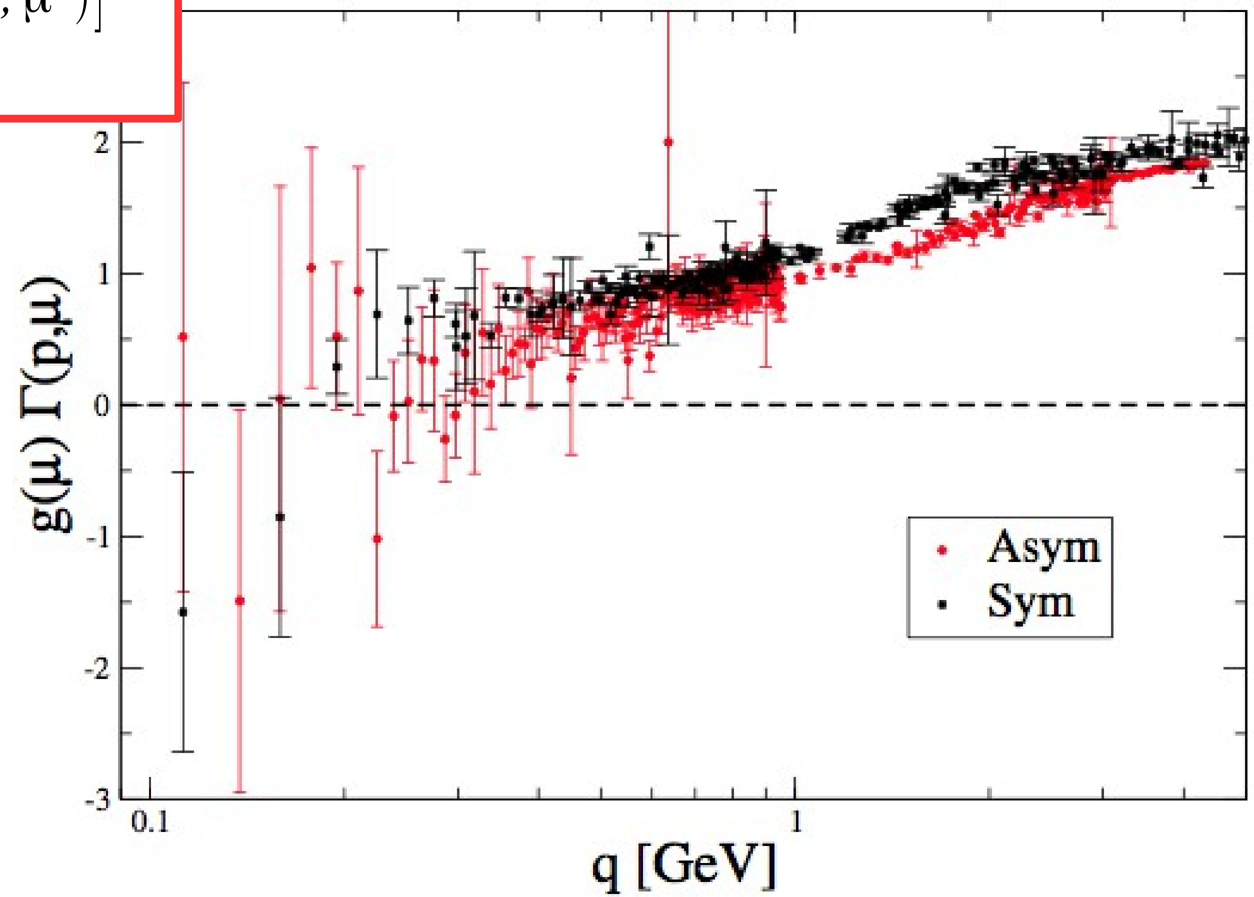
The zero-crossing of the three-gluon vertex

$$g^{i}(\mu^2)\Gamma^{i}_{T,R}(q^2;\mu^2) = \frac{g^{i}(q^2)}{\left[q^2\Delta_R(q^2;\mu^2)\right]^{3/2}}$$

$i = sym, asym.$

$$g^{sym}(q^2) = q^3 \frac{T^{sym}(q^2)}{[\Delta(q^2)]^{3/2}}$$

$$g^{asym}(q^2) = q^3 \frac{T^{asym}(q^2)}{\Delta(0)[\Delta(q^2)]^{1/2}}$$



The form factor for the tree-level tensor structure of the 1PI three-gluon vertex appear to show similar IR behavior in both symmetric and asymmetric kinematic configurations of momenta. The asymmetric case is however noisier than the symmetric one!

