Statistical combination of experimental results in ATLAS

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[a]CERN

The role of combinations in HEP

- Combinations are **increasingly important** for HEP to perform the single most powerful test of a hypothesis, e.g. for a **measurement** or assessing the **compatibility** of measurements
  - **Dedicated analyses** for each production and decay mode target specific properties
  - **Example:** Higgs boson mass and couplings
- Searches are often performed more **inclusively**, complicating statistical combinations
- Many analyses perform combinations already **implicitly**
  - **Example:** data driven background estimates, subsidiary measurements, etc.

Channel	References for individual publications		Signal strength [μ] from results in this paper (Section 5.2)		Signal significance [σ]	
	ATLAS	CMS	ATLAS	CMS	ATLAS	CMS
$H \to \gamma\gamma$	[91]	[92]	$1.14\ ^{+0.27}_{-0.25}$ $\left(^{+0.26}_{-0.24}\right)$	$1.11\ ^{+0.25}_{-0.23}$ $\left(^{+0.23}_{-0.21}\right)$	5.0 (4.6)	5.6 (5.1)
$H \to ZZ$	[93]	[94]	$1.52\ ^{+0.40}_{-0.34}$ $\left(^{+0.32}_{-0.27}\right)$	$1.04\ ^{+0.32}_{-0.26}$ $\left(^{+0.30}_{-0.25}\right)$	7.6 (5.6)	7.0 (6.8)
$H \to WW$	[95, 96]	[97]	$1.22\ ^{+0.23}_{-0.21}$ $\left(^{+0.21}_{-0.20}\right)$	$0.90\ ^{+0.23}_{-0.21}$ $\left(^{+0.23}_{-0.20}\right)$	6.8 (5.8)	4.8 (5.6)
$H \to \tau\tau$	[98]	[99]	$1.41\ ^{+0.40}_{-0.36}$ $\left(^{+0.37}_{-0.33}\right)$	$0.88\ ^{+0.30}_{-0.28}$ $\left(^{+0.31}_{-0.29}\right)$	4.4 (3.3)	3.4 (3.7)
$H \to bb$	[100]	[101]	$0.62\ ^{+0.37}_{-0.37}$ $\left(^{+0.39}_{-0.37}\right)$	$0.81\ ^{+0.45}_{-0.43}$ $\left(^{+0.45}_{-0.43}\right)$	1.7 (2.7)	2.0 (2.5)
$H \to \mu\mu$	[102]	[103]	$-0.6\ ^{+3.6}_{-3.6}$ $\left(^{+3.6}_{-3.6}\right)$	$0.9\ ^{+3.6}_{-3.5}$ $\left(^{+3.3}_{-3.2}\right)$		
ttH production	[77, 104, 105]	[107]	$1.9\ ^{+0.8}_{-0.7}$ $\left(^{+0.7}_{-0.7}\right)$	$2.9\ ^{+1.0}_{-0.9}$ $\left(^{+0.9}_{-0.8}\right)$	2.7 (1.6)	3.6 (1.3)

The work of many people

Disclaimer

- This talk will be focussed on **likelihood combinations**
  - Other approaches, e.g. BLUE, will be covered in different talks 💬 L. Lista
- Key concepts will be explained with the Higgs coupling combination as an example
- Statistical inference also covered by other speakers

💬 E. Gross, F. Matorras, S. Schmitt, S. Biondi, ...

Approaches to perform combinations

- Combina-tions in HEP
  - "Naive" combi-nations
    - For sufficiently Gaussian models
    - May neglect correlations
    - Phe-nomeno-logical
      - Using public information, e.g. from HepData
  - Likeli-hood combi-nations
    - Combine likelihood functions
    - Can be "complete" or use idealized measurements
    - Covered in this slides
  - BLUE
    - Best linear unbiased estimator
    - Popular approximation if full likelihood not available
    - Covered by L. Lista

Likelihood combinations

- Given a set of **measurements** and a **hypothesis**, a **likelihood function** is defined as the probability of the data under this hypothesis, formulated as a **probability density function**
  - Simplest example: idealised measurement

$$\theta = \tilde{\theta} \pm \sigma \rightarrow \text{Gaussian}(\tilde{\theta} | \theta, \sigma)$$

- Disjoint selections of the data are simultaneously described by a **product over the respective likelihood functions**

