Isospin – breaking effects in decay constants of heavy mesons from QCD sum rules

Dmitri Melikhov

HEPHY, Austrian Academy of Sciences, Vienna
and
D. V. Skobeltsyn Institute of Nuclear Physics, M. V. Lomonosov Moscow State University

We present a study of IB effects in decay constants of charmed and beauty pseudoscalar and vector mesons with QCD sum rules.

Our results:

$\delta f_B/f_B = (4.1 \pm 0.4)10^{-3}$**,** $\quad \delta f_{B^*}/f_{B^*} = (3.6 \pm 0.3)10^{-3}$**,**

$\delta f_D/f_D = (3.8 \pm 0.4)10^{-3}$**,** $\quad \delta f_{D^*}/f_{D^*} = (5.7 \pm 1.2)10^{-3}$**.**

In collaboration with W. Lucha and S. Simula

Supported by Austrian Science Fund (FWF): P29028

FWF Der Wissenschaftsfonds.

I. The decay constant of the pseudoscalar and the vector mesons are defined as

$$\langle 0|\bar{q}\gamma_\mu\gamma_5 Q|P_q(p)\rangle = i f_{P_q} p_\mu, \qquad \langle 0|\bar{q}\gamma_\mu Q|V_q(p)\rangle = f_{V_q} M_{V_q} \varepsilon_\mu(p).$$

We are interested in the isospin-breaking effects in the decay constants of heavy-light meson, i.e. the difference between the decay constants of $\bar{Q}d$ and $\bar{Q}u$ mesons originating from the small mass difference $\delta m = m_d - m_u$.

II. The basic object of the method—2-point function and its OPE:

$$\Pi(p^2) = i \int d^4x\, e^{ipx} \left\langle T\left(j_{Qq}(x) j^\dagger_{Qq}(0)\right)\right\rangle = \int\limits_{(m_Q+m_q)^2}^{\infty} \frac{ds}{s-p^2} \rho_{\rm pert}(s) + \Pi_{\rm power}(p^2),$$

Borel tranform $p^2 \to \tau$ gives $\Pi(\tau)$:

$$\Pi(\tau) = \int\limits_{(m_Q+m_q)^2}^{\infty} ds \exp(-s\tau)\rho_{\rm pert}(s) + \Pi_{\rm power}(\tau),$$

$\Pi_{\rm power}(\tau)$ **- polynomials in τ, calculable via condensates.**

III. Insert the full system of hadron states and obtain hadron representation for $\Pi(\tau)$

$$\Pi(\tau) = f^2 \exp(-M^2\tau) + \textbf{ excited stated},$$

IV. Duality: excited states are counterbalanced by large-s perturbative integrals, above $s_{\rm eff}$.

$$f_q^2 e^{-M_q^2\tau} = \int\limits_{(m_Q+m_q)^2}^{s_{\rm eff}^{(q)}} ds e^{-s\tau}\rho_{\rm pert}(s, m_Q, m_q, \alpha_s|m_{\rm sea}) + \Pi_{\rm power}(\tau, m_Q, m_q, \langle\bar{q}q\rangle ...) \equiv \Pi_{\rm dual}(\tau, s_{\rm eff}^{(q)}, m_Q, m_q|m_{\rm sea})$$

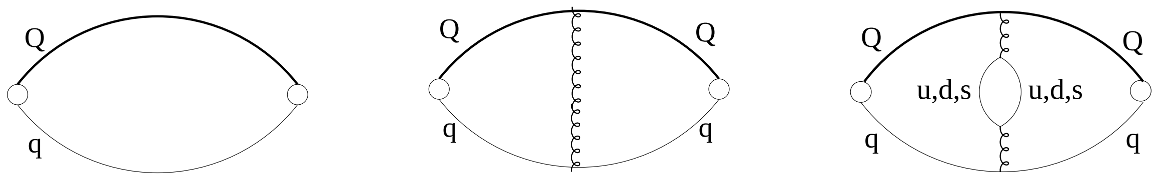
Here $m_q = m_u$, m_d, or m_s, and $m_{\rm sea}$ denotes the set of values m_u, m_d, and m_s.

The IB is related to

$$\delta\Pi_{\rm dual} = \Pi_{\rm dual}(\tau, s_{\rm eff}^d, m_d|m_{\rm sea}) - \Pi_{\rm dual}(\tau, s_{\rm eff}^u, m_u|m_{\rm sea}).$$

• **$\rho_{\rm pert}$ is calculated as expansion in powers of $a \equiv \alpha_s/\pi$:**

$$\rho_{\rm pert}(s, m_Q, m_q, \alpha_s) = \rho^{(0)}(s, m_Q, m_q) + a\rho^{(1)}(s, m_Q, m_q) + a^2\rho^{(2)}(s, m_Q, m_q|m_{\rm sea}) + \ldots$$



Order a^2 is the first order where the "sea-quark" masses appear. The second-order spectral density is known approximately, i.e. for massless quarks only, $\rho^{(2)}(s, m_Q, m_q = 0|m_{\rm sea} = 0)$.