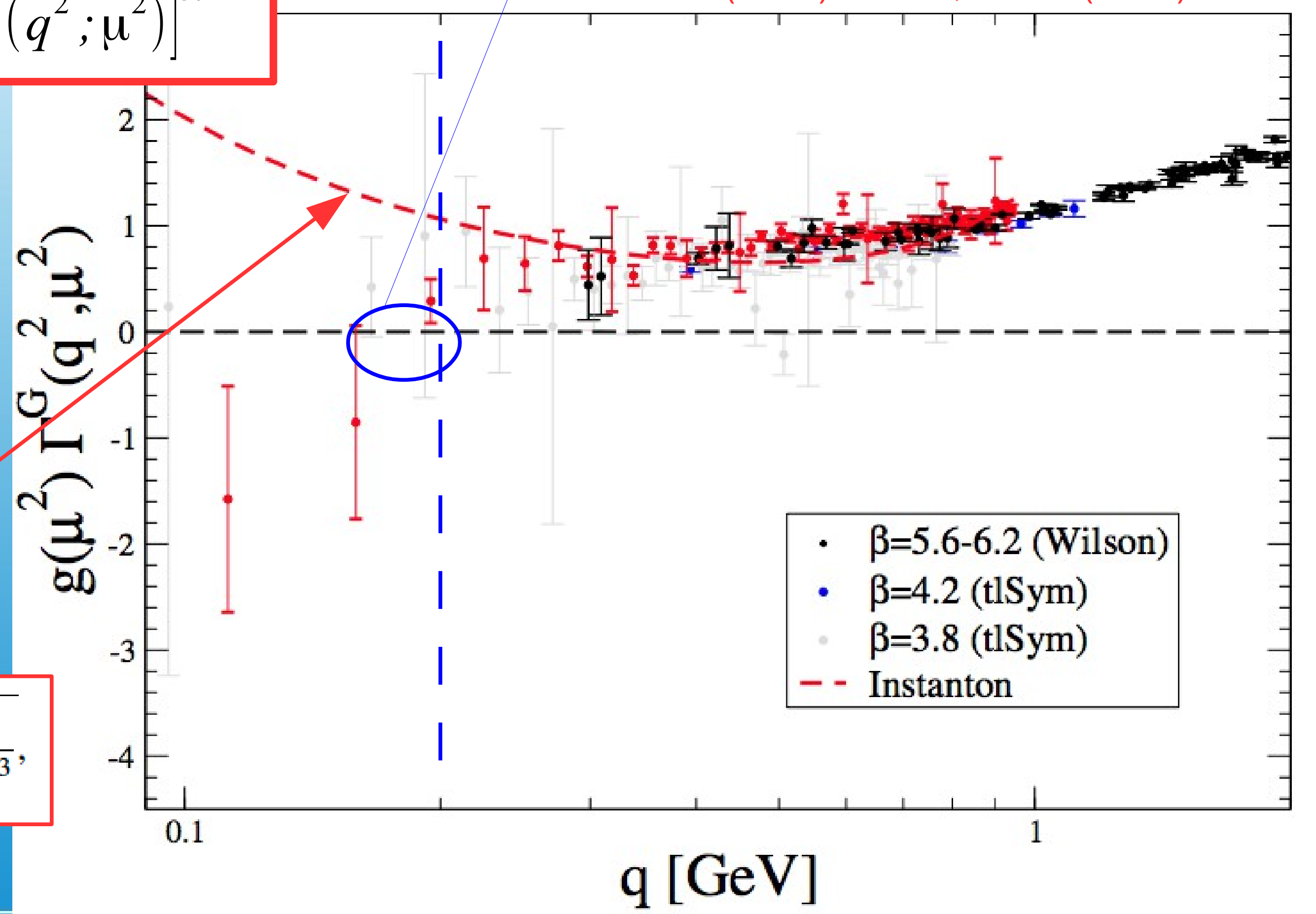
The zero-crossing of the three-gluon vertex