$$L(\alpha) = \prod_{c=1}^{c_{max}} L(\alpha)$$

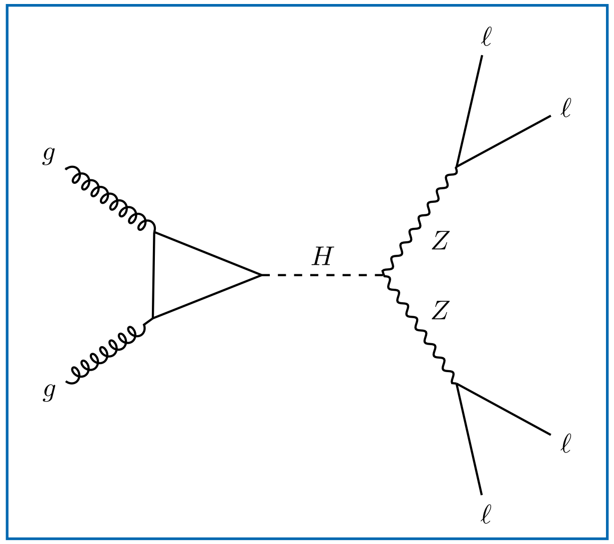
Joint likelihood model

Likelihood function for individual selections/analyses

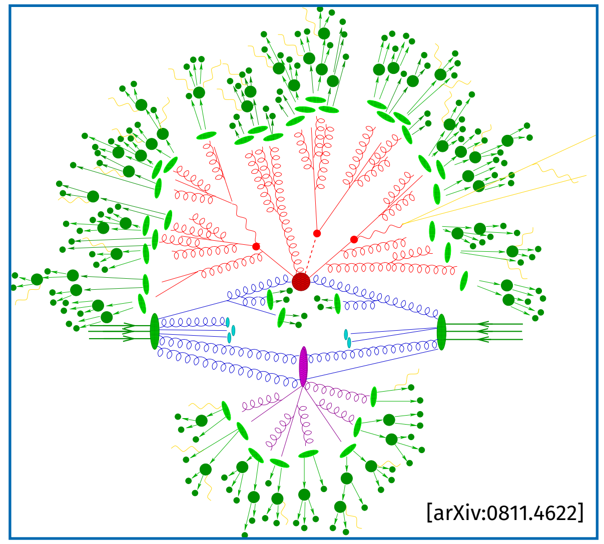
Hypothesis

HEP analysis chain

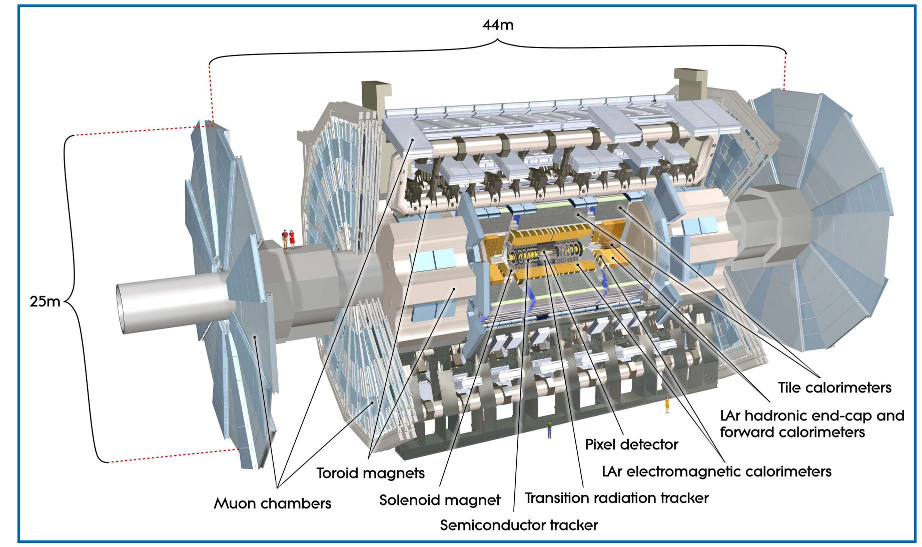
Hard scatter

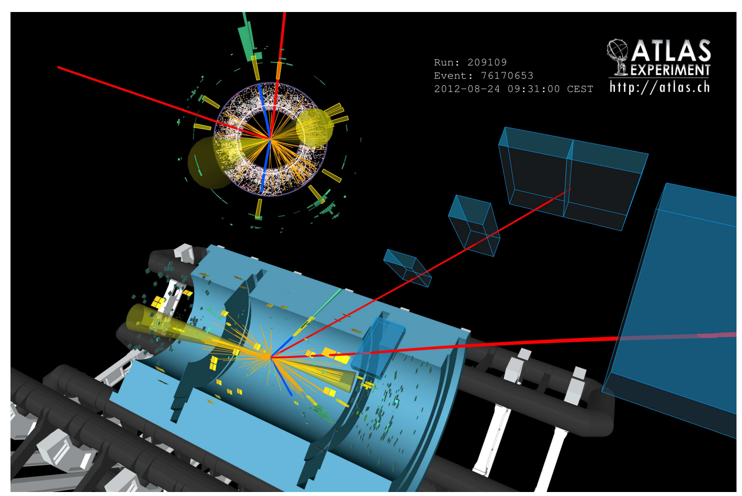


Soft physics

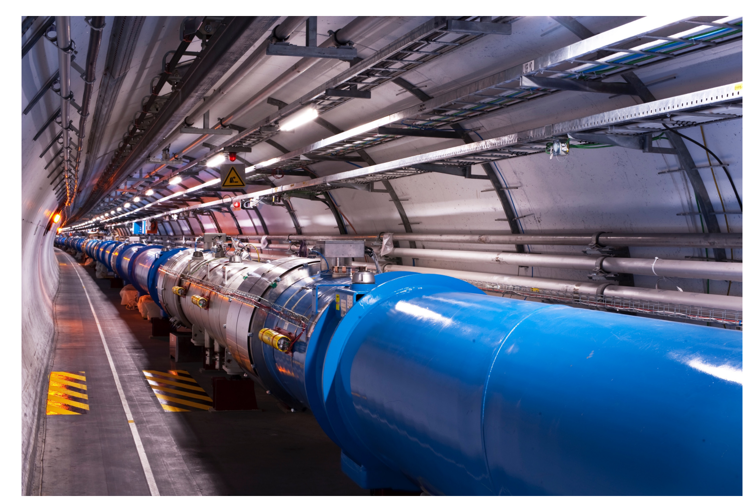


Detector simulation

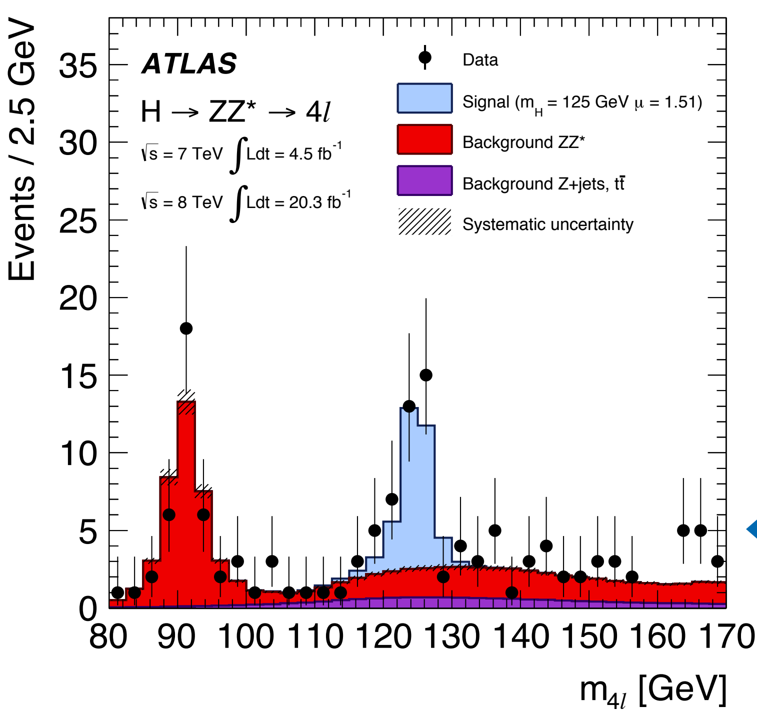




LHC data



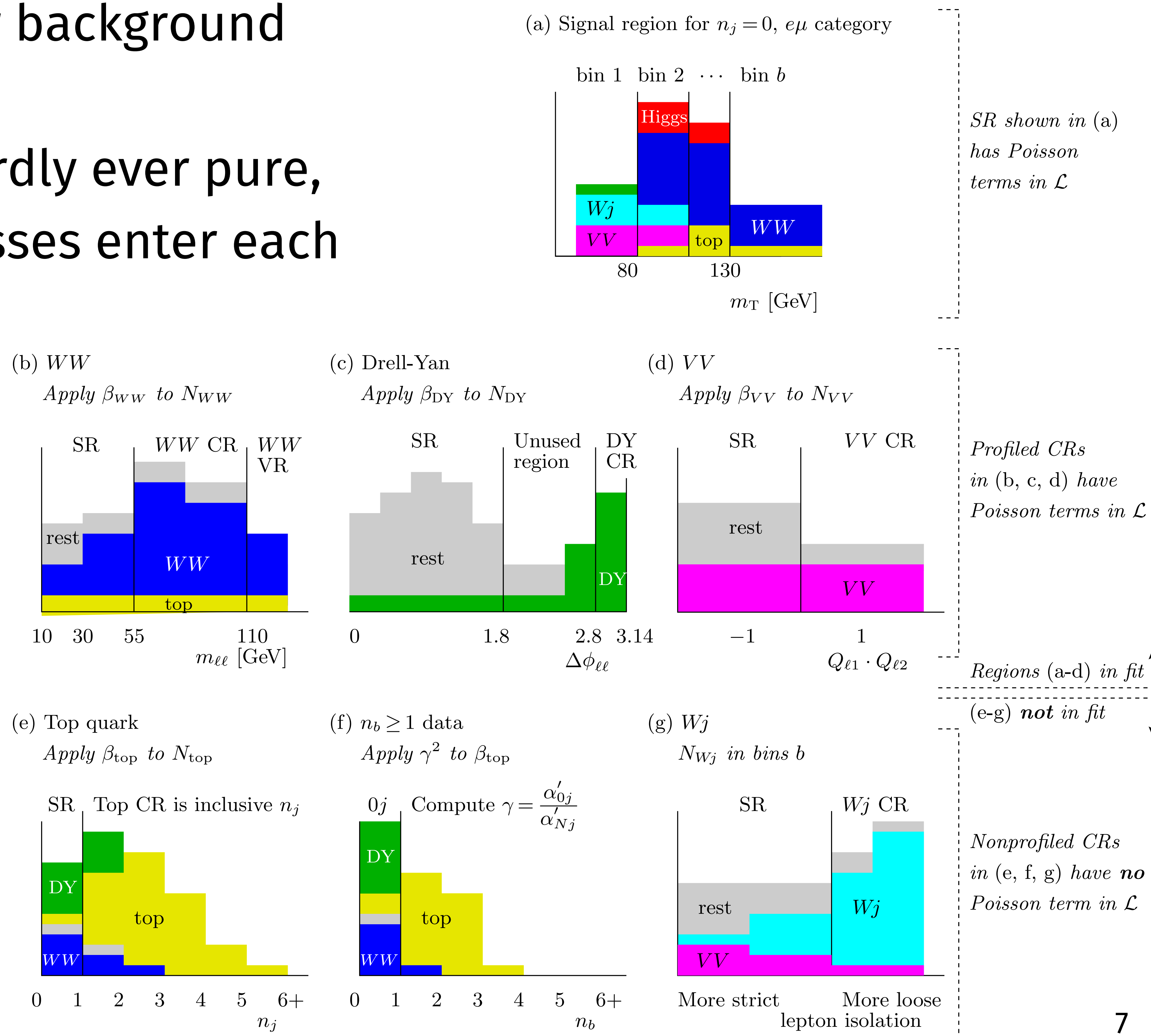
Reconstruction



Selection

Typical HEP measurements

- Typically analyses consist of one or more **disjoint selections** enriched in signal or background like events
  - Categories are hardly ever pure, i.e. various processes enter each selection
- The measurement can be a **single number** or the data can be augmented with one or more discriminating **observables**



Constructing a probability model

- Construct **likelihood** function for each event selection
  - At LHC, templates typically are derived from **histograms** with each bin effectively being a **Poisson counting experiment**

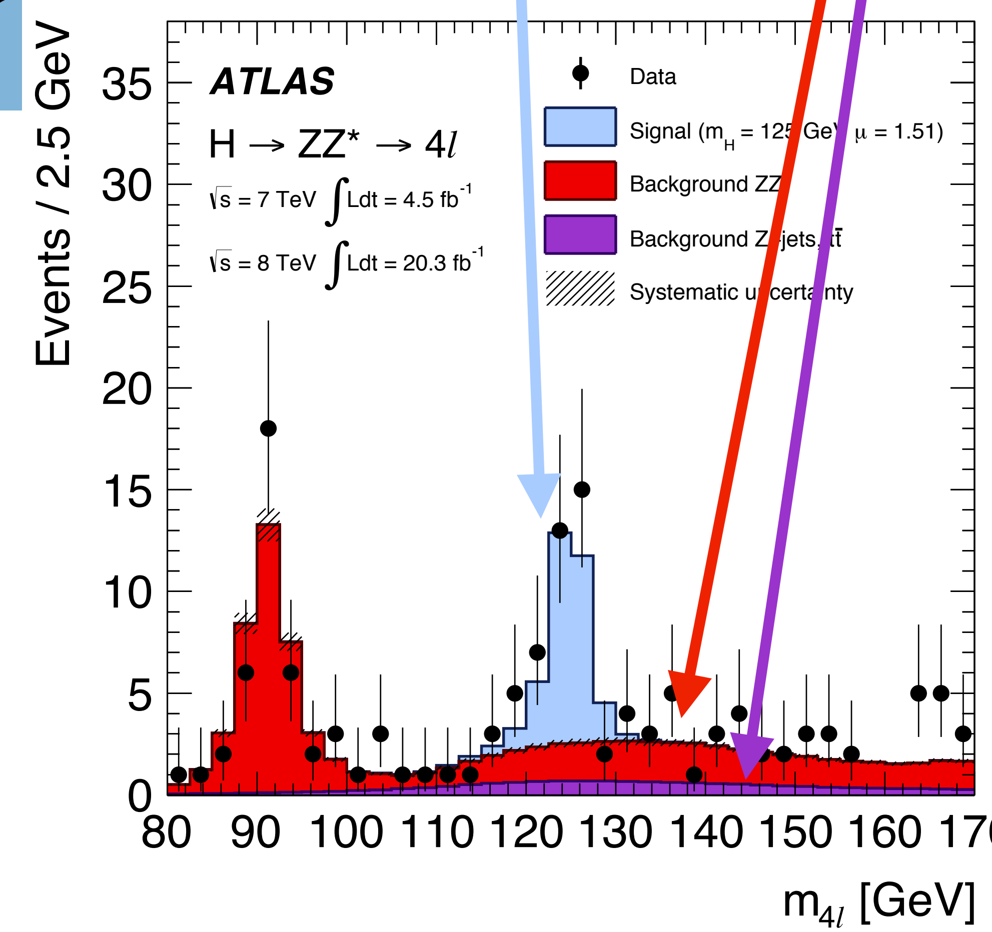
$$L(\mathcal{D}|\alpha) = \prod_{k=1}^{n_{bins}} \text{Poisson}(n_k|\nu_k(\alpha)), \qquad \nu_k(\alpha) = \mu_i \cdot \mu^f \cdot s^f_{i,k}(\theta) + \sum_m^{n_{bkg}} b_m(\theta)$$

where n_k: *observed count*, $\nu_k(\alpha)$: *expected count*

$$s^f_{i,k} = \mathcal{L}(k) \cdot \left\{ \sigma_i^{SM} \cdot A_i^f(k) \cdot \varepsilon_i^f(k) \cdot BR^f_{SM} \right\}$$

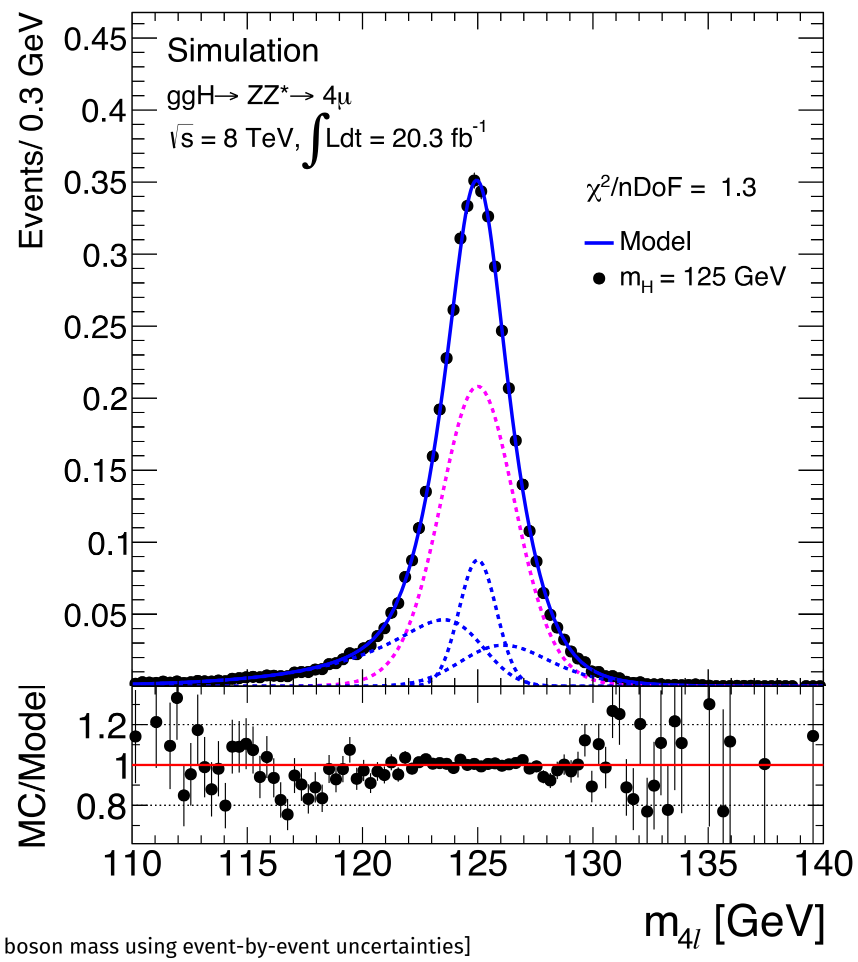
Luminosity — Incl. SM cross section — Acceptance (MC) — Efficiency (MC) — Higgs BR

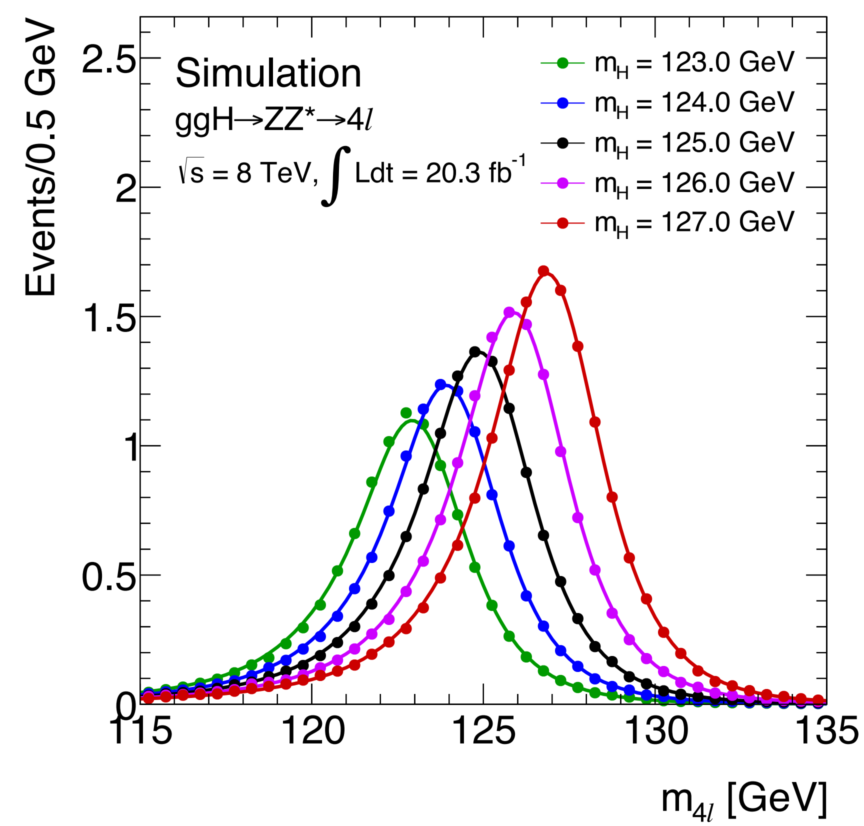
- Can generalise the approach to **continuous distributions** if analytical model is available



Continuous probability density models

- In rare cases (e.g. measuring m_H in Higgs decays to four leptons) it is possible to construct an **analytical model**
- Requires **convolving** the predicted **detector response** with the truth **lineshape** per event (typically a Breit-Wigner for resonances)
- Response varies e.g. for different lepton flavours
- Can be extended to include per-event uncertainties



