Form factors and differential distributions in rare radiative leptonic B-decays

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XIIth Quark Confinement and the Hadron Spectrum

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Motivation

Discrepancies concerning $b \to sl^+l^-$ transitions:

- $\frac{\mathcal{B}(B^+ \to K^+\mu^+\mu^-)}{\mathcal{B}(B^+ \to K^+e^+e^-)} = 0.745^{+0.090}_{-0.074}(\mathrm{stat}) \pm 0.036(\mathrm{syst})\ (2.6\sigma)$
- $\mathcal{B}(B^+ \to K^+\mu^+\mu^-)_{\mathrm{SM}} = (1.75^{+0.60}_{-0.29}) \times 10^{-7}$
  $\mathcal{B}(B^+ \to K^+\mu^+\mu^-)_{\mathrm{exp}} = (1.19 \pm 0.03 \pm 0.06) \times 10^{-7}$
- Same for $\mathcal{B}(B^+ \to \phi\mu^+\mu^-)\ (> 3\sigma)$
- $\frac{\mathcal{B}(B_s^0 \to \mu^+\mu^-)_{\mathrm{exp}}}{\mathcal{B}(B_s^0 \to \mu^+\mu^-)_{\mathrm{SM}}} = 0.76^{+0.20}_{-0.18}\ \ (1.2\sigma)$
- $\frac{\mathcal{B}(B_s^0 \to \ell^+\ell^-\gamma)}{\mathcal{B}(B_s^0 \to \ell^+\ell^-)} \sim \left(\frac{M_{B^0}}{m_\ell}\right)^2 \frac{\alpha_{em}}{4\pi}$
  $(M_{B^0}/m_\mu)^2 \sim 2.5 \times 10^3 \sim 4\pi/\alpha_{em} \Longrightarrow \mathcal{B}(B_s \to \mu^+\mu^-\gamma) \sim \mathcal{B}(B_s \to \mu^+\mu^-)$

Effective Theory

Wilson expansion:

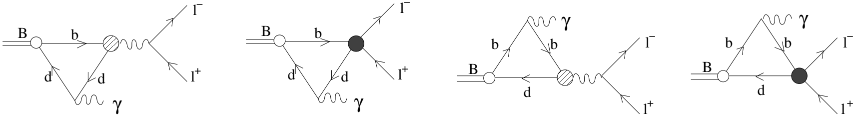
$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{tb} V_{tq}^* \sum_i c_i(\mu) O_i(\mu)$$

- W, t are integrated out
- c-quarks are dynamical

Buras,1995.

Contributions to Effective Hamiltonian

The $\ell^+\ell^-$ pair is coupled to the penguin



$$H_{\text{eff}}^{b\rightarrow d\ell^+\ell^-} = \frac{G_F}{\sqrt{2}}\frac{\alpha_{\text{em}}}{2\pi}V_{tb}V_{tq}^*\Big[-2im_b\frac{C_{7\gamma}(\mu)}{q^2}\cdot\bar{d}\sigma_{\mu\nu}q^\nu\,(1+\gamma_5)\,b\cdot\bar{\ell}\gamma^\mu\ell+$$
$$+C_{9V}^{\text{eff}}(\mu,q^2)\cdot\bar{d}\gamma_\mu\,(1-\gamma_5)\,b\cdot\bar{\ell}\gamma^\mu\ell+C_{10A}(\mu)\cdot\bar{d}\gamma_\mu\,(1-\gamma_5)\,b\cdot\bar{\ell}\gamma^\mu\gamma_5\ell\Big]$$

- Nonperturbative contribution

Form Factors

$$\langle\gamma(k,\,\epsilon)|\bar{d}\gamma_\mu\gamma_5 b|B(p)\rangle \;=\; i\,e\,\epsilon^*_\alpha\,\left(g_{\mu\alpha}\,pk - p_\alpha k_\mu\right)\,\frac{F_A(q^2)}{M_B},$$

$$\langle\gamma(k,\,\epsilon)|\bar{d}\gamma_\mu b|B(p)\rangle \;=\; e\,\epsilon^*_\alpha\,\epsilon_{\mu\alpha\xi\eta}p_\xi k_\eta\,\frac{F_V(q^2)}{M_B},$$

$$\langle\gamma(k,\,\epsilon)|\bar{d}\sigma_{\mu\nu}\gamma_5 b|B(p)\rangle\,(p-k)^\nu \;=\; e\,\epsilon^*_\alpha\,\left[g_{\mu\alpha}\,pk - p_\alpha k_\mu\right]\,F_{TA}(q^2,0),$$

$$\langle\gamma(k,\,\epsilon)|\bar{d}\sigma_{\mu\nu}b|B(p)\rangle\,(p-k)^\nu \;=\; i\,e\,\epsilon^*_\alpha\epsilon_{\mu\alpha\xi\eta}p_\xi k_\eta\,F_{TV}(q^2,0).$$