A.C Aguilar et al.; PRD89(2014)05008
A. Blum et al.; PRD89(2014)061703
G. Eichmann et al.; PRD89(2014)105014
A.K. Cyrol et al.; arXiv:1605.01856[hep-ph]
A. Cucchieri, A. Maas, T. Mendes; PRD74(2006)014503;PRD77(2008)094510

$$g^{sym}(\mu^2)\Gamma^{sym}_{T,R}(q^2;\mu^2) = \frac{g^{sym}(q^2)}{\left[q^2\Delta_R(q^2;\mu^2)\right]^{3/2}}$$

$$g^{sym}(q^2) = q^3\frac{T^{sym}(q^2)}{[\Delta(q^2)]^{3/2}}$$

$$g^{sym}(\mu^2)\Gamma^{sym}_{T,R}(q^2;\mu^2) \simeq \sqrt{\frac{2}{9np^2\,[\Delta(p^2;\mu^2)]^3}},$$

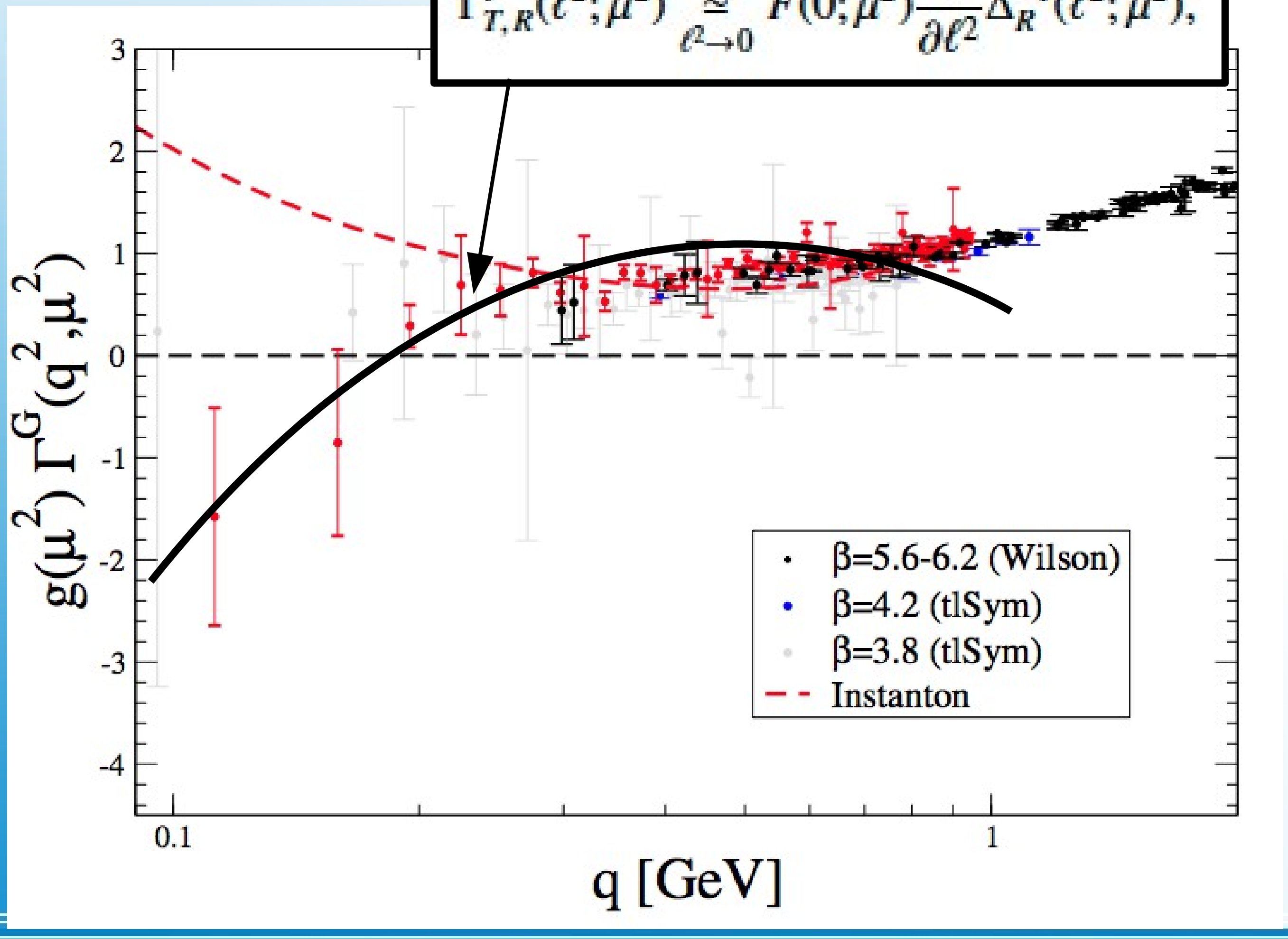


Let's then focus (again) on the symmetric case: the form factor appears to change its sign at very deep IR momenta and show then a zero-crossing. This feature, happening below ~0.2 GeV, is not accounted for by the semiclassical instanton picture.

The zero-crossing of the three-gluon vertex

A.C Aguilar et al.; PRD89(2014)05008

$$\Gamma^i_{T,R}(\ell^2;\mu^2) \underset{\ell^2\to 0}{\simeq} F(0;\mu^2)\frac{\partial}{\partial \ell^2}\Delta_R^{-1}(\ell^2;\mu^2),$$