Does this approximate OPE allow us to obtain IB and with what accuracy?

• For decay constants of heavy mesons, the OPE error is $O(a^2 m_s) \sim O($a few MeV$)$, $a \leq 0.1$

• However, for IB effects, the strange sea-quark contributions cancel in the difference and the OPE accuracy increases strongly:

$$\begin{aligned}\delta\Pi_{\rm dual} &= \Pi_{\rm dual}(\tau, s^d_{\rm eff}, m_d|m_u, m_d, m_s) - \Pi_{\rm dual}(\tau, s^u_{\rm eff}, m_u|m_u, m_d, m_s)\\ &= \Pi_{\rm dual}(\tau, s^d_{\rm eff}, m_d|m_{\rm sea} = 0) - \Pi_{\rm dual}(\tau, s^u_{\rm eff}, m_u|m_{\rm sea} = 0) + O(a^2\delta m).\end{aligned}$$

This relation suggests the following algorithm for the calculation of the IB effects:

(i) We obtain the decay constant $f(m_q)$ corresponding to the correlation function in which the light-quark mass in the LO and the NLO spectral densities is equal to m_q which we choose in the range $0 < m_q < m_s$, whereas in the NNLO spectral density the light u, d and s quarks are taken massless.

(ii) We calculates $f(m_d) - f(m_u)$; the OPE error in this quantity compared to the IB in "real" QCD (i.e. corresponding to the physical sea-quark masses) is of order $O(a^2\delta m) \simeq \delta m/100$. Noteworhty, the actual final uncertainty of the IB obtained from the sum-rule analysis is much larger as it has several other sources of uncertainties.

Nevertheless, it is important to understand that the known OPE allows one to address properly the IB effects.

OPE parameters

We must take into account the dependence of the $\langle\bar{q}q\rangle$ quark condensate on m_q. To characterize IB we introduce a scale-independent parameter $x_q = m_q/m_s$ and assume a linear interpolation between $\langle uu\rangle \sim \langle dd\rangle$ for $x_q \sim 0.04$ and $\langle ss\rangle$ condensate for $x_q = 1$.

$$m_u(2\text{ GeV}) = 2.3^{+0.7}_{-0.5}\text{ MeV},\quad m_d(2\text{ GeV}) = 4.8^{+0.5}_{-0.3}\text{ MeV},\quad m_s(2\text{ GeV}) = (93.8 \pm 2.4)\text{ MeV},$$
$$m_b(m_b) = (4.247 \pm 0.034)\text{ GeV},\quad \alpha_s(M_Z) = 0.1184 \pm 0.0020,$$
$$\left\langle \frac{\alpha_s}{\pi} GG \right\rangle = (0.024 \pm 0.012)\text{ GeV}^4,\quad \langle\bar{q}q\rangle(2\text{ GeV}) = -[(267 \pm 17)\text{ MeV}]^3,\quad \frac{\langle\bar{s}s\rangle(2\text{ GeV})}{\langle\bar{q}q\rangle(2\text{ GeV})} = 0.8 \pm 0.3.$$