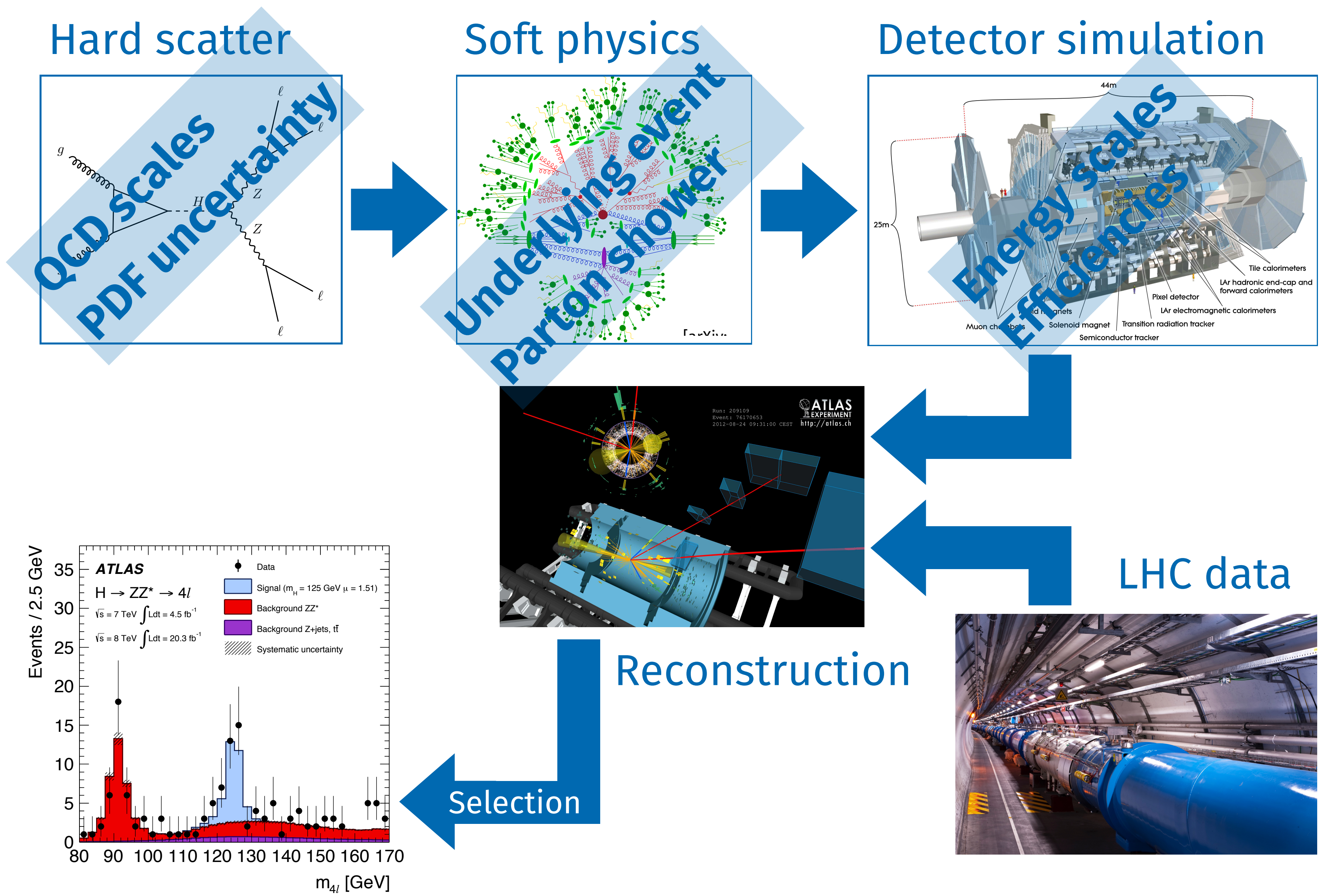


[A. Castelli: Measuring the Higgs boson mass using event-by-event uncertainties]

Incorporating systematic uncertainties

- Counts and distributions are subject to many uncertainties
- Capture the effect of each by introducing **nuisance parameters**
- Often only the maximum likelihood estimate from **subsidiary measurements** known
  - Information not sufficient to construct full likelihood model
  - **Empirically parametrise** effect of each nuisance from available simulation for discrete model parameters

💬 E. Gross



The likelihood for an analysis with different selections

Assumption: likelihood of subsidiary measurement and effect on physics measurement factorise

"Combination" of selections

"Combination" of physics and subsidiary measurements

Data

Augment data from count using observable

$$L(\mathcal{D},\mathcal{G}|\alpha) = \prod_{c=1}^{c_{max}} \left[\text{Poisson}(n_c|\nu_c(\alpha)) \prod_{e=1}^{n_c} f_c(x_{ce}|\alpha) \right] \times \prod_{p\in\mathbb{S}} f_p(\tilde{\theta}_p|\theta_p,\sigma_p)$$

Global observables, i.e. MLE of nuisances from subsidiary measurements

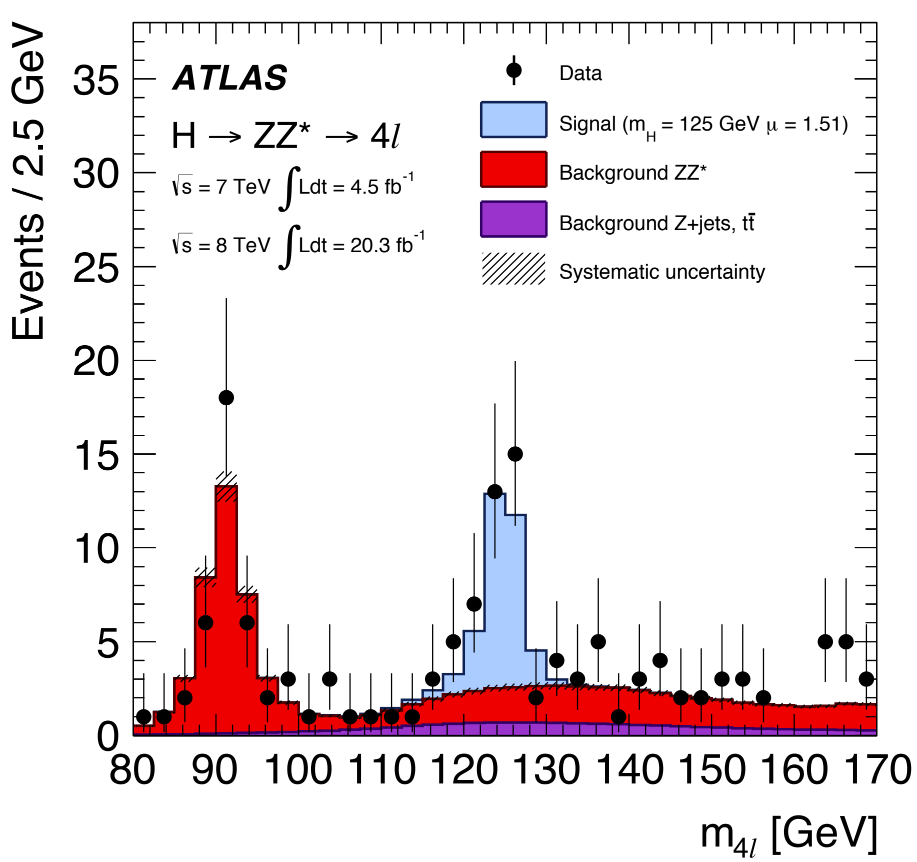
expected counts

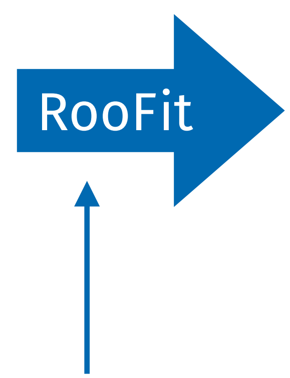
Empirically derive dependence on nuisances from **morphing** between "nominal", "up" and "down" variations, i.e. simulation for discrete model parameters

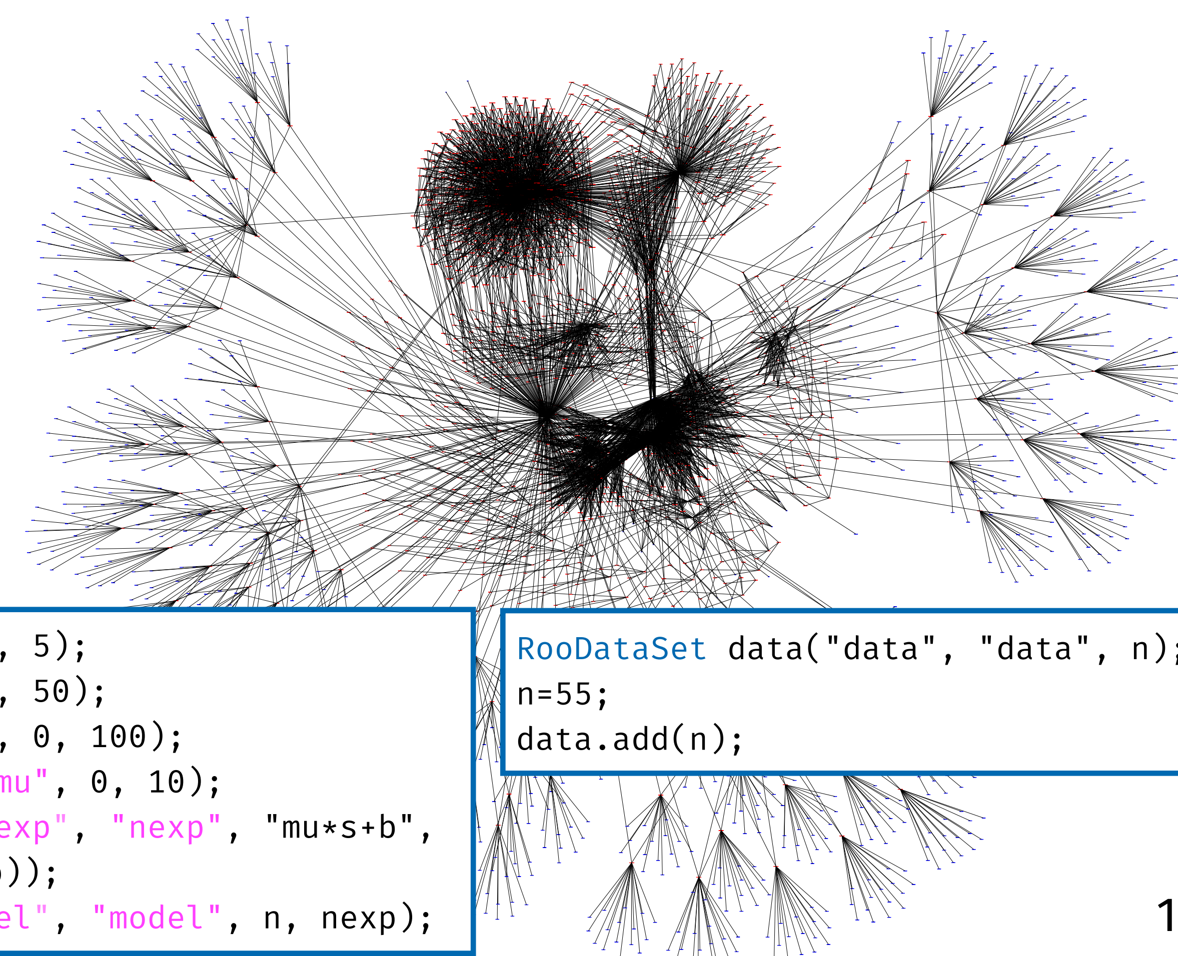
Subsidiary measurement, typically assume Gauss or Poisson distribution

In practice: formulating the PDF using RooFit

- Model building based on the **RooFit** package
  - **Key concept:** represent mathematical model by C++ objects from which the likelihood function is build
  - Support for normalisation relies on numeric techniques when analytical integrals not available
  - Support for conditional probability models, convolutions, etc.
- Commonly adopted by the **LHC experiments**





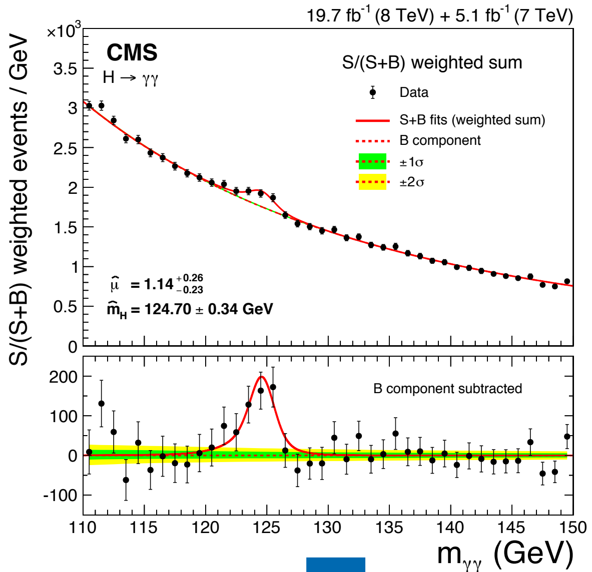


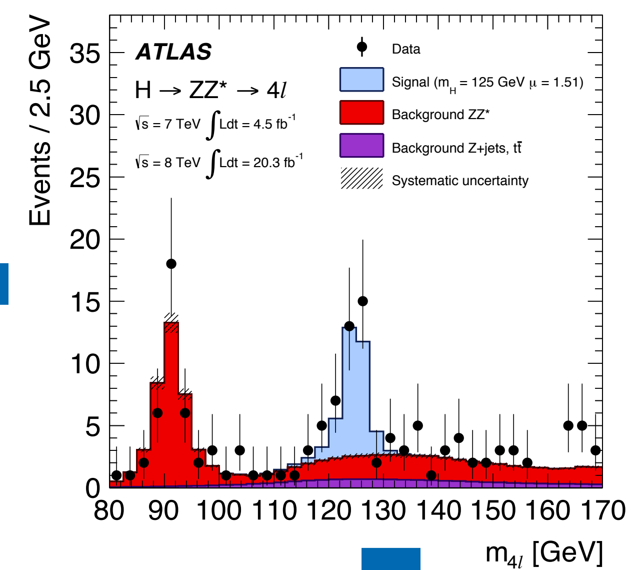
```
RooRealVar s("s", "s", 5);
RooRealVar b("b", "b", 50);
RooRealVar n("n", "n", 0, 100);
RooRealVar mu("mu", "mu", 0, 10);
RooFormulaVar nexp("nexp", "nexp", "mu*s+b",
   RooArgList(mu, s, b));
RooPoisson model("model", "model", n, nexp);
```

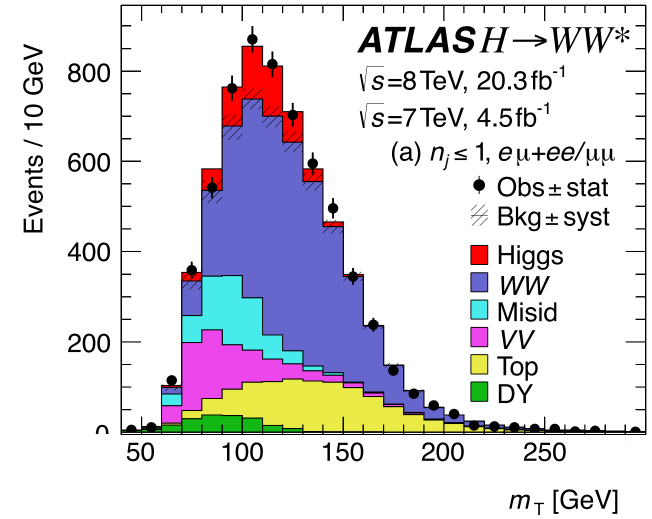
```
RooDataSet data("data", "data", n);
n=55;
data.add(n);
```

The analysis workflow

Analysers specialised in simulation, specific measurements, etc.