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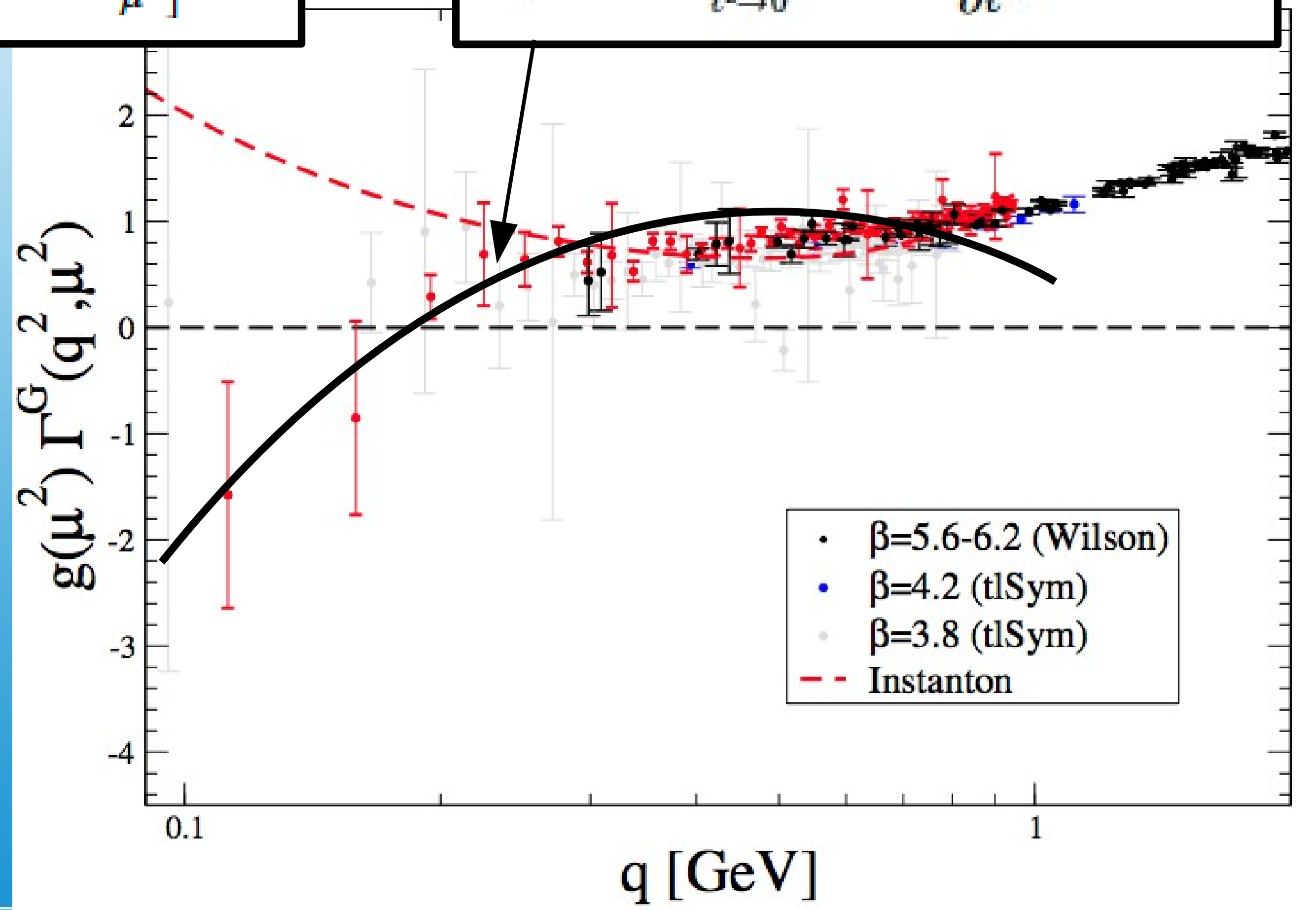
The zero-crossing of the three-gluon vertex

A.C Aguilar et al.; PRD89(2014)05008

$$\Delta_R^{-1}(q^2;\mu^2) \underset{q^2\to 0}{=} q^2\left[a + b\log\frac{q^2+m^2}{\mu^2} + c\log\frac{q^2}{\mu^2}\right] + m^2,$$

$$\Gamma^i_{T,R}(\ell^2;\mu^2) \underset{\ell^2\to 0}{\simeq} F(0;\mu^2)\frac{\partial}{\partial\ell^2}\Delta_R^{-1}(\ell^2;\mu^2),$$



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A.C Aguilar et al.; PRD89(2014)05008
M.Tissier, N.Wschebor; PRD84(2011)045018