Amplitude

$$A_\mu^{(1)} = \langle \gamma(k,\, \epsilon),\, \ell^+(p_1),\, \ell^-(p_2) \left| H_{\rm eff}^{b\to d\ell^+\ell^-} \right| B(p) \rangle \;=\; \frac{G_F}{\sqrt{2}}\, V_{tb} V_{tq}^* \, \frac{\alpha_{\rm em}}{2\pi}\, e\, \epsilon_\alpha^*$$

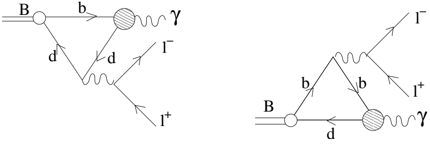
$$\times \Bigg[\frac{2\, C_{7\gamma}(\mu)}{q^2}\, m_b \, \left(\varepsilon_{\mu\alpha\xi\eta} p_\xi k_\eta \, F_{TV}(q^2, 0) \;-\; i\, (g_{\mu\alpha}\, pk \;-\; p_\alpha k_\mu)\; F_{TA}(q^2, 0) \right) \bar{\ell}(p_2)\gamma_\mu \ell(-p_1)$$

$$C_{9V}^{\rm eff}(\mu, q^2) \left(\varepsilon_{\mu\alpha\xi\eta} p_\xi k_\eta \, \frac{F_V(q^2)}{M_B} \;-\; i\, (g_{\mu\alpha}\, pk \;-\; p_\alpha k_\mu)\; \frac{F_A(q^2)}{M_B} \right) \bar{\ell}(p_2)\gamma_\mu \ell(-p_1) \;+$$

$$C_{10A}(\mu) \left(\varepsilon_{\mu\alpha\xi\eta} p_\xi k_\eta \, \frac{F_V(q^2)}{M_B} \;-\; i\, (g_{\mu\alpha}\, pk \;-\; p_\alpha k_\mu)\; \frac{F_A(q^2)}{M_B} \right) \bar{\ell}(p_2)\gamma_\mu \gamma_5 \ell(-p_1) \Bigg].$$

Contributions to Effective Hamiltonian

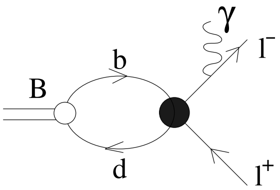
The virtual photon is emitted from the valence quark

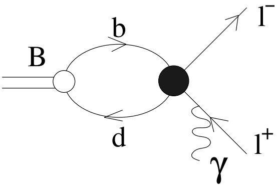


$$F_{TV,TA}(0,q^2) = F_{TV,TA}(0,0) - \sum_V 2 f_V g_+^{B\to V}(0) \frac{q^2/M_V}{q^2 - M_V^2 + iM_V\Gamma_V}$$

Contributions to Effective Hamiltonian

Bremsstrahlung:



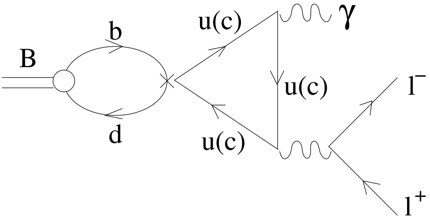


$$-i\, e\, \frac{G_F}{\sqrt{2}}\, \frac{\alpha_{\rm em}}{2\pi}\, V_{td}^* V_{tb}\, \frac{f_{B_q}}{M_B}\, 2\hat{m}_\ell\, C_{10A}(\mu)\, \bar{\ell}(p_2) \left[\frac{(\gamma\epsilon^*)\,(\gamma p)}{\hat{t}-\hat{m}_\ell^2} - \frac{(\gamma p)\,(\gamma\epsilon^*)}{\hat{u}-\hat{m}_\ell^2} \right] \gamma_5\, \ell(-p_1),$$

$f_B > 0$

Contributions to Effective Hamiltonian

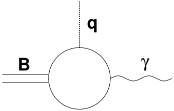
Weak annihilation diagrams:



$$H_{\text{eff}}^{B\to\bar{Q}Q} = -\frac{G_F}{\sqrt{2}}\, a_1\, V_{Qb}V^*_{Qd}\, \bar{d}\gamma_\mu(1-\gamma_5)b\,\bar{Q}\gamma_\mu(1-\gamma_5)Q, \qquad (1)$$

with $Q = \{u, c\}$, $a_1 = C_1 + C_2/N_c$

Form factors: constraints in QCD



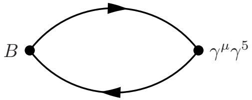
$F_i(q^2, k^2)$, $4m_\ell^2 \le q^2 \le M_B^2$