+ ...

Custom classes that are added to vanilla ROOT over time

HistFactory, HistFitter, etc.

```
RooWorkspace w("w");
w.import(model);
w.import(data);
w.writeToFile("w.root");
```

$L^{\gamma\gamma}_{CMS}(\alpha)$ $L^{ZZ}_{ATLAS}(\alpha)$ $L^{WW}_{ATLAS}(\alpha)$

RooWorkspace container

Combination

$$L(\alpha) = L^{\gamma\gamma}_{CMS}(\alpha) \times L^{ZZ}_{ATLAS}(\alpha) \times L^{WW}_{ATLAS}(\alpha) \times \ldots$$

→ Inference

RooStats, etc.

The combination step

- Combining involves more than building a **simultaneous PDF** and implementing **parameterisations**

Detector uncertainties follow strategy of individual experiments
Theoretical uncertainties on inclusive cross sections are generally correlated between experiments, but correlation between processes is often not clear and depends on phase space
Acceptance and **efficiency** related uncertainties not correlated
Monte Carlo statistics related nuisances affect a single bin only

Eigenbasis, multivariate subsidiary measurements, template morphing, ...

Corre-lations

Fully or partially correlated parameters, correlated and uncorrelated parts, ...

E.g. luminosity, jet energy scale

Cross sections, pdf sets, uncertainties, ...

Har-moni-sation

Likelihood combinations

Independent tools and analysis teams cross checking pdf, nuisance parameters and inference

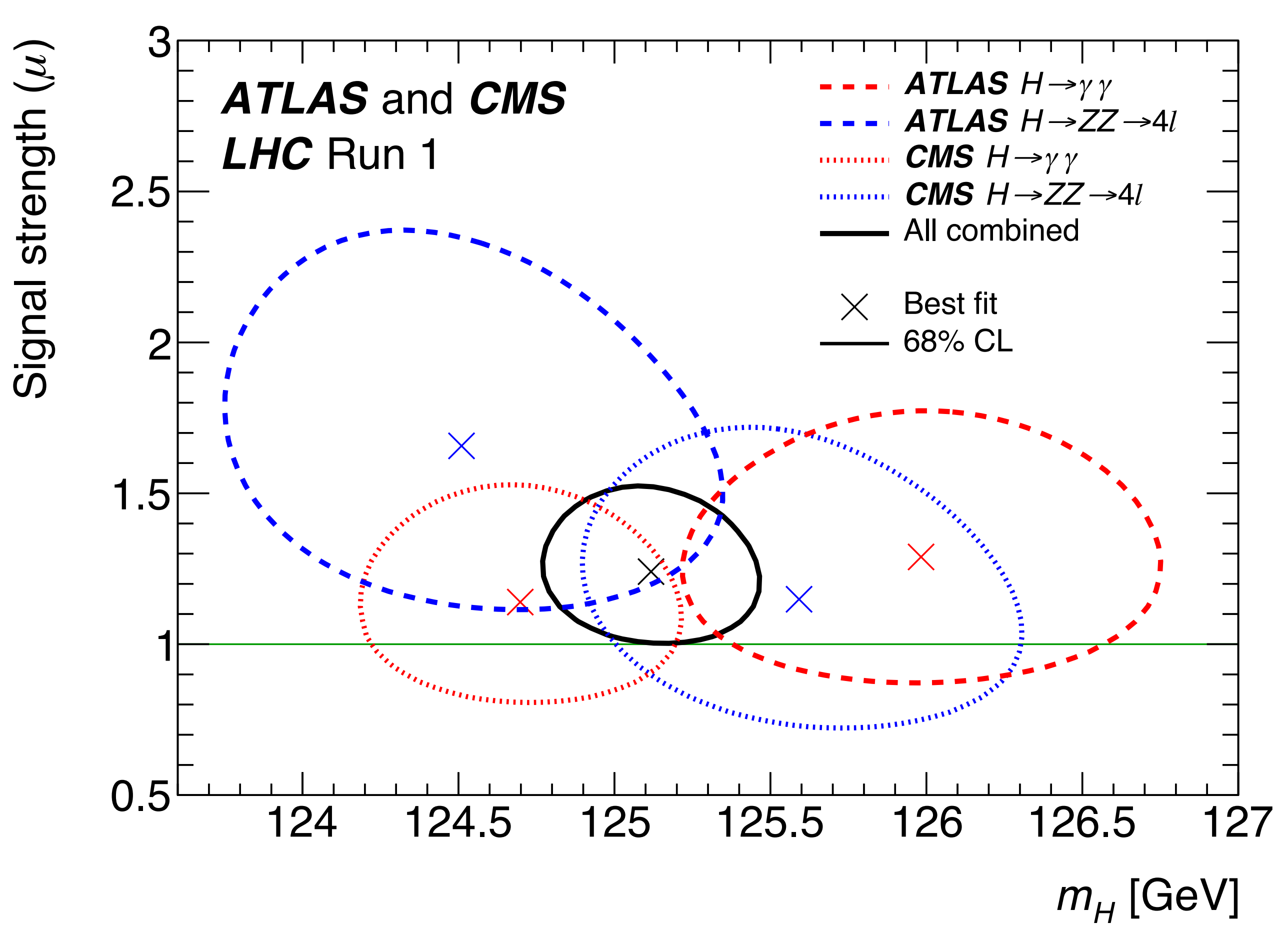
Cross checks

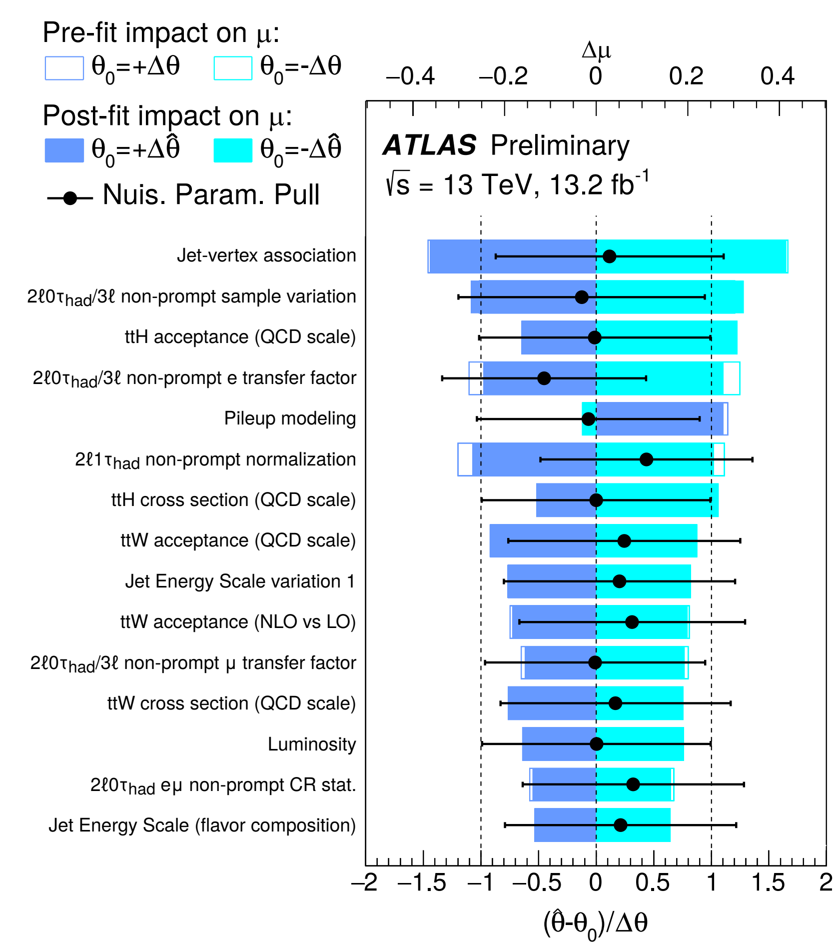
Book-keeping

Almost 600 disjoint selections with one, two, or more observables and more than 4200 nuisance parameters

Statistical inference using RooStats

- Follow standard **frequentist methods** for constructing confidence intervals, hypothesis testing and evaluating the tensions
  - Often the **asymptotic approximation** is used 💬 E. Gross
- **RooStats** provides a series of tools for this tasks
  - General concepts are completely **decoupled** from model building and work on any probability model and dataset provided
- Internally developed tools for **additional cross checks**
  - E.g. pulls, constraints, correlations and impact of nuisance parameters



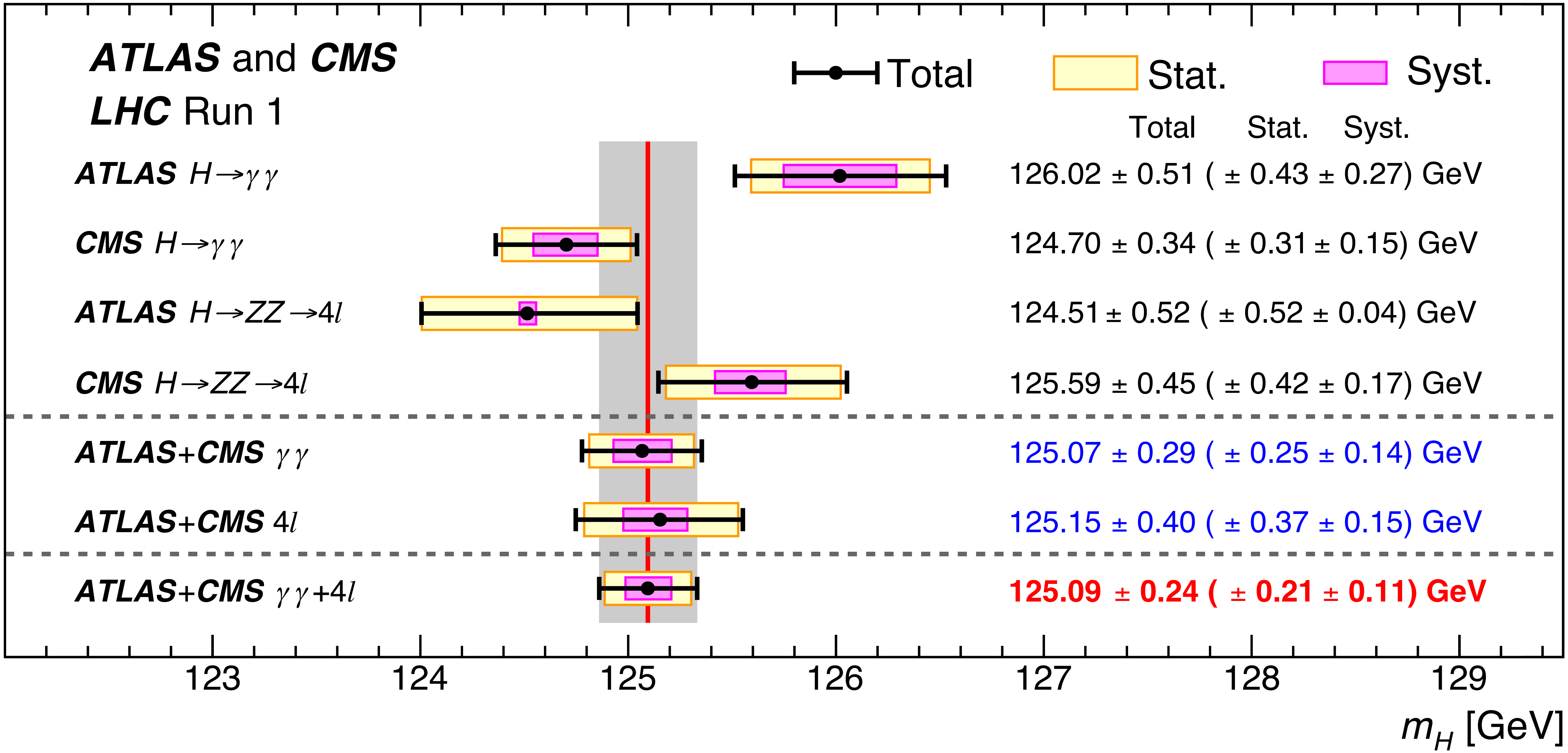


Improving the performance for complex models

- Complex likelihood function may lead to **instabilities** and **performance issues**, even though many aspects are simplified
  - Minimisation of the likelihood function takes several hours (with Hesse days) and requires large (> 7G) memory nodes
  - Largest fraction of time spend in **interpolation**
- Several strategies to **reduce number of interpolations**
  - Disentangle **“real” effect** of systematic variations from **statistical noise**, e.g. using **bootstrap replicas**
  - Performance groups provide **reduced set of variations**
  - Depending on the measurement performed, non-significant uncertainties can be **pruned** on analysis level
    - **Examples**: evaluate the variation of the counts and templates or the correlation with measurements
- Complementary **technical developments**
  - Constant term optimisation, caching, vectorisation, binned evaluation

💬 D. Börner

Naive combinations vs. full model (I)