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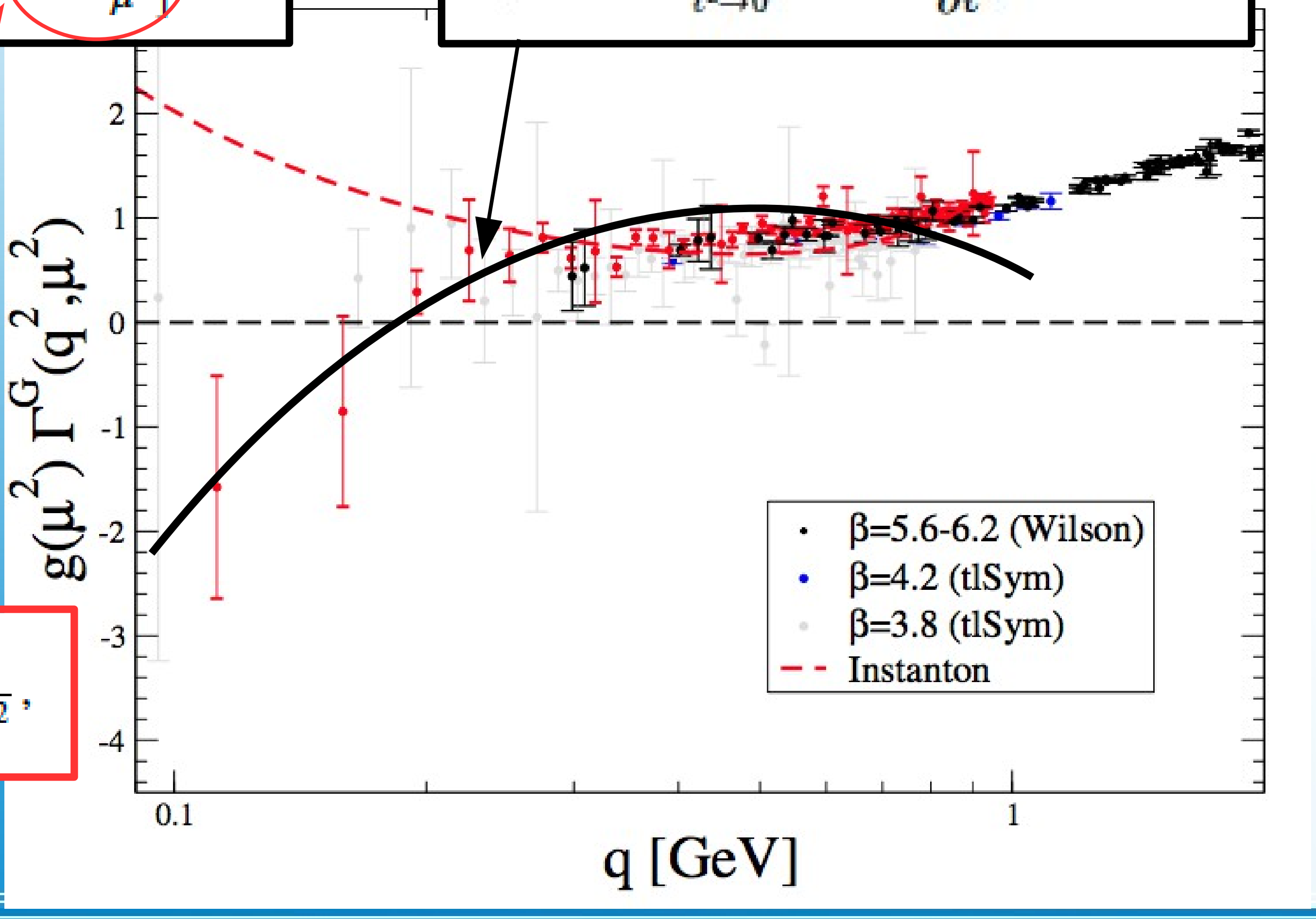

$$\Pi_c(q^2) = \frac{g^2 C_A}{6} q^2 F(q^2) \int_k \frac{F(k^2)}{k^2(k+q)^2},$$

Ghost loop contribution!!!

Let's then focus (again) on the symmetric case: the form factor appears to change its sign at very deep IR momenta and show then a zero-crossing. This feature, happening below ~0.2 GeV, is not accounted for by the semiclassical instanton picture. It's a soft quantum effect!!!

The zero-crossing of the three-gluon vertex

A.C Aguilar et al.; PRD89(2014)05008
M.Tissier, N.Wschebor; PRD84(2011)045018

$$\Delta_R^{-1}(q^2;\mu^2) \underset{q^2\to 0}{=} q^2\left[a + b\log\frac{q^2+m^2}{\mu^2} + c\log\frac{q^2}{\mu^2}\right] + m^2,$$