- Electromagnetic gauge invariance $\Longrightarrow$ constraints at $q^2 = 0$ (q^2 is the invariant mass of the lepton pair)
- Large Energy Effective Theory (LEET) $\Longrightarrow$ $F_A(E_\gamma) \approx F_V(E_\gamma) \approx F_{TA}(E_\gamma) \approx F_{TV}(E_\gamma)$ for $E_\gamma >> \Lambda_{QCD}$, $E_\gamma = \frac{M_B^2 - q^2}{2M_B}$
- Locations of meson singularities at $q^2 \ge M_B^2 \Longrightarrow$ constraints at $q^2 = M_B^2$
- Form factors related to photon emission by heavy quark are $1/m_Q$ suppressed compared to those with γ-emission by light quark

Relativistic Quark Model

We make use of dispersion approach based on constituent quark picture: All hadron observables are given by dispersion representations in terms of the hadron relativistic wave functions and the spectral densities of Feynman diagrams with constituent quarks in the loops.

- Decay constants: $f_B = \int ds \phi_B(s)\rho(s)$



- Meson-meson form factors:
  $F_{M_1\to M_2}(q^2) = \int ds_1\phi_1(s_1)ds_2\phi_2(s_2)\Delta(s_1,s_2,q^2)$

- Meson-photon transition form factors: $F(q^2,k^2) = \int ds\phi(s)\frac{ds'\Delta(s,s',q_2^2)}{s'-q_1^2}$

- Wave function ($\phi(s) \sim e^{-\frac{k^2}{2\beta^2}}$) is normalized by the condition that electromagnetic form factor is 1 at $q^2=0$: $F_{el}(q^2=0)=1$

Relativistic Quark Model

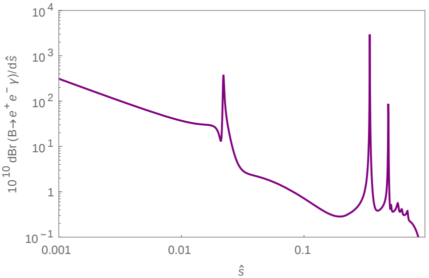
In leading and subleading order in $1/m_Q$:

- The meson-meson transition form factors satisfy constraints from HQET for heavy-to-heavy transitions
- The meson-meson and meson-photon form factors satisfy constraints from LEET for heavy-to-light transitions

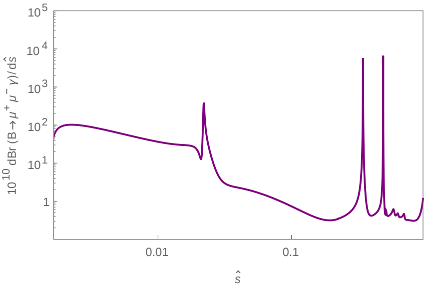
Fixing parameters:

- We use a simple Gaussian parametrization for $\phi(s)$ ($\phi(s) \sim e^{-\frac{k^2}{2\beta^2}}$)
- Constituent quark masses and the parameter of the wave function are fixed by the condition that meson decay constants reproduce well-known results from lattice QCD and sum rules.

Numerical Estimates



$B^0 \to \gamma e^+ e^-$



$B_s^0 \to \gamma \mu^+ \mu^-$

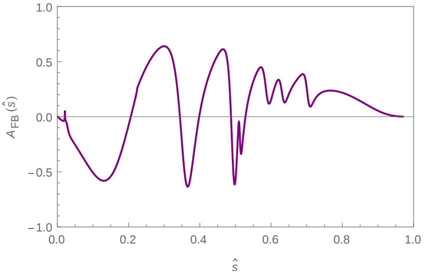
Numerical Estimates

	this work	[1]
$Br\,(B \to e^+e^-\gamma)\ \times\ 10^{10}$	4.84	3.95
$Br\,(B \to \mu^+\mu^-\gamma)\ \times\ 10^{10}$	1.60	1.31
$Br\,(B_s \to e^+e^-\gamma)\ \times\ 10^{9}$	18.8	24.6
$Br\,(B_s \to \mu^+\mu^-\gamma)\ \times\ 10^{9}$	12.2	18.8

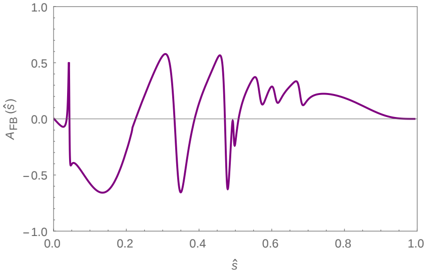
D. Melikhov and N. Nikitin, Phys. Rev. D **70**, 114028 (2004)

Numerical Estimates

Forward-Backward Asymmetry



$B^0 \to \gamma\mu^+\mu^-$



$B_s^0 \to \gamma\mu^+\mu^-$

Summary

- We obtained predictions for the branching ratios of $B_{d,s} \to \gamma l^+ l^-$ decays in the Standard Model
- We used reliable form factors that satisfy all known QCD constraints.
- We took into accout contributions of light vector resonances.
- We obtained distributions for the forward-backward asymmetry.

Thank you for your attention!