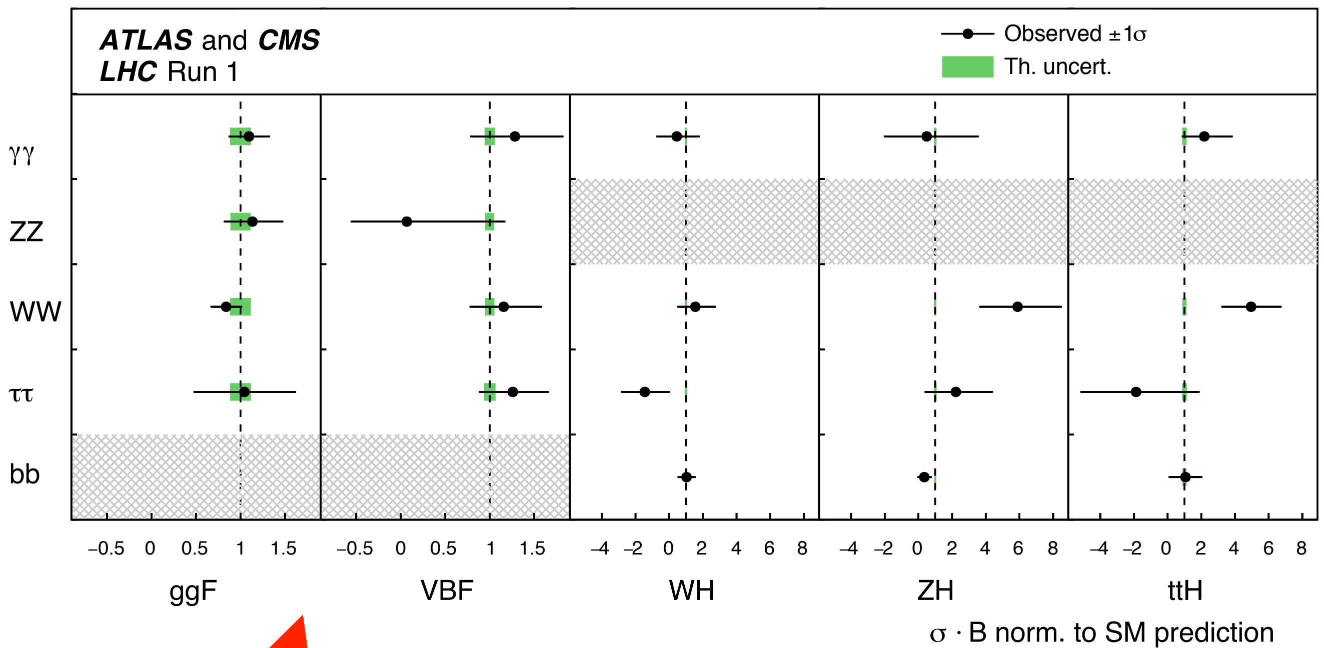


In a gaussian regime, e.g. statistically limited analyses with no complex correlations, weighted average is a good approximation:

$$m_H^{\gamma\gamma+4\ell} = 125.11 \pm 0.22\ \text{GeV}$$

Naive combinations vs. full model (II)

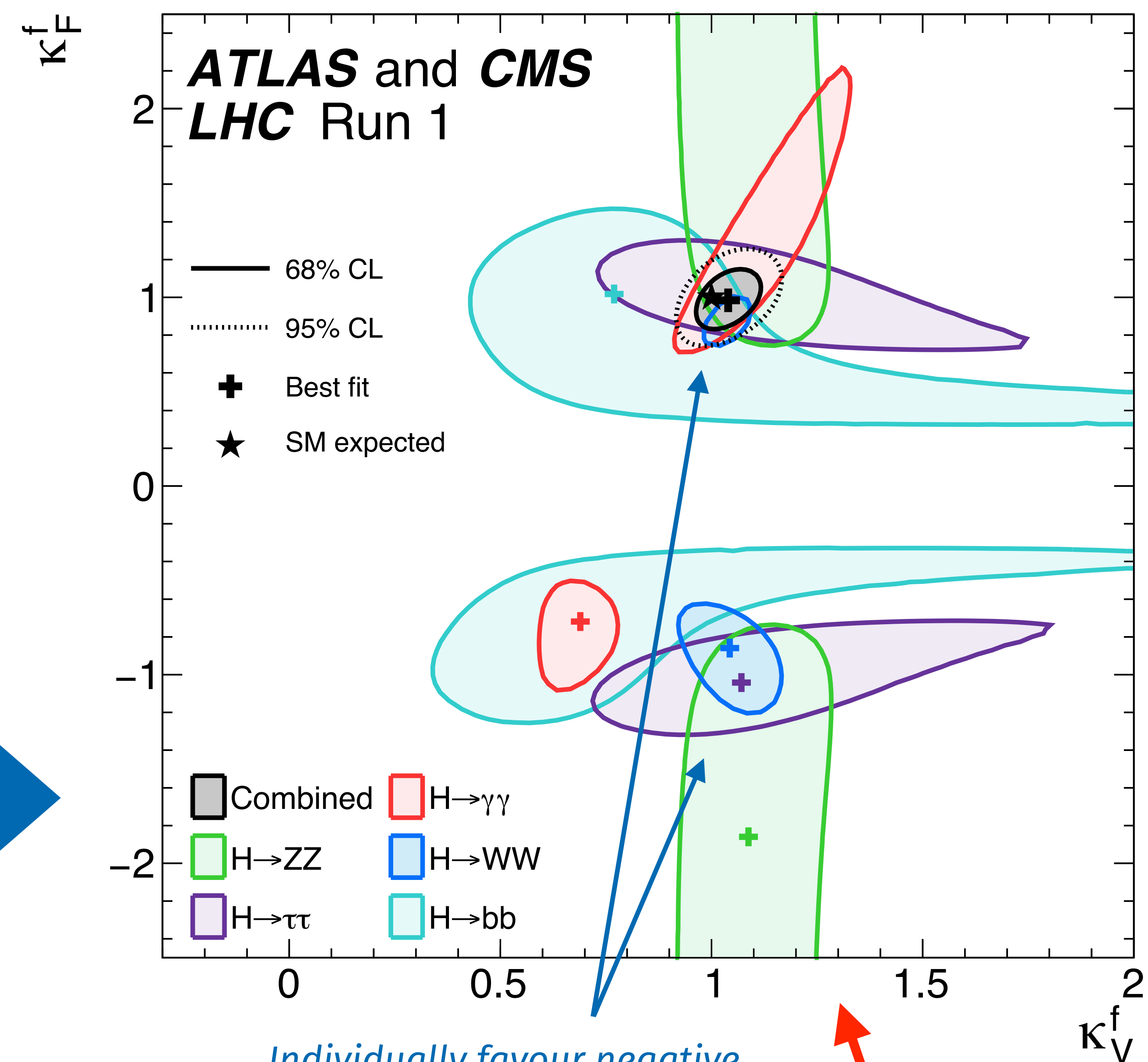
For more complex models, naive combinations are not feasible and may not reproduce results accurately



Measurements

Hides all intrinsic measurements, e.g. background rates, phase space regions, etc.

Requires knowledge of parametrisation, correlations, etc.



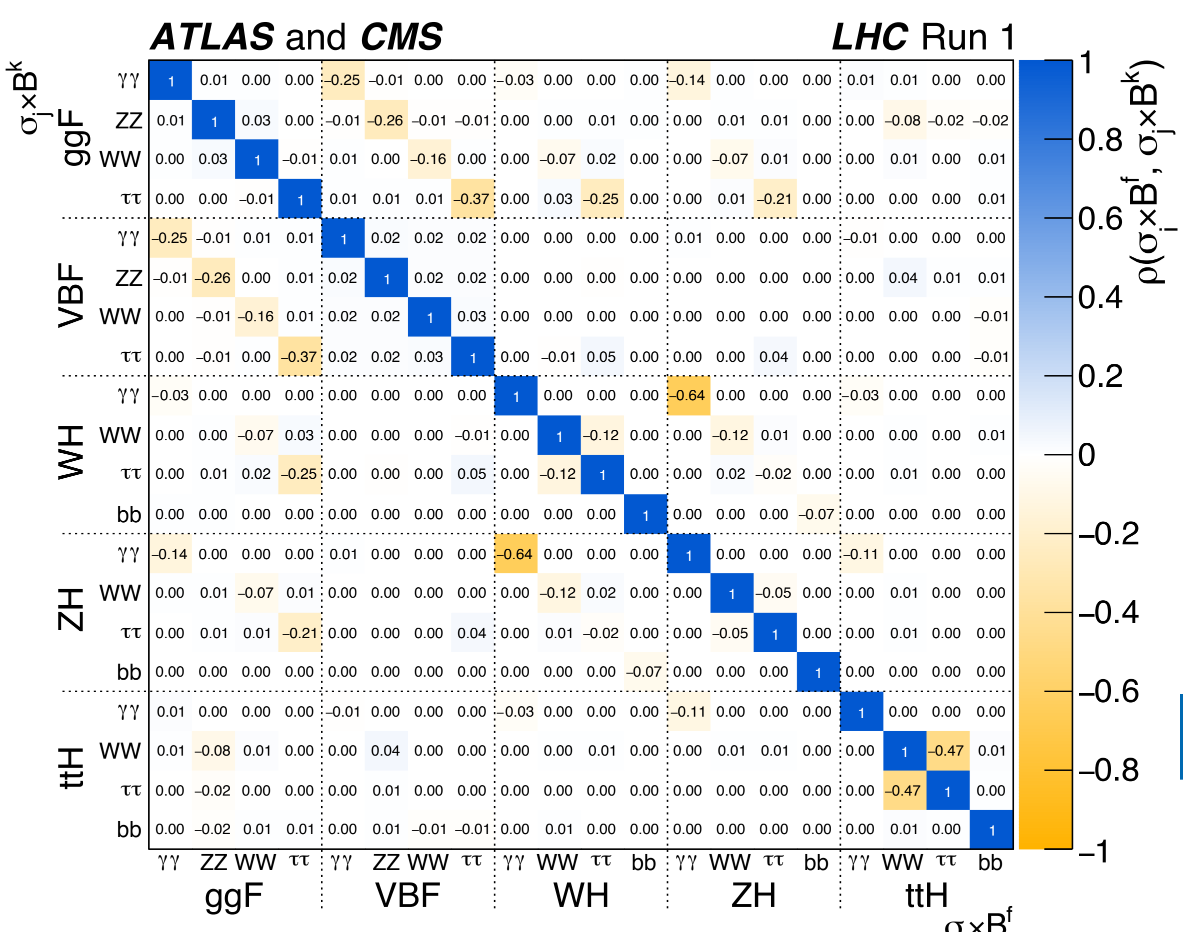
Individually favour negative relative signs, but combination favours positive

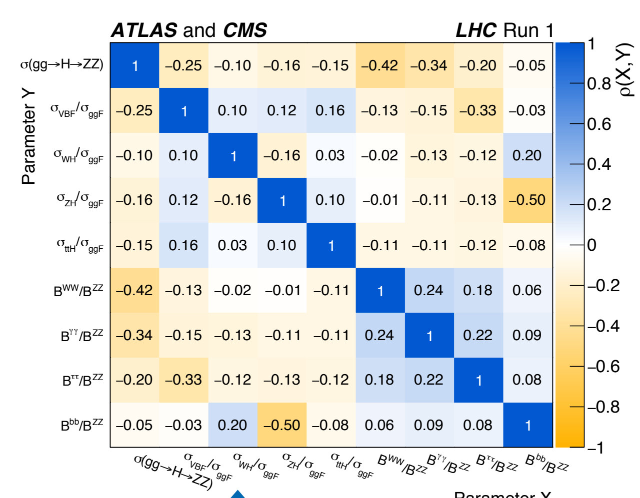
Interpretation

Making the results available (I)

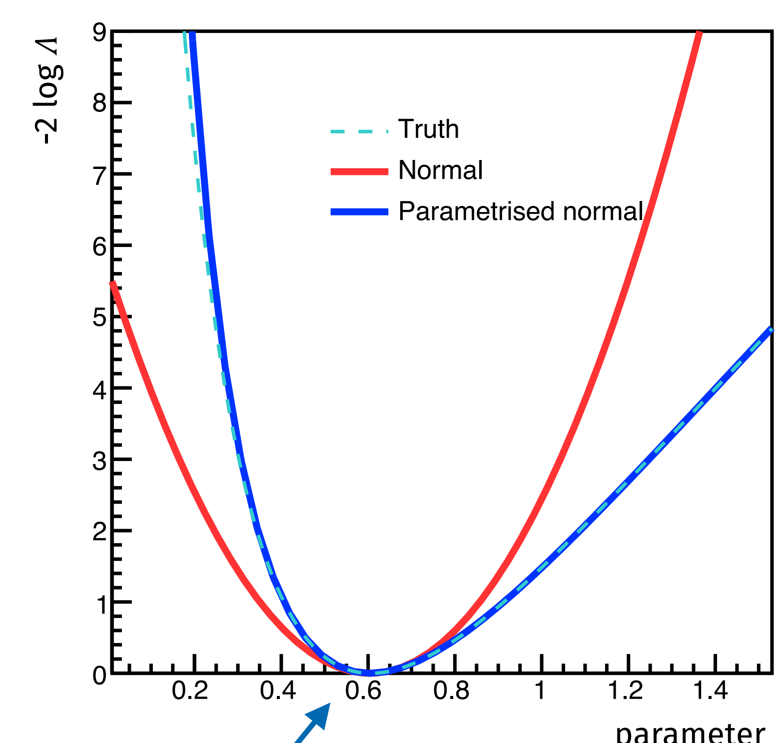
Ongoing efforts to make results from generic models available for re-interpretations

- Typically quote 1σ and 2σ **confidence intervals**
  - Sufficient to perform **naive combination** of results, but may differ significantly from full model when neglecting correlations, etc.
  - **Correlation matrix** at the minimum provides additional information
  - Can build **(parametrised) normal distributions** that reproduce likelihood scans if model sufficiently Gaussian