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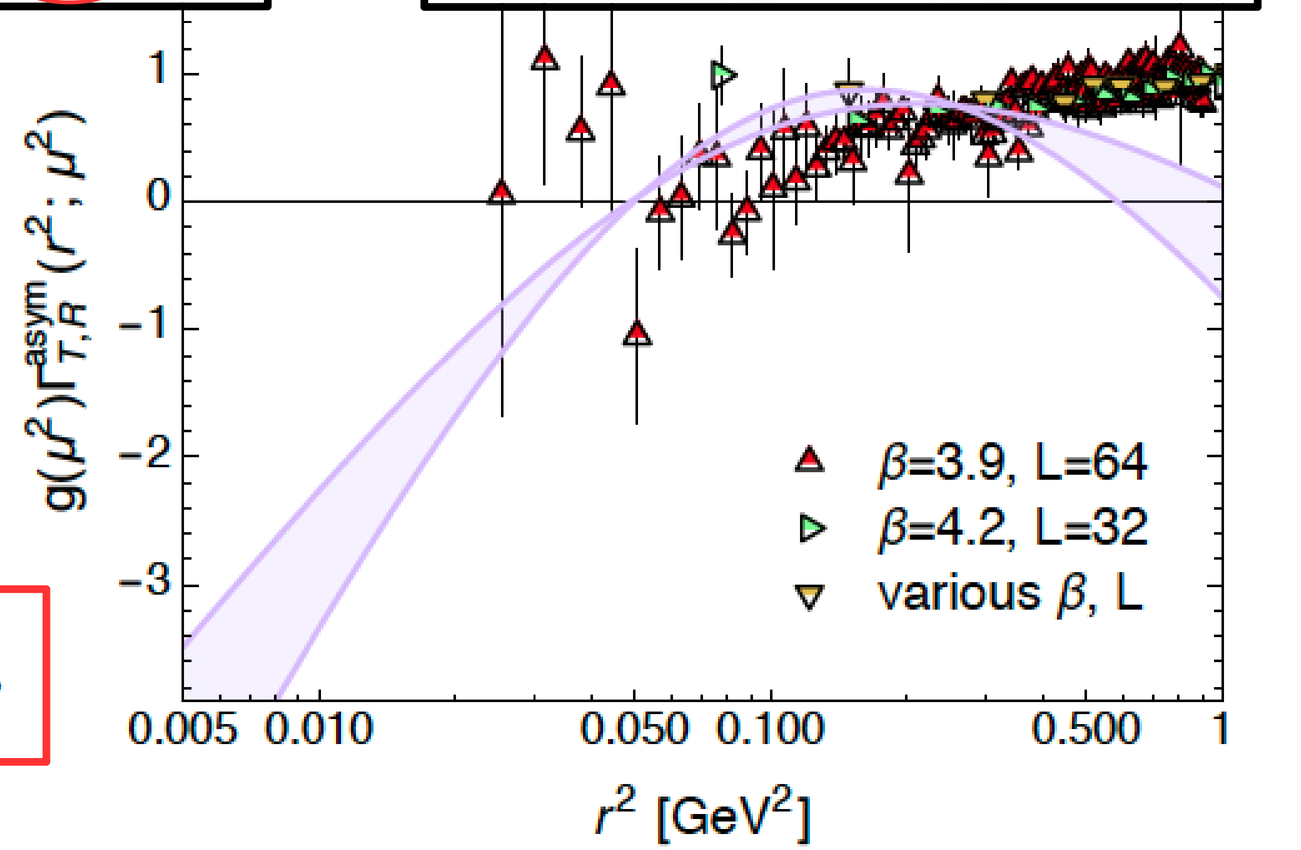

The data for the asymmetric case display a behavior much noisier... but compatible with the predicted one on the basis of the soft quantum effect that comes out from the ghost sector.

Conclusions:

- 2- and 3-gluon Green functions have been deprived from the UV quantum fluctuations by applying the Wilson flow and then shown to be well described as correlations in the field of a multi-instanton ensemble.

- The Wilson flow smoothing procedure leaves the low-momentum domain of these Green functions essentially unmodified; and gets rid of the fundamental QCD scale Λ_{QCD} (which indicates where the mechanism driving the transition from asymptotically free to confinement regimes take place).

- Nevertheless, the three-gluon Green function shows a feature at very low-momentum not fitting in the multi-instanton picture: the zero-crossing which can be explained as a soft quantum effect induced by the contribution of unprotected (by a mass) ghost-loops.