Extraction procedure

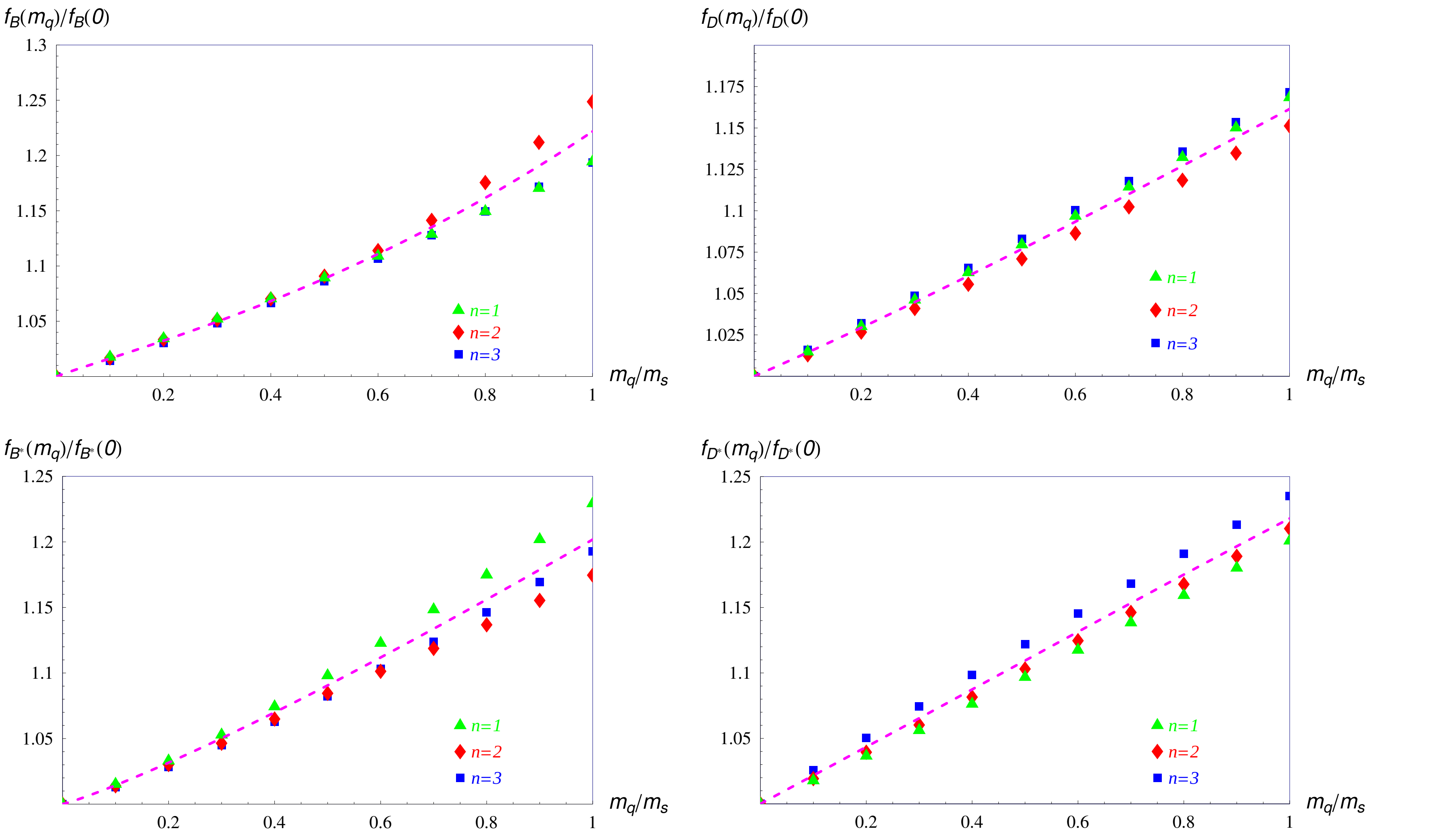
Our algorithm is based on considering the polynomial Ansaetze (the linear $n = 1$, the quadratic $n = 2$ and the cubic $n = 3$) for the τ-dependent $s_{\rm eff}$ and obtaining parameters of these thresholds numerically by the most accurate reproduction of the known meson mass in the τ-window. For a given $s_{\rm eff}$, the hadron decay constants is given by the sum rule.

As an input for our algorithm, in addition to the OPE, we need the meson mass M_{H_q}. We assume a linear interpolation in $x_q = m_q/m_s$:

$$M_{H_q}(x_q) = M_{H_{ud}} + x_q(M_{H_s} - M_{H_{ud}}),$$

H_{ud} meson containing $\bar{Q}$ and the light u- or d-quark, H_s is the $\bar{Q}s$ meson.

Precise shape of the interpolation function is not cricual; precise values of the meson masses are not important; slope is important.



$$R(x_q) = 1 + R_L x_q \log(x_q) + R_1 x_q + \ldots, \qquad \delta R = R(m_d) - R(m_u).$$

Summary and conclusions

• We present the first application of QCD sum rules to IB in the decay constants of heavy mesons and show that it is possible to obtain reasonably accurate predictions for $\delta f/f$. This was not obvious since the typical accuracy of the SR predictions for f_{B,B^*,D,D^*} is about 10-15 MeV.

• The known OPE (full m_q-dependence at LO and NLO, and massless light quarks in NNLO) allows one to access IB with $O(a^2 m_u, a^2 m_d)$ accuracy, whereas the accuracy of the individual quantities is $O(a^2 m_s)$.

• Knowing the explicit dependence of the OPE on m_q and obtaining the decay constants as a function $f(m_q)$ opens the possibility to access the IB effects.

• We report the following results:

$$\delta f_B/f_B = (4.1 \pm 0.4)10^{-3}, \qquad \delta f_D/f_D = (3.8 \pm 0.4)10^{-3},$$
$$\delta f_{B^*}/f_{B^*} = (3.6 \pm 0.3)10^{-3}, \qquad \delta f_{D^*}/f_{D^*} = (5.7 \pm 1.2)10^{-3}.$$

and

$$f_{B^0} - f_{B^+} = 0.79 \pm 0.14\,\text{MeV}, \qquad f_{B^{*0}} - f_{B^{*+}} = 0.65 \pm 0.10\,\text{MeV},$$
$$f_{D^+} - f_{D^0} = 0.78 \pm 0.13\,\text{MeV}, \qquad f_{D^{*+}} - f_{D^{*0}} = 1.41 \pm 0.42\,\text{MeV}.$$

We point out that our rather accurate finding for $f_{B^0} - f_{B^+}$ is considerably lower than the available estimate $f_{B^0} - f_{B^+} \sim 4$ MeV from lattice QCD.
The IB found in our analysis has a typical size around 1 MeV, which nicely agrees with the IB in K-mesons $f_{K^0} - f_{K^+} \sim 1.2$ MeV found in a lattice QCD study.