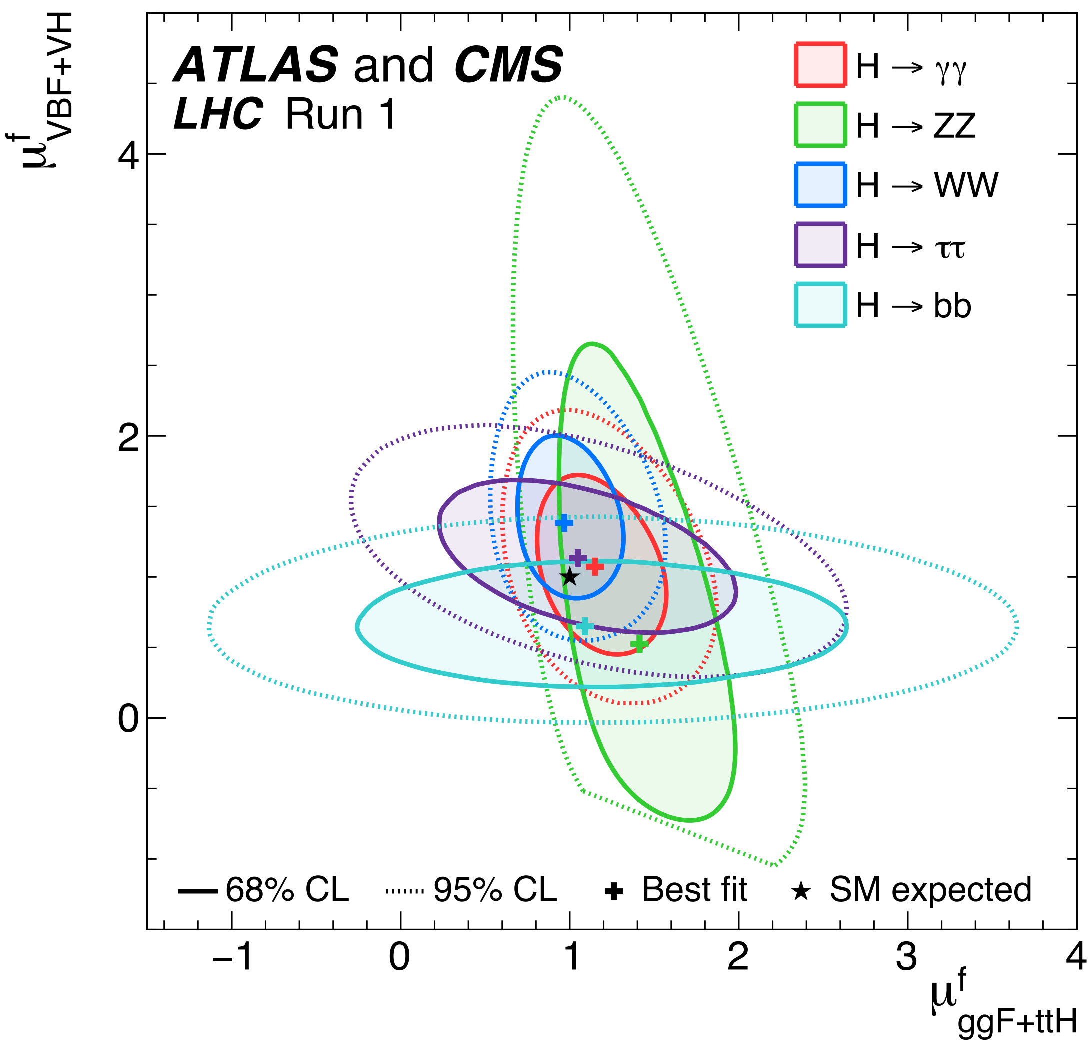
Assumptions introduce correlations

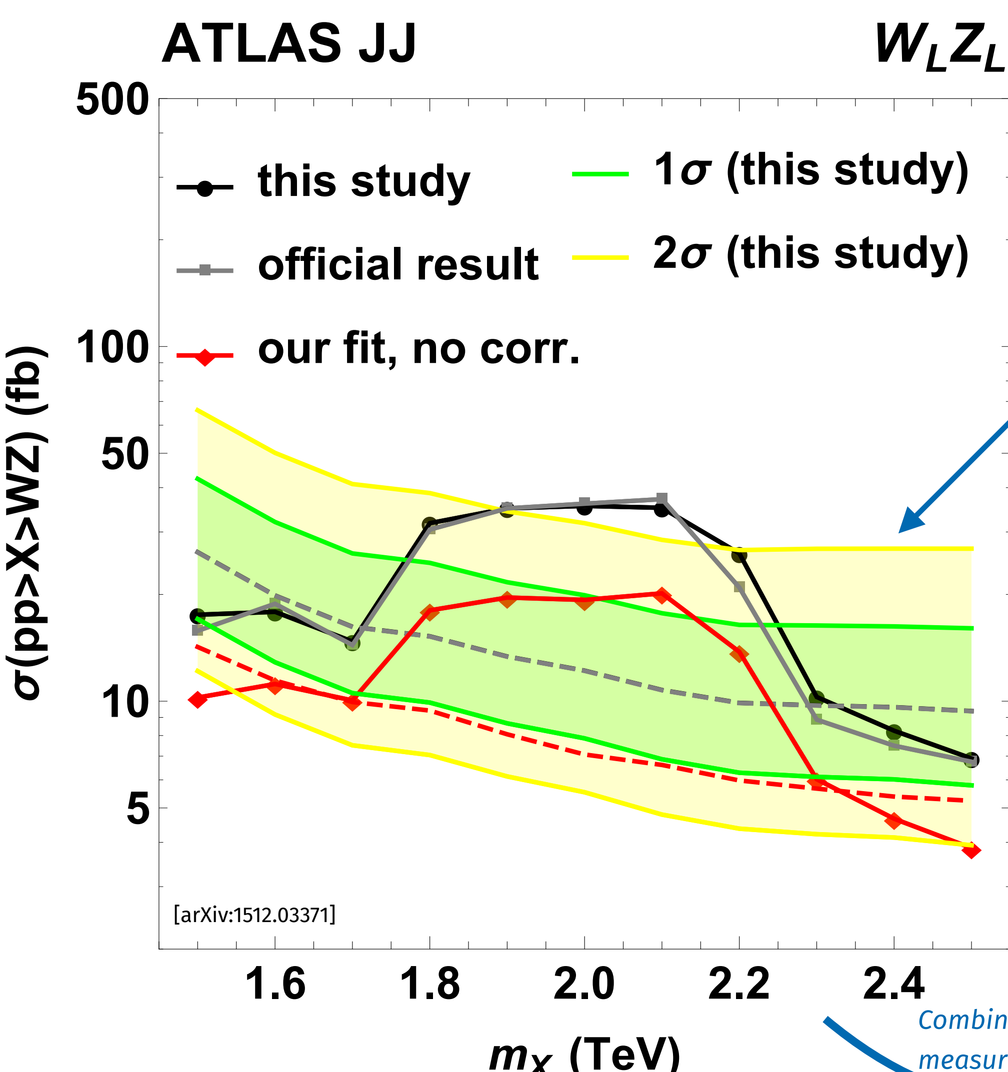


Uses just knowledge of confidence interval

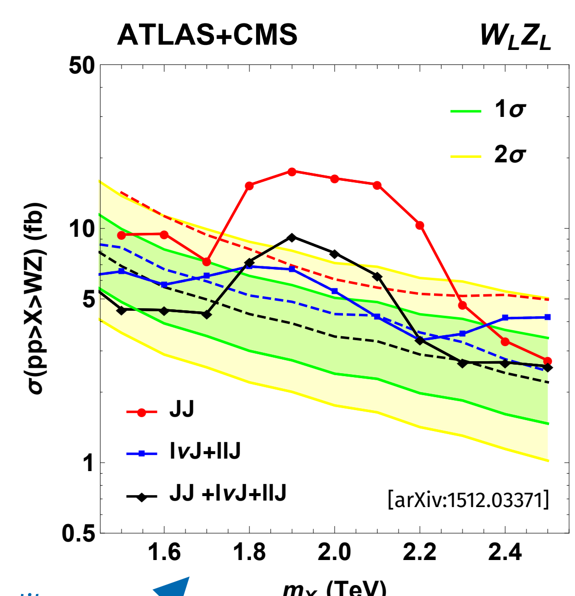
Making the results available (II)

- Often provide **(multi-dimensional) profile likelihood scans** to compare predictions, e.g. from simulating custom model
- Improving habit to make templates available, e.g. on **HepData**
  - Build simplified likelihood that can be combined
- Within the collaboration, **share the persisted pdf**, e.g. for BSM interpretations





Phenomenological parametrisation using HepData material



Combine diboson measurements

What’s next?

	Best fit μ	Uncertainty				
		Total	Stat	Expt	Thbgd	Thsig
ATLAS + CMS (measured)	1.09	$^{+0.11}_{-0.10}$	$^{+0.07}_{-0.07}$	$^{+0.04}_{-0.04}$	$^{+0.03}_{-0.03}$	$^{+0.07}_{-0.06}$
ATLAS + CMS (expected)		$^{+0.11}_{-0.10}$	$^{+0.07}_{-0.07}$	$^{+0.04}_{-0.04}$	$^{+0.03}_{-0.03}$	$^{+0.07}_{-0.06}$
ATLAS (measured)	1.20	$^{+0.15}_{-0.14}$	$^{+0.10}_{-0.10}$	$^{+0.06}_{-0.06}$	$^{+0.04}_{-0.04}$	$^{+0.08}_{-0.07}$
ATLAS (expected)		$^{+0.14}_{-0.13}$	$^{+0.10}_{-0.10}$	$^{+0.06}_{-0.05}$	$^{+0.04}_{-0.04}$	$^{+0.07}_{-0.06}$
CMS (measured)	0.97	$^{+0.14}_{-0.13}$	$^{+0.09}_{-0.09}$	$^{+0.05}_{-0.05}$	$^{+0.04}_{-0.03}$	$^{+0.07}_{-0.06}$
CMS (expected)		$^{+0.14}_{-0.13}$	$^{+0.09}_{-0.09}$	$^{+0.05}_{-0.05}$	$^{+0.04}_{-0.03}$	$^{+0.08}_{-0.06}$

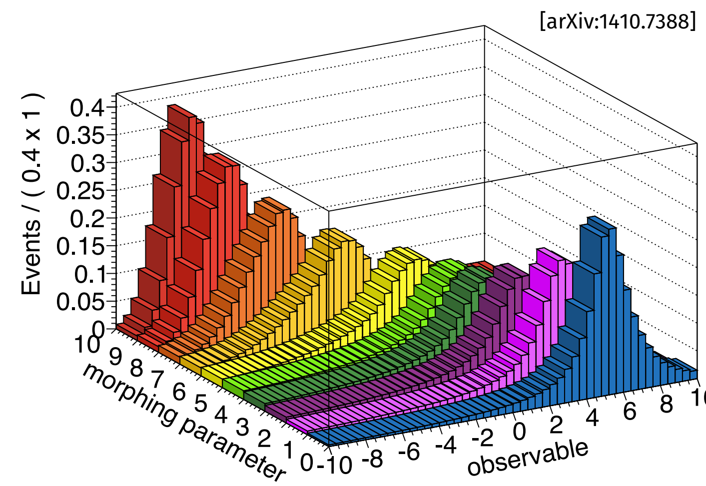
- With increasing data statistics, more sophisticated **theoretical models** can be tested, but also **systematic uncertainties** and **statistical modelling** gain in importance
  - Experimental systematics will shrink with more data being available for calibration measurements
  - Theoretical uncertainties will shrink when calculations improve
- This poses **many questions** and requires **technical developments**:
  - *What distributions of **subsidiary measurements** are correct?*
  - *Does the **parametrisation of an systematic uncertainty** capture the “real” effect, or introduces it too much detail?*
  - *Is the interpolation between sampling points for **response mapping** accurate, e.g. hadronisation/fragmentation model and non-factorisable uncertainties?*

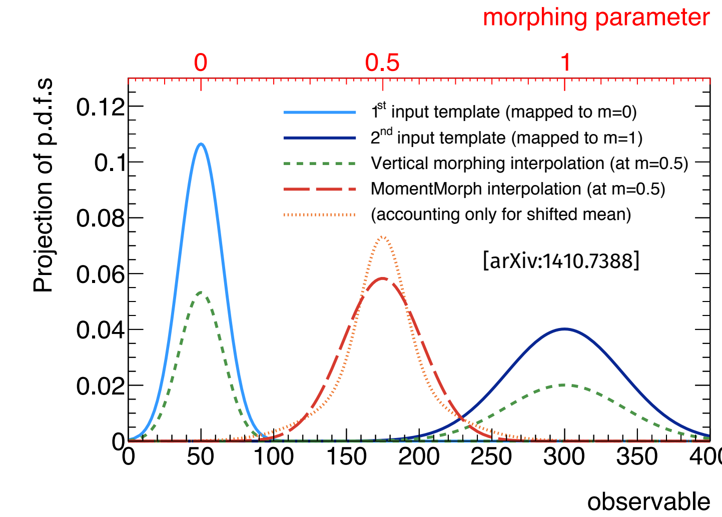
Summary

- Combinations are gaining in importance in HEP
- Combining **full likelihood preserves all available information**, i.e. correlations, composition of selections, subsidiary measurements, etc.
- Build the likelihood function using **RooFit**, persist it in a sharable **RooWorkspace** container, and perform inference using **RooStats** tools
- Combination requires **harmonisation**, poses **technical challenges** and drives **developments**
- Naive combinations can be done quickly, but do often not reproduce full results
- Ongoing efforts to make results available to community

Backup

Template morphing





Template morphing

- Analytical shapes
  - Parametrized Gaussian
  - . . .
- Empirical descriptions
  - Piecewise linear interpolation
  - Integral morphing
  - Moment morphing
  - . . .
- Physics inspired
  - Effective Lagrangian Morphing

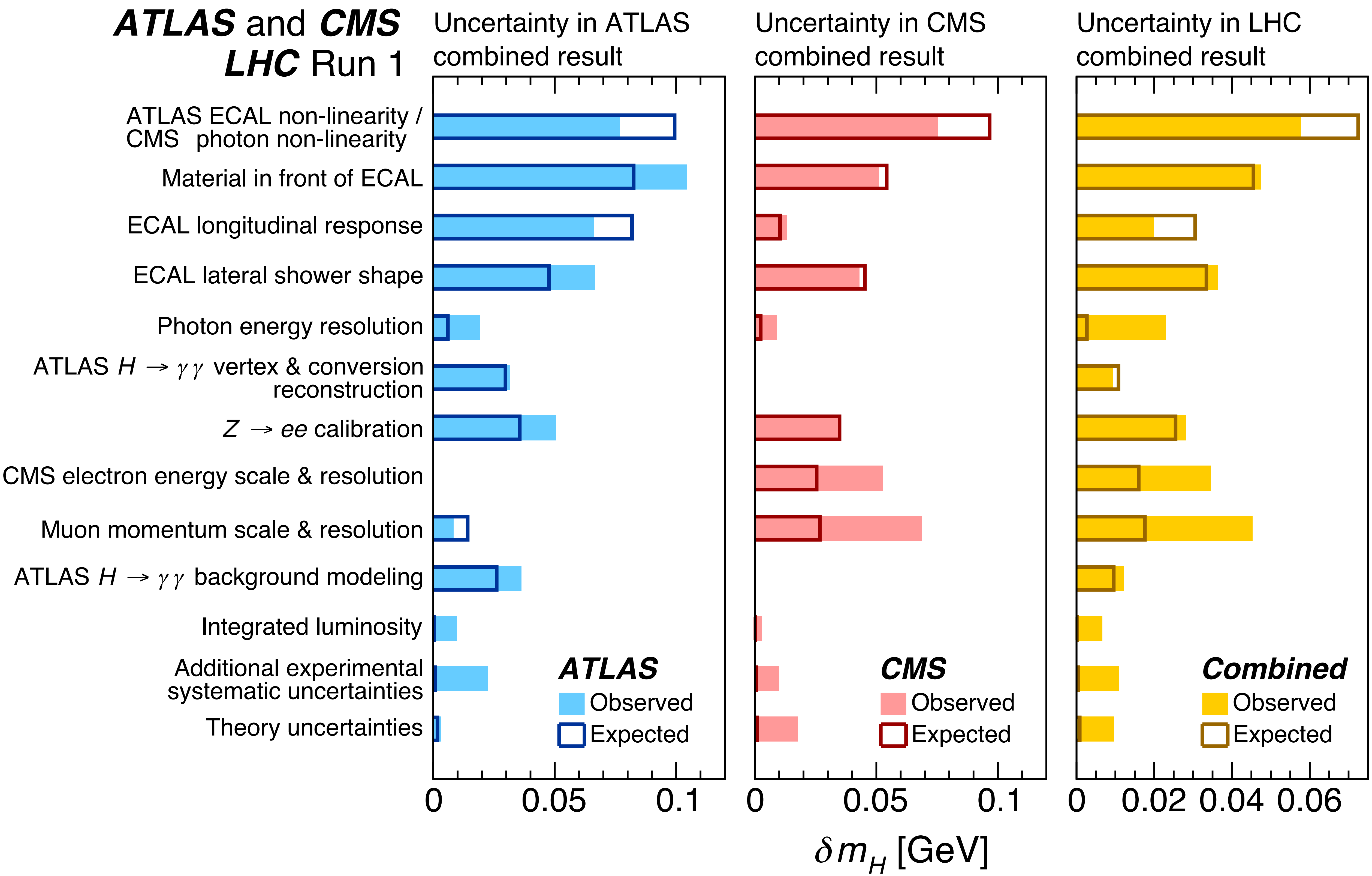
Mainly used for physics measurements, e.g. m_H, or CP mixing

Handles multidimensional morphing for systematics, but no simple interface

Typically used for modelling effect of systematic uncertainties on templates

Each algorithm has advantages and disadvantages and has specific applications

Uncertainties on the Higgs boson mass



Uncertainties in the Higgs combination

Parameter	SM prediction	ATLAS+CMS Best fit value	ATLAS+CMS Stat	ATLAS+CMS Expt	ATLAS+CMS Thbgd	ATLAS+CMS Thsig	ATLAS Best fit value	ATLAS Stat	ATLAS Expt	ATLAS Thbgd	ATLAS Thsig	CMS Best fit value	CMS Stat	CMS Expt	CMS Thbgd	CMS Thsig
$\sigma(gg \to H \to ZZ)$ [pb]	0.51 ±0.06	0.59 $^{+0.11}_{-0.10}$ ($^{+0.11}_{-0.10}$)	$^{+0.11}_{-0.10}$ ($^{+0.11}_{-0.09}$)	$^{+0.02}_{-0.01}$ ($^{+0.02}_{-0.02}$)	$^{+0.01}_{-0.01}$ ($^{+0.01}_{-0.01}$)	$^{+0.01}_{-0.01}$ ($^{+0.01}_{-0.01}$)	0.77 $^{+0.19}_{-0.17}$ ($^{+0.16}_{-0.14}$)	$^{+0.19}_{-0.16}$ ($^{+0.16}_{-0.13}$)	$^{+0.04}_{-0.03}$ ($^{+0.03}_{-0.02}$)	$^{+0.02}_{-0.02}$ ($^{+0.01}_{-0.01}$)	$^{+0.01}_{-0.01}$ ($^{+0.01}_{-0.01}$)	0.44 $^{+0.14}_{-0.12}$ ($^{+0.15}_{-0.13}$)	$^{+0.13}_{-0.11}$ ($^{+0.15}_{-0.13}$)	$^{+0.04}_{-0.03}$ ($^{+0.03}_{-0.02}$)	$^{+0.01}_{-0.01}$ ($^{+0.01}_{-0.01}$)	$^{+0.02}_{-0.01}$ ($^{+0.02}_{-0.01}$)
$\sigma_{VBF}/\sigma_{ggF}$	0.082 ±0.009	0.109 $^{+0.034}_{-0.027}$ ($^{+0.029}_{-0.024}$)	$^{+0.029}_{-0.024}$ ($^{+0.024}_{-0.020}$)	$^{+0.013}_{-0.009}$ ($^{+0.012}_{-0.009}$)	$^{+0.006}_{-0.004}$ ($^{+0.005}_{-0.003}$)	$^{+0.010}_{-0.008}$ ($^{+0.009}_{-0.007}$)	0.079 $^{+0.035}_{-0.026}$ ($^{+0.042}_{-0.031}$)	$^{+0.030}_{-0.023}$ ($^{+0.036}_{-0.028}$)	$^{+0.014}_{-0.009}$ ($^{+0.016}_{-0.010}$)	$^{+0.008}_{-0.005}$ ($^{+0.010}_{-0.006}$)	$^{+0.009}_{-0.006}$ ($^{+0.011}_{-0.008}$)	0.138 $^{+0.073}_{-0.051}$ ($^{+0.043}_{-0.033}$)	$^{+0.061}_{-0.046}$ ($^{+0.037}_{-0.029}$)	$^{+0.033}_{-0.019}$ ($^{+0.020}_{-0.012}$)	$^{+0.014}_{-0.006}$ ($^{+0.006}_{-0.003}$)	$^{+0.015}_{-0.010}$ ($^{+0.010}_{-0.008}$)
σ_{WH}/σ_{ggF}	0.037 ±0.004	0.031 $^{+0.028}_{-0.026}$ ($^{+0.021}_{-0.017}$)	$^{+0.024}_{-0.022}$ ($^{+0.019}_{-0.015}$)	$^{+0.012}_{-0.012}$ ($^{+0.008}_{-0.005}$)	$^{+0.008}_{-0.008}$ ($^{+0.007}_{-0.005}$)	$^{+0.003}_{-0.002}$ ($^{+0.002}_{-0.002}$)	0.054 $^{+0.036}_{-0.026}$ ($^{+0.033}_{-0.022}$)	$^{+0.031}_{-0.023}$ ($^{+0.029}_{-0.020}$)	$^{+0.012}_{-0.008}$ ($^{+0.010}_{-0.006}$)	$^{+0.014}_{-0.009}$ ($^{+0.011}_{-0.006}$)	$^{+0.007}_{-0.004}$ ($^{+0.005}_{-0.002}$)	0.005 $^{+0.044}_{-0.037}$ ($^{+0.032}_{-0.022}$)	$^{+0.037}_{-0.028}$ ($^{+0.027}_{-0.020}$)	$^{+0.021}_{-0.023}$ ($^{+0.014}_{-0.008}$)	$^{+0.010}_{-0.008}$ ($^{+0.009}_{-0.006}$)	$^{+0.003}_{-0.001}$ ($^{+0.003}_{-0.001}$)
σ_{ZH}/σ_{ggF}	0.0216 ±0.0024	0.066 $^{+0.039}_{-0.031}$ ($^{+0.016}_{-0.011}$)	$^{+0.032}_{-0.025}$ ($^{+0.014}_{-0.010}$)	$^{+0.018}_{-0.012}$ ($^{+0.006}_{-0.003}$)	$^{+0.014}_{-0.012}$ ($^{+0.006}_{-0.003}$)	$^{+0.005}_{-0.003}$ ($^{+0.002}_{-0.001}$)	0.013 $^{+0.028}_{-0.014}$ ($^{+0.027}_{-0.014}$)	$^{+0.021}_{-0.012}$ ($^{+0.023}_{-0.013}$)	$^{+0.013}_{-0.005}$ ($^{+0.009}_{-0.003}$)	$^{+0.013}_{-0.005}$ ($^{+0.011}_{-0.004}$)	$^{+0.003}_{-0.002}$ ($^{+0.003}_{-0.001}$)	0.123 $^{+0.076}_{-0.053}$ ($^{+0.024}_{-0.013}$)	$^{+0.063}_{-0.046}$ ($^{+0.020}_{-0.012}$)	$^{+0.038}_{-0.022}$ ($^{+0.010}_{-0.004}$)	$^{+0.019}_{-0.013}$ ($^{+0.009}_{-0.004}$)	$^{+0.009}_{-0.005}$ ($^{+0.002}_{-0.001}$)
$\sigma_{ttH}/\sigma_{ggF}$	0.0067 ±0.0010	0.0220 $^{+0.0068}_{-0.0057}$ ($^{+0.0042}_{-0.0035}$)	$^{+0.0055}_{-0.0048}$ ($^{+0.0033}_{-0.0027}$)	$^{+0.0031}_{-0.0023}$ ($^{+0.0018}_{-0.0013}$)	$^{+0.0023}_{-0.0020}$ ($^{+0.0020}_{-0.0019}$)	$^{+0.0014}_{-0.0010}$ ($^{+0.0005}_{-0.0003}$)	0.0126 $^{+0.0066}_{-0.0053}$ ($^{+0.0061}_{-0.0045}$)	$^{+0.0052}_{-0.0042}$ ($^{+0.0047}_{-0.0035}$)	$^{+0.0031}_{-0.0023}$ ($^{+0.0026}_{-0.0017}$)	$^{+0.0024}_{-0.0020}$ ($^{+0.0027}_{-0.0022}$)	$^{+0.0013}_{-0.0007}$ ($^{+0.0008}_{-0.0004}$)	0.0340 $^{+0.0158}_{-0.0116}$ ($^{+0.0066}_{-0.0054}$)	$^{+0.0121}_{-0.0097}$ ($^{+0.0051}_{-0.0038}$)	$^{+0.0085}_{-0.0051}$ ($^{+0.0027}_{-0.0016}$)	$^{+0.0048}_{-0.0036}$ ($^{+0.0032}_{-0.0034}$)	$^{+0.0026}_{-0.0015}$ ($^{+0.0006}_{-0.0002}$)
B^{WW}/B^{ZZ}	8.09 ± < 0.01	6.7 $^{+1.6}_{-1.3}$ ($^{+2.2}_{-1.7}$)	$^{+1.5}_{-1.2}$ ($^{+2.0}_{-1.6}$)	$^{+0.4}_{-0.3}$ ($^{+0.7}_{-0.5}$)	$^{+0.4}_{-0.3}$ ($^{+0.5}_{-0.4}$)	$^{+0.3}_{-0.2}$ ($^{+0.3}_{-0.2}$)	6.5 $^{+2.1}_{-1.6}$ ($^{+3.5}_{-2.4}$)	$^{+2.0}_{-1.4}$ ($^{+3.3}_{-2.2}$)	$^{+0.6}_{-0.4}$ ($^{+0.9}_{-0.6}$)	$^{+0.5}_{-0.4}$ ($^{+0.8}_{-0.6}$)	$^{+0.3}_{-0.2}$ ($^{+0.4}_{-0.3}$)	7.1 $^{+2.9}_{-2.1}$ ($^{+3.2}_{-2.2}$)	$^{+2.6}_{-1.8}$ ($^{+2.9}_{-2.0}$)	$^{+1.0}_{-0.7}$ ($^{+1.1}_{-0.8}$)	$^{+0.7}_{-0.5}$ ($^{+0.7}_{-0.5}$)	$^{+0.4}_{-0.3}$ ($^{+0.5}_{-0.4}$)
$B^{\gamma\gamma}/B^{ZZ}$	0.0854 ±0.0010	0.069 $^{+0.018}_{-0.014}$ ($^{+0.025}_{-0.019}$)	$^{+0.018}_{-0.014}$ ($^{+0.024}_{-0.019}$)	$^{+0.003}_{-0.002}$ ($^{+0.005}_{-0.003}$)	$^{+0.002}_{-0.001}$ ($^{+0.002}_{-0.001}$)	$^{+0.002}_{-0.002}$ ($^{+0.003}_{-0.002}$)	0.062 $^{+0.024}_{-0.018}$ ($^{+0.040}_{-0.027}$)	$^{+0.023}_{-0.017}$ ($^{+0.039}_{-0.027}$)	$^{+0.007}_{-0.004}$ ($^{+0.008}_{-0.005}$)	$^{+0.002}_{-0.001}$ ($^{+0.003}_{-0.002}$)	$^{+0.003}_{-0.002}$ ($^{+0.004}_{-0.003}$)	0.079 $^{+0.034}_{-0.023}$ ($^{+0.035}_{-0.025}$)	$^{+0.032}_{-0.023}$ ($^{+0.034}_{-0.024}$)	$^{+0.009}_{-0.005}$ ($^{+0.007}_{-0.004}$)	$^{+0.003}_{-0.002}$ ($^{+0.002}_{-0.001}$)	$^{+0.004}_{-0.003}$ ($^{+0.004}_{-0.003}$)
$B^{\tau\tau}/B^{ZZ}$	2.36 ±0.05	1.77 $^{+0.59}_{-0.46}$ ($^{+0.90}_{-0.68}$)	$^{+0.52}_{-0.41}$ ($^{+0.75}_{-0.58}$)	$^{+0.27}_{-0.20}$ ($^{+0.47}_{-0.33}$)	$^{+0.05}_{-0.04}$ ($^{+0.08}_{-0.06}$)	$^{+0.06}_{-0.04}$ ($^{+0.10}_{-0.06}$)	2.17 $^{+1.07}_{-0.74}$ ($^{+1.54}_{-0.98}$)	$^{+0.89}_{-0.64}$ ($^{+1.30}_{-0.86}$)	$^{+0.53}_{-0.35}$ ($^{+0.76}_{-0.44}$)	$^{+0.16}_{-0.10}$ ($^{+0.22}_{-0.12}$)	$^{+0.17}_{-0.09}$ ($^{+0.22}_{-0.10}$)	1.56 $^{+0.90}_{-0.61}$ ($^{+1.23}_{-0.86}$)	$^{+0.78}_{-0.54}$ ($^{+1.03}_{-0.73}$)	$^{+0.45}_{-0.26}$ ($^{+0.66}_{-0.44}$)	$^{+0.07}_{-0.05}$ ($^{+0.04}_{-0.03}$)	$^{+0.07}_{-0.04}$ ($^{+0.12}_{-0.07}$)
B^{bb}/B^{ZZ}	21.5 ±1.0	4.2 $^{+4.4}_{-2.6}$ ($^{+16.8}_{-9.0}$)	$^{+2.8}_{-2.0}$ ($^{+13.9}_{-7.9}$)	$^{+2.3}_{-1.1}$ ($^{+6.3}_{-2.8}$)	$^{+2.5}_{-1.2}$ ($^{+6.7}_{-3.3}$)	$^{+0.4}_{-0.2}$ ($^{+2.1}_{-0.9}$)	9.6 $^{+10.1}_{-5.7}$ ($^{+29.3}_{-11.8}$)	$^{+7.4}_{-4.4}$ ($^{+24.2}_{-10.5}$)	$^{+4.5}_{-2.4}$ ($^{+10.9}_{-3.3}$)	$^{+5.1}_{-2.7}$ ($^{+11.8}_{-4.0}$)	$^{+1.3}_{-0.5}$ ($^{+4.0}_{-1.2}$)	3.7 $^{+4.1}_{-2.4}$ ($^{+29.4}_{-11.9}$)	$^{+3.1}_{-2.0}$ ($^{+23.4}_{-10.4}$)	$^{+1.8}_{-0.9}$ ($^{+12.7}_{-3.8}$)	$^{+1.9}_{-1.1}$ ($^{+12.2}_{-4.4}$)	$^{+0.4}_{-0.2}$ ($^{+2.5}_{-0.9}$)

Parameter	ATLAS+CMS Best fit value	ATLAS+CMS Stat	ATLAS+CMS Expt	ATLAS+CMS Thbgd	ATLAS+CMS Thsig	ATLAS Best fit value	ATLAS Stat	ATLAS Expt	ATLAS Thbgd	ATLAS Thsig	CMS Best fit value	CMS Stat	CMS Expt	CMS Thbgd	CMS Thsig
κ_{gZ}	1.09 $^{+0.11}_{-0.11}$ ($^{+0.11}_{-0.11}$)	$^{+0.09}_{-0.09}$ ($^{+0.09}_{-0.09}$)	$^{+0.02}_{-0.02}$ ($^{+0.02}_{-0.02}$)	$^{+0.00}_{-0.01}$ ($^{+0.01}_{-0.01}$)	$^{+0.06}_{-0.05}$ ($^{+0.06}_{-0.05}$)	1.20 $^{+0.16}_{-0.15}$ ($^{+0.15}_{-0.15}$)	$^{+0.14}_{-0.14}$ ($^{+0.14}_{-0.13}$)	$^{+0.03}_{-0.03}$ ($^{+0.03}_{-0.03}$)	$^{+0.02}_{-0.02}$ ($^{+0.01}_{-0.02}$)	$^{+0.07}_{-0.06}$ ($^{+0.06}_{-0.05}$)	0.99 $^{+0.14}_{-0.13}$ ($^{+0.14}_{-0.14}$)	$^{+0.12}_{-0.12}$ ($^{+0.13}_{-0.12}$)	$^{+0.03}_{-0.04}$ ($^{+0.03}_{-0.03}$)	$^{+0.01}_{-0.01}$ ($^{+0.01}_{-0.01}$)	$^{+0.06}_{-0.04}$ ($^{+0.06}_{-0.05}$)
λ_{Zg}	1.27 $^{+0.23}_{-0.20}$ ($^{+0.20}_{-0.17}$)	$^{+0.18}_{-0.16}$ ($^{+0.15}_{-0.14}$)	$^{+0.10}_{-0.07}$ ($^{+0.08}_{-0.06}$)	$^{+0.06}_{-0.05}$ ($^{+0.05}_{-0.04}$)	$^{+0.10}_{-0.08}$ ($^{+0.08}_{-0.07}$)	1.07 $^{+0.26}_{-0.22}$ ($^{+0.28}_{-0.23}$)	$^{+0.21}_{-0.18}$ ($^{+0.23}_{-0.20}$)	$^{+0.10}_{-0.06}$ ($^{+0.10}_{-0.07}$)	$^{+0.07}_{-0.06}$ ($^{+0.09}_{-0.05}$)	$^{+0.09}_{-0.07}$ ($^{+0.09}_{-0.07}$)	1.47 $^{+0.45}_{-0.34}$ ($^{+0.27}_{-0.23}$)	$^{+0.35}_{-0.28}$ ($^{+0.21}_{-0.19}$)	$^{+0.22}_{-0.14}$ ($^{+0.12}_{-0.09}$)	$^{+0.11}_{-0.10}$ ($^{+0.07}_{-0.05}$)	$^{+0.13}_{-0.09}$ ($^{+0.09}_{-0.07}$)
λ_{tg}	1.78 $^{+0.30}_{-0.27}$ ($^{+0.28}_{-0.38}$)	$^{+0.21}_{-0.20}$ ($^{+0.20}_{-0.30}$)	$^{+0.13}_{-0.11}$ ($^{+0.11}_{-0.13}$)	$^{+0.09}_{-0.09}$ ($^{+0.14}_{-0.20}$)	$^{+0.14}_{-0.11}$ ($^{+0.09}_{-0.05}$)	1.40 $^{+0.34}_{-0.33}$ ($^{+0.38}_{-0.54}$)	$^{+0.25}_{-0.24}$ ($^{+0.28}_{-0.39}$)	$^{+0.14}_{-0.15}$ ($^{+0.14}_{-0.22}$)	$^{+0.12}_{-0.14}$ ($^{+0.18}_{-0.29}$)	$^{+0.14}_{-0.09}$ ($^{+0.11}_{-0.06}$)	-2.26 $^{+0.50}_{-0.53}$ ($^{+0.42}_{-0.64}$)	$^{+0.43}_{-0.39}$ ($^{+0.31}_{-0.42}$)	$^{+0.22}_{-0.23}$ ($^{+0.16}_{-0.22}$)	$^{+0.04}_{-0.18}$ ($^{+0.21}_{-0.43}$)	$^{+0.14}_{-0.21}$ ($^{+0.11}_{-0.06}$)
λ_{WZ}	0.88 $^{+0.10}_{-0.09}$ ($^{+0.12}_{-0.10}$)	$^{+0.09}_{-0.08}$ ($^{+0.11}_{-0.09}$)	$^{+0.03}_{-0.03}$ ($^{+0.04}_{-0.03}$)	$^{+0.03}_{-0.02}$ ($^{+0.03}_{-0.03}$)	$^{+0.02}_{-0.01}$ ($^{+0.02}_{-0.01}$)	0.92 $^{+0.14}_{-0.12}$ ($^{+0.18}_{-0.15}$)	$^{+0.13}_{-0.11}$ ($^{+0.17}_{-0.13}$)	$^{+0.04}_{-0.03}$ ($^{+0.04}_{-0.04}$)	$^{+0.03}_{-0.03}$ ($^{+0.04}_{-0.04}$)	$^{+0.02}_{-0.02}$ ($^{+0.02}_{-0.02}$)	-0.85 $^{+0.13}_{-0.15}$ ($^{+0.17}_{-0.14}$)	$^{+0.11}_{-0.13}$ ($^{+0.15}_{-0.13}$)	$^{+0.05}_{-0.06}$ ($^{+0.06}_{-0.05}$)	$^{+0.04}_{-0.04}$ ($^{+0.03}_{-0.03}$)	$^{+0.01}_{-0.03}$ ($^{+0.03}_{-0.02}$)
$\lvert\lambda_{\gamma Z}\rvert$	0.89 $^{+0.11}_{-0.10}$ ($^{+0.13}_{-0.12}$)	$^{+0.10}_{-0.09}$ ($^{+0.13}_{-0.11}$)	$^{+0.03}_{-0.02}$ ($^{+0.03}_{-0.02}$)	$^{+0.01}_{-0.02}$ ($^{+0.01}_{-0.01}$)	$^{+0.02}_{-0.01}$ ($^{+0.02}_{-0.01}$)	0.87 $^{+0.15}_{-0.13}$ ($^{+0.20}_{-0.17}$)	$^{+0.15}_{-0.13}$ ($^{+0.20}_{-0.17}$)	$^{+0.04}_{-0.04}$ ($^{+0.05}_{-0.03}$)	$^{+0.02}_{-0.01}$ ($^{+0.03}_{-0.01}$)	$^{+0.02}_{-0.02}$ ($^{+0.02}_{-0.02}$)	0.91 $^{+0.17}_{-0.14}$ ($^{+0.18}_{-0.16}$)	$^{+0.16}_{-0.14}$ ($^{+0.18}_{-0.15}$)	$^{+0.04}_{-0.03}$ ($^{+0.04}_{-0.03}$)	$^{+0.02}_{-0.02}$ ($^{+0.01}_{-0.01}$)	$^{+0.02}_{-0.02}$ ($^{+0.02}_{-0.02}$)
$\lvert\lambda_{\tau Z}\rvert$	0.85 $^{+0.13}_{-0.12}$ ($^{+0.17}_{-0.15}$)	$^{+0.12}_{-0.10}$ ($^{+0.14}_{-0.13}$)	$^{+0.07}_{-0.06}$ ($^{+0.09}_{-0.08}$)	$^{+0.01}_{-0.02}$ ($^{+0.02}_{-0.02}$)	$^{+0.02}_{-0.01}$ ($^{+0.02}_{-0.02}$)	0.96 $^{+0.21}_{-0.18}$ ($^{+0.27}_{-0.23}$)	$^{+0.18}_{-0.15}$ ($^{+0.23}_{-0.19}$)	$^{+0.10}_{-0.09}$ ($^{+0.13}_{-0.11}$)	$^{+0.04}_{-0.03}$ ($^{+0.04}_{-0.03}$)	$^{+0.03}_{-0.02}$ ($^{+0.04}_{-0.02}$)	0.78 $^{+0.20}_{-0.17}$ ($^{+0.23}_{-0.20}$)	$^{+0.17}_{-0.15}$ ($^{+0.19}_{-0.17}$)	$^{+0.10}_{-0.08}$ ($^{+0.12}_{-0.11}$)	$^{+0.01}_{-0.02}$ ($^{+0.02}_{-0.01}$)	$^{+0.02}_{-0.01}$ ($^{+0.02}_{-0.02}$)
$\lvert\lambda_{bZ}\rvert$	0.58 $^{+0.16}_{-0.20}$ ($^{+0.25}_{-0.22}$)	$^{+0.12}_{-0.17}$ ($^{+0.21}_{-0.20}$)	$^{+0.07}_{-0.06}$ ($^{+0.09}_{-0.07}$)	$^{+0.07}_{-0.07}$ ($^{+0.08}_{-0.05}$)	$^{+0.03}_{-0.04}$ ($^{+0.06}_{-0.05}$)	0.61 $^{+0.24}_{-0.24}$ ($^{+0.36}_{-0.29}$)	$^{+0.20}_{-0.19}$ ($^{+0.31}_{-0.26}$)	$^{+0.09}_{-0.12}$ ($^{+0.12}_{-0.09}$)	$^{+0.10}_{-0.10}$ ($^{+0.11}_{-0.08}$)	$^{+0.04}_{-0.03}$ ($^{+0.08}_{-0.05}$)	0.47 $^{+0.26}_{-0.17}$ ($^{+0.38}_{-0.37}$)	$^{+0.23}_{-0.13}$ ($^{+0.32}_{-0.34}$)	$^{+0.06}_{-0.07}$ ($^{+0.15}_{-0.11}$)	$^{+0.11}_{-0.08}$ ($^{+0.11}_{-0.09}$)	$^{+0.00}_{-0.04}$ ($^{+0.07}_{-0.07}$)